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ENERGETIC PROPERTIES OF RECTANGULAR SOUND SOURCES WITH LARGE DIRECTIONALITY

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In this paper the author investigated energetic properties of rectangular sound sources with the following amplitude distributions of the vibration velocity: uniform, HAMMING'S, HANNING'S and BLACKMAN'S. The frequency characteristics of the active power, reactive power and apparent power of these sources was determined, as well as their power factor. It was found that sources under investigations effectively radiate vibration energy into the far field (i.e. with the power factor equal to one) in the wave length range, in which they exhibit large directionality. The energy of vibrations radiated by a source into the far field in a unit of time is by an order of magnitude smaller in the case of HAMMING'S, HANNING'S and BLACKMAN'S distributions than in the case of a uniform distribution. Therefore, an increase of the directivity of radiation of the vibration energy into the far field by rectangular sound sources is accompanied by a decrease of the value of radiated energy.

1. Introduction

There is a need for sound sources with large directionality of vibration energy radiated into the far field in many domains: metrology, diagnostics, hydrolocation and ultrasonic technology. They are applied to obtain an adequate beam of ultrasonic waves or to obtain a required concentration of energy in a certain area of the medium. In these applications sound sources have to radiate energy of vibrations effectively. The far field of a rectangular sound source with uniform, HANNING'S and BLACKMAN'S amplitude distributions of the vibration velocity have been investigated in paper [5]. It was stated that the directional characteristic of such a source with HANNING'S distribution has a relatively narrow main maximum (much more narrow than for a Gaussian distribution [10]) and sufficiently strongly damped side maxima. It was also found that a sound source, which radiates energy of vibrations into the far field with a sufficiently large directionality, can be in practice realized by a mosaic system of plane sound sources with discrete HANNING'S distributions of their relative bulk efficiencies. This paper analyses energetic properties of rectangular sound sources with large directionality of energy radiate into the far field. A developed by the author new method of determining frequency characteristics of the reactive power of sound sources with the application of a fast Fourier transform was used in investigations.

2. Acoustic field of plane sound sources

Let us accept (Fig. 1) that a plane sound source σ_0, which vibrates with a simple periodic motion with frequency f_0, is situated in plane $z = 0$, which is a perfectly rigid baffle S_0. It was also assumed that the distribution of the normal component of the amplitude of vibration velocity produced by source σ_0 in the baffle S_0 is defined by the following function

$$\begin{aligned} &\varkappa(x_0, y_0) \neq 0 \quad \text{for surface of the source } \sigma_0; \\ &\varkappa(x_0, y_0) = 0 \quad \text{for the rest of the surface of the baffle } S_0. \end{aligned} \tag{1}$$

Let us assume that source σ_0 radiates the energy of vibrations into the half-space $z > 0$ filled with a lossless and homogeneous fluid medium with density ϱ, in which a sound wave propagates with velocity c. The amplitude distribution of the acoustic potential in this half-space is determined by the solution of Helmholtz's equation [2, 6]

$$\Delta\Phi(x, y, z) + 4\pi^2 \nu^2 \Phi(x, y, z) = 0 \tag{2}$$

which satisfies Neumann's boundary condition

$$\frac{\partial}{\partial z}\Phi(x, y, z)\Big|_{z=0} = -\varkappa(x_0, y_0) \tag{3}$$

and Sommerfeld's condition of finity

$$\lim_{r\to\infty} \Phi(x, y, z) = 0 \tag{4}$$

and radiation

$$\lim_{r\to\infty} r\left[\frac{\partial}{\partial r}\Phi(x, y, z) + j2\pi\nu\Phi(x, y, z)\right] = 0 \tag{5}$$

where Δ is a Laplacian, $\nu = f_0/c$ — spatial frequency of a sound wave with frequency f_0, which propagates with velocity c in the direction of radius r (Fig. 1).

Let us consider an arbitrary plane S_z, situated in half-space $z > 0$ and parallel to the baffle S_0 (Fig. 1). The distance between these two planes is denoted by z. Function $\Phi(x, y, z)$ defines the amplitude distribution of the acoustic potential produced by source σ_0 in plane S_z. Now we will define components of the spatial frequency of a plane sound wave propagating in the direction of radius r:

