ARCHIVES OF ACOUSTICS
13, 1-2, 109-126 (1988)

A NONINVASIVE ULTRASONIC METHOD FOR THE ELASTICITY EVALUATION OF THE CAROTID ARTERIES AND ITS APPLICATION IN THE DIAGNOSIS OF THE CEREBRO-VASCULAR SYSTEM

TADEUSZ POWAŁOWSKI, BOGUMIŁ PEŃSKO

Ultrasonics Department, Institute of Fundamental Technological Research, Polish Academy of Sciences.
(00-049 Warsaw, ul. Świętokrzyska 21)

This study presents a comparative evaluation of the methods applied so far to describe the elasticity of the blood vessel walls. It was shown that one of the possible solutions in the description of the dependence between the vessel cross-section and the blood pressure is a logarithmic function. The authors assumed the logarithmic wall rigidity coefficient α resulting from this function as the index of the mechanical properties of the walls of the carotid arteries examined. The value of this coefficient was determined noninvasively from the ultrasonic measurement of the instantaneous diameter of the common carotid artery and the systolic and diastolic pressures measured with a manometer in the brachial artery. The studies were carried out for a group of 43 healthy persons aged between 9 and 64, and for a group of 9 persons aged between 53 and 62 in whom arteriosclerotic changes were found by the X-ray arteriography in the extracranial carotid arteries. The results obtained indicate a linear increase in the coefficient α with age of healthy persons. For the ill group the mean value of the coefficient α was about 50% higher than that for the healthy in the same age group.

W pracy przeprowadzona została ocena porównawcza stosowanych dotychczas metod opisu elastyczności ścianek naczyń krwionośnych. Wykazano, że jednym z możliwych rozwiązań w opisie zależności między przekrojem naczynia i ciśnieniem krwi jest funkcja logarytmiczna. Wynikający z niej logarytmiczny współczynnik sztywności ścianki α przyjęty został przez autorów jako wskaźnik własności mechanicznych ścianek badanych tętnic szyjnych. Wartość tego współczynnika wyznaczano nieinwazyjnie na podstawie ultradźwiękowego pomiaru chwilowej średnicy tętnicy szyjnej wspólnej oraz ciśnień: skurczowego i rozkurczowego mierzonych manometrem w tętnicy ramiennej. Badania przeprowadzono dla grupy 43 osób zdrowych w wieku od 9 do 64 lat oraz dla grupy 9 osób w wieku od 53 do 62 lat, u których stwierdzono metodą arteriografii rentgenowskiej zmiany miażdżycowe w tętnicach szyjnych pozaczaszkowych. Uzyskane wyniki wskazują na liniowy wzrost współczynnika α z wiekiem badanych osób zdrowych. Dla grupy osób chorych średnia wartość współczynnika α była o około 50% większa niż u osób zdrowych w tej samej grupie wiekowej.

1. Introduction

Studies on the blood vessel elasticity are very significant in the diagnosis of human vascular systems. Changes which occur in the vessel walls as a result of the organism's aging or diseases of the vessel walls (arteriosclerosis) cause an increase in their rigidity. This has a negative effect on the blood circulation mechanism in which the vessel elasticity plays a very important role. An increase in the rigidity of the arterial vessel walls is also a factor which contributes to the development of arteriosclerosis, the most serious disease of human vascular system.

Depending on the measurement methods applied, various indices are used to evaluate the vessel wall elasticity. Commonly they are: the pulse wave velocity [1, 5, 13, 28], the relative vessel diameter change [25], the elastic (pressure-strain) modulus E_p [1, 8, 19, 20, 21, 26] BERGEL's incremental modulus E_{inc} [2, 3, 4, 12, 18, 32]. All the above mentioned indices are functions of blood pressure [2, 5, 13, 16, 32]. The effect of blood pressure on the vessel wall rigidity makes difficult the comparative evaluation of the mechanical properties of the walls in persons with different blood pressure.

To take into account the effect of the blood pressure on the vessel wall rigidity, it is necessary to know the functional dependence between the cross-section of the vessel and the blood pressure inside it. This dependence was studied by many authors [12, 14, 15, 17, 30], however, the functions proposed by them cannot be used in noninvasive vessel elasticity measurements. This is due to the fact that their description requires coefficients whose values can be determined only by invasive methods.

This study carried out a comparative evaluation of the methods applied so far to describe the blood vessel wall elasticity. By analysing the dependence between the vessel cross-section area and the blood pressure, it was shown that one of the possible solutions is a logarithmic function. Its determination requires two reference points which can be found by the noninvasive method. This function was applied by the authors in the noninvasive measurement of the input vessel impedance [23, 24], to determine the blood pressure from the instantaneous values of the diameter of the common carotid artery.

Taking the logarithmic dependence between the artery cross-section and the blood pressure as the basis of analysis of the wall elasticity in the common carotid arteries, a new elasticity index was defined. It is a logarithmic wall rigidity coefficient α. This coefficient was applied by the authors in the comparative evaluation of the elastic properties of the walls of the common carotid arteries examined. Its determination requires knowledge of the artery diameter and the blood pressure in systole and diastole. These quantities were measured noninvasively. The instantaneous diameter of the common carotid artery was measured with a pulsed ultrasonic tracing system, constructed by the authors, which was connected "on line" with a computer. The systolic and diastolic blood pressures were measured with a manometer in the brachial artery. The examinations were performed on healthy persons of different age and on persons with pathological changes in the extracranial carotid arteries.

Apart from the coefficient α, for comparison, the relative diameter change $\Delta d/d$ and the elastic modulus E_p were determined.

