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EVALUATION OF A COMPUTER-MODEL FOR PVDF-TRANSDUCERS OF ARBITRARY CONFIGURATION

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The electro-mechanical behaviour of PVDF can be described by the modified Mason equivalent circuit. It is possible to extract all necessary parameters and their frequency dependence from electrical input-impedance measurements and subsequent fitting-procedure with the computer-model. A chain-parameter matrix can readily be obtained for the equivalent circuit and matrix-multiplication enables the computation of any transfer function.

Elektromechaniczne zjawiska zachodzące w folii PVDF mogą być opisane przez zmodyfikowany układ zastępczy Masona. Pomiar elektrycznej impedancji wejściowej, a następnie dopasowanie wyników pomiarów do modelu komputerowego umożliwia wyznaczenie wszystkich żądanych parametrów i ich zależności od częstotliwości. Macierz parametrów może być z łatwością wyznaczona z otrzymanego układu zastępczego, a metoda macierzowa pozwala na obliczenie dowolnej funkcji przenoszenia.

1. Introduction

For many years transducers for medical ultrasound and other purposes were based on the application of piezoelectric ceramics. Particularly in using these ceramics for medical diagnostic purposes, where very short pulses are required, the high acoustic impedance has always caused problems.

Moreover, especially in more sophisticated constructions such as phased and linear arrays, suppressing the effects of vibrational modes and coupling coefficients other than just wanted has always been difficult.

Probably due to these shortcomings of piezoelectric ceramics the welcoming of the new piezoelectric plastic PVDF (polyvinylidenefluoride) has been accompanied with a lot of optimism. The much lower acoustic impedance of this plastic would naturally match much better to that of water and biological tissues, resulting in more efficient transmission of acoustic energy into the body, automatically yielding an effective damping which is desired for wide-band behaviour. It was obviously expected that the better matching would sufficiently compensate for the unfortunately much lower coupling factor of PVDF as compared with e.g. PZT-5A. Even if this were true, still the relatively low dielectric constant is responsible for a high electrical input impedance of PVDF-transducers, which may amount 100 to 200 times as much as for comparable PZT-transducers. This results in very high voltages required for generating a practically useful acoustic output power. In other words, the transmission sensitivity in terms of the ratio of acoustic output power to squared input voltage is considerably lower than for comparable PZT-transducers.

Of course, in the light of the semi-conductor technology high driving voltages are most inconvenient. As a remedy against this trouble it was suggested to construct transducers by stacking several PVDF-layers with alternating poling directions [1, 2] as shown in Fig. 1. Then, when connected as shown, these layers are acoustically

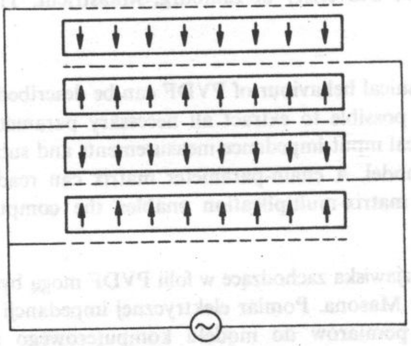

FIG. 1. Stacked transducer configuration with alternating poling directions. Electroded in such a way that the piezoelectric layers are electrically in series and acoustically in parallel

in series and electrically in parallel. In this way we can lower the electrical input impedance with a factor n^2, where n is the number of active layers, as compared with a one-layer PVDF-transducer of similar overall thickness. In this way, more electrical input power is achieved at a lower voltage and, consequently, a corresponding acoustical output power, depending on the coupling factor and the matching to the medium. This idea is by no means new, since already in 1926 it was LANGEVIN who patented such a system using quarts layers [3, 4].

Unfortunately, such a technique to improve the transmision sensitivity will lower the reception sensitivity by a factor n. Complicated electrode-switching may perhaps lead to a compromise yielding a practical transmission sensitivity combined with a best possible reception sensitivity [2].

It will be appreciated that finding the best configuration for a particular purpose would be an almost impossible task since the number of possible combinations is very high, especially when also matching layers and backing are involved. When realizing that bonding several thin metallized PVDF films together, with bonding layers of let us say 1 μm and of perfect quality, which can hardly be tested, the above conclusion seems more than justified.

