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Research paper

Active Cancellation of the Tonal Component of Sound Using a Discrete Fourier Transform of Variable Length

Michał ŁUCZYŃSKI*, Andrzej DOBRUCKI, Stefan BRACHMAŃSKI

Wroclaw University of Science and Technology, Chair of Acoustics and Multimedia
Wroclaw, Poland; e-mail: {andrzej.dobrucki, stefan.brachmanski}@pwr.edu.pl
*Corresponding Author e-mail: michal.luczynski@pwr.edu.pl

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The paper presents a method of eliminating the tonal component of an acoustic signal. The tonal component is approximated by a sinusoidal signal of a given amplitude and frequency. As the parameters of this component: amplitude, frequency and initial phase may be variable, it is important to detect these parameters in subsequent analysis time intervals (frames). If the detection of the parameters is correct, the elimination consists in adding a sinusoidal component with the detected amplitude and frequency to the signal, but the phase shifted by 180 degrees. The accuracy of the reduction depends on the accuracy of parameters detection and their changes.

Detection takes place using the Discrete Fourier Transform, whose length is changed to match the spectrum resolution to the signal frequency. The operation for various methods of synthesis of the compensating signal as well as various window functions were checked. An elimination simulation was performed to analyze the effectiveness of the reduction. The result of the paper is the assessment of the method in narrowband active noise control systems. The method was tested by simulation and then experimentally with real acoustic signals. The level of reduction was from 6.9 to 31.5 dB.

Keywords: active noise control; tonal signal; Discrete Fourier Transform.

1. Introduction

Tonal signals are most often defined as signals in which energy is concentrated in a narrow frequency band and tonality is assessed by comparing levels in adjacent bands (ISO 2017). The definition applies to both signals that contain more than one tonal component and to signals for which the nature of the tonal component is different. In (ZIVANOVIC *et al.*, 2004), the authors indicate that the definition given above includes both pure tones (time-invariant and modulated) and narrowband noise. The classification of tonal components has been extended (ŁUCZYŃSKI, 2019a) to include further signals showing tonal nature under certain conditions, e.g. sweep or signals from the engine, where the rotational speed increases or decreases. In this work, the algorithm will be tested for stationary periodic signals and periodic signals modulated by a random function according to the classification proposed in (ŁUCZYŃSKI, 2019a). An input signal with a tonal component whose level will be reduced is called the original signal. The added tonal component is called a compensation signal. The result of the process (the output signal) is called the compensated signal.

Signals containing tonal components include sound produced by the transformer (QUIA *et al.*, 2002; ZHANG *et al.*, 2012), mains hum (ŁUCZYŃSKI, 2019b), fan noise (WANG *et al.*, 2005), music and speech signals (YOSHIZAWA *et al.*, 2011; ROCHA, 2014; ŁUCZYŃSKI, 2018) or internal combustion engine noise (ŁUCZYŃSKI, BRACHMAŃSKI, 2017). These signals can be modeled as sinusoidal signals with added noise. Attenuation or reduction of these signals is important in noise control and in signal processing. Narrow band active reduction (ANC) systems have been developed for this purpose. The effectiveness of such systems is largely influenced by the adjustment of the original signal and compensating signal parameters. In ANC systems in free field, the adjustment highly depends on the position of the reference microphone and secondary source (Dąbrowski, 2013), but also on accurate detection of parameters (Łuczyński, 2017). These systems have a certain effectiveness, but it should be remembered that total noise reduction is not always desirable, as e.g. in the case of a car cabin, where we do not want to get rid of the sound of the engine but only reduce its level (active equalization (Kuo, 1997)). General requirements for ANC systems are described in 6 points: high attenuation, wide attenuation band, internal stability, numerical stability, convergence is case of adaptive systems, high robustness to changes of disturbance (Pawełczyk, 2008).

In narrowband ANC, both different signal parameter detection algorithms and different methods of creating the compensating signal are used. Sometimes notch filters (Kim, Park, 1999; Górski, Morzyński, 2013) are used for frequency estimation. Some papers use high resolution spectral analysis to determine the parameters of tonal components (Łuczyński, 2019b; 2018; Yoshizawa *et al.*, 2011). The compensating signal can be the filter compensated signal with appropriate gain and phase shift (Wang *et al.*, 2005). Another solution used is the synthesis of the compensating signal (Quia *et al.*, 2002; Zhang *et al.*, 2012; Xiao *et al.*, 2009).

Initially, narrowband ANC systems were intended for signals in which the parameters of tonal components were stable (Quia *et al.*, 2002; Qio, Hansen, 2000) and it was demonstrated that the effectiveness of these systems decreases with the increase of the length of the compensating signal (Zhang *et al.*, 2012). In next step, the systems were developed that can work when both the signal and the ambient acoustic conditions change over time (Xiao *et al.*, 2009; Rout *et al.*, 2019). A common feature of narrowband ANC systems is the high efficiency of reducing tonal components under appropriate operating conditions.

