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Ranking of the Nonlinearities of Electrodynamic Loudspeakers

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The aim of this paper is to present a way of ranking the nonlinearities of electrodynamic loudspeakers. For this purpose, we have constructed a nonlinear analytic model which takes into account the variations of the small signal parameters. The determination of these variations is based on a very precise measurement of the electrical impedance of the electrodynamic loudspeaker. First, we present the experimental method to identify the variations of these parameters, then we propose to study theoretically the importance of these nonlinearities according to the input level or the input frequency. We show that the parameter which creates most of the distortions is not always the same and depends mainly on both the input level and the input frequency. Such results can be very useful for optimization of electrodynamic loudspeakers.

Keywords: electrodynamic loudspeaker, nonlinearities, small signal parameters, nonlinear modeling.

1. Introduction

The reference model describing the electrodynamic loudspeaker was designed by Thiele and Small (1978). Their model was adapted to describe an electrodynamic loudspeaker as a linear system. This model is very useful because it is very simple to use and its parameters can be conveniently presented in terms of an electric analog circuit (Klippel, 1990). However, a loudspeaker shows nonlinearities that produce distortions. These nonlinearities have three major sources (Borwick, 2001; Klippel, 2006): the suspensions, the diaphragm (Suzuki, Tichy, 1981; Quaegebeur *et al.*, 2009; Lemarquand, Bruneau, 2007) and the motor (Dobrucki, 1994; Wright, 1990; Ravaud *et al.*, 2009b). They have been largely studied and there are many attempts to model them (Kaizer, 1987;

Gander, 1981; 1986; Merit *et al.*, 2009b; Leach, 2002; Clark, 1997; Vanderkooy, 1989).

However, most of the papers dealing with the nonlinearities of electrodynamic loudspeakers do not take into account the variations of the mechanical damping R_{ms} or the equivalent mass M_{ms}. But these parameters are also nonlinear and their variations must be taken into account in order to precisely characterize the distortions created by electrodynamic loudspeakers. It can be noted that alternative loudspeakers structure without iron have been proposed and studied (Lemarquand, 2007; Merit *et al.*, 2009a; Ravaud *et al.*, 2008; Remy *et al.*, 2008), which use ferrofluid suspensions in order to delete the classical distortions generated by electrodynamic loudspeakers (Ravaud, Lemarquand, 2009a; 2009b; Remy *et al.*, 2009; Ravaud *et al.*, 2009a).

This paper has two objectives. In the first part, we present an experimental method which allows us to determine precisely the variations of the small signal parameters according to the input current. Our experimental method is based on a very precise measurement of the electrical impedance of the electrodynamic loudspeaker. Indeed, the measurement accuracy is about 10^{-4} Ohm for both the real and imaginary parts of the electrical impedance.

However, our approach does not use the classical definition of an electrical impedance. An electrical impedance, commonly defined by its ratio U/I (in the frequency domain) does not depend on the input current. Indeed, this electrical impedance is only defined for linear systems.

However, our experimental measurement shows that this ratio U/I depends on I. This can be easily verified experimentally by measuring the experimental impedance for different input currents. Such a result is in fact known for scientists involved in modeling of nonlinear systems: a nonlinear system depends generally on the input level. In the case of an electrodynamic loudspeaker, the electrical impedance (that is to say, the ratio U/I) depends on the input level.

We also show that there exists a bijective relation between the input current I and the position of the voice coil X. This result is of great importance because it shows that the nonlinear parameters describing an electrodynamic loudspeaker ($Bl(x)$, $R_{ms}(x)$ and $L_e(x)$) can also be described as parameters depending on the input current (in the time-domain or in the frequency-domain). It can be noted that, strictly speaking, the force factor Bl which is commonly used in analog circuits, represents the mean induction field times the length l of the voice coil inside the air gap. Therefore, when the voice coil position $x(t)$ is sinusoidal, in complex notations, the force factor should be seen as a mean force factor $\widetilde{Bl}$.

In the second part, we discuss the behaviour of each nonlinearity according to both the input level and the input frequency. It should be noted that all this part is treated theoretically. Indeed, a very good agreement between our analytical model and experimental measurements has confirmed that our model can be used for modelling of the distortions created by an electrodynamic loudspeaker.

We show here that the lumped parameter whose relative variation according to input current is the most important, is not always the same and depends greatly on both the input level and the input frequency.

Another drawback known in electrodynamic loudspeakers is that it is a time-varying system. In fact, the electrical resistance increases with time and the mechanical compliance of the outer rims depends also on time. These dependences have been studied with a similar approach in previous papers. In this paper, some care has been taken in the experimental measurements for omitting these temporal effects.