The purpose of this study is to evaluate a reliable computer model describing the electro-acoustic behaviour of piezo-electric PVDF-film. Then, using the method of chain-parameter matrix multiplication the performance of any desired transducer-configuration can be computed as a function of e.g. frequency. In this way we might be able to find the theoretical optimum for any transducer-system before trying to realize it in practice.

2. Equivalent circuit for a single layer

It would be most attractive if the values of all parameters, necessary to fully describe the transducer's behaviour, could be extracted from simple measurements of the electrical impedance as a function of frequency.

For this purpose test objects were used consisting of nominally 25 μm thick PVDF-film electroded in such a way that an accurately defined circular area of 6 mm diameter was poled. The measured electrical input impedance as a function of frequency showed remarkable features. Fig. 2 shows the frequency dependence of such a test object where the impedance is assumed to consist of a parallel circuit of a capacitance and a resistance as given in Fig. 3.

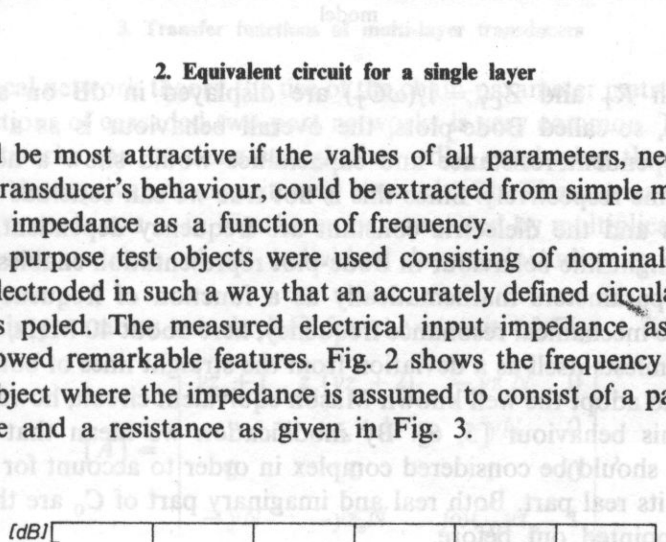

FIG. 2. The behaviour of 20 log R_T and 20 log Z_{CT} as a function of frequency at logarithmic frequency scale (Bode-plots). R_T (asterisks) and Z_{CT} (circles) represent the measured elecrical input-impedance Z_T of a single-layer PVDF-transducer in air, with $Z_{CT} = 1/(\omega C_T)$, according to Fig. 3. The parameters y_{R1}, y_{R2}, y_{C1} and y_{C2} are the coordinates at x_1 and x_2 respectively, of the straight lines through the impedance curves. The constants A and B can be obtained from the data in Fig. 5 and are chosen such that $A \log(Bf) = 400$ corresponds to $f = 40$ MHz

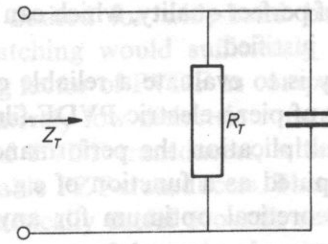

FIG. 3. The electrical impedance circuit representation used for both the measurements and the computer model

When both R_T and $Z_{CT} = 1/(\omega C_T)$ are displayed in dB on a logarithmic frequency scale, so-called Bode-plots, the overall behaviour is as a straight line. Frequency-independent resistance and capacitance would show a horizontal and a 6 dB/octave line respectively. Since this is not true we can conclude that both the dielectric losses and the dielectric constant are frequency dependent. Fortunately enough this straight line behaviour in Bode-plot representation enables us to readily describe these parameters mathematically as a function of frequency.

Around the mechanical resonance frequency, here about 40 MHz, the piezoelectric activity manifests itself as a deviation from the straight lines of both C_T and R_T. This invites us to adopt the well known Mason equivalent circuit, however modified, to represent this behaviour [5, 6]. By modification we mean that the clamped capacitance C_0 should be considered complex in order to account for the dielectric losses through its real part. Both real and imaginary part of C_0 are then frequency dependent as pointed out before.

