Research Paper

Magnetosonic Excitation of the Entropy Perturbations in a Plasma with Thermal Conduction Depending on Temperature

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Nonlinear excitation of the entropy perturbations by magnetosonic waves in a uniform and infinite plasma model is considered. The wave vector of slow or fast mode forms an arbitrary angle θ ($0 \le \theta \le \pi$) with the equilibrium straight magnetic field, and all perturbations are functions of the time and longitudinal coordinate. Thermal conduction is the only factor which destroys isentropicity of wave perturbations and causes the nonlinear excitation of the entropy mode. A dynamic equation is derived which describes excitation of perturbation in the entropy mode in the field of dominant magnetosonic mode. Effects associatiated with temperature dependent and anisotropic thermal conduction are considered and discussed.

Keywords: nonlinear magnetohydrodynamics; magnetosonic heating; thermal conduction of a plasma.

1. Introduction

The understanding of magnetohydrodynamic (MHD) perturbations and associated nonlinear phenomena has a cornerstone role in the astrophysical applications (e.g. BALLAI, 2006). They are of key importance in the solar hydrodynamics, fluid flows in thermonuclear reactors, planetary nebulae, interstellar gases, and laboratory plasma. Gaseous plasma is often an open system. This may be modelled by some heating-cooling function which incorporates radiation losses and release of energy (VESECKY *et al.*, 1979; DAHLBURG, MARISKA, 1988; IBÁŃEZ, PARRAVANO, 1994).

In this study, we focus on plasma flows with thermal conduction exclusively. This is the most important reason for the magnetosonic waves damping compared to electrical resistivity and mechanical attenuation of a plasma (CHIN *et al.*, 2010; ZAVERSHINSKII *et al.*, 2020). The thermal conduction has impact both on the evolution of the fast and slow magnetosonic modes and on nonlinear phenomena. In many cases, the linear theory can not explain the experimental data on plasma's heating. The nonlinear transfer of the wave energy into microscopic energy of chaotic motion of molecules leads to magnetosonic heating, that is, to the excitation of the entropy mode. This happens to nonlinear flows due to impact of the damping factors and heating-cooling function. In particular, WANG (2011)

and AFANASYEV, NAKARIAKOV (2014) pointed to importance of the nonlinear effects in the coronal heating. DE MOORTEL, HOOK (2004) and ZAVERSHINSKII *et al.* (2020) have concluded that the thermal conduction is the most significant damping factor in the dynamics of magnetosonic waves and relative nonlinear phenomena. Nonlinear heating due to passage of a wave should be distinguished from that due to impact of an external source of energy. Waves transport energy and momentum, but the excited entropy mode is not the wave motion. It is characterised by isobaric variations in temperature in the areas through which the wave passes. Observation of dynamics of the entropy mode is more convenient than observation of the wave process and may confidently indicate the wave exciter, equilibrium parameters of a plasma and transport factors. Since the magnetosonic heating is a slow process, it is of great importance in the remote observations. This concerns astrophysical applications first of all.

The theory of magnetosound heating is not yet sufficiently developed. Probably, the first results on the way of analytical description of the magnetosonic heating caused by any kind of magnetosonic exciter due to impact of the heating-cooling function and thermal conduction, were obtained by the author (PERELOMOVA, 2018a; 2020). They concern constant thermal conduction. The transport parameters in a plasma are essentially anisotropic (BRAGINSKII,

1965). This study considers anisotropicity of thermal conduction. It considers also variations of parallel and perpendicular to the magnetic field compounds of thermal conduction with temperature. In description of nonlinear interaction of modes, we face mathematical difficulties which are much severe than in the case of nonlinear dynamics of individual mode. Following Nakariakov *et al.* (2000), Chin *et al.* (2010), we consider weakly nonlinear planar flow of completely ionised gas with constant angle between the wave vector and straight equilibrium magnetic field. Weak nonlinearity imposes the smallness of the Mach number in a flow: $M \ll 1$. Nonlinearity has crucial impact on the wave dynamics and may lead to discontinuities in the waveform. While perturbations of infinitely small magnitude develop independently, the nonlinearity and deviation from adiabaticity of any kind condition interaction of different types of a fluid's motion.

The mathematical method is projecting of the initial PDE system onto the governing equations for interacting modes. It has been applied by Leble, Perelomova (2018), Perelomova (2006) in studies of nonlinear interaction of wave and non-wave modes in a number of different fluid flows. The essence of the method is to specify some linear combination of equations which reduces all linear terms containing nonspecific perturbations. This leads to a set of dynamic equations describing interaction of different modes. In the linear part, they include perturbations of the excited mode exclusively, and the nonlinear terms form the stimulative "sources". Actually, projecting relies on the links of specific perturbations which represent every dispersion relation. In the context of magnetosonic heating, the quadratic terms including magnetosonic perturbations are of major importance among all quadratic terms forming the "source" exciting the entropy mode. The nonlinear dynamic equation for the excitation of the secondary entropy perturbations is valid with the accepted accuracy up to quadratic nonlinear terms, that is, up to terms proportional to the squared Mach number, M^2. That concerns also terms originating from the thermal conduction. The perturbations which specify the entropy mode, enlarge in absolute value over time. In the frames of a model, their magnitudes should not exceed magnitudes of exciting magnetosonic perturbations. When these magnitudes equalise, the entropy mode may have impact on the wave process. Hence, the equations are valid over some temporal and spatial domains, where the wave perturbations remain dominant. The effects of plasma's boundaries are not considered.