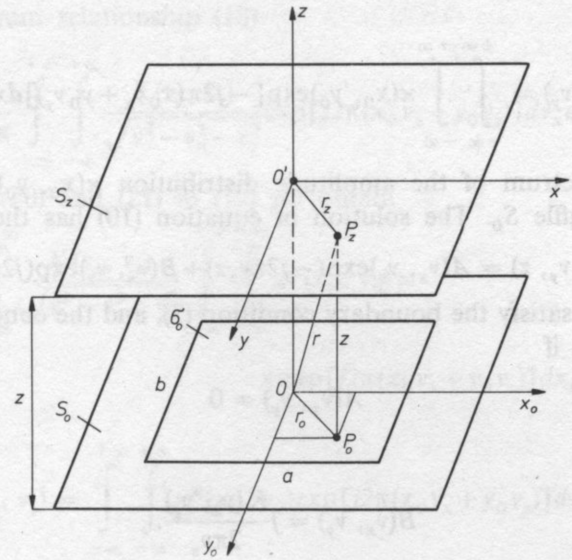

Fig. 1. Plane sound source σ_0 located in baffle S_0, which radiates energy of vibrations into a medium, which fills the half-space above the baffle

$$v_x = v\cos(x, r), \tag{6}$$

$$v_y = v\cos(y, r), \tag{7}$$

$$v_z = v\cos(z, r). \tag{8}$$

The spatial spectrum of the amplitude distribution of the acoustic potential in plane S_z is determined with the application of a simple two-dimensional Fourier transform [1]. Namely,

$$F(v_x, v_y\ z) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \Phi(x, y, z)\exp[-j2\pi(xv_x + yv_y)]dxdy. \tag{9}$$

In accordance to this, equation (2) and the boundary condition (3) can be written as follows

$$\frac{d^2}{dz^2}F(v_x, v_y, z) + 4\pi^2 v_z^2 F(v_x, v_y, z) = 0, \tag{10}$$

where

$$v_z = \sqrt{v^2 - v_x^2 - v_y^2} \tag{11}$$

and

$$\left.\frac{d}{dz}F(v_x, v_y, z)\right|_{z=0} = -K(v_x, v_y), \tag{12}$$

while

$$K(\nu_x, \nu_y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \varkappa(x_0, y_0)\exp[-j2\pi(x_0\nu_x + y_0\nu_y)]\,dx_0\,dy_0 \tag{13}$$

is the spatial spectrum of the amplitude distribution $\varkappa(x_0, y_0)$ of the vibration velocity in the baffle S_0. The solution of equation (10) has the following from

$$F(\nu_x, \nu_y, z) = A(\nu_x, \nu_y)\exp(-j2\pi\nu_z z) + B(\nu_x, \nu_y)\exp(j2\pi\nu_z z). \tag{14}$$

This solution will satisfy the boundary condition (3), and the conditions of finity (4) and radiation (5), if

$$A(\nu_x, \nu_y) = 0 \tag{15}$$

and

$$B(\nu_x, \nu_y) = j\frac{K(\nu_x, \nu_y)}{2\pi\nu_z}. \tag{16}$$

Hence the solution of (14) has the following form

$$F(\nu_x, \nu_y, z) = jK(\nu_x, \nu_y)\frac{\exp(j2\pi\nu_z z)}{2\pi\nu_z}. \tag{17}$$

With the application of the inverse Fourier transform [1], we can determine the interesting to us amplitude distribution of the acoustic potential in an arbitrary plane S_z, parallel to the baffle S_0 on the basis of dependence (17). Namely

$$\Phi(x, y, z) = j\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} K(\nu_x, \nu_y)\frac{\exp(j2\pi\nu_z z)}{2\pi\nu_z}[j2\pi(x\nu_x + y\nu_y)]\,d\nu_x\,d\nu_y. \tag{18}$$

3. Sound power of a plane source

The sound power of a plane source σ_0, situated in baffle S_0 can be derived from [2, 6]

$$N = (1/2)\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \varkappa^*(x_0, y_0)P(x_0, y_0)\,dx_0\,dy_0, \tag{19}$$

where function $\varkappa^*(x, y)$ denotes the distribution of the complex conjugate amplitude of the vibration velocity in the baffle S_0 and function $P(x, y)$ determines the amplitude distribution of the acoustic pressure in this plane. Because [2]

$$P(x_0, y_0) = -j2\pi\nu\varrho c\Phi(x_0, y_0), \tag{20}$$

while for $z = 0$ from relationship (18)

$$\Phi(x_0, y_0) = j\frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{K(\nu_x, \nu_y)}{\sqrt{\nu^2-\nu_x^2-\nu_y^2}}\exp[j2\pi(x_0\nu_x+y_0\nu_y)]\,d\nu_x d\nu_y \tag{21}$$

thus substituting (20) and (21) in (19) we obtain

$$N = \frac{\nu\varrho c}{2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{K(\nu_x, \nu_y)}{\sqrt{\nu^2-\nu_x^2-\nu_y^2}}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\varkappa^*(x_0, y_0)\times$$

$$\times\exp[j2\pi(x_0\nu_x+y_0\nu_y)]\,dx_0dy_0d\nu_xd\nu_y. \tag{22}$$

For [1]

$$K^*(\nu_x, \nu_y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\varkappa^*(x_0, y_0)\exp[j2\pi(x_0\nu_x+y_0\nu_y)]\,dx_0dy_0 \tag{23}$$

is the complex conjugate spatial spectrum of the amplitude distribution of the vibration velocity in the baffle S_0, thus

$$N = \frac{\nu\varrho c}{2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{|K(\nu_x, \nu_y)|^2}{\sqrt{\nu^2-\nu_x^2-\nu_y^2}}d\nu_xd\nu_y. \tag{24}$$