2. Elasticity of the blood vessel

It is generally assumed that human blood vessels are elastic. This means that each change in the blood pressure inside the vessel is accompanied by a change in its dimensions, depending on the mechanical properties of the blood vessel walls. In a living organism blood vessels are strongly longitudinally extended (about 1.5 times with respect to their length outside the organism) and fixed to the surrounding tissue. This excludes almost completely the possibility of wall motion along the vessel axis (longitudinal motion). Therefore, it is generally assumed [5, 32, 1], that the only mechanical reaction of the vessel to a change in the blood pressure is a change in its transverse dimensions. Hence, there result, variously defined, elasticity coefficients often applied in the literature to characterise the properties of the blood vessel walls.

In 1960 PETERSON [21] introduced the elastic modulus E_p defined as:

$$E_p(\text{acc. to Peterson}) = \Delta p/(\Delta d/d), \tag{1}$$

where Δp is a pressure change causing a change in the external diameter d of the vessel by Δd.

Analogously to formula (1), the coefficient E_p is also used for the internal diameter of the vessel, according to the dependence [26]:

$$E_p = \frac{(p_s - p_d)\, d_d}{(d_s - d_d)}, \tag{2}$$

where d_s and d_d denote the internal vessel diameters for the systolic p_s and diastolic p_d pressures, respectively.

In the 1961 BERGEL introduced [2] an incremental elastic modulus in the form

$$E_{\text{inc}} = 2(1-\sigma^2)\frac{R_i^2 R_e \Delta p}{(R_e^2 - R_i^2)\Delta R_e}, \tag{3}$$

where R_i and R_e are the internal and external radii of the blood vessel, respectively.

Table 1 lists the values of the moduli E_p and E_{inc} published by different authors. The quite large discrepancy of the results presented in the table results mainly from the different conditions of the experiments, (in vitro, in vivo – with exposed artery, in vivo – studied from the surface of the body) and very different techniques of the measurements of the blood pressure. The results cited were obtained for pressure close to 100 mmHg. In two cases they apply to a wider pressure range.

Most of experimental studies carried out for years in the world indicate that the

Table 1. Elasticity moduli E_p and E_{inc} determined in common carotid artery by different authors

Author	Conditions of experiment	Subject	Blood pressure [mmHg]	Age (years)	E_p (10^6 dyn/cm^2)	E_{inc} (10^6 dyn/cm^2)
PETERSON 1960 [21]	in vivo	dog	—	—	2.3–4.2*	—
BERGEL 1961 [2]	in vitro	dog	40–220			1–12.2
			100	—	—	6.4
PATEL 1963 [19]	in vivo	dog	< 120	—	2.88*	—
PATEL 1964 [20]	in vivo	man	100	28–69 mean value 45	6.0*	—
GOW 1968 [8]	in vivo	dog	~ 100	—	2.1*	—
ARNDT 1968 [1]	in vivo*	man		24–34	0.32–0.58*	—
			~ 100	mean value 28	0.46**	—
HAYASHI 1980 [12]	in vitro	man	100	< 40	—	4.5
			100	40–50	—	3.0
NEWMAN 1982 [18]	in vitro	rabbit	100			7.0
WEIZSACKER 1982 [32]	in vitro	rat	45–112			2.5–25
			100	—	—	15
RILEY 1984 [26]	in vivo(*)	man		7–25	0.32–1.61**	—
			~ 80	mean value 16	0.64**	—
BOROVETZ 1986 [4]	in vitro	dog	~ 100	—	—	5.0

* acc. to formula (1), ** acc. to formula (2)

(*) ultrasonic measurements from the body surface reaction of the walls to a change in the blood pressure is nonlinear, or even strongly nonlinear. This means that the elastic indices given by formulae (1), (2), (3) are function of pressure – this greatly reduces their usefulness in evaluating the state of blood vessels.

So far there is no unambiguous agreement as to the analytical form of the nonlinear function connecting the blood pressure with a change in the transverse dimensions of the blood vessel. Most functions proposed in the literature are purely empirical, as they are based on the authors' own experimental data. These functions are greatly different from one another and often have a very complex mathematical form. Table 2 lists formulae describing recent attempts to represent analytically the relations between the dimensions of the blood vessel and the blood pressure. The table also includes the exponential relation, proposed in 1985 by the present authors [23, 24] between the blood pressure and the squared vessel radius, and the equivalent logarithmic function between the vessel cross-section area and the blood pressure. The exponential form was applied for noninvasive determination of the course of the blood pressure in the carotid arteries from ultrasonic measurements of the instantaneous diameters of these arteries. With reference to cross-section area S of a cylindrical vessel in which the blood pressure p is greater than zero, this function can be represented in the following form

$$p = p_0 \exp(\gamma S), \tag{4}$$

where p_0 and γ are constant coefficients.

Earlier suggestions regarding the exponential character of the stress – strain dependeces in blood vessel can be found e.g. in studies by FUNG [6], SIMON [29], TANAKA [31] or GHISTA [7].

Formula (4) can be justified by using GREEN's theory [9, 10] which describes the behaviour of bodies under elastic strains with large amplitudes. The mathematical model of the blood vessel assumed an axially symmetric cylinder in the state of plain strain with large (finite) amplitudes. The cylinder is built of homogenous, elastic and incompressible material. Assuming moreover that the deformation of such a cylinder can be treated as constant axial elongation and homogenous transverse inflation causing a change in the internal and external vessel radii from their initial values a_2 and a_1 to the current values r_2 and r_1, it can be shown [29] that the pressure p inside the vessel is given by the formula

$$p = 2A \int_{r_1}^{r_2} e^{kI} [Q^2/\lambda^2 - 1/Q^2]\, dr/r, \tag{5}$$

where I is the first invariant of the strain tensor given by the expression

$$I = \lambda^2 + Q^2/\lambda^2 + 1/Q^2.$$

λ is the ratio between the deformed and undeformed axial coordinates of the cylinder and Q is the ratio between the undeformed and deformed radial coordinates of the cylinder.