We do not consider mechanical losses in a plasma and its finite electrical conductivity whose impact is well understood. These factors introduce additional attenuation of the magnetosonic perturbations and enhance the nonlinear interactions. The magnetoacoustic heating/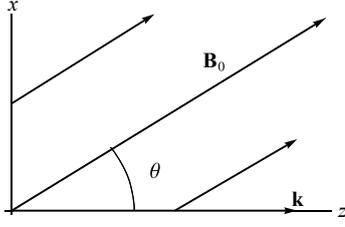cooling due to exclusively heating-cooling function has been studied by Perelomova (2018a; 2018b). Some kinds of heating-cooling function may overbalance thermal conduction and lead to acoustical activity and magnetosound cooling of a plasma. In this study, we analyse the instantaneous dynamic equation describing excitation of the entropy mode in the field of planar magnetosonic wave (fast or slow) and discuss it in the cases of periodic and impulsive wave perturbations. The only damping mechanism is the thermal conduction of a plasma which depends on its temperature and on the geometry of a flow.

2. Evolutionary equations in the finite-magnitude flow with thermal losses

We start from the MHD equations describing uniform, quasi-neutral, and fully ionised plasma with infinite electrical conductivity. Ideal magnetohydrodynamics is a reasonable approximation in astrophysical applications. The mechanical viscosity and electrical resistivity of a plasma are discarded. We make use of the continuity equation, the momentum equation, the energy balance equation, and electrodynamic equations in the differential form (Krall, Trivelpiece, 1973; Callen, 2003):

$$
\begin{aligned}
&\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \\
&\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \frac{1}{\mu_0}(\nabla \times \mathbf{B}) \times \mathbf{B}, \\
&\frac{Dp}{Dt} - \gamma \frac{p}{\rho}\frac{D\rho}{Dt} = (\gamma - 1)\nabla \cdot (\chi \nabla T), \\
&\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}), \\
&\nabla \cdot \mathbf{B} = 0,
\end{aligned}
\tag{1}
$$

where p, ρ, $\mathbf{v}$ are thermodynamic pressure, mass density of a plasma, and its velocity, $\mathbf{B}$ designates the magnetic field, μ_0 is permeability of the free space, χ is the thermal conduction of a plasma. The third equation in the set (1) incorporates the continuity and energy equations. We consider plasma as an ideal gas with the ratio of specific heats (per unit mass) under constant pressure and constant density γ, $\gamma = C_P/C_V$. T designates the temperature of a plasma (it obeys an ideal gas state $T = \frac{p}{(C_P - C_V)\rho}$). The equation of state for an ideal gas is valid for weakly coupled plasmas. The two last equations are the induction equation and the Maxwell's equation reflecting solenoidal character of $\mathbf{B}$. We accept the geometry used in the studies by Nakariakov *et al.* (2000) and Chin *et al.* (2010): the wave vector of a planar flow $\mathbf{k}$ is directed along axis z and forms a constant angle θ ($0 \leq \theta \leq \pi$) with the straight equilibrium magnetic field $\mathbf{B}_0$. The figure displays the geometry of a flow.

The thermal conduction coefficient depends on θ and T:

$$\chi = \chi_\perp(T)\sin^2(\theta) + \chi_\parallel(T)\cos^2(\theta).$$

The classical transport theory concludes that the thermal conduction parallel to the magnetic field is much larger than the perpendicular one, that is, $\chi_\parallel \gg \chi_\perp$, $\chi_\parallel \sim T^{5/2}$, $\chi_\perp \sim T^{-5/2}$ (BRAGINSKII, 1965). This study considers $\chi_\perp$ bearing in mind its impact at $\theta \approx \pi/2$. The equilibrium thermal conduction at $T = T_0$ sounds as

$$\chi_0 = \chi_{0,\perp}\sin^2(\theta) + \chi_{0,\parallel}\cos^2(\theta)$$

($\chi_{0,\parallel} = \chi_\parallel(T_0)$, $\chi_{0,\perp} = \chi_\perp(T_0)$). All thermodynamic perturbations are functions of t and z. The y-component of $\mathbf{B}_0$ equals zero, and

$$B_{0,x} = B_0\sin(\theta), \qquad B_{0,z} = B_0\cos(\theta), \qquad B_{0,y} = 0.$$

The system (1) displays the nonlinearity of MHD flows. For further analysis, its leading-order form is considered, valid with accuracy up to quadratic nonlinear terms (CHIN et al., 2010; PERELOMOVA, 2018a), completed with the nonlinear terms originating from the temperature-dependent behaviour of χ:

$$\frac{\partial \rho'}{\partial t} + \rho_0\frac{\partial v_z}{\partial z} = -\rho'\frac{\partial v_z}{\partial z} - v_z\frac{\partial \rho'}{\partial z},$$

$$\frac{\partial v_x}{\partial t} - \frac{B_{0,z}}{\rho_0\mu_0}\frac{\partial B_x}{\partial z} = -v_z\frac{\partial v_x}{\partial z} - \frac{B_{0,z}}{\rho_0^2\mu_0}\rho'\frac{\partial B_x}{\partial z},$$