It results from this relationship that the acoustic power of a plane sound source σ_0 is a complex quantity. Let

$$N = N_R+jN_I, \tag{25}$$

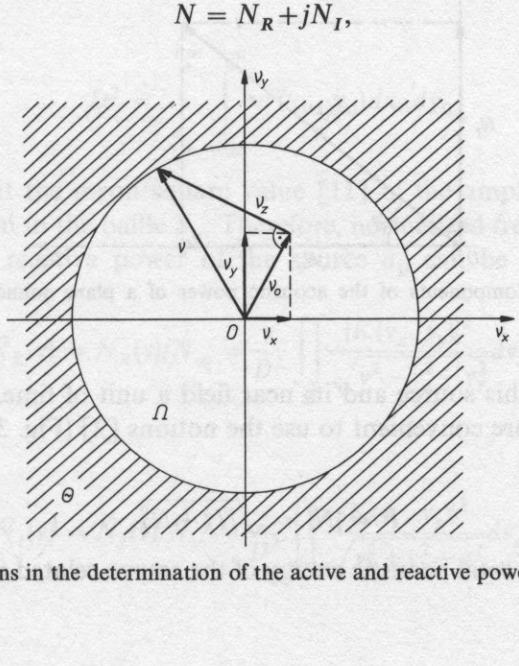

Fig. 2. Integration regions in the determination of the active and reactive power of a plane sound source

where N_R is the active power and N_I is the reactive power of such a source. We will consider the plane of components v_x and v_y of spatial frequency (Fig. 2). It results from (24) that the active power of the sound source σ_0 can be determined from relationship

$$N_R = \frac{v\varrho c}{2} \iint_{\Omega} \frac{|K(v_x, v_y)|^2}{\sqrt{v^2 - v_x^2 - v_y^2}} dv_x dv_y \tag{26}$$

by integrating in the region Ω, of spatial components v_x and v_y, in which

$$v_x^2 + v_y^2 \leqslant v^2. \tag{27}$$

Whereas the reactive power of this source can be determined from

$$N_I = -\frac{v\varrho c}{2} \iint_{\theta} \frac{|K(v_x, v_y)|^2}{\sqrt{v_x^2 + v_y^2 - v^2}} dv_x dv_y, \tag{28}$$

where the integration in done in the region θ of spatial frequencies v_x and v_y, in which

$$v_x^2 + v_y^2 > v^2. \tag{29}$$

The active power of source σ_0 determines its energy of vibrations, which is radiated in a unit of time into the far field, while the reactive power determines the energy

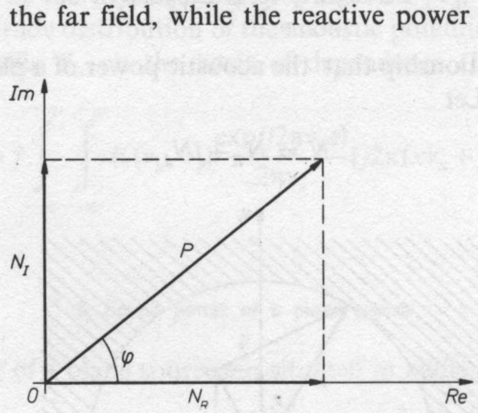

Fig. 3. Components of the acoustic power of a plane sound source

exchanged between this source and its near field a unit of time. From the practical point of view it is more convenient to use the notions [2] (Fig. 3) of apparent power of the sound source

$$P = |N| = \sqrt{N_R^2 + N_I^2}, \tag{30}$$

which determines the total acoustic energy of the source related to a unit of time, and power factor

$$\cos\varphi = N_R/P, \tag{31}$$

which informș what part of this energy is radiated by the source into the far field in a unit of time.

4. Energetic characteristics of a sound source

It should be noticed that the acoustic power of the sound source σ_0 (and hence its active power, reactive power, apparent power and power factor) depend on the vibration frequency f_0 of its surface through the spatial frequency ν. Functions presenting the relationship between the spatial frequency ν and these quantities are called energetic characteristics of sound source σ_0 [2]. It is more convenient to use normalized energetic characteristics when comparing energetic properties of sound source. We have

$$\lim_{\nu\to\infty} N(\nu) = \frac{\varrho c}{2} D^2 = N_\infty, \tag{32}$$

where

$$D^2 = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} |K(\nu_x, \nu_y)|^2 d\nu_x d\nu_y. \tag{33}$$

With the application of Parseval's theorem [1] the above expression can be written as

$$D^2 = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \varkappa^2(x_0, y_0) dx_0 dy_0. \tag{34}$$