This paper proposes a way of ranking the nonlinearities of electrodynamic loudspeakers. To our knowledge, this way of characterizing these nonlinearties has never been used for electrodynamic loudspeakers. However, many new phenomena can be seen by using such an experimental approach. This is why such results are very interesting for many manufacturers involved in the design of electrodynamic loudspeakers. We precise here that we use in the rest of this paper the small letter notations $x(t)$ and $i(t)$, the displacement of the voice coil and the input current in the voice coil in the time-domain, and the capital letters (X, I, U) in the frequency domain. We also precise that U is the input voltage.

2. Classical description of a loudspeaker and its limits

2.1. The small signal model using lumped parameters

A common way of characterizing an electrodynamic loudspeaker is to measure its electrical impedance Z_e. Its theoretical expression is well-known and is given by Eq. (1):

$$Z_e = R_e + \frac{jL_e w R_\mu}{jL_e w + R_\mu} + \frac{Bl^2}{R_{ms} + jM_{ms}w + \dfrac{1}{jC_{ms}w}}, \tag{1}$$

where all the parameters are defined below.

Bl – electrodynamic driving parameter [T· m],
R_{ms} – mechanical damping, drag force [N· s· m^{-1}],
C_{ms} – mechanical compliance of suspension (spider, outer rim) [N^{-1}· m],
L_e – inductance of voice coil [H],
M_{ms} – equivalent mass of moving voice coil, cone and air [kg],
R_e – electrical resistance of voice coil [Ω],
R_μ – eddy current resistance [Ω],
w – angular frequency [rad· s^{-1}].

These parameters can be represented in terms of an analog circuit in Fig. 1.

According to (1), the electrical impedance Z_e does not depend on I. However, an experimental measurement shows the contrary. Indeed, the real part and the imaginary part of this electrical impedance are represented by using the

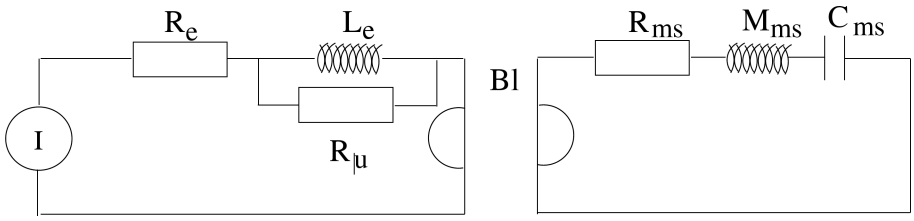

Fig. 1. Representation of the lumped parameters in terms of an analog circuit.

Nyquist diagram in Fig. 2 and we see that this electrical impedance depends on the input current. Consequently, this eletrical impedance can be seen as a nonlinear electrical impedance. We denote it $Z_e^{(NL)}(I)$. In short, we can say that the classical electrical impedance Z_e characterizing an electrodynamic loudspeaker is not constant when the input current varies. Consequently, we use the notation $Z_e^{(NL)}(I)$ to describe the electrical impedance. However, strictly speaking, the term "electrical impedance" should not be used to describe a nonlinear system like an electrodynamic loudspeaker.

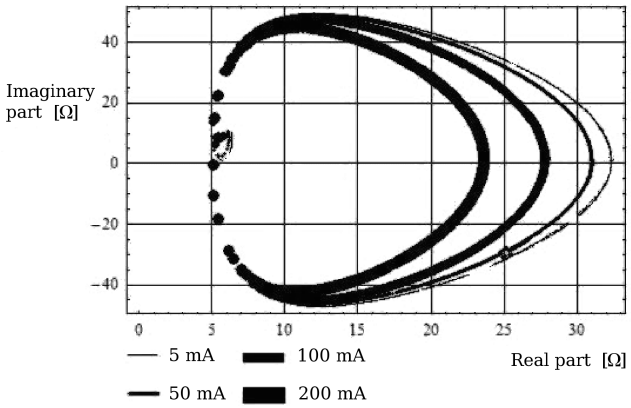

Fig. 2. Experimental representation in the complex plane of the electrical impedance of the electrodynamic loudspeaker for four coil current levels (5 mA, 50 mA, 100 mA, 200 mA).

As a consequence, the lumped parameters defined in Eq. (1) depend on the input current. The main problem is thus to know how to characterize their dependence according to the input current. According to the current state of art, cited in the introduction of this paper, the dominant nonlinearities in electrodynamic loudspeakers are $Bl(x)$, $C_{ms}(x)$ and $L_e(x)$. However, there exists a relation between the voice coil position x and the input current i. This relation is shown in the next section. Consequently, it can be noted that these nonlinearities can depend on x or i: it is in fact equivalent. In this paper, we choose to work directly with the input current i because $Z_e^{(NL)}(I)$ is easier to determine experimentally. Consequently, thanks to this very precise experimental measurement, the

variations of the lumped parameters can be determined according to the input current i.

