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TUTORIAL

REVIEW OF THE TRANSMISSION OF SOUND FROM AIR TO WATER

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This paper reviews the physical principles relating to the transmission of sound from air to water. The major emphasis is on the effect of the air-water interface on the transmission of sound to an underwater receiver. A limited consideration only is given to non-flat surfaces and the transmission at grazing angles of incidence.

Artykuł przedstawia fizyczne podstawy transmisji dźwięku z powietrza do wody. Główny nacisk położony na efekty graniczne i ich wpływ na rejestrację przechodzącej fali dźwiękowej pod wodą. Pewne problemy związane z pofalowaną powierzchnią wody są też dyskutowane.

1. Introduction

The transmission of sound from air to water has been the subject of many publications as is shown by references [1-5]. The physical problem relates to two situations; that in which the interface is supposed to be flat i.e. by comparison to the wave length of sound and that in which the surface is supposed to be disturbed by waves. The emphasis in this paper is on the consideration of the first case. There has been much discussion in the literature of the general problem of a situation which relates to a curved wavefront incident on a boundary. This problem has been considered in various circumstances for electromagnetic waves [6, 7] as well as acoustic waves [13]. The topic of interfacial effects has been fashionable in recent years as it has a connection with transmission of noise over the ground. In this regard there is a substantial literature, of which, ATTENBOROUGH's work [8] might be representative and also convenient for its listing of the major references.

2. The transmission of sound into water from a point source

If we follow URICK's approach [2] then we see (Fig. 1) that sound can arrive at the receiver by several paths. First, there is the direct, refracted, path which is shown as *OAR* in Fig. 1. Second, there is the indirect path from bottom reflection, i.e. *OBCR*. Finally, there is a path which may be associated with surface, layer or inhomogeneous waves following paths such as *ODR*. This surface wave might be important at depths small compared with the wavelength of the sound.

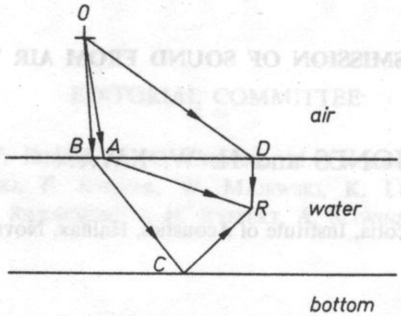

FIG. 1. The contribution to the sound field at a point in the Sea made by a source O in air

If we concern ourselves first with the direct path then we note that there has been considerable discussion of this topic in the literature. HUDIMAC [9] proposed a consideration based on the assumption that the sound reaching the particular point underwater could be obtained from a ray treatment; see Fig. 2. Essentially it was assumed that an element of the wavefront at some point on the surface can be assumed to be plane and the transmission coefficient at some angle (θ_0 in Fig. 2) can then be used to obtain the contribution to the sound pressure at an element dR. It is shown on these assumptions that:

$$\frac{I}{E} = \frac{\varrho_1 c_1 \varrho_2 c_2 \sin^2\theta_0}{\pi(\varrho_2 c_2 + \varrho c_1 B)^2 (h + dc_2/Bc_1)(h + dc_2/B^3 c_1)}, \tag{1}$$

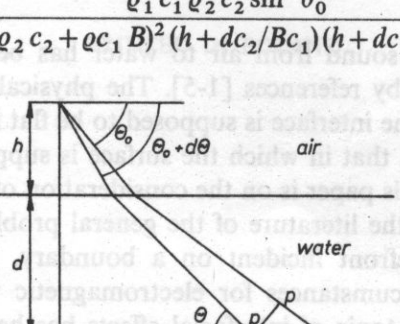

FIG. 2. The geometry of the ray paths. The source is at a height h above the water surface

where

$$B = \{1 + [1 - (c_2^2/c_1^2)] \operatorname{ctn}^2 \theta_0)\}^{1/2},$$

I, is the sound intensity at R, and E is the output power of the source. HUDIMAC calculated the intensity for a particular case, see Fig. 3.