$$\frac{\partial v_y}{\partial t} - \frac{B_{0,z}}{\rho_0\mu_0}\frac{\partial B_y}{\partial z} = -v_z\frac{\partial v_y}{\partial z} - \frac{B_{0,z}}{\rho_0^2\mu_0}\rho'\frac{\partial B_y}{\partial z},$$

$$\frac{\partial v_z}{\partial t} + \frac{1}{\rho_0}\frac{\partial p'}{\partial z} + \frac{B_{0,x}}{\rho_0\mu_0}\frac{\partial B_x}{\partial z} = \frac{\rho'}{\rho_0^2}\frac{\partial p'}{\partial z} + \frac{B_{0,x}}{\rho_0^2\mu_0}\rho'\frac{\partial B_x}{\partial z}$$
$$- \frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\frac{B_x^2 + B_y^2}{2\mu_0}\right) - v_z\frac{\partial v_z}{\partial z},$$

$$\frac{\partial p'}{\partial t} + c^2\rho_0\frac{\partial v_z}{\partial z} - \frac{\chi_0}{\rho_0 C_P}\frac{\partial^2\gamma p'}{\partial z^2} + \frac{\chi_0 c_0^2}{\rho_0 C_P}\frac{\partial^2\rho'}{\partial z^2}$$
$$= -\gamma p'\frac{\partial v_z}{\partial z} - v_z\frac{\partial p'}{\partial z} - \frac{\chi_0}{\rho_0^2 C_P}\frac{\partial^2(\gamma p'\rho' - c_0^2\rho'^2)}{\partial z^2}$$
$$+ 5(\chi_{0,\parallel}\cos^2(\theta) - \chi_{0,\perp}\sin^2(\theta))$$
$$\cdot \frac{(\gamma p' - c_0^2\rho')}{2c_0^2 C_P\rho_0^2}\frac{\partial^2(\gamma p' - c_0^2\rho')}{\partial z^2},$$
$$\quad (2)$$

$$\frac{\partial B_x}{\partial t} + \frac{\partial}{\partial z}(B_{0,x}v_z - B_{0,z}v_x) = -B_x\frac{\partial v_z}{\partial z} - v_z\frac{\partial B_x}{\partial z},$$

$$\frac{\partial B_y}{\partial t} - \frac{\partial}{\partial z}(B_{0,z}v_y) = -B_y\frac{\partial v_z}{\partial z} - v_z\frac{\partial B_y}{\partial z}.$$

The linear terms proportional to χ_0 are responsible for the linear damping. Equations (2) describe small signal MHD perturbations and refer to the combined effects of weak nonlinearity and weak damping due to thermal conduction.

2.1. Linear analysis

The linear analysis of the flow properties always precedes the nonlinear one. This is the case of small magnitude perturbations which are described by Eqs (2) with zero nonlinear terms on the right of equations. The dispersion relations follow from Eqs (2), if one looks for a solution in the form of a sum of planar waves proportional to $\exp(i\omega(k_z)t - ik_z z)$,

$$f'(z,t) = \int_{-\infty}^{\infty} \widetilde{f}(k_z)\exp(i\omega(k_z)t - ik_z z)\,dk_z,$$

where k_z designates the wave number. We remind four relations inherent to the magnetosonic modes which rely on compressibility:

$$\omega_j = C_j k_z + i\frac{A_j}{2}k_z^2, \qquad j = 1,...,4, \qquad (3)$$

where

$$A_j = \frac{C_j^2 - C_A^2}{2C_j^2 - c_0^2 - C_A^2}\frac{\chi_0\left(\frac{1}{C_V} - \frac{1}{C_P}\right)}{\rho_0}, \qquad (4)$$

C_j is the magnetosonic speed, a root of the equation

$$C_j^4 - C_j^2(c_0^2 + C_A^2) + c_0^2 C_{A,z}^2 = 0, \qquad (5)$$

C_A and c_0

$$C_A = \frac{B_0}{\sqrt{\mu_0\rho_0}}, \qquad c_0 = \sqrt{\frac{\gamma p_0}{\rho_0}}$$

designate the Alfvén speed and the acoustic speed in unmagnetised gas in equilibrium, $C_{A,z} = C_A\cos(\theta)$. There are two dispersion relations $\omega = \pm C_A k_z$ which specify the Alfvén waves with the links

$$v_y = \mp\frac{B_y' C_A}{B_0}, \qquad \rho' = 0, \qquad p' = 0, \qquad v_x = 0,$$
$$v_z = 0, \qquad B_x' = 0,$$

and one relation corresponding to the entropy mode. Equations (3) are leading order, they are valid with accuracy up to terms proportional to the first power of χ_0. The dispersion relations (3), Eq. (5) and the dynamic Eq. (7) have been derived by CHIN et al. (2010) for the more general case taking into account the heating-cooling function. As for the entropy nonwave mode, it is specified by the dispersion relation

$$\omega_{\text{ent}} = ik_z^2\frac{\chi_0}{C_P\rho_0} \qquad (6)$$

and the links

$$v_x = -\frac{C_{A,x}\chi}{C_{A,z}C_P\rho_0^2}\frac{\partial\rho'}{\partial z}, \qquad v_z = -\frac{\chi}{C_P\rho_0^2}\frac{\partial\rho'}{\partial z},$$

$$p' = 0, \qquad v_y = 0, \qquad B_x' = 0, \qquad B_y' = 0.$$

An attenuation of the wave modes given by Eq. (4) depends on the kind of sound (fast or slow), on the angle θ and the ratio $\alpha = \frac{C_A}{c_0}$. Figure 1 shows the dimensional attenuation in the case of nearly parallel or perpendicular propagation of the magnetosonic wave (the lower index j is omitted).