In this work, the authors focus on the ANC element, which is responsible for the correct acquisition of data on the parameters of tonal components based on signal analysis and preparation of the compensating signal. The difference in sound levels before and after reduction was assumed as the measure of evaluation of the algorithm quality. The algorithm for detecting tonal components is based on the use of a discrete Fourier transform. In general, there is a trade-off between frequency resolution and signal length for DFT. To obtain a high signal resolution, the long analysis window should be selected. In the case of time-varying signals, however, it is necessary to operate on as few signal samples as possible. For this reason, solutions are being sought that increase the frequency resolution of DFT (Yoshizawa *et al.*, 2011; Ueda *et al.*, 2013) or allow the identification of the frequency spectrum of tonal components parameters with the highest possible precision. This work presents the use of one of the DFT properties that allow accurate identification of parameters so that they can be used for effective reduction of the tonal components, even if the parameters of these components vary over time, as e.g. in the case of car engine rotation speed fluctuation (Dąbrowski *et.al.* 2017).

The rest of this article is organized as follows. Section 2 introduces the tonal component reduction algorithm. The methods for parameter detection are discussed in detail. Sections 3 and 4 present the results of experiments: in Sec. 3 for a pure tone and in Sec. 4 for an actual signal, i.e. a single tonal component of the combustion engine sound signal. The conclusions are presented in Sec. 5.

2. Tonal signal reduction algorithm

The algorithm for reducing the level of tonal components in acoustic signals consists of three stages: detection of the parameters of the tonal component (amplitude, frequency and initial phase), synthesis of the compensating signal (pure tone) and addition of the compensating signal to the original signal in accordance with the assumptions of active noise reduction algorithms.

For the detection of tonal component parameters, the discrete Fourier transform (DFT) property was used, which means that the most accurate readings occur when the signal frequency corresponds to an integral multiple of the DFT resolution, which is related to the sample rate and DFT length (formula (1)). In such situations, a single frame (window) contains the entire signal periods

$$f = k \cdot \frac{F_s}{N}, \tag{1}$$

where f is signal frequency [Hz], F_s is sampling rate [S/s], N is DFT length, $k \in \{1, 2, ...\}$.

To observe this, calculations were made on how the detection of parameters changes depending on the length of the DFT. The tested signal is expressed by the formula (2):

$$x(t) = A\cos(2\pi f t - \varphi), \tag{2}$$

where the parameters are: $A = 0.7$, $f = 100$ Hz, $\varphi = 0$, $t \in \left\{0, \frac{1}{F_s}, \frac{2}{F_s}, \cdots\right\}$, and A is amplitude of a tonal component, f is frequency of a tonal component, φ is initial phase of a tonal component.

The calculations were done in a loop for the DFT length $N \in \{i, I+1, I+2, ...\}$ samples, where k is natural number. The DFT length corresponds to the signal length, expressed in samples, which has been subjected to a discrete transform. The DFT transform was calculated for each number of samples and the maximum

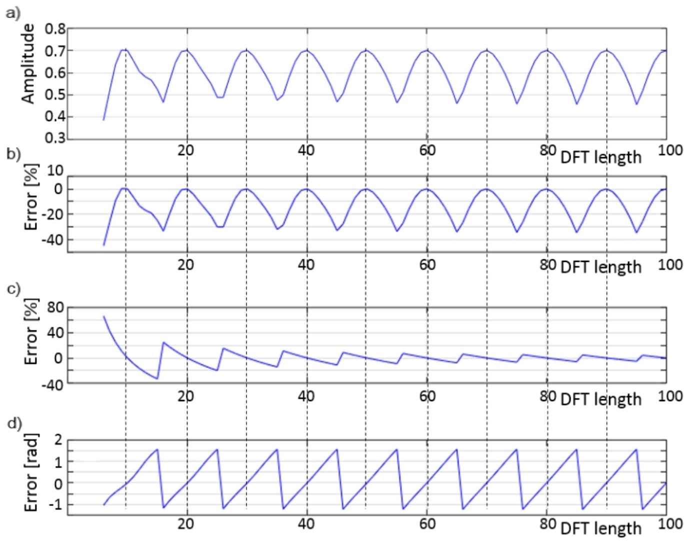


Fig. 1. Impact of DFT length on the detected parameters: a) value of the detected amplitude, b) relative error of amplitude detection, c) relative error of frequency detection, d) absolute error of initial phase detection.

amplitude was determined from the spectrum. The corresponding frequency and the initial phase were read for the determined amplitude. Knowing the amplitude, frequency and initial phase of the signal and the corresponding detected parameters, detection errors were calculated. A summary of the read out amplitude value and parameter detection errors as a function of DFT length is shown in Fig. 1.