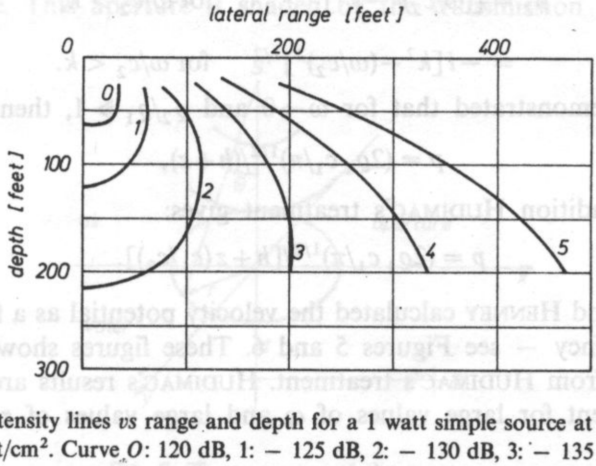

FIG. 3. Plot of iso-intensity lines *vs* range and depth for a 1 watt simple source at 25 feet. Intensities are given in dB re 1 watt/cm^2. Curve O: 120 dB, 1: − 125 dB, 2: − 130 dB, 3: − 135 dB, 4: − 140 dB, and 5: − 145 dB (After HUDIMAC)

WEINSTEIN and HENNEY [1], about eight years later attempted the problem using wave theory to obtain sound pressure at a receiver in the cordinate system in Fig. 4. Its is assumed that there is a point source in air with a velocity potential given by:

$$\phi = (1/R) \exp(-i\omega R/c_1), \tag{2}$$

and in the water the velocity potential is given by:

$$\phi = \int^\infty [2\beta_2/(\sigma\beta_1 + \beta_2)] \{\exp[-i(\beta_2 z + \beta_1 h)]\} [J_0(kr) k/(i\beta_2)] \, dk, \tag{3}$$

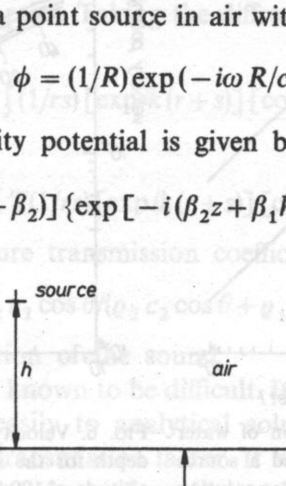

FIG. 4. Coordinate system

where

$$\delta = \varrho_2/\varrho_1,$$

$$\beta_1 = [(\omega/c_1)^2 - k^2]^{1/2} \quad \text{for } \omega/c_1 > k,$$

$$= -i[k^2 - (\omega/c_1)^2]^{1/2} \quad \text{for } \omega/c_1 < k,$$

$$\beta_2 = [(\omega/c_2)^2 - k^2]^{1/2} \quad \text{for } \omega/c_2 > k,$$

$$= -i[k^2 - (\omega/c_2)^2]^{1/2} \quad \text{for } \omega/c_2 < k.$$

These authors demonstrated that for $\omega \to 0$ and $\varrho_2/\varrho_1 \gg 1$, then:

$$p = (2\varrho_1 c_1/\pi)^{1/2}/(h+z), \tag{4}$$

for the same condition HUDIMAC's treatment gives:

$$p = (2\varrho_1 c_1/\pi)^{1/2}/[h + z(c_1/c_2)]. \tag{5}$$

WEINSTEIN and HENNEY calculated the velocity potential as a function of water depth and frequency – see Figures 5 and 6. These figures show a curve for the results obtained from HUDIMAC's treatment. HUDIMAC's results are consistent with the wave treatment for large values of ω and large values of z.

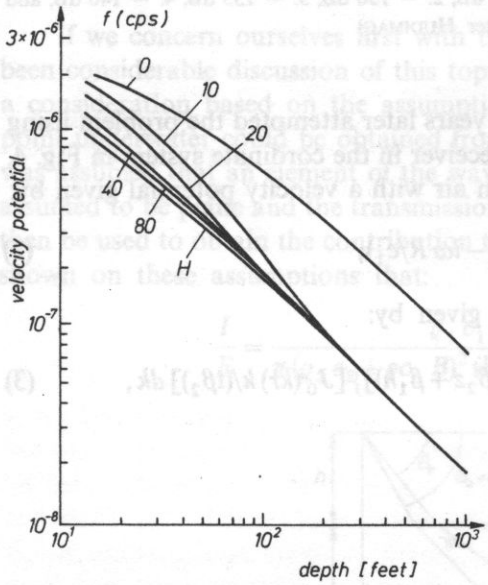

FIG. 5. Velocity potential as a function of water depth for the indicated frequencies and a source altitude of 25 ft. *H* refers to the Hudimac solution and the remaining curves to the wave theoretical solution for a point source in air with a velocity potential of unity at 1 cm (After WEINSTEIN and HENNEY)