The appropriate evolutionary nonlinear equation for an excess density in any magnetosonic mode ρ_{ms} takes the form (CHIN *et al.* (2010); the lower index is omitted):

$$\frac{\partial\rho_{ms}}{\partial t} + C\frac{\partial\rho_{ms}}{\partial z} + \frac{C\varepsilon}{\rho_0}\rho_{ms}\frac{\partial\rho_{ms}}{\partial z} - \frac{A}{2}\frac{\partial^2\rho_{ms}}{\partial z^2} = 0, \quad (7)$$

where ε is responsible for nonlinear distortions,

$$\varepsilon = \frac{3c_0^2 + (\gamma+1)C_A^2 - (\gamma+4)C^2}{2(c_0^2 - 2C^2 + C_A^2)}.$$

The case $\theta = 0$ and $C = c_0 = C_A$ is special. In fact, two magnetosound modes exist in this case, and two roots $(\omega = \pm C_A k_z)$ degenerate into the Alfvén modes with the links

$$v_x = \mp\frac{B_x'C_A}{B_0}, \qquad \rho' = 0, \qquad p' = 0, \qquad v_y = 0,$$

$$v_z = 0, \qquad B_y' = 0.$$

The dynamic equation for the velocity in the degenerate modes is as follows:

$$\frac{\partial v_x}{\partial t} + C\frac{\partial v_x}{\partial z} = 0. \qquad (8)$$

These wave forms do not subject to distortion due to nonlinear effects and thermal conduction. They do not contribute to magnetosonic heating.
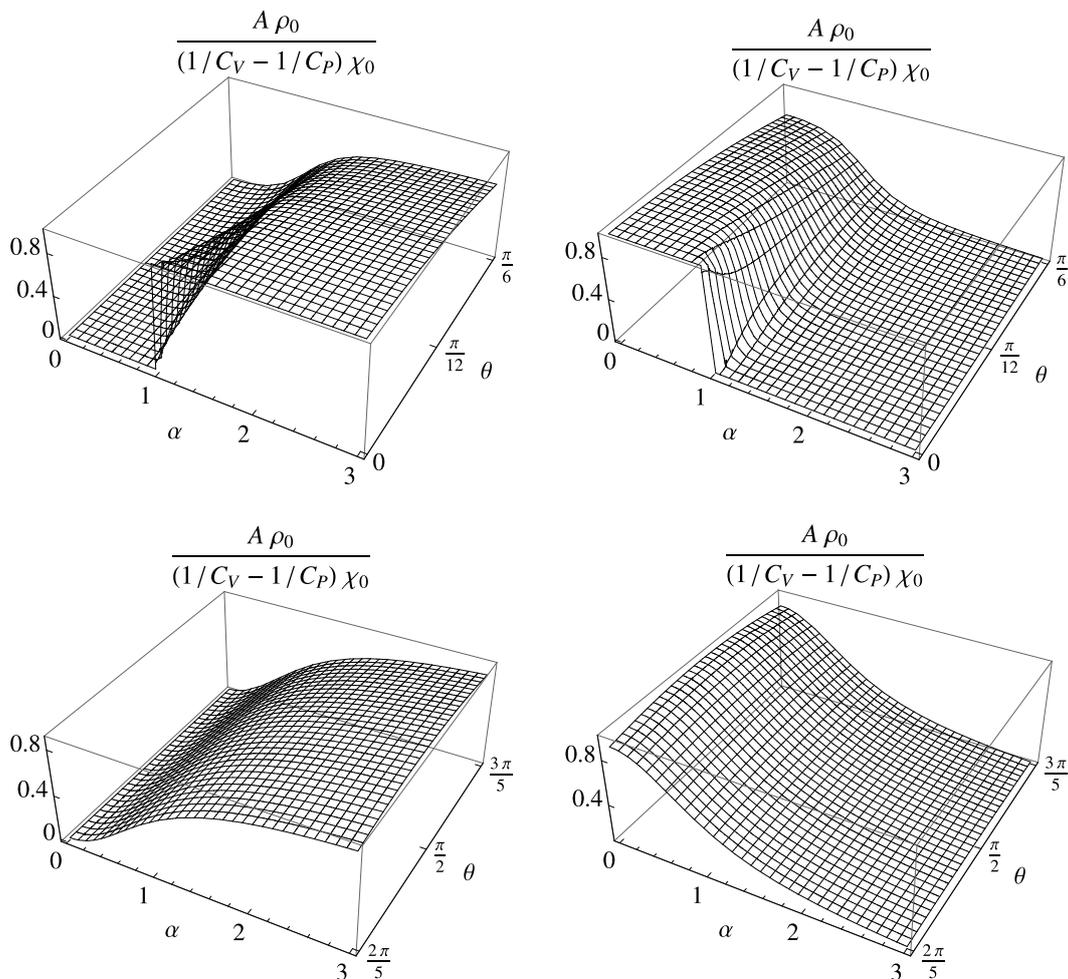

Fig. 1. Dimensionless attenuation $\frac{A\rho_0}{(1/C_V-1/C_P)\chi_0}$ in the case of a nearly parallel (upper row, $\chi_0 \approx \chi_{0,\parallel}$) and nearly perpendicular (bottom row, $\chi_0 \approx \chi_{0,\perp}$) magnetic field and the wave vector of the magnetosonic mode as a function of $\alpha = \frac{C_A}{c_0}$ and θ. Left panels: slow waves, right panels: fast waves.