It results that D^2 it the meon square value [11] of the amplitude of the vibration velocity distribution in the baffle S_0. Therefore, normalized frequency characteristics of the active and reactive power of the source σ_0 can be defined as

$$\bar{N}_R(\nu) = N_R(\nu)/N_\infty = \frac{\nu}{D^2}\iint_\Omega \frac{|K(\nu_x, \nu_y)|^2}{\sqrt{\nu^2-\nu_x^2-\nu_y^2}} d\nu_x d\nu_y, \tag{35}$$

where as

$$\bar{N}_I(\nu) = N_I(\nu)/N_\infty = \frac{\nu}{D^2}\iint_\theta \frac{|K(\nu_x, \nu_y)|^2}{\sqrt{\nu_x^2+\nu_y^2-\nu^2}} d\nu_x d\nu_y, \tag{36}$$

while

$$\lim_{\nu\to\infty} \bar{N}_R(\nu) = 1 \tag{37}$$

and

$$\lim_{\nu\to\infty} \bar{N}_I(\nu) = 0. \tag{38}$$

5. Methods of determining energetic characteristics of sound sources

Energetic characteristics of sound sources are most frequently determined directly form relationships (35), (36) and (30), (31) (e.g. [6, 7]). It was proved in papers [9] and [10] that the characteristics of the reactive power of a sound source can be determined on the basis of its frequency characteristic of the active power, with the application of a simple Hilbert transformation. Namely,

$$\bar{N}_I(\nu) = \frac{1}{\pi\nu} * \bar{N}_R(\nu) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{\bar{N}_R(\eta)}{\nu-\eta} d\eta. \tag{39}$$

Now we will prove that the frequency characteristic of the reactive power of a sound source can be determined on the basis of its frequency characteristic of the active power, with the application of a simple and inverse Fourier transform. Fourier transform of both sides of the relationship (39) were determined. Taking advantage [11] of the theorem about the Fourier transform of a convolution and the theorem about the Fourier transform of function $1/(\pi\nu)$ we obtain

$$\bar{\mathcal{N}}_{I\mu} = -j\operatorname{sgn}(\mu)\bar{\mathcal{N}}_R(\mu), \tag{40}$$

where distribution

$$\operatorname{sgn}(\mu) = \begin{cases} 1 & \text{for } \mu > 0, \\ 0 & \text{for } \mu = 0, \\ -1 & \text{for } \mu < 0. \end{cases} \tag{41}$$

Hence, on the basis of the inverse Fourier transform [11] we can note

$$\bar{N}_I(\nu) = \int_{-\infty}^{+\infty} \bar{\mathcal{N}}_I(\mu) \exp(j2\pi\nu\mu) d\mu, \tag{42}$$

where

$$\bar{\mathcal{N}}_I(\mu) = -j\operatorname{sgn}(\mu)\bar{\mathcal{N}}_R(\mu), \tag{43}$$

while

$$\bar{\mathcal{N}}_R(\mu) = \int_{-\infty}^{+\infty} \bar{N}(\nu)\exp(-j2\pi\mu\nu)d\nu. \tag{44}$$

The resulting from above considerations algorithm of determining the frequency characteristic of the reactive power of a sound source on the basis of its frequency characteristic of the active power with the application of the Fourier transform is presented in Fig. 4.

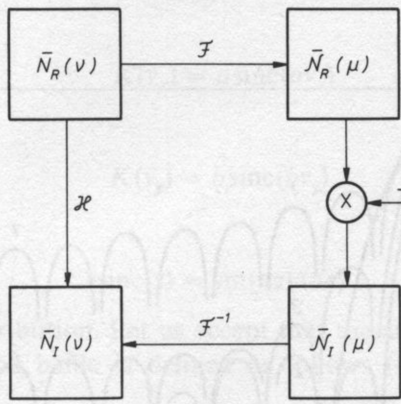

Fig. 4. Algorithms of determining the frequency characteristic of the reactive power of a sound source: $\mathscr{H}$ — Hilbert's transform, $\mathscr{F}$ — Fourier transform, $\mathscr{F}^{-1}$ — inverse Fourier transform

The frequency characteristics of the active power of sound sources with large directionality have been determined in this paper from relationship (35) with the application of the trapezoid method of calculating the values of definite integrals. While frequency characteristics of the reactive power of these sources were determined in accordance with the algorithm presented in Fig. 4 with the application of a simple and inverse discrete Fourier transform [3, 4]. The Cooley-Tukey algorithm of the fast Fourier transform [3, 4] was used in the course of calculations. Calculations were performed on a minicomputer.