Table 2. Nonlinear functions between vessel dimensions and blood pressure, proposed in the recent literature

LOON et al. [15]: V – volume, p – pressure

$$V = V_0 + (V_m - V_0)(1 - e^{-ap})$$

V_0 by $p = 0$ mmHg, V_m – max. valve

HAYASHI et al. [12]: R – radius, p – pressure

$$R/R_0 - 1 = \frac{1}{\beta}\ln(p/p_0)$$

R_0 for $p_0 = 100$ mmHg

STETTLER et al. [30]: S – cross-section, p – pressure, z – distance from the heart

$$S(p, z) = S_0(z)\exp\left[\frac{p - p_0}{c(p_0, z)\,c(p, z)\,\varrho}\right]$$

$$c(p, z) = (c_0 + Bp)\,g(z)$$

$p_0 = 100$ mmHg

MEISTER [17]: S – cross-section, p – pressure, z – distance from the heart

$$S(p, z) = S(p_0, z)\exp\frac{1}{\varrho g^2}\left[\frac{2a_3p + a_2}{\Delta c(p)} + \frac{4a_3}{\Delta^{3/2}}\operatorname{arc\,tg}\frac{2a_3p + a_3}{\Delta^{1/2}}\right]$$

$$c(p) = a_1 + a_2p + a_3p^2$$

$$g(z) = c_1 + c_2z, \quad \Delta = 4a_1a_3 - a_2^2, \quad p_0 = 100 \text{ mmHg}$$

LANGEWOUTERS et al. [14]: S – cross-section, p – pressure

$$S(p) = S_m\left(\frac{1}{2} + \frac{1}{\pi}\operatorname{arc\,tg}\frac{p - p_0}{p_1}\right)$$

p_0 determined from the half-width value of S_m
p_1 determined from the half-width value of vessel compliance

POWAŁOWSKI et al. [24]: p – pressure, R – radius

$$p(R) = p_d\exp\left\{\frac{R^2 - R_d^2}{R_s^2 - R_d^2}\ln\frac{p_s}{p_d}\right\}$$

S – cross-section, p – pressure

$$S(p) = S_d\left[1 + \left(\ln\frac{p}{p_d}\right)\Big/\left(\frac{S_d}{S_s - S_d}\ln\frac{p_s}{p_d}\right)\right]$$

S_d, R_d – for diastolic pressure p_d; S_s, R_s – for systolic pressure p_s

In the formula (5) use was made of the exponential form of the constitutive equation, which, according to SIMON [29], can be written in the form

$$\partial W/\partial I = A\, e^{kI}, \tag{6}$$

where A and k are parameters to be determined from the boundary conditions and W is the density function of the strain energy introduced by GREEN [10].

Expression (5) can be reduced to a form more convenient for numerical calculations:

$$p = -C \int_{x_1}^{x_2} e^{\beta(x+1/x)} [1+1/x]\, dx, \tag{7}$$

where

$$x = Q^2/\lambda = 1+\mathrm{const}/r^2, \tag{7a}$$

$$\beta = k/\lambda, \tag{7b}$$

$$C = (A/\lambda)\, e^{-2\beta}, \tag{7c}$$

The incompressibility of the material of the blood vessel walls can be expressed by the equation

$$a_1^2 - a_2^2 = \lambda (r_1^2 - r_2^2), \tag{8}$$

where a_1 and a_2 are the external and internal radii of the vessel for $p = 0$, r_1 and r_2 are its external and internal radii for the pressure p, and λ is the relative elongation which is constant as a function of pressure (according to SIMON [29] $\lambda = 1.532$).

The dependence of the variable x in formula (7) on the squared radius only (formula (7a)) suggests that blood pressure is a function of the cross-section area of the blood vessel. It follows from the incompressibility condition (8) that the function $p = f(S)$ has the same form for the external and internal cross-section of the vessel.

The course of the function $p(S)$ calculated numerically from formula (7) is shown in Fig. 1. The marked experimental points come from a study by SIMON et al. [29], who measured the external radius of the vessel r_1 as a function of pressure for a canine abdominal aorta in vitro. The reference points for the calculations were $S_1 = 60.9$ mm^2, $p_1 = 46.2$ mm Hg, $S_2 = 84{,}9$ mm^2, $p_2 = 199$ mm Hg. The exponential function described by formula (4) was plotted through the same two points. The course of the exponential function distinctly agreed with numerical calculations, and the maximum difference between the values of p calculated from formulae (4) and (7) did not exceed 0.7%.

To show if the exponential function (4) approximates the real experimental data well, the coefficient of determination of the curve R^2 was calculated for data in Fig. 1, using the definition [27]

$$R^2 = 1 - \frac{\sum_i (\tilde{y}_i - y_i)^2}{\sum_i (\bar{y} - y_i)^2}, \tag{9}$$

where y_i is the value of y measured experimentally for the independent variable x equal to x_i, $\bar{y}$ is the mean of N measurements, and $y_i = f(x_i)$ if $f(x)$ denotes the approximating function applied. The coefficient R^2 for the exponential curve in Fig. 1 was 0.9868.