3. The dynamic equation for the magnetosonic heating

The details of linear projecting and its application to the magnetosonic heating/cooling in a weakly non-linear plasma flow may be found in (PERELOMOVA, 2018a; 2018b). The ideas of projecting in the various fluid flows have been developed and discussed by LEBLE, PERELOMOVA (2018). In the context of excitation of the entropy perturbations in the field of magnetosonic dominant mode, the dynamic equation for the perturbation in density in the entropy mode may be selected from the Eqs (2) by applying of the projection operator P_{ent}. It distinguishes an excess density in the entropy mode from the total vector of perturbations

$$\psi = \begin{pmatrix} \rho' & v_x & v_y & v_z & p' & B_x & B_y \end{pmatrix}^{\text{T}}$$

and may be readily established from the system of seven algebraic equations (ρ_{ent} is an excess density specifying the entropy mode)

$$P_{\text{ent}}\psi = \rho_{\text{ent}}, \qquad (9)$$

bearing in mind linear relations between perturbations specifying every mode. Its leading order form is

$$P_{\text{ent}} = \begin{bmatrix} 1 \\ -\dfrac{(\gamma-1)C_{A,x}\chi_0}{C_{A,z}c_0^2 C_P}\dfrac{\partial}{\partial z} \\ 0 \\ -\dfrac{(\gamma-1)\chi_0}{c_0^2 C_P}\dfrac{\partial}{\partial z} \\ -\dfrac{1}{c_0^2} \\ 0 \\ 0 \end{bmatrix}^{\text{T}}. \qquad (10)$$

The equation which governs an excess density in the entropy mode, has been obtained and analysed by the author in a number of studies in the case of constant thermal conduction of unmagnetised gases (LEBLE, PERELOMOVA, 2008; PERELOMOVA, 2006). The magnetosonic heating/cooling due to some heating-cooling function has been described and discussed by PERELOMOVA (2018a; 2018b). Including in the consideration $\chi_\parallel$, $\chi_\perp$ and their dependence on temperature in the magnetised plasma, we arrive to the diffusion equation with the magnetosonic source Q:

$$\frac{\partial \rho_{\text{ent}}}{\partial t} - \frac{\chi_{0,\parallel}\cos^2(\theta) + \chi_{0,\perp}\sin^2(\theta)}{C_P \rho_0}\frac{\partial^2 \rho_{\text{ent}}}{\partial z^2} = Q, \quad (11)$$

where

$$Q = \frac{(\gamma-1)(\chi_{0,\parallel}\cos^2(\theta) + \chi_{0,\perp}\sin^2(\theta))}{2C_P \rho_0^2 c_0^2 C_{A,x}^3(C^4 - c_0^2 C_{A,z}^2)}\Big[(C^6(c_0^2 C_{A,x}$$

$$+ 2(C_{A,x}^2 C_{A,z} + C_{A,x}C_A^2 - C_{A,z}^3)) - C^4 c_0^2(2c_0^2 C_{A,x}$$

$$+ 2C_{A,x}^2 C_{A,z} + 7C_{A,x}C_{A,z}^2 - 6C_{A,z}^3 + 4(\gamma-1)C_{A,x}^3)$$

$$+ C^2 c_0^2(c_0^4 C_{A,x} + c_0^2(8C_{A,x}C_{A,z}^2 - 6C_{A,z}^3 + (\gamma-1)C_{A,x}^3)$$

$$+ 2C_{A,x}^2 C_{A,z}^2(C_{A,x}(\gamma-2) - \gamma C_{A,z}))$$

$$+ c_0^2(C_{A,z}^2(c_0^2(2C_{A,z} - 3C_{A,x})$$

$$+ C_{A,x}^2((\gamma-1)C_{A,x} + 2\gamma C_{A,z})))\Big]\frac{\partial}{\partial z}\left(\rho_{ms}\frac{\partial \rho_{ms}}{\partial z}\right)$$

$$+ \frac{(\gamma-1)^2(\chi_{0,\parallel}\cos^2(\theta) + \chi_{0,\perp}\sin^2(\theta))}{C_P \rho_0^2}\rho_{ms}\frac{\partial^2 \rho_{ms}}{\partial z^2}$$

$$- 5(\chi_{0,\parallel}\cos^2(\theta) - \chi_{0,\perp}\sin^2(\theta))\frac{(\gamma-1)^2}{2C_P \rho_0^2}\frac{\partial}{\partial z}\left(\rho_{ms}\frac{\partial \rho_{ms}}{\partial z}\right),$$
$$(12)$$

where ρ_{ms} designates a perturbation of density which individuates some dominant magnetosonic mode. The magnetosonic mode is dominant if magnitudes of its perturbations are much larger than those of other modes. We consider one dominant mode and hold only its quadratic terms in the source Q on the right of Eq. (12). The source is associated with the thermal conduction. The source Q coincides with that obtained in the context of acoustic heating of unmagnetised gases and constant thermal conduction, where $C_A = 0$, $C = c_0$, $\rho_{ms} = \rho_s$ (PERELOMOVA, 2008):

$$Q = (\gamma-1)\frac{\chi_0}{\rho_0^2 C_P}\left(\frac{5-3\gamma}{2}\left(\frac{\partial \rho_s}{\partial z}\right)^2 - \frac{\gamma-3}{2}\rho_s\frac{\partial^2 \rho_s}{\partial z^2}\right)$$