6. Rectangular sound sources with large directionality

Let us accept that a sound source σ_0 has a shape of a rectangular with sides a and b (Fig. 1). We will analyse the following amplitude distributions of the vibration velocity in the baffle S_0: uniform, HAMMING'S, HANNING'S and BLACKMAN'S. It was proved in paper [5] that for HAMMING'S, HANNING'S and BLACKMAN'S distributions, the directional characteristic of a rectangular sound has a relatively narrow main

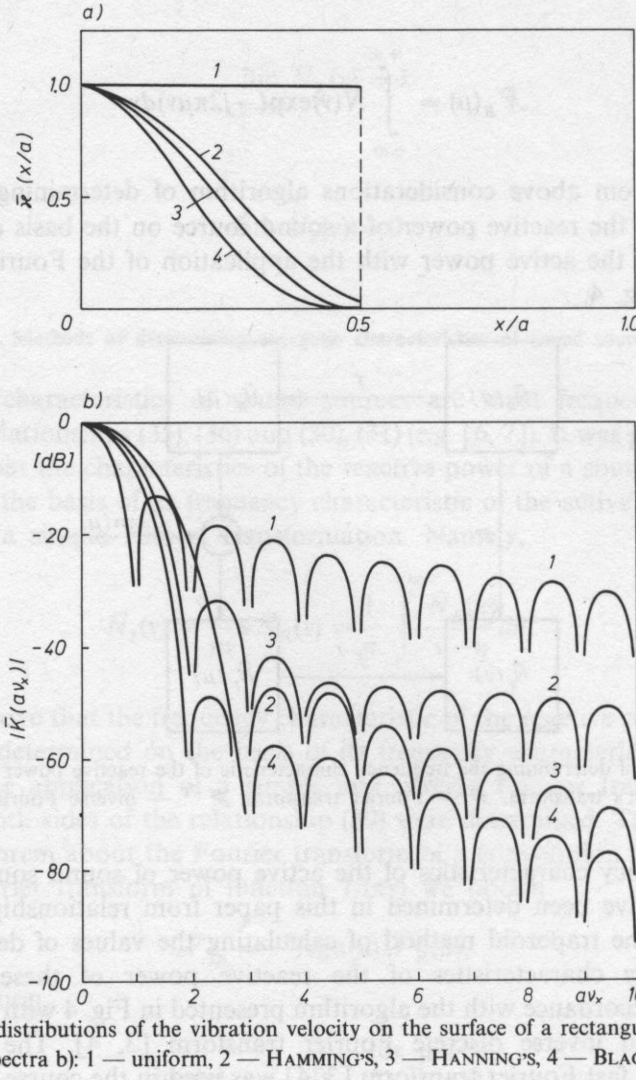

Fig. 5. Amplitude distributions of the vibration velocity on the surface of a rectangular sound source a) and their spatial spectra b): 1 — uniform, 2 — HAMMING'S, 3 — HANNING'S, 4 — BLACKMAN'S distribution

maximum and sufficiently strongly damped side maxima, if the dimensions of the source are sufficiently large in relation to the wave length.

a) Uniform distribution. Let us accept that the amplitude distribution in the baffle S_0 is determined as follows (Fig. 5)

$$\varkappa(x_0, y_0) = \varkappa_0 \varkappa(x_0) \varkappa(y_0), \tag{45}$$

where

$$\varkappa(x_0) = \begin{cases} 1 & \text{for } |x_0| \leqslant a/2, \\ 0 & \text{for } |x_0| > a/2 \end{cases} \tag{46}$$

and

$$\varkappa(y_0) = \begin{cases} 1 & \text{for } |y_0| \leqslant b/2, \\ 0 & \text{for } |y_0| > b/2. \end{cases} \tag{47}$$

The mean square value of this distribution is equal to

$$D^2 = \varkappa_0^2 ab, \tag{48}$$

while its spatial spectrum (Fig. 5) is given as

$$K(v_x, v_y) = \varkappa_0 K(v_x) K(v_y), \tag{49}$$

where

$$K(v_x) = a\,\mathrm{sinc}(av_x) \tag{50}$$

and

$$K(v_y) = b\,\mathrm{sinc}(bv_y), \tag{51}$$

while

$$\mathrm{sinc}(z) = \sin(\pi z)/(\pi z). \tag{52}$$

b) HAMMING'S distribution. Let us accept that the amplitude distribution of the vibration velocity in the baffle is defined as follows (Fig. 5)

$$\varkappa(x_0, y_0) = \varkappa_0 \varkappa(x_0) \varkappa(y_0), \tag{53}$$

where

$$\varkappa(x_0) = \begin{cases} 0.54 + 0.46\cos(2\pi x_0/a) & \text{for } |x_0| \leqslant a/2, \\ 0 & \text{for } |x_0| > a/2 \end{cases} \tag{54}$$

and

$$\varkappa(y_0) = \begin{cases} 0.54 + 0.46\cos(2\pi y_0/b) & \text{for } |y_0| \leqslant b/2, \\ 0 & \text{for } |y_0| > b/2. \end{cases} \tag{55}$$