When the condition of formula (1) is not met, i.e. there is no signal frequency match to the DFT resolution, there is an underestimation of the amplitude value. On the other hand, the largest amplitude values read from the spectrum correspond to the search value. The smallest parameter detection errors (amplitude, frequency and initial phase) occur for these DFT lengths when the amplitude value read is the largest. As part of the algorithm is indication of the optimal DFT length based on finding the number of samples for which the amplitude for the selected DFT band is the highest. This means that by performing DFT repeatedly for a different number of samples, you can determine when the DFT resolution matches the signal frequency. The algorithm has been tested for various operating conditions, for different window functions used at the detection stage, and for different methods of creating the compensating signal. The block diagram of the algorithm is shown in Fig. 2. The number n corresponds to the local maximum of the function of the maximum value of the amplitude-frequency spectrum. This corresponds to the number of periods of the tonal component of the original signal which are included in a signal with a length such as the DFT for this maximum of the function.

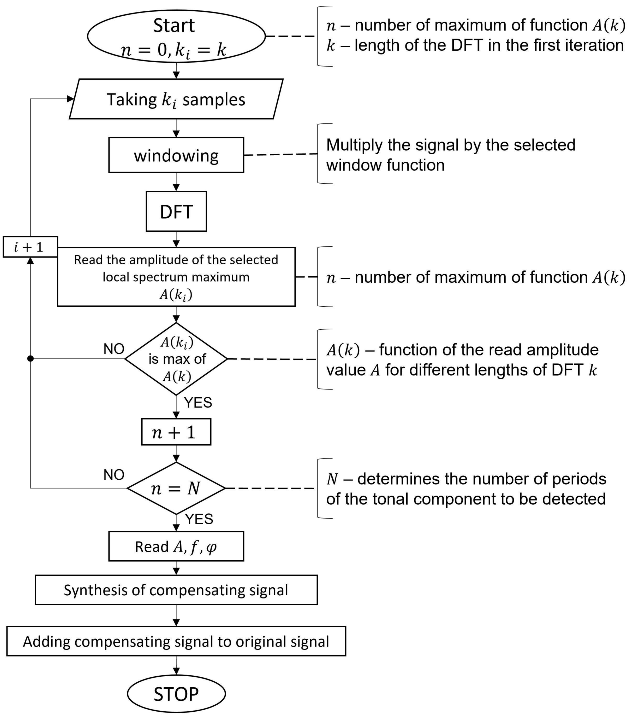


Fig. 2. Block diagram of the tonal components elimination algorithm in single frame (window).

3. Elimination of stationary tonal component

3.1. The impact of the initial phase

One of the factors that affects the detection efficiency and thus the reduction of the component level is the value of the initial phase in the signal frame. A case with a signal frequency of 100 Hz and a sampling rate of 1000 samples per second is a situation where it is possible to accurately match the DFT resolution to the signal frequency using the number of samples that corresponds to the full number of signal periods. For example, when the DFT length is 20 samples, the signal frequency is an integer multiple of the DFT resolution. However, in most cases it is difficult or even impossible to accurately match the number of samples to meet the condition described in formula (1). Even if we choose the optimal DFT length, we are not sure that the frequency and resolution match will be perfect. This occurs, for example, for a signal with a frequency of 127 Hz and a sampling rate constant of 44 100 samples per second. The signal length subjected to the DFT transform would have to be about 347.2 or its multiple samples, which of course is impossible because the number of samples must be an integer. Therefore, parameter detection will be affected by an error and under certain conditions it may happen that the value of the amplitude read from the complex spectrum exceeds the actual value of the amplitude. In such a situation, the developed algorithm will consider the given number of signal samples as adequate to read the other parameters, although this is not the optimal DFT length. It is affected by the initial phase of the signal, which in some cases increases the effective value of the signal fragment. The shorter the signal, the more noticeable this relationship is.

To prove this, an experiment was carried out for two sinusoidal signals described by formula (2). In both cases the amplitude was 0.7, the frequency was 127 Hz and the sampling rate was 44 100 samples per second. The signals differed in the initial phase, which was 0.8 radians in the first case and 0 in the second. For both signals, the effect of DFT length on the value of the detected amplitude was determined, and then according to the algorithm for which the DFT lengths were determined the largest values (search for local extremes). The results are shown in Fig. 3. The crosses indicate the amplitude maxima, which are related to the signal lengths, which approximately correspond to the full signal periods.

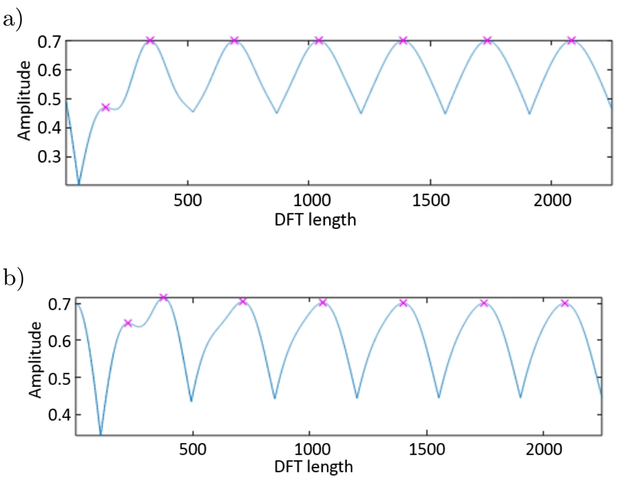


Fig. 3. The effect of DFT length on the read maximum amplitude for a signal with an initial phase: a) 0.8 radians, b) 0 radians.