2.2. Relation between the input current I and the voice coil position X according to the Thiele and Small model

This section presents the relation between the input curreent I and the voice coil position X with the linear approximation of the Thiele and Small model. It is noted here that the aim of this section is merely to show that there exists a bijective relation between the input current I and the voice coil position X. To find this relation, we use one differential equation describing the electrodynamic loudspeaker and we use complex notations. We find:

$$\left(M_{ms}(jw)^2 + R_{ms}(jw) + \frac{1}{C_{ms}}\right) X = BlI. \tag{2}$$

So, we can write:

$$X = \frac{Bl}{\left(M_{ms}(jw)^2 + R_{ms}(jw) + \frac{1}{C_{ms}}\right)} I. \tag{3}$$

Therefore, we see that Eq. (3) is the bijective relation between the input current I and the voice coil position X, in complex notation. This relation shows that we can work either with the input current I or with the voice coil position X, so as to describe the nonlinear variations of the Thiele and Small parameters.

2.3. Relation between the input current I and the voice coil position X according to the nonlinear model used in this paper

Strictly speaking, the relation between the input current I and the voice coil position is nonlinear. Indeed, all the small signal parameters depend on the input current. It is emphasized here that the transfer function describing the ratio U/I is considered as the ratio of the part of response with fundamental frequency w to the excitation. We call it a nonlinear electric impedance in this paper but it is useful to precise that nonlinear systems cannot be described with complex notations.

We describe in the next section a way of determining the nonlinear laws of all the small signal parameters. Therefore, we precise here for the rest of this paper that (3) can be written in the frequency-domain as follows:

$$X = \frac{Bl(I)}{\left(M_{ms}(jw)(I)^2 + R_{ms}(I)(jw) + \frac{1}{C_{ms}(I)}\right)} I. \tag{4}$$

It is noted that the determination of the small signal parameters can be determined in the frequency domain. We explain in the next section how to find these variations.

3. Determination of the nonlinear variations of the lumped parameters

3.1. Principle of the measurement

We have shown in the previous section that the nonlinear variations of the lumped parameters describing an electrodynamic loudspeaker depend on the input current. We explain now how to find these variations.

The electrodynamic loudspeaker used is a boomer (mark: Eminence, number: beta 15). We use the electrical impedance of the electrodynamic loudspeaker because its measurement is very precise. Our measurement device is a Wayne–Kerr bridge which has an excellent precision (10^{-4} Ω). This experimental devices is dedicated to the impedance measurement and functions as a classical impedance bridge.

In order to measure the electrical impedance of a loudspeaker, it is mounted in a normalized plane in an anechoic room (I.E.C, 2007). Dalmont (2001) has shown that the acoustical impedance in these conditions is the same as the one when the loudspeaker is mounted in an infinite baffle in an anechoic room. We measure the electrical impedance by varying the frequency and the coil current. So, we build an experimental impedance layer by using a Runge–Kutta algorithm to determine all the experimental measured points. In other words, this measurement algorithm chooses the best measurement point according to the gradient method. Such a measurement method allows us to detect all the subtle effects due to the nonlinearities in electrodynamic loudspeakers. This algorithm has been improved in relation to the one used in a previous paper. In the previous paper, the algorithm took some experimental points by using constant intensities. In other words, our measurement system used the algorithm of gradient only in two dimensions for one intensity and then it was repeated for another intensity. For example, an intensity was fixed (for example 50 mA), and a two-dimensional algortihm allowed us to determine the measurement points, that is to say, our algorithm automatically settled one intensity and took some experimental points by using a method based on the gradient method. In this paper, the algorithm uses a Runge–Kutta algorithm in three dimensions (Z_e, I, f) and we can rapidly obtain an impedance layer which is very precise. Consequently, the temporal effects can be neglected. A two-dimensional representation of the electrical impedance is shown in Fig. 2: the imaginary part is a function of the real part (it is a Nyquist diagram for different coil currents). We can say that the Wayne Kerr bridge cannot supply currents greater than 0.2 A. Consequently, the parameter variations are determined in this current interval.

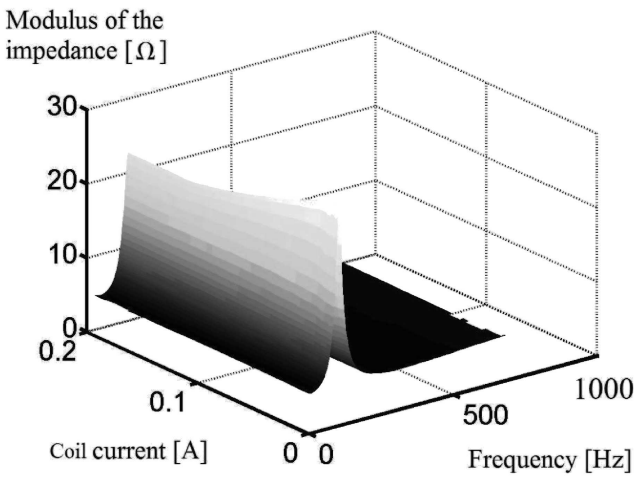

Fig. 3. Experimental three-dimensional representation of the modulus of electrical impedance of the electrodynamic loudspeaker (x: coil current 0–0.2 A) (y: frequency 0–1000 Hz) (z: impedance 0–30 Ω).