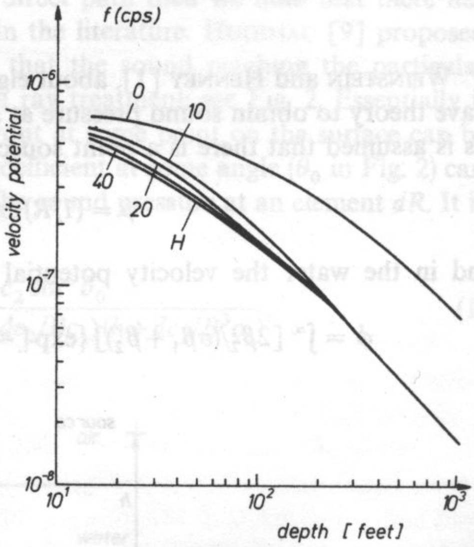

FIG. 6. Velocity potential as a function of water depth for the indicated frequencies and a source altitude of 100 ft. H refers to the Hudimac solution and the remaining curves to the wave theoretical solution for a point source in air with a velocity potential of unity at 1 cm (After WEINSTEIN and HENNEY)

3. Discussion of ray and wave theory results

The findings of the authors of references [1 and 9] are to be expected in that the problem which is being considered is essentially a diffraction problem – see Fig. 7. The incident sound field consists of spherical (or near spherical in the presence of wind) wave fronts impinging on an aperture shown somewhat diagrammatically at the water-surface. This aperture is shaded by the transmission coefficient of the

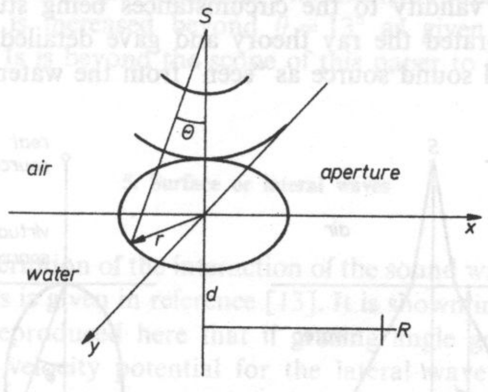

FIG. 7. The geometry of the aperture

sound at the interface; the transmission is a function of and is symmetrical about a vertical axis. Seen in these terms we are left with the solution of the Fresnel-Kirchhoff integral as in optics. Taking the diffraction integral as given in BORN and WOLF's book [10]

$$U(P) = -iA/(2\pi) \int\int (1/rs)\,[\exp ik(r+s)]\,[\cos(n,r) - \cos(n,s)]\,dS, \tag{6}$$

then for this case,

$$U(P) = -iA/(2\pi) \int\int T(1/rs)\,[\exp ik(r+s)]\,[\cos(n,r) - \cos(n,s)]\,dS, \tag{7}$$

where T is the sound pressure transmission coefficient at the interface i.e.:

$$T = 2\varrho_2 c_2 \cos\theta / (\varrho_2 c_2 \cos\theta + \varrho_1 c_1 \cos\phi),$$

ϕ being the angle of refraction of the sound.

The solution of Eq. (7) is known to be difficult. It is a form of Fresnel diffraction which does not lend itself easily to analytical solution. The situation is further complicated by the variable transmission factor. It would seem to be convenient to use a numerical solution. If we do so then the results must coincide with those of WEINSTEIN and HENNEY unless an arithmetical error is made.

At this point it is to be observed that it must be expected that the HUDIMAC solution converges to that of the WEINSTEIN and HENNEY solution because as the wavelength becomes short it is in the nature of the diffraction integral that its result will be similar to ray theory. This point is well demonstrated in the general literature. If further demonstration of the point is necessary, then to the extent that data are available, reference [3] supports the conclusion that the wave or diffraction solution is, in general, the required solution. It should be observed however, that the diffraction solution is only valid for the circumstance that the coherence length of the sound is sufficient for it to encompass all the phase changes required by the geometry of the aperture. This assumption is often made in noise studies without the necessary demonstration of its validity to the circumstances being studied.

URICK [2] elaborated the ray theory and gave detailed consideration of the position of the virtual sound source as "seen" from the water, see Fig. 8. He notes

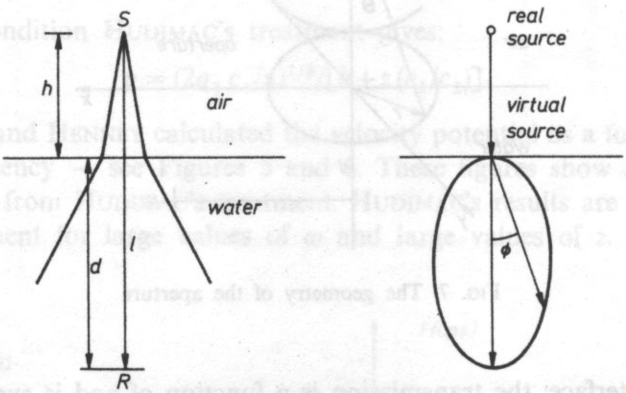

FIG. 8. The receiver (a) beneath the source and (b) in the farfield at distances much greater than the height of the source