The mean square value of this distribution is equal to

$$D^2 = 0.158\varkappa^2 ab, \tag{56}$$

while its spatial spectrum (Fig. 5) is given as

$$K(v_x, v_y) = \varkappa_0 K(v_x) K(v_y), \tag{57}$$

where

$$K(v_x) = a[0.54\,\mathrm{sinc}(av_x) + 0.23\,\mathrm{sinc}(av_x - 1) + 0.23\,\mathrm{sinc}(av_x + 1)] \tag{58}$$

and

$$K(v_y) = b[0.54\,\mathrm{sinc}(bv_y) + 0.23\,\mathrm{sinc}(bv_y - 1) + 0.23\,\mathrm{sinc}(bv_y + 1)]. \tag{59}$$

c) HANNING'S distribution. Let us accept that the amplitude distribution of the vibration velocity in the baffle is defined as follows (Fig. 5)

$$\varkappa(x_0, y_0) = \varkappa_0 \varkappa(x_0)\varkappa(y_0), \tag{60}$$

where

$$\varkappa(x_0) = \begin{cases} 0.5+0.5\cos(2\pi x_0/a) & \text{for } |x_0| \leqslant a/2, \\ 0 & \text{for } |x_0| > a/2 \end{cases} \tag{61}$$

and

$$\varkappa(y_0) = \begin{cases} 0.5+0.5\cos(2\pi y_0/b) & \text{for } |y_0| \leqslant b/2, \\ 0 & \text{for } |y_0| > b/2. \end{cases} \tag{62}$$

The mean square value of this distribution is equal to

$$D^2 = 0.141\varkappa_0^2 ab, \tag{63}$$

while its spatial spectrum (Fig. 5) is given as

$$K(v_x, v_y) = \varkappa_0 K(v_x)K(v_y), \tag{64}$$

where

$$K(v_x) = a[0.5\operatorname{sinc}(av_x)+0.25\operatorname{sinc}(av_x-1)+0.25\operatorname{sinc}(av_x+1)] \tag{65}$$

and

$$K(v_y) = b[0.5\operatorname{sinc}(bv_y)+0.25\operatorname{sinc}(bv_y-1)+0.25\operatorname{sinc}(bv_y+1)]. \tag{66}$$

d) BLACKMAN'S distribution. Let us accept that the amplitude distribution of the vibration velocity in the baffle S_0 is defined as follows (Fig. 5)

$$\varkappa(x_0, y_0) = \varkappa_0 \varkappa(x_0)\varkappa(y_0), \tag{67}$$

where

$$\varkappa(x_0) = \begin{cases} 0.42+0.5\cos(2\pi x_0/a)+0.08\cos(4\pi x_0/a) & \text{for } |x_0| \leqslant a/2, \\ 0 & \text{for } |x_0| > a/2 \end{cases} \tag{68}$$

and

$$\varkappa(y_0) = \begin{cases} 0.42+0.5\cos(2\pi y_0/b)+0.08\cos(4\pi y_0/b) & \text{for } |y_0| \leqslant b/2, \\ 0 & \text{for } |y_0| > b/2. \end{cases} \tag{69}$$