An exact match occurs for hypothetical DFT lengths equal to: 173.6, 347.2, 694.5, 1041.7, 1389.0, 1736.2, and 2083.4. These numbers, rounded to the nearest integer, correspond to the numbers of samples read in example a) (except for the first DFT length). For this case, subsequent local extremes of function correspond to numbers of samples 164, 347, 694, 1042, 1389, 1736, and 2083. Therefore, it can be assumed that subsequent local extremes of function correspond to integer multiples of full signal periods except for the first extremum, which corresponds to half signal period. In this case, however, the amplitude value is significantly underestimated. A DFT length of 347 samples corresponds to a spectrum resolution of 127.09 Hz. This means that the DFT resolution did not perfectly match the signal frequency. However, the number of samples 347 is an integer that with the smallest error approximates the sought value. For comparison, the DFT length of 346 samples corresponds to a resolution of 127.46 Hz and the length of 348 samples corresponds to a resolution of 126.72 Hz.

In the case of b) successive local extremes of function correspond to the number of samples 224, 376, 715, 1057, 1401, 1746, and 2092. One signal period corresponds to approximately 347 samples. The amplitude value read for these frame lengths is: 0.6469, 0.7164, 0.7055, 0.7027, 0.7016, 0.7010, and 0.7007. It can be seen that in this case the maximum amplitude is shifted. The longer the signal, the smaller the shift and it corresponds to: 60, 29, 21, 15, 12, 10, and 9 samples.

The only difference in the signals presented in both examples is the value of the initial phase. Figure 4 shows the signal waveforms that correspond to one and two signal periods indicated by the algorithm for both initial phase cases (see also Table 1).

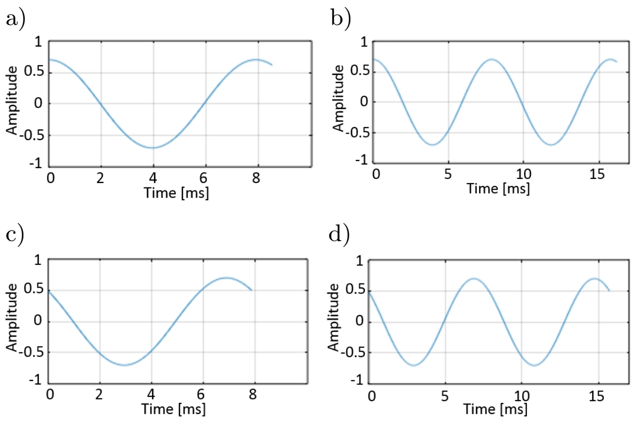


Fig. 4. Signal waveforms with a length of about 1 period (a and c) and two periods (b and d) for the initial phase of the signal equal to 0.8 (a and b) and the initial phase equal to 0 radian (c and d).

The signal lengths, relative errors of detection of the tonal components parameters (dA amplitude er-

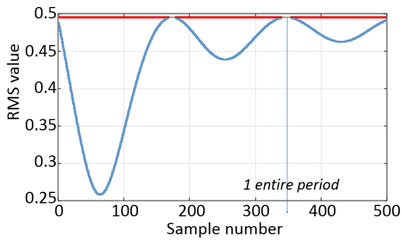
Table 1. Analysis of signal parameters shown in Fig. 4.

Case	Signal length [number of samples]	RMS value	Detection error		
			Amplitude (dA) [%]	Frequency (df) [%]	Phase (dφ) [rad]
a	347	0.495	0.00016	0.07	0.0019
b	694	0.495	0.00024	0.07	0.0041
c	376	0.511	−2.34	7.65	0.56
d	715	0.501	−0.79	2.87	0.62

ror, df frequency error, dφ initial phase error) were compared and the RMS values of both signals were calculated. The theoretical RMS value of a sinusoidal signal should be $0.7/\sqrt{2} = 0.495$.

The RMS value of the second signal is greater than the theoretical value for the sinusoidal signal. This is due to the high values of samples with indexes above 347. For this reason, the algorithm indicated this DFT length as the optimal, i.e. one for which the amplitude value read is the largest. The amplitude value read from the spectrum is related to the RMS value of the signal. Figure 5 shows how the RMS value of the signal depends on the signal length for two cases differing in the initial phase. In the first case, when the signal starts with the initial phase 0.8, the effective value of the sinusoidal signal fragment does not exceed the theoretical value, i.e. $A/\sqrt{2}$ in any case. At the same time, it has the highest value for the number of samples corresponding to entire periods (and half of the period). In the second case, when the initial phase was 0, so the signal began with high values, it happens that the effective value of the signal is greater than the theoretical, therefore for these lengths DFT the largest amplitude values read from the signal spectrum were found. This phenomenon is becoming less noticeable when the length of the signal increases.

a)



b)