Further modification also includes that the propagation constant γ is assumed

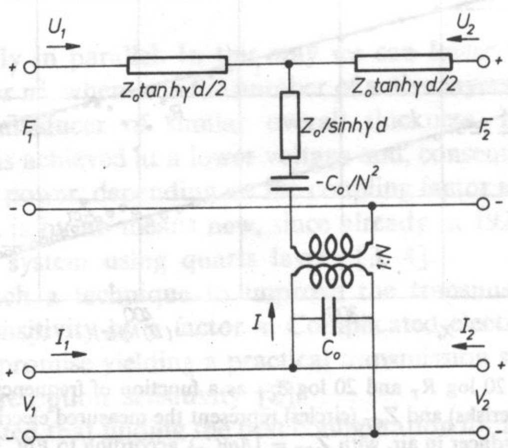

FIG. 4. Four-port equivalent circuit for a single-layer transducer, based on the Mason-model and adapted for application of chain-parameter matrices. Since the real part α of the propagation coefficient γ can not be neglected, the hyperbolic functions will not reduce to trigonometric functions

complex, $\gamma = \alpha + j(\omega/v)$, since mechanical losses, accounted for through α, should also be taken into account. Since C_0 is complex, also $N = hC_0$ is complex. For the time being h, the piezoelectric constant is assumed real. Later on it will be shown that both h and the mechanical loss coefficient α are also frequency dependent, although again obeying simple mathematical expressions.

Fig. 4. represents this equivalent circuit, drawn as a four-port rather than the usual three-port configuration [7].

3. Transfer functions of multi-layer transducers

In classical network theory the use of the chain-parameter metrix for calculating transfer functions of cascaded two-port networks is very common. The extension to four-port networks can be done straightforwardly, employing the basic equations from which the equivalent circuit was derived [7]. Then, transfer functions of multilayered transducers as in Fig. 1, can be computed by multiplication of the 4×4 chain-parameter matrices of all individual layers, including possible matching (passive) layers.

The chain-parameter matrix $[A]$ for the circuit of Fig. 4 takes the form

$$[A] = \begin{bmatrix} yz_s+1 & z_s(yz_s+2) & -yz_sN & 0 \\ y & yz_s+1 & -yN & 0 \\ 0 & 0 & 1 & 0 \\ -yN & -yz_sN & j\omega C_0 yz_p & 1 \end{bmatrix},$$

with

$z_s = Z_0 \tanh \gamma d/2$, $\quad z_p = Z_o/\sinh \gamma d$,

$y = 1/[z_p - N^2/(j\omega C_0)]$, $\quad N = hC_0$,

$Z_0 = A_{pe} \cdot j\omega\varrho/\gamma$, $\quad A_{pe}$ = effective piezoelectric area,

d = transducer thickness, $\quad \varrho$ = density of transducer material.

It can be observed that the matrix coefficients are made to fulfil the electrical input requirements as dictated by the equivalent circuit of Fig. 4. Firstly the input voltage of the transducer is equal to both V_1 and V_2, secondly its input current I equals the sum of I_1 and I_2.

In the case of a passive layer we simply have to put both N and C_0 equal to zero to find the proper matrix.

Now, any overall transfer function can be obtained from the expression

$$\begin{bmatrix} F_1 \\ U_1 \\ V_1 \\ I_1 \end{bmatrix} = [A_t] \cdot \begin{bmatrix} F_2 \\ -U_2 \\ V_2 \\ -I_2 \end{bmatrix}$$

with F = force, U = particle velocity, V = electrical voltage, I = electrical current, where

$$[A_t] = [A_b]\cdot[A_n]\cdot\ldots\cdot[A_i]\cdot\ldots\cdot[A_1]\cdot[A_f],$$

with

$[A_b]$ = chain-parameter matrix of back matching layer,
$[A_i]$ = chain-parameter matrix of i^{th} active layer,
$[A_f]$ = chain-parameter matrix of front matching layer.

4. Estimation of transducer parameters

The crucial step in the whole process is the estimation of the parameters h, α, v_s (sound velocity in transducer material), and the co-ordinates determining the straight lines through R_M and Z_{CM} (the computed impedance curves for the model), similar to the measured values R_T and Z_{CT} as in Fig. 2.

A computer programme has been created which enables us to display both the model-impedance curves and the measured ones simultaneously. Then each of the above mentioned parameters can be altered as desired and again all curves are displayed. After some trials a set of values for h, α, v_s and the co-ordinates y_{C1}, y_{C2}, y_{R1}, y_{R2}, x_1 and x_2 (see Fig. 2) will be obtained, which produce model-curves for R_M and Z_{CM} coinciding with the measured R_T and Z_{CT} as well as possible. We learn from Fig. 5 that such a procedure can lead to a remarkably close fit between measured and model impedance curves. Fig. 5a shows in detail the fit-accuracy around the resonance region whereas Fig. 5b gives the overall view showing a fit within some tenths of a decibel.

