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ANALYSIS OF SOUND ABSORPTION BY A THREE-LAYER RESONANT AND ABSORBING STRUCTURE

KRZYSZTOF WERNEROWSKI

Faculty of Mechanics, Academy of Technology and Agriculture
(85-763 Bydgoszcz, ul. Prof. S. Kaliskiego 7)

The paper presents an analogous method of calculating resonance frequencies of a three-layer perforated structure. It was found that a model of vibrations with three degrees of freedom can be applied here. The possibility of widening frequency bands by using absorbing layers was developed and investigated. Effective attenuation of harmful noise with relatively low frequencies was stated.

W pracy przedstawiono analogową metodę obliczania częstotliwości rezonansowych trójwarstwowego ustroju perforowanego. Stwierdzono możliwość zastosowania modelu drgań o trzech stopniach swobody. Opracowano i zbadano możliwości rozszerzenia pasm częstotliwości przez zastosowanie warstw pochłaniających. Stwierdzono możliwość skutecznego tłumienia szkodliwego dźwięku o stosunkowo niskich częstotliwościach.

1. Introduction

Practical attenuation of harmful noise of machines and machinery is a very difficult problem. Spectral analysis determines characteristic frequencies and the white noise share.

Only a single resonant-absorbing structure has been worked out theoretically and experimentally in detail [8, 11]. The solution for one frequency with definite attenuation in the surroundings (e.g. in the range of 2.5 octave) is often not sufficient.

Hence, the application of multiple perforated resonant and absorbing structures is necessary. A solution (Fig. 1) effective for three frequencies with an adequately expanded band is frequently sufficient. A relatively high noise suppression effectiveness is ensured by axially distributed decreasing holes.

Calculations of structures with several layers concern multilayer baffles without perforations, mainly [8, 10]. In literature [1–7, 9] referring to analysis of sound absorption of resonant and absorbing structures the problem of multiple solutions is limited just to advise of general character.

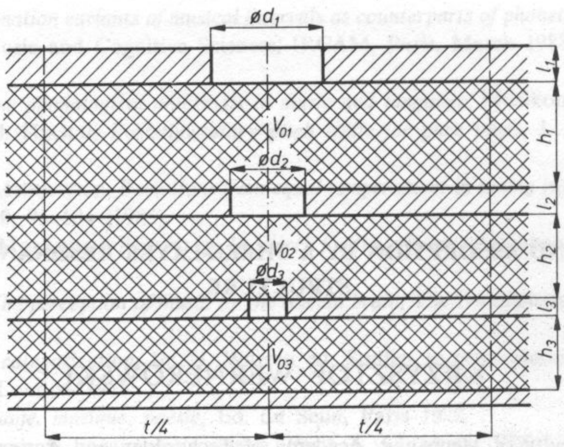

Fig. 1

It is very difficult to silence noise components of machines with several frequencies distinct in the spectrum and relatively low. Only a precisely calculated multiple resonant and absorbing structure secures effective attenuation of very strenuous noise.

There is an analogy between a three-layer perforated structure and a vibrating system with three degrees of freedom.

Air in the holes can be defined as "acoustic masses", while compliances of the gas mixture in the space between plates are inverses of acoustic "elasticities". Of course damping takes place also in the porous material.

2. Calculations

An accurate determination of the size of the vibrating system is the basis for modelling. The following quantities were calculated from the presented in Fig. 1 system's geometry:

acoustic "masses"

$$m_{ai} = \frac{4\varrho_0(l_i + \Delta l_i)}{\pi d_i^2}, \tag{1}$$

corrections for co-vibrating masses

$$\Delta l_i = 0.7 d_i, \tag{2}$$

acoustic stiffnesses

$$k_{ai} = \frac{\varrho_0 \cdot c^2}{v_{0i} - \Delta v_{0i}}, \tag{3}$$

$$V_{0i} = \frac{h_i \cdot t^2}{4} \tag{3a}$$

corrections for the existing co-vibrating mass

$$V_{oi} = 0.14\, d_i^3, \tag{3b}$$

coefficient of "attenuation" i. e. "dissipation" of acoustic energy

$$c_{ai} = \frac{\tilde{\mathbf{P}}_i}{\tilde{\mathbf{V}}_i}, \tag{4}$$

where: ϱ_0 – density of air, in kg m^{-3}, c – speed of sound in the medium, in m s^{-1}, $\tilde{\mathbf{P}}_i$ – effective pressure expressed with the method of complex numbers in the analysed part of the structure, in Pa, $\tilde{\mathbf{V}}_i$ – effective volume velocity expressed with the method of complex numbers in an element of the silencing construction, in m^3 s^{-1}.