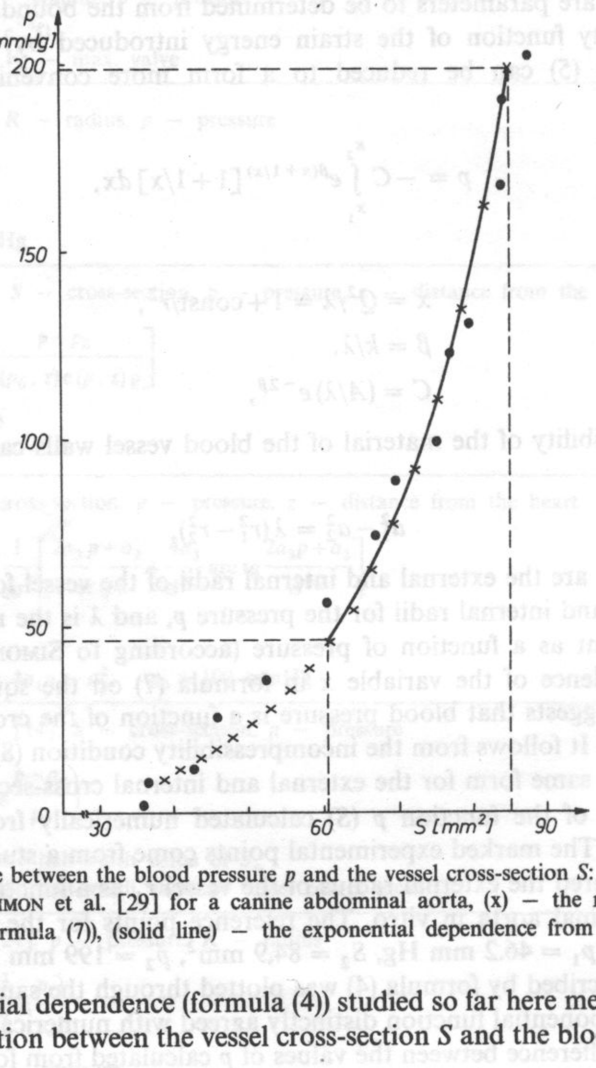

FIG. 1. The dependence between the blood pressure p and the vessel cross-section S: (o) – experimental points according to SIMON et al. [29] for a canine abdominal aorta, (x) – the results of numerical calculations (formula (7)), (solid line) – the exponential dependence from formula (4)

The exponential dependence (formula (4)) studied so far here means that there is a logarithmic function between the vessel cross-section S and the blood pressure p, in the form

$$S = (1/\gamma)\ln(p/p_0). \tag{10}$$

On the basis of the results of experimental studies presented by LOON et al [15] for human common carotid artery and by SIMON et al [29] for a canine abdominal aorta, comparative analysis was carried out of the nonlinear functions $S = f(p)$ described in the literature, including the logarithmic function proposed here. For comparison, a linear dependence between the vessel cross-section and pressure was also assumed. The results of the comparative analysis are shown in Fig. 2 and 3 and in Table 3. Using the least squares method it was shown that, apart from the linear one, all the functions studied describe very well the results of experimental studies in physiological pressure range from 25 mmHg to 200 mmHg.

There are only slight differences between the courses of particular nonlinear functions. Accordingly, it seems fully justified to assume the function $S = f(p)$ in the logarithmic form proposed here.

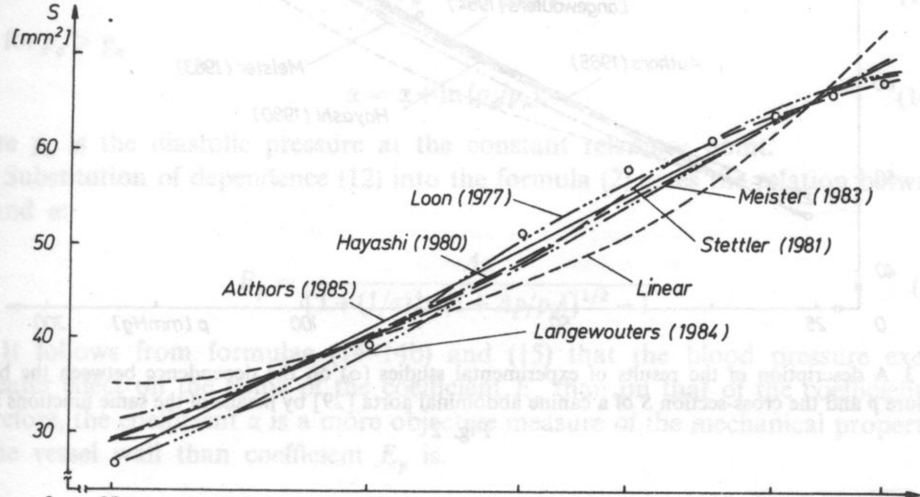

FIG. 2. A description of the results of experimental studies (o) on the dependence between the blood pressure p and the cross-section S in human common carotid artery [15] by means of the functions proposed in the literature (Table 2)

As only one of studied nonlinear functions $S = f(p)$, the logarithimc function can be determined on the basis of any two measurement points. The same applied to the equivalent exponential dependence between the blood pressure and the blood vessel cross-section. Substitution in formulae (4) and (10) the two pairs of values: p_d, S_d and p_s, S_s, corresponding successively to these points, gives the following expressions

$$p = p_d \exp[\alpha(S/S_d - 1)], \tag{11}$$

and

$$S = S_d[1 + (1/\alpha)\ln(p/p_d)]. \tag{12}$$

The coefficient α which occurs in formulae (11) and (12) is defined in the following way

$$\alpha = \frac{S_d \ln(p_s/p_d)}{S_s - S_d} = \frac{S_d \ln(1 + \Delta p/p_d)}{S_s - S_d}, \tag{13}$$

where $\Delta p = p_s - p_d$.