It should be realized that no information whatsoever can be extracted from this procedure about the values of h and α outside the resonance region. In order to estimate any frequency dependence a further investigation is necessary. In order to avoid uncertainties, already measured and fitted samples had to be used for this purpose. Several two- and four-layer transducers were realized by bonding with epoxy-resin whereby bonding thicknesses of ca. 1μm were obtained. As expected the new resonance frequencies were one half and one quarter of the original respectively. Again the actual impedance curves were measured and through the matrix-multiplication method corresponding models were computed. Then, by the fitting-method the new h and α values were determined for these composite samples at the pertinent resonance frequencies.

In Fig. 6 the right side scale shows A_1 rather than α. However, since $A_1 = \alpha d/2$, where d is the single layer thickness, there is only a constant factor involved, which does not influence the frequency dependence.

Through each (averaged) single layer value of both A_1 and h, and the corresponding values for the composite transducers, straight lines have been drawn

v_s	2.14550E+03	A	3.32193E+02
h	2.08039E+09	B	0.40000E−06
$\alpha d/2$	0.66277E−01	d	0.26000E−04
Y_{C1}	4.31350E+01	D	0.60000E−02
Y_{C2}	3.47960E+01	ϱ	1.80000E+03
Y_{R1}	5.59200E+01	x_1	3.00000E+02
Y_{R2}	4.74340E+01	x_2	4.60000E+02

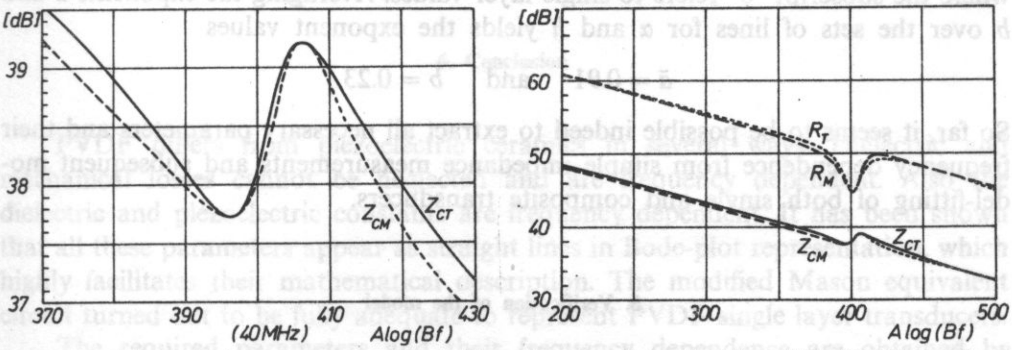

FIG. 5. Final fit-result of experimental and model impedance functions of a 25 μm thick PVDF-transducer of 6 mm diameter in air. Left: In the resonance frequency region (about 40 MHz) right: Frequency range from 10 to 80 MHz. The parameters at the right side are varied until the best fit has been obtained. These are: the sound propagation velocity v_s in m/sec, the piezoelectric constant h ($= h_{33}$) in V/m, the mechanical loss coefficient α in Neper/m multiplied by half the thickness and y-coordinates (see text and Fig. 2). The other parameters are all determined or chosen in advance like the constants A and B, used for the logarithmic frequency scale, the thickness d and the diameter D, both in m, the density ϱ in kg/m^3 and the x – coordinates (see text and Fig. 2)