The model of a vibrating system with three degrees of freedom is shown in Fig. 2.

Including attenuation (energy dissipation) Lagrange equations of the second type were supplemented to the form

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} + \frac{\partial \Phi}{\partial \dot{q}_i} = 0, \tag{5}$$

$$L = T - U\Phi = \frac{1}{2}\sum_{i,j=1}^{f} c_j\, \dot{q}_i\, \dot{q}_j, \tag{5a, 5b}$$

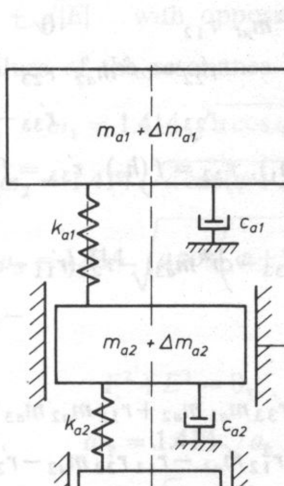

Fig. 2

where: T – kinetic energy, U – potential energy, Φ – Rayleigh's dissipation function, q_1 – generalized coordinates $x_1, x_2, x_3, \dot{q}_1$ – generalized velocities, $x_1 \ldots$ The actual mechanical system is described by the following system of differential equations

$$\begin{aligned} m_{a1}\ddot{x}_1 + c_{a1}\dot{x}_1 + r_{11}x_1 + r_{12}x_2 &= 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ m_{a3}\ddot{x}_3 + c_{a3}\dot{x}_3 + r_{32}x_2 + r_{33}x_3 &= 0, \end{aligned} \tag{6}$$

$$r_{11} = k_{a1}, \tag{6.1}$$

$$r_{12} = -k_{a1} = r_{21}, \tag{6.2}$$

$$r_{22} = k_{a1} + k_{a2}, \tag{6.3}$$

$$r_{23} = -k_{a2} = r_{32}, \tag{6.4}$$

$$r_{33} = k_{a2} + k_{a3}, \tag{6.5}$$

$$r_{13} = r_{31} = 0. \tag{6.6}$$

We seek solutions in the following form

$$x_i(t) = e^{-h_i t} f_i(t), \tag{7}$$

$$h_i = \frac{c_{ai}}{2m_{ai}} \tag{8}$$

The determinant of the characteristic system

$$D(\omega^2) = \begin{vmatrix} r'_{11} - \omega^2 m_{ai} & r_{12} & 0 \\ r_{12} & r'_{22} - \omega^2 m_{a2} & r_{23} \\ 0 & r_{23} & r'_{33} - \omega^2 m_{a3} \end{vmatrix} = 0, \tag{9}$$

$$r'_{11} = f(h_1),\ r'_{22} = f(h_2),\ r'_{33} = f(h_3), \tag{9a, b, c}$$

are with expansion

$$(r'_{11} - \omega^2 m_{a1})(r'_{22} - \omega^2 m_{a2})(r'_{33} - \omega^2 m_{a3}) - r_{23}^2(r'_{11} - \omega^2 m_{a1}) - r_{12}^2(r'_{33} - \omega^2 m_{a3}) = 0. \tag{10}$$

Equation

$$\begin{aligned} -m_{a1}m_{a2}m_{a3}\omega^6 &+ (r'_{33}m_{a1}m_{a2} + r'_{11}m_{a2}m_{a3} + r'_{22}m_{a1}m_{a3})\omega^4 \\ &+ (r_{23}^2 m_{a1} + r_{12}^2 m_{a3} - r'_{11}r'_{33}m_{a2} - r'_{22}r'_{33}m_{a1} \\ &- r'_{11}r'_{22}m_{a3})\omega^2 + r'_{11}r'_{22}r'_{33} - r_{23}^2 r'_{11} - r_{12}^2 r'_{33} = 0, \end{aligned} \tag{11}$$

is the result of transformations.