Figure 3 confirms that the electrical impedance is a function of the input current.

3.2. Determination of the nonlinear electrical impedance

The next step is thus to determine the dependence of the small signal parameters with the coil current so as to construct the nonlinear transfer function. For this purpose, we use the Nyquist diagram constructed previously. Five parameters ($Bl(I)$, $R_{ms}(I)$, $C_{ms}(I)$, $M_{ms}(I)$, $L_e(I)$) are assumed to vary with the coil current. Indeed, the electrical resistance does not depend on input current because our experimental measurements have been done with a stabilized temperature (the electrodynamic loudspeaker has been run during 24 hours before the experiment) and the eddy current resistance does not seem to vary with the input current I. In the first approximation, we use a polynomial writing to represent the dependence of the parameters on the coil current. We write:

$$Bl(I) = Bl(1 + \mu_{Bl} I + \mu_{Bl}^2 I^2 + ... + \mu_{Bl}^n I^n), \tag{5}$$

$$R_{ms}(I) = R_{ms}(1 + \mu_{R_{ms}} I + \mu_{R_{ms}}^2 I^2 + ... + \mu_{R_{ms}}^n I^n), \tag{6}$$

$$C_{ms}(I) = C_{ms}(1 + \mu_{C_{ms}} I + \mu_{C_{ms}}^2 I^2 + ... + \mu_{C_{ms}}^n I^n), \tag{7}$$

$$M_{ms}(I) = M_{ms}(1 + \mu_{M_{ms}} I + \mu_{M_{ms}}^2 I^2 + ... + \mu_{M_{ms}}^n I^n), \tag{8}$$

$$L_e(i) = L_e(1 + \mu_{L_e} I + \mu_{L_e}^2 I^2 + ... + \mu_{L_e}^n I^n). \tag{9}$$

So the electrical impedance becomes:

$$Z_e(I) = R_e + \frac{\sum_{s=0}^{n} jL_e w R_\mu (\mu_{L_e} I)^s}{R_\mu + \sum_{s=0}^{n} jL_e w (\mu_{L_e} I)^s} + \frac{\left(\sum_{s=0}^{n} Bl(\mu_{Bl} I)^s\right)^2}{\sum_{s=0}^{n} jM_{ms} w (\mu_{M_{ms}} I)^s + \sum_{s=0}^{n} R_{ms} (\mu_{R_{ms}} I)^s + \frac{1}{\sum_{s=0}^{n} jwC_{ms} (\mu_{C_{ms}} I)^s}}. \quad (10)$$

We use the least square method to identify all the parameters; this method is based on the Simplex algorithm. The Simplex method is a systematic procedure which selects the variable that will produce the largest change towards the minimum solution. This algorithm selects the best choice at each iteration, without needing information from previous and future iterations. In our case, the principle of this algorithm is to minimize the difference D between the experimental and the theoretical Nyquist diagrams. Consequently, the two parameters which are minimized are the real part $Real(I)$ and the imaginary part $Imag(I)$ of the electrical impedance defined by Eq. (11):

$$Z_e^{(NL)}(I) = Real(I) + jImag(I). \quad (11)$$

When the algorithm converges, we obtain the values of the parameters of the Eqs. (5)–(9).

4. Experimental results

When we take into account the variations of the small signal parameters with the coil current, the mean the difference between the experimental and the theoretical values is 0.4 Ω whereas the mean difference is 2.0 Ω with the Thiele and Small model with constant parameters. We present in Table 1 the laws of variations of the five parameters that vary according to the coil current, and we give for each parameter the sensitivity to the least square. We propose a ranking of these parameters based on the criterion D. To obtain this ranking, we proceed as follows: we write that one parameter (L_e, R_{ms}, M_{ms}, Bl or C_{ms}) is a function of the coil current. We input the polynomial in the theoretical impedance and we use the least square algorithm to determine the value of the mean difference between the experimental impedance and the theoretical impedance. We obtain in this case the ranking shown in Table 1. Moreover, the value of the eddy current resistance is 2.2 Ω and the value of the electrical resistance is 3.3 Ω. The Table 2 shows the laws of variations of the Thiele and Small parameters. The representations of these variations are given in Figs. 4–8.

Table 1. Ranking of the parameters according to their sensitivity to the least square algorithm.