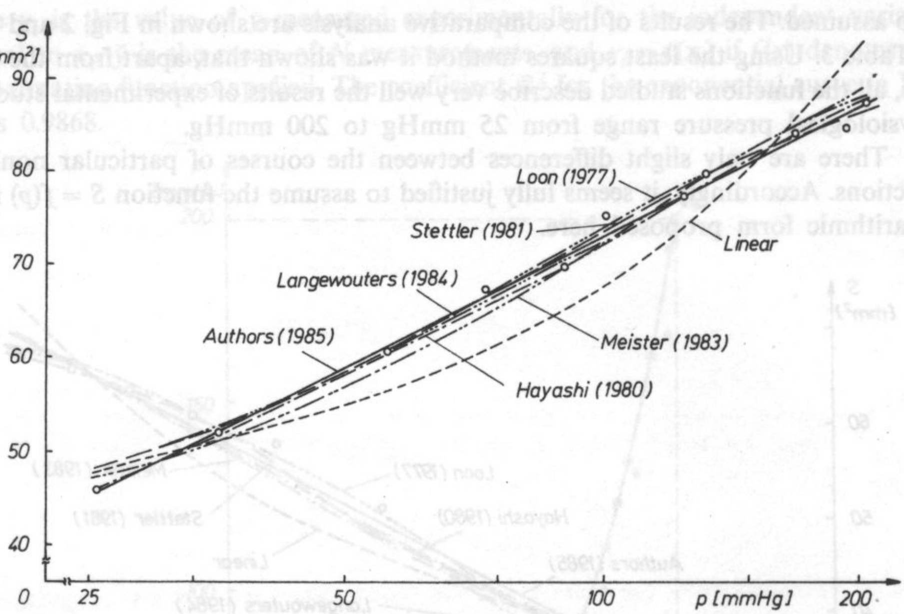

FIG. 3. A description of the results of experimental studies (o) on the dependence between the blood pressure p and the cross-section S of a canine abdominal aorta [29] by means of the same functions as in Fig. 2

Table 3. The coefficient of determination R^2 calculated for experimental data approximated by different non-linear functions

Functions acc. to:	Common carotid man (LOON, 1977)	Abdominal aorta dog (SIMON, 1971)
LOON* [15]	0.9974	0.9950
HAYASHI* [12]	0.9722	0.9873
STETTLER [30]	0.9613	0.9917
MEISTER [17]	0.9635	0.9894
LANGEWOUTERS [14]	0.9862	0.9945
Authors [23]	0.9868	0.9942
Linear	0.8653	0.9185

*) after transformation to the form $S = f(p)$

In noninvasive studies, the values p_d and S_d represent the pressure and the vessel diameter in the diastole, and p_s and S_s are the corresponding quantities in the systole (Table 2) [24].

The coefficient α can be called a logarithmic rigidity coefficient of the blood vessel wall. It follows from the formulae (10) and (13) that the value of the coefficient α depends on the choice of the reference point (p_d, S_d). This means that it is a function of the diastolic pressure p_d. The effect of the pressure p_d on the value of the coefficient α can be described in the following way:

for $p_d = p_n$

$$\alpha = \alpha_n, \tag{14}$$

for $p_d < p_n$

$$\alpha = \alpha_n - \ln(p_n/p_d), \tag{14a}$$

for $p_d > p_n$

$$\alpha = \alpha + \ln(p_d/p_n), \tag{14b}$$

where p_n is the diastolic pressure at the constant reference point.

Substitution of dependence (12) into the formula (2) gives the relation between E_p and α:

$$E_p = \frac{\Delta p}{[1 + (1/\alpha)\ln(1 + \Delta p/p_d)]^{1/2} - 1}. \tag{15}$$

It follows from formulae (14–14b) and (15) that the blood pressure exerts a greater effect on the value of the coefficient E_p than on that of the coefficient α. Therefore, the coefficient α is a more objective measure of the mechanical properties of the vessel wall than coefficient E_p is.

3. Results and discussion

The elasticity measurements were carried out in human common carotid artery using ultrasonic equipment constructed by the authors [22]. It permits the simultaneous transcutaneous measurement of the instantaneous blood velocity and the instantaneous internal diameter in the blood vessel examined. The vessel diameter was determined by the echo method. Its instantaneous value was measured using a digital system tracking echos from both walls of the blood vessel [11, 22].

The accuracy of the measurements of the vessel wall displacement was 0.03 mm. The data were registered and analysed on a PDP-11 computer connected "on line" with the ultrasonic device.

The studies were carried out on two groups of persons. The first group (43 persons) did not show any pathological changes in the region of the arteries examined. The second group (9 persons – 12 arteries) included patients with arteriosclerosis changes identified by X-ray angiography in the region of the carotid arteries. In all the persons examined, the maximum and minimum values of the vessel diameters d_s and d_d were measured, and then subordinated respective to the values of blood pressure is systole and diastole. The values of systolic and diastolic pressure p_s and p_d were measured with a manometer in the brachial artery, at the height of the neck, with the patient in supine position. The diameter of the common carotid artery was measured transcutaneously from a point located about 2–3 cm away from bifurcation into the internal and external carotid arteries.

The measurements results were grouped depending on the age of the persons examined (Tables 4 and 5). This age varied in the healthy group from 9 to 64 years, and in the pathological group, from 53 to 62 years. For each patient, from measurements of the diameter and pressure, the following quantities were determined: the relative diameter change $\Delta d/d = (d_s - d_d)/d_d$: the elasticity coefficient E_p (formula (2)); and the logarithmic rigidity coefficient α (formula (13)).