(ρ_s is the acoustic excess density). Its averaged over the exciter's period form for the nearly harmonic acoustic exciter is

$$\langle Q \rangle \approx -(\gamma-1)^2\frac{\chi_0}{\rho_0^2 C_P}\left\langle\left(\frac{\partial \rho_s}{\partial z}\right)^2\right\rangle.$$

Generally, the source Q depends on $\chi_{0,\parallel}$ and $\chi_{0,\perp}$ individually and includes the term

$$5(\chi_{0,\parallel}\cos^2(\theta) - \chi_{0,\perp}\sin^2(\theta))\frac{(\gamma-1)^2}{2C_P \rho_0^2}\frac{\partial}{\partial z}\left(\rho_{ms}\frac{\partial \rho_{ms}}{\partial z}\right)$$

which refers to the power dependence of the thermal conduction on temperature. Typically, $\chi_{0,\parallel} \gg \chi_{0,\perp}$. In the coronal case, $\chi_{0,\parallel} \approx 10^{11}\chi_{0,\perp}$, and in the chromospheric case, $\chi_{0,\parallel} \approx 10^8 \chi_{0,\perp}$ (HEYVAERTS, 1974). Thus, the individual impact of $\chi_{0,\perp}$ might be significant at the angles θ very close to $\pi/2$.

4. Excitation of the entropy mode

For definiteness, we consider the dominant magnetosonic mode (slow or fast) with $C > 0$.

4.1. Nearly parallel to the magnetic field wave vector of the dominant wave

This is the case $C_{A,z} = C_A$, $C_{A,x} = 0$, $C = c_0$

$$Q = -\frac{(\gamma-1)(3\gamma-5)\chi_{0,\|}}{2C_P\rho_0^2}\frac{\partial}{\partial z}\left(\rho_{ms}\frac{\partial\rho_{ms}}{\partial z}\right)$$
$$+\frac{(\gamma-1)^2\chi_{0,\|}}{C_P\rho_0^2}\rho_{ms}\frac{\partial^2\rho_{ms}}{\partial z^2}$$
$$-5\chi_{0,\|}\frac{(\gamma-1)^2}{2C_P\rho_0^2}\frac{\partial}{\partial z}\left(\rho_{ms}\frac{\partial\rho_{ms}}{\partial z}\right).$$

In the case $\gamma = 5/3$, the first term equals zero.

4.1.1. Harmonic exciter

The harmonic magnetosonic exciter takes the leading order form

$$\rho_{ms} = M\rho_0\sin\left(\frac{z-Ct}{\Lambda}\right)\equiv\rho_A\sin(Z), \qquad (13)$$

where Λ^{-1} designates the wave number, and $\rho_A = M\rho_0$ is the amplitude of an exciting perturbation in density. In this case,

$$Q = -\frac{(\gamma-1)M^2\chi_{0,\|}}{2\Lambda^2C_P}((\gamma-1)+(7\gamma-9)\cos(2Z)).$$

The approximate evaluation of ρ_{ent} is an integral of Q over time (we suppose that the nonlinear interaction starts at $t = 0$):

$$\rho_{ent} = -\frac{\rho_0}{T_0}T_{ent} = \int_0^t Q\,dt = -\frac{(\gamma-1)^2M^2\chi_{0,\|}}{2\Lambda^2C_P}$$
$$\cdot\left(t+\frac{(7\gamma-9)\Lambda}{(\gamma-1)C}\cos\left(\frac{Ct}{\Lambda}+2Z\right)\sin\left(\frac{Ct}{\Lambda}\right)\right).$$

It is negative on average and does not depend on α:

$$\langle\rho_{ent}\rangle = -\frac{(\gamma-1)^2M^2\chi_{0,\|}t}{2\Lambda^2C_P}.$$

This corresponds to the positive excess temperature of the background T_{ent} due to transform of the wave energy into the energy of chaotic motion of molecules.