Ranking	Parameter	Law of variation (100 Hz)	D [Ω](100 Hz)
1	R_{ms}	$1.1(1+4.09I-8.36I^2)$	1.24
2	Bl	$5.5(1+0.33I-1.02I^2)$	1.67
3	M_{ms}	$0.009(1+0.56I-0.22I^2)$	1.74
4	C_{ms}	$0.00013(1+2.02I-9.3I^2)$	1.86
5	L_e	$0.0017(1-1.68I+7.58I^2)$	1.98

Table 2. Ranking of the parameters according to their created distortions (the frequency of excitation is 50 Hz).

Ranking (50 Hz)	Parameter (harmonic 2)	log [x]
1	L_e	−3.2
2	M_{ms}	−4.9
3	R_{ms}	−6.8
4	C_{ms}	−7.7
5	Bl	−7.8

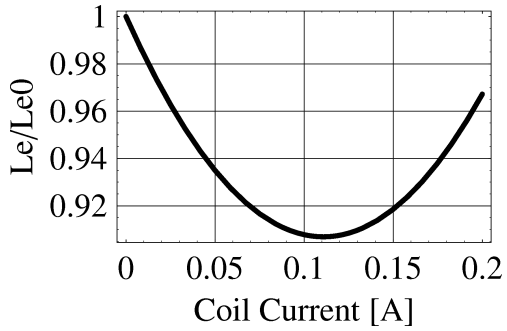

Fig. 4. The ratio L_e/L_{e0} is a function of the coil current (x: coil current 0–0.2 A) ($f = 100$ Hz); $L_{e0} = 0.0017$ H.

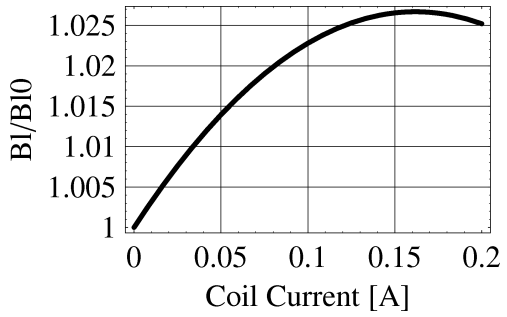

Fig. 5. The ratio Bl/Bl_0 is a function of the coil current (x: coil current 0–0.2 A) ($f = 100$ Hz); $Bl_0 = 5.5$ Tm.

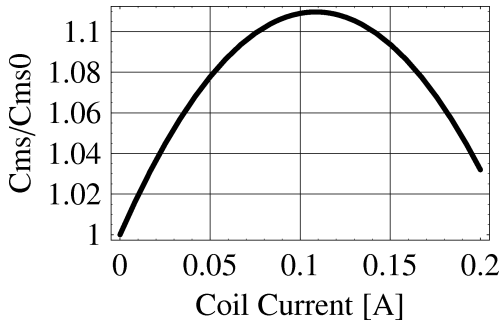

Fig. 6. The ratio C_{ms}/C_{ms0} is a function of the coil current (x: coil current 0–0.2 A) ($f = 100$ Hz); $C_{ms0} = 0.00013$ m/N.

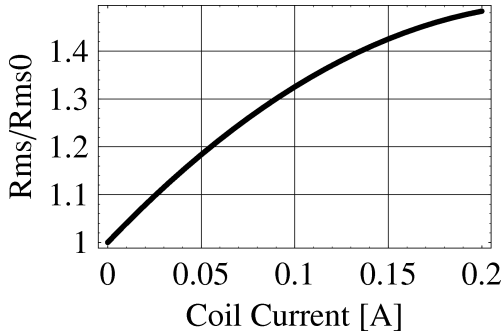

Fig. 7. The ratio R_{ms}/R_{ms0} is a function of the coil current (x: coil current 0–0.2 A) ($f = 100$ Hz); $R_{ms0} = 1.1$ kg/s.

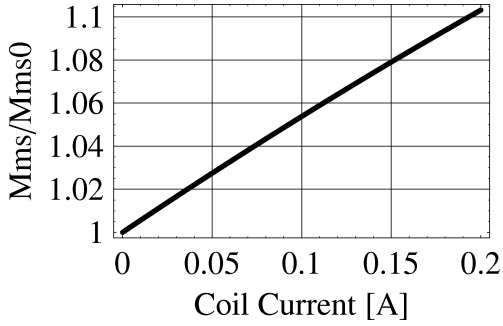

Fig. 8. The ratio M_{ms}/M_{ms0} is a function of the coil current (x: coil current 0–0.2 A) ($f = 100$ Hz); $M_{ms0} = 0.009$ kg.

4.1. Discussion

The most sensitive parameter to the variations of the coil current is the equivalent damping parameter R_{ms}. The parameter which is less sensitive to the variations of the coil current is the inductance L_e of the voice coil. However, though this inductance sensitiveness is rather weak, the relative variation of the inductance

L_e is rather important. In other words, even if the inductance L_e is a parameter that contributes just a little to the variation of its electrical impedance, we see that its relative variation is important. Another interesting point can be adressed. With the values found in this paper, at 200 Hz, we see that the eddy current resistance R_μ equals roughly wL_e. We can conclude that the effects of the eddy currents become important from 200 Hz for this electrodynamic loudspeaker.