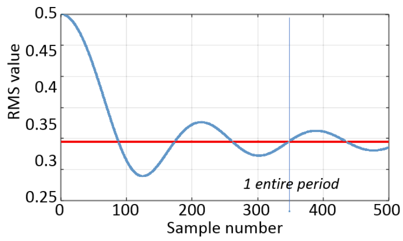
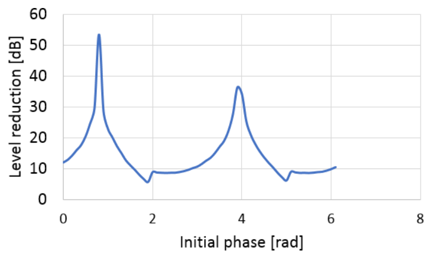


Fig. 5. Dependence of the RMS value of the signal on the DFT length expressed in the number of samples: a) the initial phase equal to 0.8 rad; b) initial phase equal to 0. Theoretical effective value of the sinusoidal signal is marked with a red line.

To indicate how the developed method is effective for detection and elimination of tonal components, the least favourable conditions should be determined. These cases mainly depend on the value of the initial phase in the signal frame. It should be assumed that the initial phase of the signal in the frame can take any value in the range of 0 to 2π. To find the least favourable conditions, it was checked how the reduction efficiency for a pure tone depends on the initial phase. Figure 6 shows this relationship for signals with approximately one and two signal periods. The data regarding the analysis of the algorithm operation for the DFT length corresponding to the first six periods are presented in Table 2. The conditions when the lowest reduction was achieved are assumed as the least favourable and for these conditions the effectiveness of the method is determined.

a)



b)

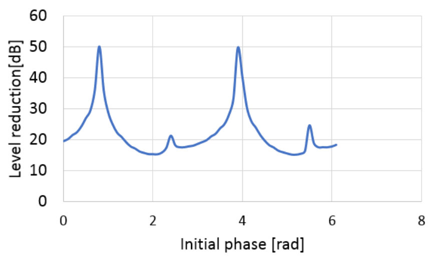


Fig. 6. The influence of the initial phase of the signal on the value of the reduction of the tonal component level for the DFT length corresponding to: a) one signal period, b) two signal periods.

In all the cases analysed, the largest reduction occurred for the initial phase of 0.8. The smallest reduction occurred for the initial phase from 1.8 to 2.1 depending on the number of signal periods, and this value

Table 2. The results of the algorithm for signals with different initial phase. Time window shape: rectangular.

	Number of signal periods					
	1	2	3	4	5	6
The shortest DFT length	301	664	1023	1375	1725	2075
The longest DFT length	416	722	1060	1402	1747	2092
Difference	115	58	37	27	22	17
The largest reduction [dB]	58.2	52	57.1	78.2	58.8	52.3
Initial phase for the largest reduction	0.8	0.8	0.8	0.8	0.8	0.8
The smallest reduction [dB]	10.3	16.3	20.3	22.8	24.7	27.1
Initial phase for the smallest reduction	2	2.1	2	1.8	1.8	1.8
Average level of reduction*	19.7	23.1	26.5	29.9	31.3	32.7

*For initial phase between 0 and 2π

of the initial phase is considered the least favourable condition for a rectangular window.

Additionally, based on Table 2, the following conclusions can be drawn:

- To make sure that the maximum corresponding to one period of the analysed signal is captured, regardless of the value of the initial phase, at least 416 samples must be taken, which is about 20% higher than the theoretical value for which the frequency should be adjusted to the DFT resolution.
- With the increase of signal length and with successive periods, the difference between the largest and smallest DFT lengths that the algorithm chooses as optimal decreases. This confirms that the longer the signal, the harder it is to change the effective value of the signal with single samples (even of high value).
- The longer the signal and the more signal periods, the greater the minimum and average tonal component reduction. There was no clear trend to change the largest reduction value.
- The longer the signal and the more signal periods, the less influence the initial phase of the signal is.

The calculations were repeated for different frequencies and the observations were similar. Therefore, the effectiveness of the method for a rectangular window is determined on the basis of the value of the initial phase for which the smallest reduction of the stationary tonal signal occurs.

3.2. Impact of window function

Next calculations were carried out for different time windows (Fig. 7). Time windows were used at the stage of parameter detection (according to the algorithm in Fig. 2). For each time window shape, calculations were made with an initial phase in the range of 0 to 2π in steps of 0.1 and for different frequencies in the range of 100 Hz to 150 Hz in steps of 1 Hz. In order to determine the reduction efficiency for the stationary tonal signal, the least favourable conditions for each time window were determined.