Fig. 4 shows the values of the coefficient α obtained as a function of the age of the persons examined. Circles represent healthy arteries, and crosses mark those with

Table 4. The logarithmic function parameters p_0, γ, α (formulae (16) and (18)) determined on the basis of two measured points (for diastolic p_d and systolic p_s pressures) for healthy and ill persons

Age group (years)	Healthy persons 9–16	19–30	32–40	41–50	52–64	Ill persons 53–62
Meam Age	12.3	24.4	36.8	44.9	56.2	57.2
S.D. (years)	2.3	4.6	3.2	2.9	3.9	3.0
Number of persons	7	9	8	8	11	9
p_d	60.7	77.2	80.0	70.6	80.0	83.1
S.D. (mmHg)	6.1	7.1	5.9	8.6	8.7	4.6
p_s	104.3	117.8	116.9	110.0	122.7	150.0
S.D. (mmHg)	10.6	7.9	7.0	10.0	13.8	15.4
p_0	10.53	12.87	7.64	3.30	1.72	0.87
S.D. (mmHg)	4.32	5.49	4.79	1.63	0.90	0.86
γ	7.65	5.09	6.28	8.22	10.10	12.15
S.D. (cm^{-2})	2.10	1.12	2.28	2.57	2.55	4.40
α	1.84	1.87	2.55	3.18	3.98	6.16
S.D.	0.43	0.37	0.68	0.51	0.62	2.29

S. D. – standard deviation

pathological changes. The rigidity coefficient increased lineary with the patient's age in the healthy group, according to the simple regression formula:

$$\alpha = 0.858 + 0.0523x, \tag{16}$$

where x is the age in years.

Table. 5. Elasticity modulus E_p (formula 2) for common carotid arteries for healthy persons in different age groups

Age group (years)	9–16	19–30	32–40	41–50	52–64
Modulus E_p	0.423	0.499	0.681	0.776	1.091
S.D.	0.118	0.075	0.158	0.118	0.207
(10^6 dyn/cm^2)					

S.D. – standard deviation

The coefficient of the linear correlation was 0.832. The values of the coefficient α varied from (1.84 ± 0.43) on average in the age group 9–16 years to (3.88 ± 0.62) on average in the group 52–64 years. The values of the rigidity coefficient α in the pathological group were significantly different ($p < 0.01$) from those in the healthy group (older than 50), namely 6.16 ± 2.29 on average.

The mean values of diastolic pressures in particular age group of adults were similar. The same applied to the healthy and ill person in the same age group (Table 4).

It follows from the formulae (14)-(14b) and the values of p_d in Table 4 that the differences in the diastolic pressure for particular age groups influence only slightly the value of the coefficient α (not more than 15%). This effect was practically negligible (< 1%) for the compared groups of healthy and ill persons. The large difference in the coefficient α between these groups can result only from structural changes in the walls of the arteries examined.

Fig. 5 shows the results of measurements of the relative diameter change in the common carotid artery as a function of the patient's age. The ratio $\Delta d/d$ decreases with increasing age of the persons examinated, from the mean value 13.9% for the are group 9–16 years to the mean value of 5.32% for the group 52–64 years. The functional dependence of ratio $\Delta d/d$ on the age can be determined from the previously discussed logarithmic rigidity coefficient α. By assuming the linear dependence of the coefficient α on age: $ax + b$ (see formula (16)) and substituting it in formula (13), the following expression is obtained:

$$\frac{\Delta d}{d} = \frac{\ln(p_s/p_d)}{2\alpha} = \frac{\ln(p_s/p_d)}{2(ax+b)}, \tag{17}$$

where x is age in years.

Evaluation of the pressure ratio p_s/p_d and coefficient α as a function of age for the healthy persons (formula (16) and Table 4) indicates that the relative diameter change in the carotid artery is mainly affected by age. Therefore in the first approximation, the following regression function can be assumed for this group of persons:

$$\Delta d/d = 1/(3.8885+0.2370\,x), \tag{18}$$

where x is age in years.

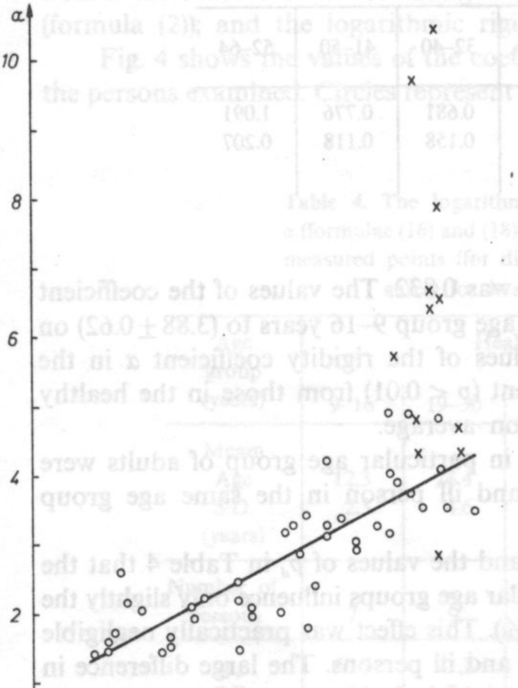

FIG. 4. The logarithmic rigidity coefficient α formula (13) determined for the common carotid arteries of healthy (o) and ill (x) persons of different age. The solid line represents the regression function describing the course of the values of the coefficient α for healthy persons, formula (16)

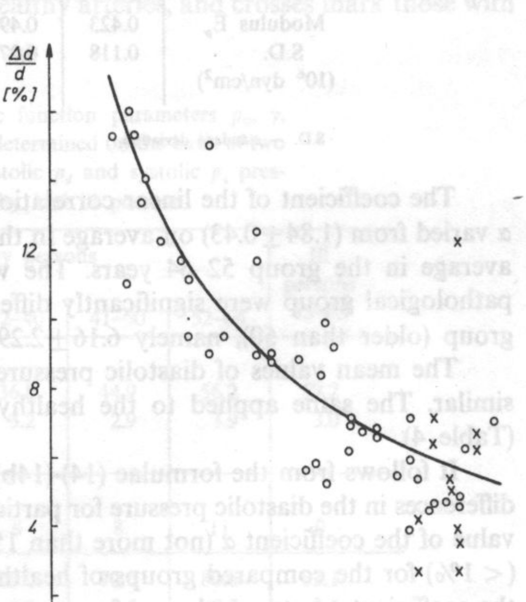

FIG. 5. The relative change $\Delta d/d$ in the diameter of the common carotid artery as function of the age of healthy (o) and ill (x) persons examined. The solid line represents the regression function describing the distribution of experimental points for healthy persons, formula (18)

The determination coefficient R^2 found for this function was 0.7713.