5. Study of the distortions created by the electrodynamic loudspeaker

5.1. Obtaining the nonlinear differential equation

The nonlinear differential equation describing the electrodynamic loudspeaker is given by Eq. (12). We use the parameter $k = 1/C_{ms}$ to solve it.

$$a(i)\frac{\mathrm{d}^3x(t)}{\mathrm{d}t^3} + b(i)\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2} + c(i)\frac{\mathrm{d}x(t)}{\mathrm{d}t} + \mathrm{d}(i)x(t) = u(t), \tag{12}$$

with

$$a(i) = \frac{\left(M_{ms}(1+\mu_{M_{ms}}i+\mu_{M_{ms}}^2 i^2)\right)\left(L_e(1+\mu_{L_e}i+\mu_{L_e}^2 i^2)\right)}{Bl(1+\mu_{Bl}i+\mu_{Bl}^2 i^2)}, \tag{13}$$

$$b(i) = \frac{\left((M_{ms}(1+\mu_{M_{ms}}i+\mu_{M_{ms}}^2 i^2)R_e\right)}{Bl(1+\mu_{Bl}i+\mu_{Bl}^2 i^2)} + \frac{\left(R_{ms}(1+\mu_{R_{ms}}i+\mu_{R_{ms}}^2 i^2)R_e\right)}{Bl(1+\mu_{Bl}i+\mu_{Bl}^2 i)}, \tag{14}$$

$$c(i) = \frac{R_e\left(R_{ms}(1+\mu_{R_{ms}}i+\mu_{R_{ms}}^2 i^2)\right)+\left(Bl(1+\mu_{Bl}i+\mu_{Bl}^2 i^2)\right)^2}{Bl(1+\mu_{Bl}i+\mu_{Bl}^2 i^2)} + \frac{\left(L_e(1+\mu_{L_e}i+\mu_{L_e}^2 i^2)\right)k(1+\mu_k i+\mu_k^2 i^2)}{Bl(1+\mu_{Bl}i+\mu_{Bl}^2 i^2)}, \tag{15}$$

$$d(i) = \frac{R_e\left(k(1+\mu_k i+\mu_k^2 i^2)\right)}{Bl(1+\mu_{Bl}i+\mu_{Bl}^2 i^2)}. \tag{16}$$

5.2. Solving the nonlinear differential equation

The relation between the coil current i and the position of the voice coil x is nonlinear (Eq. (4)). Each small signal parameter is a function of the voice coil position and we obtain, for example, for one parameter:

$$R_{ms}(x) = R_{ms}(1 + \widetilde{\mu}_{R_{ms}} x + \widetilde{\mu}^2_{R_{ms}} x^2) \tag{17}$$

with $\left|\widetilde{\mu}^2_{R_{ms}}\right| \ll |\widetilde{\mu}_{R_{ms}}| \ll 1$.

If we assume that the input voltage $u(t)$ is sinusoidal, in this case, the solution of the nonlinear differential equation is a trigonometric expansion. The solution can be developed until the order 2 (μ^2):

$$x(t) = x_0(t) + \mu x_1(t) + \mu^2 x_2(t) + \ldots , \tag{18}$$

where $x_0(t)$ is the solution of the nonlinear differential equation of the electrodynamic loudspeaker when we neglect the terms with orders higher than zero, $x_1(t)$ is the solution of the nonlinear differential equation when we omit the terms with orders higher than one and smaller than one, $x_2(t)$ is the solution of the nonlinear differential equation of the electrodynamic loudspeaker when we neglect the terms with orders smaller than two and higher than two.

It can be noted that some methods like the Volterra Series are interesting but do not show which parameters are really nonlinear. The way of solving the nonlinear differential equation is very important but we think that it is not the most important thing for characterizing the electrodynamic loudspeakers. The real problem is to know what parameters vary and how. Furthermore, the ranking of these parameters according to the input current or frequency is of great importance. Indeed, we can think that if the structure of an electrodynamic loudspeaker must be improved, we must know all the defects in the motor or the suspensions.