4.1.2. Gaussian impulse

The magnetosonic excitation in the form of the Gaussian impulse

$$\rho_{ms} = \rho_A\exp\left(-\frac{(z-Ct)^2}{\Lambda^2}\right)\equiv\rho_A\exp\left(-Z^2\right) \qquad (14)$$

results in

$$Q = -\frac{(\gamma-1)M^2\chi_{0,\|}}{\Lambda^2C_P}(8-36Z^2+\gamma(28Z^2-6))e^{-2Z^2}$$

and achieves a maximum

$$Q_{max} = \frac{(\gamma-1)(6\gamma-8)M^2\chi_{0,\|}}{\Lambda^2C_P}$$

at $Z = 0$. The simple evaluations of an excess density associating with the entropy mode after passing of the impulse, yields the negative quantity

$$\rho_{ent} = -\frac{\rho_0}{T_0}T_{ent} = \int_{-\infty}^{\infty}Q\,dt = -\frac{(\gamma-1)^2\sqrt{\pi}M^2\chi_{0,\|}}{\sqrt{2}\Lambda CC_P}.$$

4.2. Nearly perpendicular to the magnetic field wave vector of the dominant wave

This is the case $C_{A,z} = 0$, $C_{A,x} = C_A$, $C = \sqrt{c_0^2+C_A^2}$.

$$Q = -\frac{\chi_{0,\perp}}{C_P\rho_0^2}$$
$$\cdot\frac{(\gamma-1)(\gamma(3+4\alpha^2)-5-9\alpha^2-2\alpha^4)}{2(1+\alpha^2)}\frac{\partial}{\partial z}\left(\rho_{ms}\frac{\partial\rho_{ms}}{\partial z}\right)$$
$$+\frac{(\gamma-1)^2\chi_{0,\perp}}{C_P\rho_0^2}\rho_{ms}\frac{\partial^2\rho_{ms}}{\partial z^2}$$
$$+5\chi_{0,\perp}\frac{(\gamma-1)^2}{2C_P\rho_0^2}\frac{\partial}{\partial z}\left(\rho_{ms}\frac{\partial\rho_{ms}}{\partial z}\right),$$

where the last term dominates over the first one if $\alpha < 1.25$ at $\gamma = 5/3$.

4.2.1. Harmonic exciter

The harmonic excitation (13) results in

$$\rho_{ent} = -\frac{(\gamma-1)^2M^2\chi_{0,\perp}}{\Lambda^2C_P}$$
$$\cdot\left(\frac{t}{2}-a^*\cos\left(\frac{Ct}{\Lambda}+2Z\right)\sin\left(\frac{Ct}{\Lambda}\right)\right),$$

where

$$a^* = \frac{\Lambda(\gamma(7+8\alpha^2)-9-13\alpha^2-2\alpha^4)}{2C(\gamma-1)(1+\alpha^2)}.$$

The averaged over the exciter's period quantity is negative:

$$\langle\rho_{ent}\rangle = -\frac{(\gamma-1)^2M^2\chi_{0,\perp}t}{2\Lambda^2C_P}.$$

4.2.2. Gaussian impulse

The source associating with the Gaussian impulse, Eq. (14), takes the form

$$Q = \frac{(\gamma-1)^2M^2\chi_{0,\perp}}{\Lambda^2C_P}\frac{b^*}{1+\alpha^2}e^{-2Z^2},$$

where

$$b^* = (36Z^2 - 8 + \alpha^4(8Z^2 - 2) + 4\alpha^2(13Z^2 - 3)$$

$$+ \gamma(\alpha^2(7 - 32Z^2) - 28Z^2 + 6)).$$

Approximately, an excess density associating with the entropy mode after passing of the impulse, equals

$$\rho_{\text{ent}} = \int\limits_{-\infty}^{\infty} Q\, \mathrm{d}t = -\frac{(\gamma - 1)^2 \sqrt{\pi} M^2 \chi_{0,\perp}}{\sqrt{2} \Lambda C C_P}$$

and is independent from α. Figure 2 shows the source as a function of α and Z.
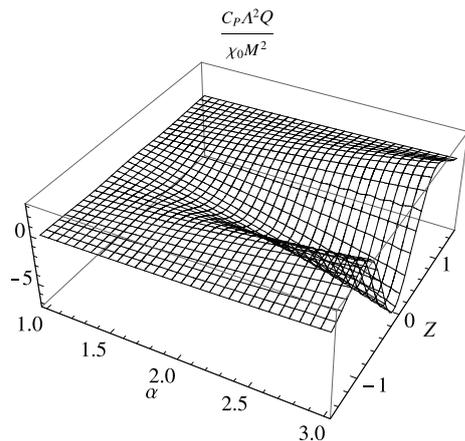

Fig. 2. Dimensionless magnetosonic source $\frac{C_P \Lambda^2 Q}{\chi_0 M^2}$ in the case of nearly perpendicular magnetic field and the wave vector of the dominant mode $\chi_0 = \chi_{0,\perp}$ as a function of $\alpha = \frac{C_A}{c_0}$ and $Z = \frac{z - Ct}{\Lambda}$.

5. Summary and remarks

In this study, we consider the nonlinear excitation of the entropy mode exclusively due to thermal conduction which depends on plasma's temperature and an angle between the equilibrium magnetic field and the wave vector, that is, is anisotropic. In the MHD theory, only parallel compound of the thermal conduction $\chi_\parallel$ is usually considered (e.g., NAKARIAKOV *et al.*, 2000; OFMAN, WANG, 2002; DE MOORTEL, HOOD, 2004; CHIN *et al.*, 2010). The transport parameters in a plasma flow depend on the temperature and are anisotropic in essence. The electrical resistivity is out of attention in this study. It originates from collisions between electrons and ions and depends on temperature as $T^{-3/2}$ (SPITZER, 1962). It is anisotropic with at most $\sigma_\perp^{-1} \approx 3.4\sigma_\parallel^{-1}$ (the electrical conductivity along the magnetic field $\sigma_\parallel$ is the Spitzer value (CALLEN, 2003)). RUDERMAN *et al.* (1996) concluded that the electrical resistivity is almost irrelevant in the solar corona applications. In the context of magnetosonic heating, implication of dependence of σ on temperature introduces nothing into equations which gov-