The values of the ratio $\Delta d/d$ obtained from the present measurements are very close to those published by RENEMAN et al. [25]. However, the value of $\Delta d/d$ cannot be recognized as the index of changes occurring in the vessel wall, in view of the dependence between the relative change in the blood vessel diameter and the blood pressure in the systolic and diastolic phases. An example is the lack of significant difference between the relative in the diameter change of the common carotid artery between the ill and the healthy in the same age group from 52 to 64 years (see Fig. 5). The mean value of the ratio $\Delta d/d$ was respectively 5.48% for ill persons and 5.32% for healthy persons. The values of the pressure difference between the systolic and diastolic phases were different between the two groups. For the healthy group the mean systolic pressure p_s was (122.7 ± 13.8) mmHg and the mean diastolic pressure p_d was (80.0 ± 8.7) mmHg. For ill persons these pressures were respectively $p_s = (150 \pm 15.4)$ mmHg and $p_d = (83.1 \pm 4.6)$ mmHg (Table 4).

In the available literature, the blood vessel elasticity is most often characterized by the elasticity coefficient E_p (formula (1) or (2)) or by E_{inc} (formula(3)).

The values of the coefficient E_p (formula (2)) calculated from the present authors' measurements for 5 age groups are shown in Table 5. Comparison of them with Table 1 shows that the present results are close to those obtained by Arndt et al. [1] and Riley et al. [26], who applied a measurement method resembling the one used here.

It follows from Table 5 that E_p increases as a function of the patient's age, just as the coefficient α does (Fig. 4, Table 4). The mutual relation between the values of these two coefficients depends on the diastolic pressure (formula (15)).

For the group of healthy persons with different diastolic pressure, the coefficient α correlated better with age than the coefficient E_p did. The correlation coefficient R determined for the linear dependence of E_p and α on age was for the examined group 0.809 for E_p and 0.8193 for α.

4. Conclusions

1. The evaluation of the elasticity of blood vessel walls requires the description of the functional dependence between the vessel cross-section S and the blood pressure p. On the basis of the results of experimental research presented by Loon et al. [15] for human common carotid artery and by Simon et al. [29] for canine abdominal aorta, comparative analysis was carried out of the function $S = f(p)$ described in the literature, including a logarithmic function proposed by the present authors. Using the least squares method, it was shown that, apart from the linear one, all the functions examined describe very well ($R^2 > 0.96$) the results of experimental research, over the pressure range from 25 mmHg to 200 mmHg.

2. As the only one of the nonlinear functions studied, the logarithmic function between the vessel cross-section and the blood pressure, proposed by the present authors can be determined by the noninvasive method. Its determination requires two pairs of values: pressure – cross-section, which can be found from ultrasonic measurements of the vessel diameter for the systolic and diastolic pressures, which pressures are determined noninvasively with a cuff.

3. The evaluation of the elasticity of blood vessel walls involved the application of the logarithmic rigidity coefficient α (formula (13)) proposed by the present

authors. For the logarithmic function between the vessel cross-section and the blood pressure, it is a more objective index of the mechanical properties of the vessel wall than the previously used coefficient E_p (formula (2)).

4. The noninvasive studies carried out in the common carotid artery in healthy persons aged between 9 and 64 years, indicate an increase in the rigidity of the artery wall as a function of age. The value of the coefficient α increased linearly with age (formula (22)). The coefficient of linear correlation was equal to 0.832.

5. For ill persons with arteriosclerotic changes in the extracranial carotid arteries, the value of the coefficient α was significantly different ($p < 0.01$) than that for healthy persons in the same age group (from 52 to 64 years). The results obtained for healthy and ill persons confirm the view that a factor which favours the development of sclerosis is an increase in the vessel wall rigidity. The mean value of the coefficient α for ill persons was 6.16 ± 2.29, whereas for healthy persons it was 3.98 ± 0.62.

6. The measurements of the relative diameter change $\Delta d/d$ carried out for healthy persons also indicate the dependence of this quantity on age (Fig. 5, formula (24)).

7. Between healthy and ill persons in the same age group there was no significant difference in the relative diameter change of common carotid artery. The mean value of the systolic pressure in ill persons was greater by about 27 mmHg than that for healthy persons. In both groups the diastolic pressures had similar values.

8. The preliminary results of clinical studies indicate that the method used for evaluating the elasticity of blood vessel walls can be highly significant in the diagnosis of extracranial carotid arteries, in particular in examinations of arteriosclerosis.