Let us consider now Eq. (12). The denominator in Eq. (12) contains a nonlinear term. Consequently, it is very difficult to solve this nonlinear differential equation with a nonlinear denominator. One possible solution is to approximate this denominator as follows:

$$\frac{1}{Bl(x)} = \widetilde{Bl}_0 + \widetilde{Bl}_1 x + \widetilde{Bl}_2 x^2 + \ldots \tag{19}$$

The previous relation is used as a simplification for solving numerically the nonlinear differential equation. Moreover, we can use a classical trigonometric expansion to solve Eq. (12). In short, the solution of the nonlinear differential equation of the electrodynamic loudspeaker is

$$x(t) = A\cos(wt) + B\sin(wt) + C\cos(2wt) + D\sin(2wt) + \ldots \tag{20}$$

All the terms A, B, C, D,... are found numerically but an analytical solution is possible if the force factor is approximated. Indeed, the terms A and B can be found by inserting $A\cos(wt) + B\sin(wt)$ in the Eq. (12) with an excitation $u(t)$ equal to $P\sin(wt)$ where P is an amplitude. The terms C and D can be found by taking the terms with orders higher than one into account, etc...

5.3. Theoretical results and position of the small signal parameters according to their created distortions

We present the position of the small signal parameters according to their created distortions. For this purpose, we solve the nonlinear differential equation by using the serial expansion presented in the previous section but we take into account only one variation of a parameter at a time. Figure 9 shows the created distortions by each Thiele and Small parameter. The level of the input voltage is 10 V and the frequency of excitation is 50 Hz. We can see that for this electrodynamic loudspeaker, the nonlinear parameter which creates the most important second-harmonic is the inductance L_e of the voice coil, and the nonlinear parameter which creates the most important third-harmonic is the equivalent damping parameter R_{ms}. However, when the frequency of excitation increases, the nonlinear parameters which create more distortions are not the same. Figure 10 presents the created distortions by each Thiele and Small parameter when the

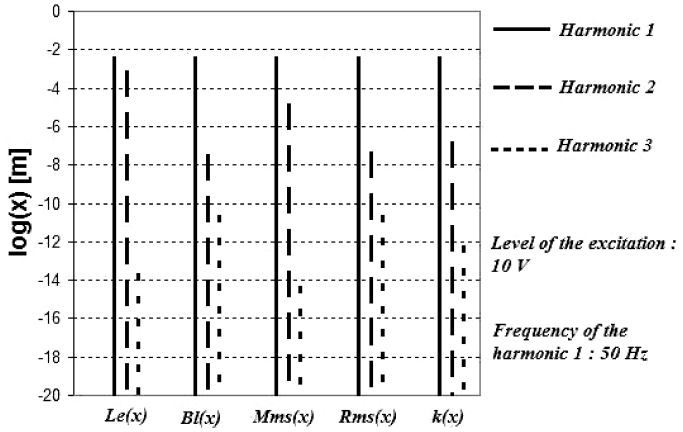

Fig. 9. Theoretical spectrums of Thiele and Small parameters: the frequency of excitation is 50 Hz and the input voltage is 10 V.

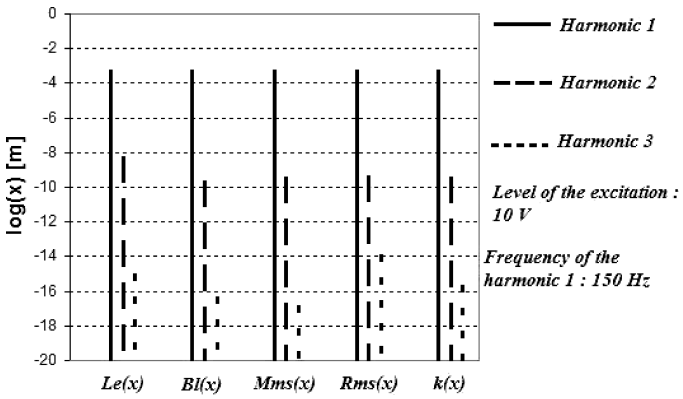

Fig. 10. Theoretical spectrums of Thiele and Small parameters: the frequency of excitation is 150 Hz and the input voltage is 10 V.

frequency of excitation is 150 Hz. As shown in the previous figure, the level of the input voltage is 10 V and the frequency of excitation is 150 Hz. We see that the nonlinear parameter which creates the most important second-harmonic is the equivalent mass M_{ms} and the nonlinear parameter which creates the more important third-harmonic is the damping parameter R_{ms}. We sum up all the results in Tables 2–5.

Table 3. Ranking of the parameters according to their created distortions (the frequency of excitation is 50 Hz).

Ranking (50 Hz)	Parameter (harmonic 3)	log [x]
1	R_{ms}	−11
2	Bl	−12.5
3	C_{ms}	−12.6
4	L_e	−12.9
5	M_{ms}	−14.6

Table 4. Ranking of the parameters according to their created distortions (the frequency of excitation is 150 Hz).

Ranking (150 Hz)	Parameter (harmonic 2)	log [x]
1	M_{ms}	−5.3
2	L_e	−6.4
3	R_{ms}	−7.7
4	C_{ms}	−8.2
5	Bl	−8.4

Table 5. Ranking of the parameters according to their created distortions (the frequency of excitation is 150 Hz).