ern magnetosonic dominant perturbations and associated entropy perturbations. That is due to the form of projector P_{ent} which includes zero order terms 1 and $-1/c_0^2$ only in the first and fifth columns, respectively. For the same reason, it is of no use to take into consideration the temperature dependent viscosity of a plasma which could be a more important damping term compared with finite electrical conductivity (see, e.g. HOLLWEG, 1985). Five viscosity coefficients $\eta_0, ..., \eta_4$ contributing to the viscous stress tensor depend on temperature (BRAGINSKII, 1965). In particular, $\eta_0 = 10^{-17}\mathrm{T}^{5/2}$ kg·m^{-1}·s^{-1} in the coronal conditions.

The main result of this study is the dynamic equation (11) with the magnetosonic source of heating Q (12). It considers dependence of the thermal conduction on temperature and its anisotropic character. Equation (11) is instantaneous and fits any kind of exciters, including impulsive ones. The example of flow in a magnetised plasma is special due to variety of fast and slow wave modes, nonlinear interactions, and dependence of the plasma's dynamics on an angle between the magnetic field and the wave vector θ and on the ratio $\alpha = C_A/c_0$. The periodic excitation leads to an excess background temperature which grows linearly with time on the average. The theory considers fast and slow magnetosound waves as exciters of the entropy perturbations and concludes that the secondary perturbation of density in the entropy mode is proportional to C^{-1}. That concerns the residual quantity after the passage of an impulse. Since C depends on θ and α and looks differently in the case of fast and slow magnetosonic waves, there is a variety in behaviour. Figure 3 shows $\frac{c_0}{C}$ in the case of the fast dominant magnetosonic mode.
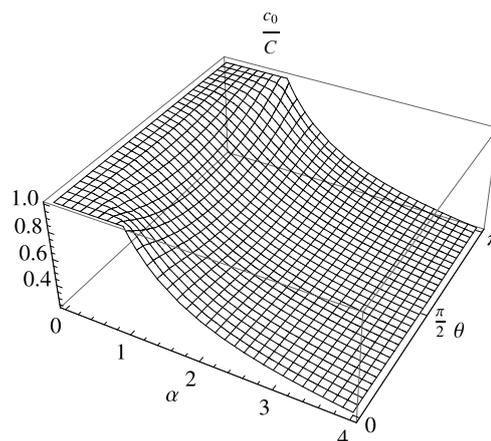

Fig. 3. $\frac{c_0}{C}$ as a function of $\alpha = \frac{c_0}{C_A}$ and θ. The case of a fast magnetosonic wave.

In spite of smallness of the part of thermal conduction perpendicular to the magnetic field $\chi_\perp$, only this part contributes to the dynamics of the magnetosonic mode and associated nonlinear phenomena at $\theta \approx \pi/2$.

In general, the magnetosonic source of heating depends on $\chi_\parallel$ and $\chi_\perp$ individually in accordance to Eq. (12). This may be of importance in the cases of non-periodic exciters. In general, the magnetosonic source of heating due to thermal conduction depends on θ, C_A/c_0 and individually on $\chi_{0,\parallel}$ and $\chi_{0,\perp}$. There are no restrictions on the magnitude of the equilibrium magnetic field and an angle between the equilibrium magnetic field and the wave vector in this study. The theory may potentially find application in laboratory and astrophysical plasmas. It may indicate physical processes and conditions of a flow and equilibrium parameters of a plasma. The results may be addressed both to cold plasma of inner atmosphere and to rarefied plasma of the outer atmosphere. In particular, studies of MHD waves and magnetosonic heating in the coronal loops are important since the mechanism of dynamic flows in a corona and coronal heating is still an unresolved problem (KUMAR *et al.*, 2006).

Magnetosonic heating/cooling associated with the heating-cooling function L was considered in detail by PERELOMOVA (2018a; 2018b). This sample is special because the absolute values of magnetosonic perturbations may grow unusually in the case af acoustical activity, that is, if

$$c_0^2 \frac{\partial L}{\partial p} + \frac{\partial L}{\partial \rho} > c_0^2 k^2 \frac{\chi}{C_P \rho_0}.$$

The entropy perturbations behave unusually under the condition of thermal instability (FIELD, 1965; PARKER, 1953; SOLER *et al.*, 2012):

$$\frac{(\gamma - 1)}{c_0^2} \frac{\partial L}{\partial \rho} + \frac{\chi}{C_P \rho_0} k^2 < 0.$$

Nonlinear interactions may also occur unusually. In particular, the background may get cooler due to excitation of negative excess temperature associated with the entropy mode. Taken alone, the damping mechanisms (thermal conduction, electric resistivity, mechanical viscosity of a plasma) lead to dissipation of magnetosonic modes and excitation of the negative perturbation in density which is inherent to the entropy mode. Magnetoacoustic heating may indicate wave processes and parameters of plasma since it is a slow readily observable process. In turn, it has impact on the propagation of the MHD perturbations. The temperature of a plasma and c_0 increase. This leads to enlargement of the magnetosonic speed C for both slow and fast MHD modes.

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