The mean square value of the distribution is equal to

$$D^2 = 0.093\varkappa_0^2 ab, \tag{70}$$

while its spatial spectrum (Fig. 5) is given as

$$K(v_x, v_y) = \varkappa_0 K(v_x)K(v_y), \tag{71}$$

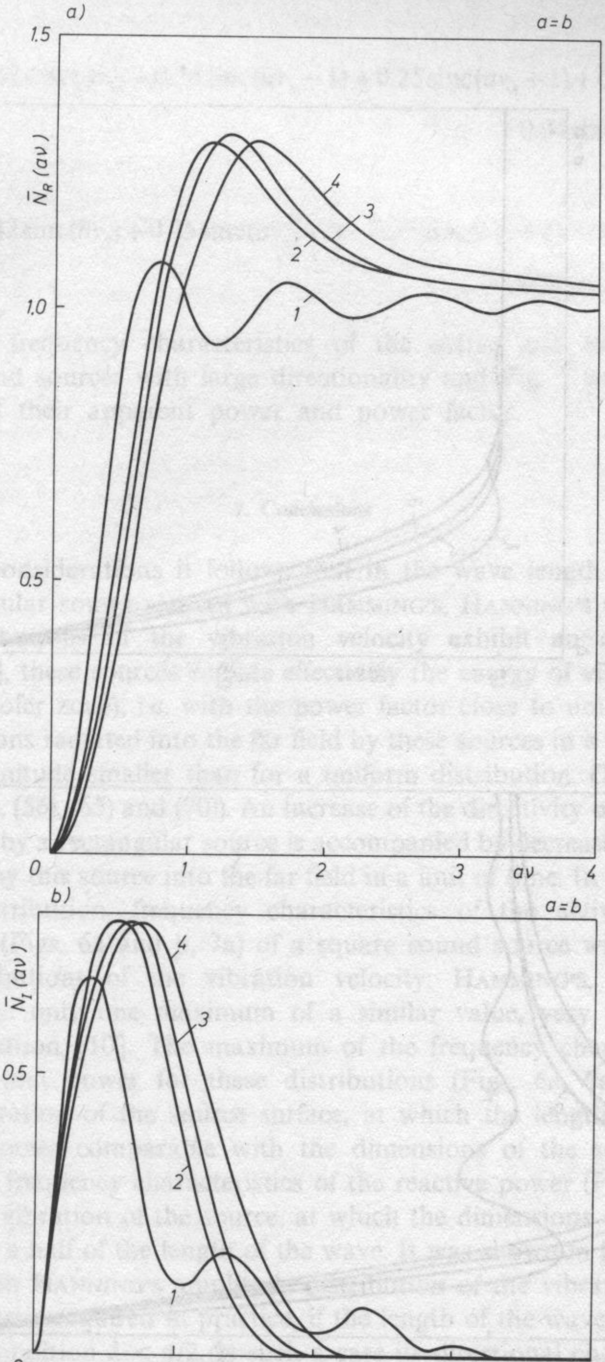

Fig. 6. Normalized frequency characteristics of the active power a) and reactive power b) of a square sound source with the following distributions: 1 — uniform, 2 — HAMMING'S, 3 — HANNING'S, 4 — BLACKMAN'S

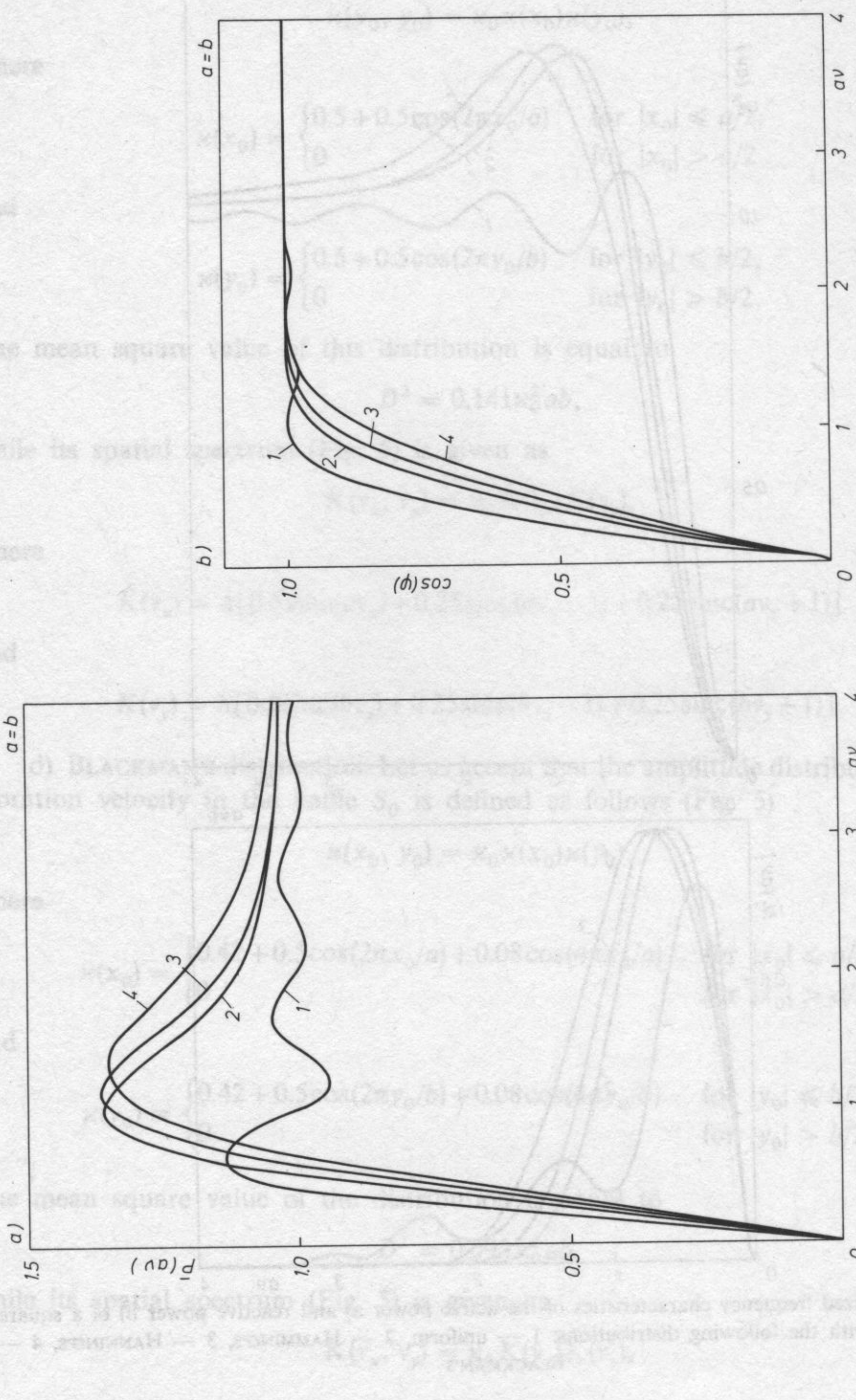

Fig. 7. Normalized frequency characteristics of the apparent power a) and the power factor b) of a square sound source with the following distributions: 1 — uniform, 2 — HAMMING'S, 3 — HANNING'S, 4 — BLACKMAN'S