that the apparent source height is the same as the apparent depth which is used in elementary optical theory. As the treatment URICK uses is in principle that advanced by HUDIMAC it provides an approximation similar to his. As URICK points out this approximation (accepting $\theta_{\max}$ is 13°) can be valuable in that it allows a simpler calculation of the sound pressure at a point in the water. His analysis arrives at the equation

$$I_\omega = I_a [4n^2/(d+nh)^2], \tag{8}$$

where I is the intensity, d the depth in the water, h the height of the source and $n = c_1 \cos\theta / c_2 \cos\phi$. URICK shows that for far field, Eq. (8) becomes

$$I_\omega = 4 I_a n^2 \cos^2 \phi / 1^2, \tag{9}$$

after some approximation. This means the source can ce replaced by a dipole located at the surface and radiating as $\cos^2 \phi$. The equivalent source has an intensity of 4 n^2 times the real source. As URICK points out this is a valuable approximation in that it allows rapid calculation of the sound intensity under the water.

4. The transmission of sound through a rough interface

This is essentially a problem relating to effect of the change of the angle of incidence associated with a non-flat surface. This is a problem possibly statistical in which the shape of the water surfaces is required. This allows a suitable generalization to be made [11, 12]. The rough surface effect is two-fold. First, the transmission factor (T) is changed locally by the surface shape and the modified value is required in the Fresnel-Kirchhoff integral equation. The effective radius of the aperture of the surface (see Fig. 7) is increased beyond $\theta = 13°$ as given by the total external reflection condition. Is is beyond the scope of this paper to deal with the details of this situation.

5. Surface or lateral waves

An excellent description of the interaction of the sound wave at the surface when total reflection occurs is given in reference [13]. It is shown in an argument which is too detailed to be reproduced here that if grazing angle geometry (see Fig. 9) is considered then the velocity potential for the lateral wave is given by

$$|\phi_{lat}| = 2\exp[-kd(\sin^2\theta - n^2)^{1/2}]/[(\varrho_2/\varrho_1)(r^2+h^2)^{1/2}], \quad (10)$$

subject to:

$$\sin\theta = r/[(r^2+h^2)^{1/2}] > n, \text{i.e. } \theta > 13°,$$

where $n = c_1/c_2$.

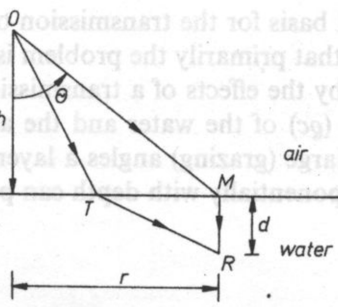

FIG. 9. Paths of wave penetration into the lower medium for the case: $n < 1$

This shows that for a given θ the surface wave decays as $\exp-(d/\lambda)$ below the surface. Similarly, the wave decays as θ increases. Urick evaluated the sound pressure at different depths and his data are reproduced below (see Fig. 10). It is to be noted that roughness of the waves on the water surface will change the situation so that

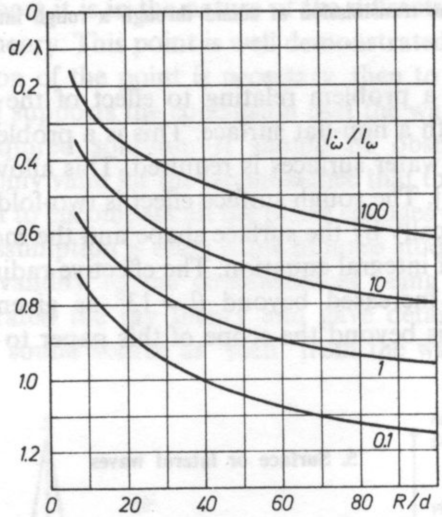

FIG. 10. Ratio of the intensity I_ω' of the lateral wave to the intensity I_ω of the refracted wave, using coordinates of d/λ and R/d, where d = receiver depth, R – receiver horizontal range, and λ – wavelength

$\sin\theta > n$ condition, will be satisfied in "unusual" regions on the surface. A satisfactory theory for the maintenance, transmission and decay of surface waves in these circumstances does not appear to exist.

6. Conclusion

The essential physical basis for the transmission of sound from air to water has been reviewed. It is noted that primarily the problem is one of diffraction through an aperture which is shaded by the effects of a transmission coefficient which is related to the physical properties (ϱc) of the water and the air, the angle of incidence and surface perturbations. At large (grazing) angles a layered or surface wave exists and this wave which decays exponentially with depth can provide significant sound levels close to the surface.

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