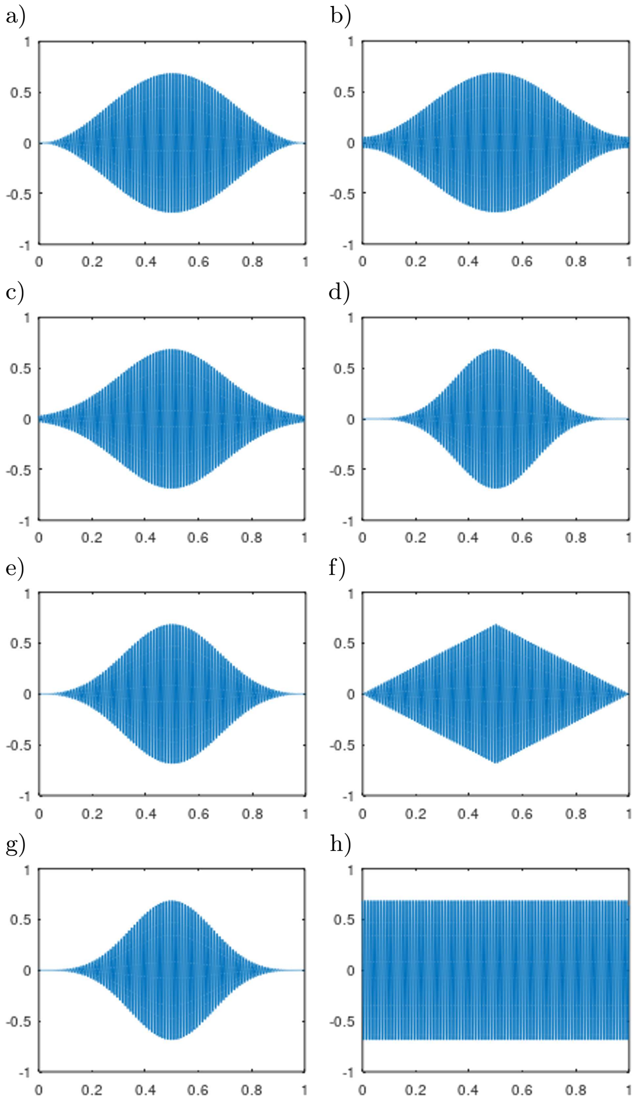


Fig. 7. Applied window functions: a) Hanning, b) Hamming, c) Gauss, d) Blackman-Harris, e) Blackman, f) Bartlet, g) Chebyshev, h) rectangular.

When a time window on the signal is applied and then a DFT transformation on this signal is performed, the amplitude of the spectrum reading will be at a lower level than in the case of a rectangular window, as shown in Fig. 8 below. Therefore, the correction depending on the window type is needed. DFT calculations of the pure tone with known amplitude and frequency as well as DFT length such that the DFT resolution was adjusted to the frequency were performed. The value of the amplitude read in the presence of individual time window was compared with the known value of the pure tone amplitude and then the correction factors were calculated on this basis. The calculated correction factors for different time windows are shown in Table 3.

a)

b)

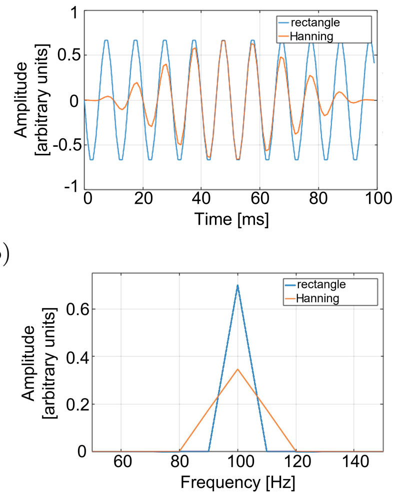


Fig. 8. (a) Graph in time domain and (b) graph in frequency domain of single tonal component with superimposed rectangular window (blue) and Hanning window (orange).

Table 3. Spectral correction factors for various time windows.

Window function	The value of the read amplitude	Correction factor
Hanning	0.35	2
Hamming	0.38	1.85
Blackman	0.29	2.38
Bartlett	0.35	2
Gauss	0.35	2.02
Blackman-Harris	0.25	2.79
Dolph-Chebyshev	0.26	2.7
rectangle	0.7	1

The results of the minimum reduction efficiency of the stationary tonal component by means of the DFT variable length algorithm in the presence of various time windows are presented in Table 4. The results are presented for DFT lengths corresponding from 1 period to 6 signal periods.

Table 4. The smallest reduction of the level of tonal component for various time windows and signal lengths corresponding to signal lengths from one to six periods.

Window function	Number of signal periods					
	1	2	3	4	5	6
Hanning	6.6	31.3	39.9	44.1	46.7	48.2
Hamming	17.4	30.8	31.7	33.2	34.6	37.1
Blackman	1.9	38.5	44.7	46.1	48.2	48.5
Bartlett	37.9	46.6	48.4	50.4	50.4	50.4
Gauss	1.2	28.8	32.5	35.1	36.9	41
Blackman-Harris	−1	45.8	48.4	49.8	50.4	50.4
Chebyshev	−0.2	37.9	41.7	45.2	47.1	48.5
rectangle	10.2	16.1	20.1	22.7	24.6	27

Conclusions from the elimination for stationary tonal components:

- The longer the signal, the greater the reduction level. The best results for short signals (about 1 signal period) were obtained for the Bartlett window.
- The worst results for short signals were obtained for the Blackmann-Harris window and the Chebyshev window. It may happen that the compensated signal will have a higher level than the original signal.
- Best results for long signals were obtained for Bartlett and Blackmann-Harris windows.
- The worst results for long signals were obtained for the rectangular window.