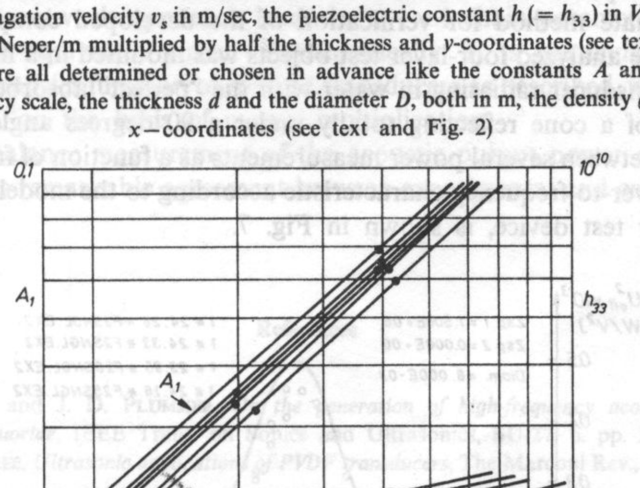

FIG. 6. Values for $A_1 = \alpha d/2$ (dots, left side scale) and h (circles, right side scale), all normalized to single layer conditions 1) around 10 MHz: values found for 4-layer transducers; 2) around 20 MHz: values found for 2-layer transducers; 3) around 40 Mz: averaged values of the single layers constituting the corresponding composite ones

on a double logarithmic scale. The parallelism between these lines indicates that h and α can readily be described as follows

$$\alpha(f) = \alpha_0 (f/f_0)^a \quad \text{and} \quad h(f) = h_0 (f/f_0)^b,$$

where the subscript "0" refers to single layer values. Averaging the exponents a and b over the sets of lines for α and h yields the exponent values

$$\bar{a} = 0.91 \quad \text{and} \quad \bar{b} = 0.23.$$

So far, it seems to be possible indeed to extract all necessary parameters and their frequency dependence from simple impedance measurements and subsequent model-fitting of both single and composite transducers.

5. Verification of the model

In order to avoid sources of error like radiation patterns and their artefacts, acoustic output power measurement with a radiation force balance seems to be the most appropriate method for verification of the developed computer-model.

One of the analyzed four-layer test objects was mounted in a holder to form an air-backed transducer radiating in water with the "perfect" absorber of the balance (in the form of a cone reflecting totally under a 90 degrees angle) in front of it. Comparison between several power measurements as a function of frequency and the calculated power-to-frequency characteristic according to the model as evaluated for this particular test device, is shown in Fig. 7.

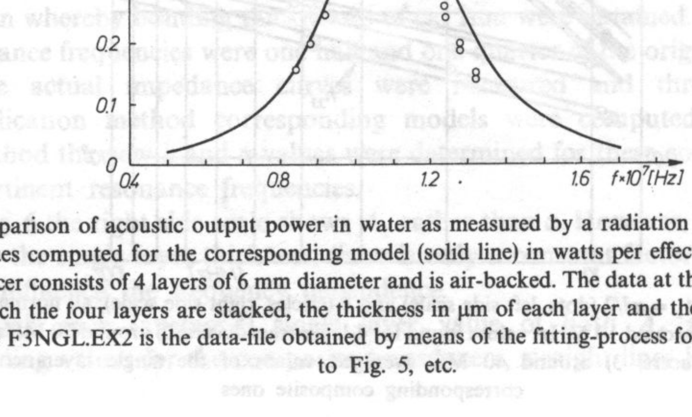

FIG. 7. Comparison of acoustic output power in water as measured by a radiation force balance circles and the values computed for the corresponding model (solid line) in watts per effective voltage squared. The transducer consists of 4 layers of 6 mm diameter and is air-backed. The data at the right side show the order in which the four layers are stacked, the thickness in μm of each layer and the pertinent data-files. For instance F3NGL.EX2 is the data-file obtained by means of the fitting-process for sample 3 according to Fig. 5, etc.

The agreement between measured and calculated results is most satisfactory. Of course, power measurement of only one test object cannot be considered a sufficiently reliable verification. Nevertheless, our approach to model-evaluation seems very promising.

6. Conclusion

PVDF differs from piezoelectric ceramics in several ways. Dielectric and mechanical losses cannot be neglected and are frequency dependent. Also the dielectric and piezoelectric constants are frequency dependent. It has been shown that all these parameters appear as straight lines in Bode-plot representation, which highly facilitates their mathematical description. The modified Mason equivalent circuit turned out to be fully adequate to represent PVDF single layer transducers.

The required parameters and their frequency dependence are obtained by finding the best possible fit between the input-impedance-to-frequency characteristics of the device and the computer model.

A four-port network approach leads to a 4×4 chain-parameter metrix fully representing this equivalent circuit mathematically. It is particularly suited for calculating any transfer function of stacked transducers as described, with or without matching layers, on the basis of matrix multiplication.

Radiation balance measurement of the acoustic output power of a four-layer transducer shows remarkable agreement between experimental and computer model results.

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