Ranking (150 Hz)	Parameter (harmonic 3)	log [x]
1	R_{ms}	−11.7
2	L_e	−12.6
3	C_{ms}	−14.2
4	Bl	−15
5	M_{ms}	−15.1

5.4. Discussion

The previous section shows that the nonlinear parameters which create the most distortions are not those which are the most nonlinear according to the coil current. For example, Fig. 7 shows that the equivalent damping parameter R_{ms}

is the most nonlinear parameter (R_{ms} varies a lot with the coil current), but it is not the parameter which creates the most distortions. On the contrary, the inductance L_e of the voice coil varies very little with the coil current, but at low frequency this parameter creates important distortions. Moreover, we see that the mechanical compliance C_{ms} is not an important nonlinear parameter since it does not create important distortions. In addition, ranking of these nonlinearities depends also on the input frequency. Such results are interesting for the design of electrodynamic loudspeakers.

6. Behavior of the created distortions according to the variations of the input voltage

The previous section presents the ranking of the nonlinear Thiele and Small parameter according to the variation of the frequency excitation. The aim of this section is to show that the distortions created by the nonlinear parameters are more sensitive than the fundamental component according to the variation of the input voltage. To show this, we solve the nonlinear differential equation of the electrodynamic loudspeaker (12). We solve this equation by taking four different amplitudes for the excitation (1 V, 5 V, 10 V, 50 V) and we represent the spectrum in Fig. 11. The frequency of excitation is 100 Hz.

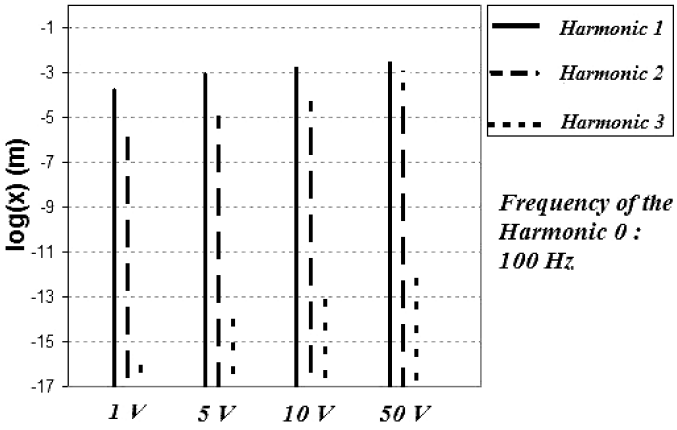

Fig. 11. Theoretical spectrum: influence of the input voltage on the created harmonics.

Table 6. Ranking of the parameters according to their created distortions (the input frequency is 100 Hz).

Harmonics	1 V	5 V	10 V	50 V
first-harmonic	−3.8	−3	−2.8	−2.7
second-harmonic	−5.9	−5.6	−4.2	−3.4
third-harmonic	−15.8	−13.5	−13	−11.4

7. Conclusion

The aim of this paper was the study of the spectrum of the electrodynamic loudspeaker. The experimental method, based on the impedance measurement of an electrodynamic loudspeaker, allows us to find all the variations of the Thiele and Small parameters. We can say that this experimental method can be used to characterize many transducers which are described with their electrical impedance. Indeed, an electrical impedance can be seen as a nonlinear transfer function which varies with the input current or the input voltage (Gille *et al.*, 1981). In Sec. 2, we have presented a method to derive the coefficients of the nonlinear parameters based on the Simplex algorithm. It is noted that a simplifying method was used in a previous paper (Ravaud *et al.*, 2009b) and has been improved in this paper.

The small signal coefficients, inserted in the differential equation of the electrodynamic loudspeaker, enable us to find the generated harmonics. Many new results are discussed. The equivalent damping parameter is the parameter which is the most nonlinear if we look at its relative variation according to the input current. However, it is not the parameter which creates the most important distortions. This result is important because it gives information about the way of manufacturing an electrodynamic loudspeaker.

More generally, when we take into account the variations of the small signal parameters with the coil current, the mean difference between the experimental and the theoretical values is 0.4 Ω, whereas the mean difference is 2.0 Ω with Thiele and Small model with constant parameters.

Another interesting result is the weak variation of the electrical inductance with the input current. It is noted that the electrodynamic loudspeaker characterized in this paper is a good one and is less nonlinear than a bad loudspeaker. However, we see that this weak variation creates important distortions. This is why it can be very important to try to build electrodynamic loudspeakers with constant inductance.

Furthermore, we have seen that these generated harmonics become more and more important when the input voltage increases. This result is in fact consistent with all the studies dealing with modeling of nonlinear systems.

This paper is a first step to derive and class the defects of electrodynamic loudspeakers. The experimental approach taken is certainly more precise for characterizing the variation of a nonlinear transfer function according to the input level. We can say that such defects are very important to determine because they lower the quality of loudspeakers.

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