4. Elimination of the tonal component in actual signals

4.1. Reduction of the tonal signal in combustion engine

The signal which was analysed in the previous section is an ideal signal model. However, these considerations were made to indicate the algorithm parameters for which the reduction of the tonal component may be the largest. Real signals are characterized by variability of parameters over time such as small random parameter modulations or an increase or decrease in amplitude and/or frequency.

In this section, the operation of the developed algorithm will be checked for a periodic signal modulated by a random function. An example of such a signal could be the sound of an internal combustion engine. Recordings of the two-stroke internal combustion engine with a constant speed of about 7200 RPM were carried out in the acoustic chamber under controlled acoustic conditions. A single tonal component was separated by means of frequency filtration. The tested signal is not an ideal pure tone with fixed parameters. Time-varying is both the amplitude and frequency, which oscillate around the average value. Figure 9 shows the signal's time course with the envelope marked in red. In the analysed range, the average amplitude value is 0.68, the maximum value is 0.82, and the minimum value is 0.55. The standard deviation of the amplitude value is 0.06. The average frequency is 122.9 Hz, the maximum value is 129.7 Hz and the minimum value is 120.5 Hz. The standard deviation of the frequency value is 1.41.

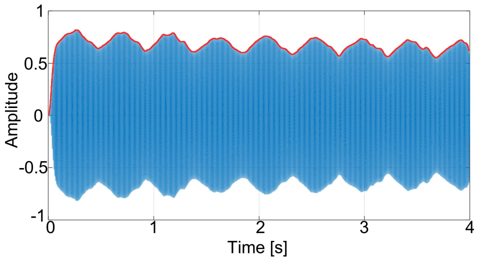


Fig. 9. Time course of tonal component with modulated amplitude and frequency.

Elimination was performed for the length of the compensating signal in a single frame corresponding from 1 to 6 periods. The results of the tonal component level reduction are shown in Fig. 10.

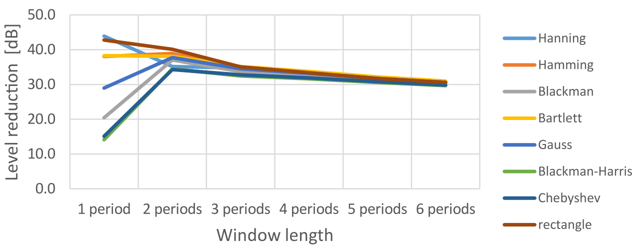


Fig. 10. The results of the reduction of the tonal component of the combustion engine signal.

With short signals of about one period, large differences in level reduction can be observed depending on the shape of the time window. The best results were obtained for Hanning, Hamming, Bartlett and rectangular windows.

It can also be observed that the longer the compensating signal, the lower the level of component reduction. This is due to the variability of the original signal parameters. As the compensating signal is a sinusoidal signal, then larger mismatches between the original signal and the compensating signal occur. The results of the algorithm using single frame length of 1 and 6 periods of given tonal component and Bartlett window are shown in Fig. 11.

For a long compensating signal, a larger mismatch between the compensating signal and the original signal can be observed.

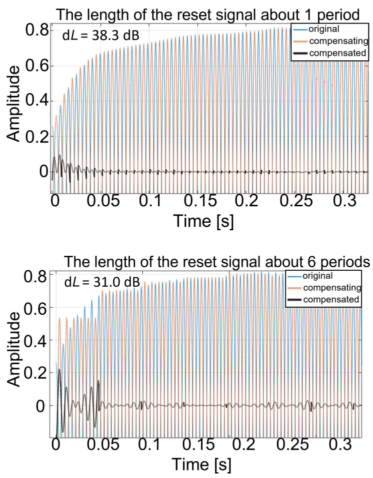


Fig. 11. The initial fragment of the signal before and after the elimination of the tonal component using the Bartlett window.

4.2. Application of overlapping windows for synthesis of the compensating signal

The elimination results presented so far have been made using the elimination method according to the diagram described in Fig. 12. This means that subsequent analysis windows do not overlap and have the shape corresponding to a rectangular function. It is simply reduced to collecting the appropriate number of samples, within these samples the parameters of the compensating component are detected, and then the compensating signal is added to the original signal. Then another set of samples is taken. This way, the compensating signal is created.

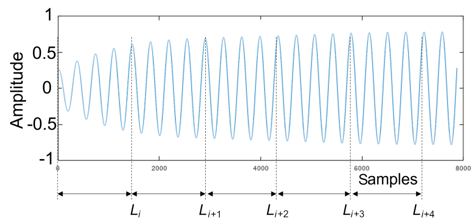


Fig. 12. Elimination scheme without windows overlapping.

In Fig. 12, L_i is a number of samples indicated by the algorithm for which the tonal frequency and DFT resolution are tuned.

The consequence of the use of rectangular time windows that do not overlap are the discontinuities of the compensating signal in the case when the successive windows (frames) have a difference between the final phase in the previous frame and the initial phase in the subsequent frame. A way to avoid this type of phenomenon is to use an overlapping of windows of the right shape. The idea of such overlapping is shown in Fig. 13.