A resultant biquadratic equation of the third degree can be noted in the following form

$$\omega^6 + A\omega^4 + B\omega^2 + C = 0, \tag{12}$$

$$A = \frac{r'_{33}\, m_{a1}\, m_{a2} + r'_{11}\, m_{a2}\, m_{a3} + r'_{22}\, m_{a1}\, m_{a3}}{-m_{a1}\, m_{a2}\, m_{a3}}, \tag{12a}$$

$$B = \frac{r'_{11}\, r'_{33}\, m_{a2} + r'_{22}\, r'_{33}\, m_{a1} + r'_{11}\, r'_{22}\, m_{a3} - r^2_{23}\, m_{a1} - r^2_{12}\, m_{a3}}{m_{a1}\, m_{a2}\, m_{a3}} \tag{12b}$$

$$C = \frac{r^2_{23}\, r'_{11} + r^2_{12}\, r'_{33} - r'_{11}\, r'_{22}\, r'_{33}}{m_{a1}\, m_{a2}\, m_{a3}}. \tag{12c}$$

Substituting

$$\omega^2 = x - \frac{A}{3}, \tag{13}$$

we have

$$x^3 + 3Ex - 2F = 0. \tag{14}$$

Real roots have vibroacoustic sense, for

$$F^2 + E^3 > 0, \tag{15}$$

and

$$\varphi = \frac{1}{3} \arccos \frac{F}{a^3}. \tag{16}$$

$$a = \pm\sqrt{|E|} \quad \text{with opposite sign } F \tag{17}$$

there are three positive values of the resonance frequency

$$\omega_1 = 1.414\sqrt{a \cos\varphi}, \tag{18a}$$

$$\omega_2 = 1.414\sqrt{a \cos(\varphi + 2/3\pi)}, \tag{18b}$$

$$\omega_3 = 1.414\sqrt{a \cos\left(\varphi + 1\frac{1}{3}\pi\right)}, \tag{18c}$$

and in the case of

$$F^2 + E^3 = 0, \tag{19}$$

$$\omega_1 = 1.414\sqrt{a}, \tag{19a}$$

$$(\omega_2 = \omega_3 = i\sqrt{a}), \quad (i = \sqrt{-1}). \tag{19b, c}$$

Model of acoustic vibrations have the following general form

$$f_i(t) = e^{-h_i t}\left[a_{ij} \sin(\omega_{it} + \varphi_i) + \ldots\right], \quad i, j = 1, 2, 3. \tag{20}$$

The structure has a relatively wide frequency range of the noise attenuation.

Naturally, constructions filled with sound absorbing material (Fig. 1) are effective in a wider range of frequencies.

The width of a band within which the absorption coefficient does not drop below half the value it has at resonance is important.

Calculations of a three-layer resonant and absorbing structure were carried out on the basis of results of analysis of the analogue model (15 – 19b, c) and the methodology of insulating-sound absorbing structures [8, 11].

The curve (Fig. 3) of the absorption coefficient of the three-layer resonant and absorbing structure depends on the frequency of absorbed sound. Theoretical curves

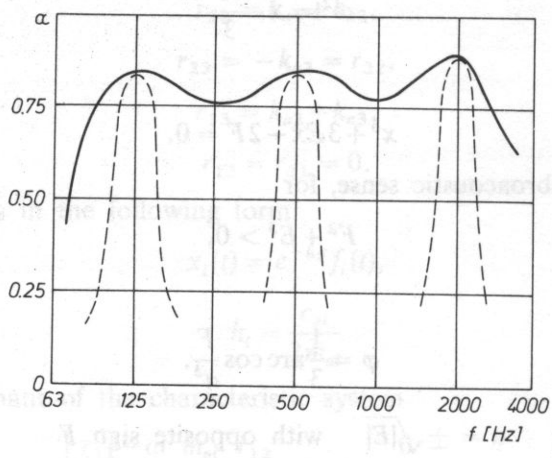

Fig. 3

for pure resonance of structure's sub-assemblies are denote with dashed lines. Continuous attenuation is ensured by the porous material and co-operation of individual Helmholtz resonators.

Of course extremal values are a result of free vibration frequencies of the system. A filled resonant and absorbing structure acts effectively on acoustic waves inciding under various angles.

Simplified model testing was performed and the literature was analysed [7, 11]. The analogous method was found sufficiently consistent with actual functioning.

The three-layer resonant and absorbing structure can function effectively in a band up to ~ 7.5 octave for $a_{rez} = 0.6$.

3. Conclusions

1. The axial distribution of decreasing openings ensures a relatively high effectiveness of noise suppression.

2. An analogy exists between acoustic functioning of a three-layer perforated structure and a vibrating system with three degrees of freedom.

3. A three-layer resonant and absorbing structure functions in a wide band of sound frequencies.

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