where

$$K(v_x) = a[0.42\operatorname{sinc}(av_x)+0.25\operatorname{sinc}(av_x-1)+0.25\operatorname{sinc}(av_x+1)+0.04\operatorname{sinc}(av_x-2)+ +0.04\operatorname{sinc}(av_x+2)] \quad (72)$$

and

$$K(v_y) = b[0.42\operatorname{sinc}(bv_y)+0.25\operatorname{sinc}(bv_y-1)+0.25\operatorname{sinc}(bv_y+1)+0.04\operatorname{sinc}(bv_y-2)+ +0.04\operatorname{sinc}(bv_y+2)]. \quad (73)$$

Fig. 6 presents frequency characteristics of the active and reactive power of investigated sound sources with large directionality and Fig. 7 presents frequency characteristics of their apparent power and power factor.

7. Conclusions

From our considerations it follows that in the wave length range, in which analysed rectangular sound sources with HAMMING'S, HANNING'S and BLACKMAN'S amplitude distributions of the vibration velocity exhibit an adequately large directionality [5], these sources radiate effectively the energy of vibrations into the far field (Fraunhofer zone), i.e. with the power factor close to unity. However, the energy of vibrations radiated into the far field by these sources in a unit of time is by an order of magnitude smaller than for a uniform distribution. (This results from relationships (48), (56), (65) and (70)). An increase of the directivity of radiation of the vibration energy by a rectangular source is accompanied by decrease of the vibration energy radiated by this source into the far field in a unit of time. In comparison with the uniform distribution, frequency characteristics of the active, reactive and apparent power (Figs. 6a and b, 7a) of a square sound source with the following amplitude distributions of the vibration velocity: HAMMING'S, HANNING'S and BLACKMAN'S have only one maximum of a similar value, very much like for a Gaussian distribution [10]. The maximum of the frequency characteristic of the active and apparent power for these distributions (Figs. 6a, 7a) occurs at the frequency of vibration of the source surface, at which the length of the radiated sound wave becomes comparable with the dimensions of the source; while the maximum of the frequency characteristics of the reactive power (Fig. 6b) occurs at the frequency of vibration of the source, at which the dimensions of the source are comparable with a half of the length of the wave. It was shown in [5] that a square sound source with HANNING'S amplitude distribution of the vibration velocity has directivity properties required in practice, if the length of the wave radiated by this source satisfies condition $\lambda < a/2$. Is such a case its directional characteristic has a relatively narrow main maximum (its width is equal to $\cos(2\theta) = 1.4\ \lambda/a$ at the level of -3 dB) and sufficiently strongly damped ($\geqslant 32$ dB) and quickly decreasing (60 dB/decade) side maxima. At wave length $\lambda < a/2$ this source radiates its whole energy into the far field (Fig. 7b).

The method of determining energetic characteristics of plane sound sources, which was presented in this paper, can be applied in the estimation of energetic properties of various real plane sound sources on the basis of experimentally determined directional characteristics. It can be proved that the directional characteristic of a sound source is a fragment of the spatial spectrum of the amplitude distribution of the vibration velocity, produced by a given source in the baffle. A detailed analysis of this problem is presented in paper [5]. Measurements of the level of the directional characteristics of a sound source at a given spatial frequency ν can be used for the calculation of its active power (26) with a chosen method of numerical integration. The frequency characteristic of the active power of an investigated sound source in the interesting to us range of spatial frequency ν can be achieved by repeating these calculations for following spatial frequencies. In turn, on this basis the frequency characteristic of the reactive power of this source (42) can be calculated with the application of a chosen algorithm of a discrete Fourier transform. The sampling interval of the frequency characteristic of the active power and the truncate function of this characteristic has to be adequately chosen. These problems have been analysed in detail in papers [3] and [4].

The basic advantage of this method of determining energetic characteristics of sound sources is the possibility of its application in investigations of energetic properties of sources and systems of sound sources with arbitrary shape and arbitrary amplitude distribution of the vibration velocity on their surface. The simplicity and speed of obtaining neccessary results with the application of a computer is another advantage.

The limited applicability of this method is the disadvantage of this method. Namely — it can be used in investigations of energetic properties of plane sources and systems of sound sources situated in a baffle.

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