References

[1] J. O. Arndt, J. Klauske, F. Mersch, *The diameter of the intact carotid artery in man and its change with pulse pressure*, Pflugers Arch. ges. Physiol., **301**, 230–240 (1968).
[2] D. H. Bergel, *The static elastic properties of the arterial wall*, J. Physiol., **156**, 445–457 (1961).
[3] D. H. Bergel, *The dynamic elastic properties of the arterial wall*, J. Physiol., **156**, 458–469 (1961).
[4] H. S. Borovetz, A. M. Brant, T. K. Hung, *Arterial wall biomechanics*, Proceedings of Fifth International Congress on Mechanics in Medicine and Biology, Bologna 1986, pp. 67–70.
[5] F. J. Callaghan, L. A. Geddes, C. F. Babbs, J. D. Bourland, *Relationship between pulse-wave velocity and arterial elasticity*, Med. and Biol. Eng. and Comput., **24**, 248–254 (1986).
[6] Y. C. B. Fung, *Stress-strain history relations of soft tissues in simple elongation*, in: Biomechanics, Its Foundations and Objectives, Prentice Hall, 1972, 181–208.
[7] D. N. Ghista, G. Jayaraman, H. Sandler, *Analysis for the non-invasive determination of arterial properties and for the transcutaneous continuous monitoring of arterial blood pressure*, Med. and Biol. Eng. and Comput., **16**, 715–726 (1978).
[8] B. S. Gow M. G. Taylor, *Measurements of viscoelastic properties of arteries in the living dog*, Circulation Research, **23**, 1, 111 (1968).
[9] A. E. Green, W. Zerna, *Theoretical elasticity*, Clarendon Press, Oxford 1963.
[10] A. E. Green, J. E. Adkins, *Large elastic deformations*, Clarendon Press, Oxford 1970.

[11] D. Groves, T. Powałowski, D. N. White, *A digital technique for tracking moving interfaces*, Ultrasound in Med. and Biol., **8**, 2, 185–190 (1982).

[12] K. Hayashi, H. Handa, S. Nagasawa, A. Okumura, K. Moritake, *Stiffness and elastic behaviour of human intracranial and extracranial arteries*, J. Biomechanics, **13**, 2, 175–184 (1980).

[13] M. B. Histand, M. Anliker, *Influence of flow and pressure on wave propagation in the canine aorta*, Circulation Research, **32**, (1973).

[14] G. J. Langewourters, K. Wesseling, W. Goedhard, *The static elastic properties of 45 human thoracic and 20 abdominal aortas in vitro and the parameters of a new model*, J. of Biomechanics, **17**, 6, 425–435 (1984).

[15] P. Loon, W. Klip, E. Bradley, *Length-force and volume-pressure relationships of arteries*, Biorheology, 14, 181–201 (1977).

[16] J. Megerman, J. Hasson, D. Varnock, G. Litalien, W. Abbot, *Noninvasive measurements of non linear arterial elasticity*, Am. J. Physiol., 250, 181–188 (1986).

[17] J. J. Meister, *Mesure par échographie Doppler et modèlisation théorique de l'effet de troubles cardiaques sur la pression et le débit artériels*, Thèsis, EPFL, Lausanne 1983.

[18] D. Newman, J. Greenwald, *The effect of smooth muscle activity on the static and dynamic elastic properties of the rabbit carotid artery*, in: Cardiovascular System Dynamics, Plenum Press, NY-London 1982.

[19] D. J. Patel, F. M. Freitas, J. C. Greenfield, D. L. Frey, *Relationship of radius to pressure along the aorta in living dogs*, J. Appl. Physiol. **18**, 6, 1111–1117 (1963).

[20] D. J. Patel, J. C. Greenfield, D. L. Fry, *In vivo pressure-length-radius relationship of certain blood vessels in man and dog*, Pulsatile Blood Flow Proceedings, Editor E. O. Attinger, New York 1964.

[21] L. Peterson, R. Jensen, J. Parnel, *Mechanical properties of arteries in vivo*, Circulation Res., **8**, 622–639 (1960).

[22] T. Powałowski, *An ultrasonic apparatus for noninvasive measurement of hemodynamical parameters of human arterial system*, Archives of Acoustics, **13**, 1–2 (1988).

[23] T. Powałowski, B. Pensko, Z. Trawiński, L. Filipczyński, *An ultrasonic transcutaneous method of simultaneous evaluation of blood pressure, flow rate and pulse wave velocity in the carotid artery*, Abstracts of Satellite Symp. of the XIIIth World Congress of Neurol., Aachen, M-68, 1985.

[24] T. Powałowski, B. Pensko, *Noninvasive ultrasonic method of pressure and flow measurements for estimation of hemodynamical properties of cerebrovascular system*, Archives of Acoustic, **10**, 3, 303–314 (1985).

[25] R. Reneman, T. Merode, P. Hick, A. Muytjens, A. Hoeks, *Age related changes in carotid artery wall properties in men*, Ultrasound in Med. and Biology, **12**, 6, 465–471 (1986).

[26] W. A. Riley, R. W. Barnes, H. M. Schey, *An approach to the noninvasive periodic assessment of arterial elasticity in the young*, Preventive Medicine, **13**, 169–184 (1984).

[27] J. T. Roscoe, *Fundamental research statistics for the behavioural sciences*, Holt Rinehard Winston, New York.

[28] W. Schimmler, *Untersuchungen zu Elastizitäts – Problemen der Aorta*, Archiv für Kreislaufforschung, Band 47, Heft 3–4 (1965).

[29] B. Simon, A. Kobayashi, D. Strandness, C. Wiederhielm, *Large deformation analysis of the arterial cross section*, J. of Basic Eng., **93D**, 2 (1971).

[30] J. Stettler, P. Niederer, M. Anliker, *Theoretical analysis of arterial hemodynamics including the influence of bifurcations*, Annals of Biomedical Engineering, **9**, 145–164 (1981).

[31] T. Tanaka, Y. Fung, *Elastic and inelastic properties of the canine aorta and their variation along the aortic tree*, J. Biomechanics, **7**, 389–395 (1974).

[32] H. Weizsacker, A. Pascale, *Anisotropic passive properties of blood vessel walls* in: Cardiovascular System Dynamics, Plenum Press, NY-London 1982, pp. 147–362.

Received on November 5, 1987.