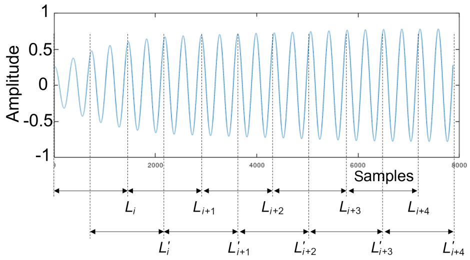


Fig. 13. Elimination scheme with windows overlapping.

L_i is the number of samples indicated by the algorithm for which the tonal component frequency and DFT resolution are tuned. L_i' is the number of samples indicated by the algorithm for windows shifted in accordance with the assumed window overlap.

The compensating signal obtained for the windows not shifted and shifted are multiplied by the function of the window of the appropriate shape and then these signals are added together. Comparison of results without overlapping and with overlapping is shown in Fig. 14.

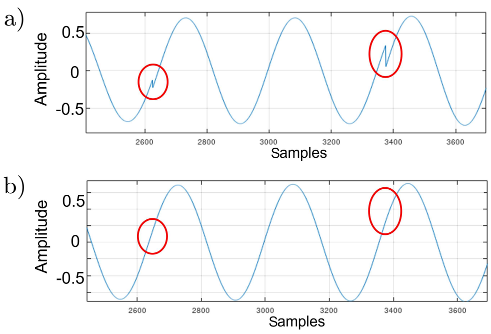


Fig. 14. Fragment of the compensated signal: a) without overlapping b) with overlapping.

It can be seen that in the signal with overlapping signal discontinuities have been eliminated (places marked in red).

Table 5. Reduction of the level of the tonal component of the internal combustion engine signal for a signal length of about 1 period with and without overlapping (expressed in dB).

Window function	Reduction with overlapping	Reduction without overlapping	Difference
Hanning	27.3	21.2	6.1
Hamming	29.8	29.7	0.1
Blackman	11.8	12.4	−0.6
Bartlett	31.5	28.2	3.2
Gauss	20.3	12.8	7.5
Blackman-Harris	6.9	6.9	0
Chebyshev	7.8	7.6	0.2
rectangle	18.1	30.1	−12

Recalculations were performed using window overlap for the length of the compensating signal in a single window approximately equal to 1 period. The Hanning window was used for overlapping. The results of the tonal component reduction with the use of overlapping and without overlapping are presented in Table 5.

5. Discussion and conclusions

An algorithm proposal for using discrete Fourier transform for reducing the level of tonal components of acoustic signals has been presented. It consists of three stages: detection of tonal component parameters (amplitude, frequency and initial phase), synthesis of the compensating signal (pure tone) and addition of the compensating signal to the original signal in accordance with the assumptions of active noise reduction algorithms.

The detection of tonal component parameters uses the discrete Fourier transform (DFT) feature, which means that the most accurate readings occur when the signal frequency corresponds to an integral multiple of the DFT resolution that is related to the sample rate and DFT length. The optimal DFT length is indicated based on finding the number of samples for which the amplitude for the selected DFT band is the highest. Only in the case of shorter signals (e.g. with a length close to one period) may additional errors appear. This is due to the fact that, depending on the initial phase of the signal, the RMS value of a signal that is a little longer than one period may exceed the theoretical RMS value of a sinusoidal signal. For this reason, calculations were made for various initial phases to find the least favourable conditions.

For the least favourable conditions and using various time window functions, the minimum tonal component reduction efficiency was determined for an ideal sinusoidal signal. A simulation was also carried out for an example signal whose parameters change over time. It was a signal of a single tonal component of the combustion engine sound at constant speed. The length of the synthesized compensating signal was dependent on the length of the DFT as indicated by the algorithm at the stage of parameter detection.

The elimination of the tonal component, which in the analysed cases was an incomplete elimination, was carried out for two cases: when the subsequent windows did not overlap and when they overlapped in 50%. In the case of overlapping windows, the Hanning window was additionally used.

The method presented in the article differs from the previously known algorithms for reducing tonal noise in that the frequency of the tonal component is not known a priori and its precise detection is an element of the reduction algorithm. The use of a variable length DFT adjusts the analysis length to the signal frequency so as to obtain the lowest possible frequency detection error. In such a situation, the signal frequency may vary.

The main disadvantage of the presented method of detecting tonal components is the long computational time, which does not allow the algorithm to work in real time due to the need to repeatedly calculate the DFT transform, whose length in the vast majority of cases will not take the value of 2^n (where n is natural number) which would allow for the use of Fast Fourier Transform in optimal conditions.

The high minimal efficiency of the tonal component level reduction even for short signals with a duration of 1 period (31.5 dB for the Bartlett window), indicates that the method can be used not only for active noise reduction, but also wherever an algorithm for accurate parameter detection is needed even with short time windows (computing of spectrograms, analysing non-stationary signals).

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