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ACOUSTIC FILTERS WITH A PERFORATED TUBE

GERARD BRZÓZKA

Institute of Machine Construction and Operation of the Engineering College in Zielona Góra
(65–246 Zielona Góra, ul. Podgórna 50)

A method of calculating acoustic properties of a low-pass filter with a perforated side baffle separating two wave-guides with constant cross-section areas is proposed. It is based on the principle of segmentation of the wave-guide into a series of elementary models which describe properties of acoustic element related to individual rows of perforation and segments of acoustic wave-guides, and on the description of acoustic properties of these models with the application of a transmission matrix with its properties of a chain matrix. Also relationships are given from which acoustic properties of a perforated baffle can be determined for a discrete parameter model at a laminar flow through orifices of the perforation.

Moreover, a calculation model for determining generally applied measures — insertion loss and transmission loss — was proposed on the basis of the transmission matrix of an acoustic filter determined with the presented method.

1. Introduction

Acoustic filters with perforated tubes are generally used to attenuate noise in systems with motion of a medium. They are constructed as analogues of low-pass elctric wave filters which as a rule operate in misfit conditions.

During the last period a very convenient and univocal description of filters transmission properties with the use of a chain matrix [1, 12, 16], in foreign literature called also a transmission matrix, was introduced. The term — transmission matrix — will be used in this paper. It is simple to determine hitherto applied attenuation measures, such as transmission loss TL or insertion loss IL, for a given transmission matrix at definite filter installation conditions [16, 17]. A description with a chain matrix is the most convenient description due to general usage of a cascade connections of individual filter elements [11].

Hitherto applied models have been developed for conditions of sound propagation in a medium at rest [7, 8]. However in practice the velocity of the motion of the medium is substantial and hence measurement results stray considerably from so obtained theoretical estimations.

On the basis of experimental and theoretical studies performed during the last several years [14] SULLIVAN formulated a physical and mathematical model which made it possible to determine elements of the transmission matrix of a filter with perforated channel at known value of the acoustic impedance of the perforated surface. In his earlier paper [2] SULLIVAN formulated propositions of calculating this impedance for a discrete parameter model. This would make it possible to estimate analytically elements of the transmission matrix, as a consequence.

This paper presents an outline of a physical model and mathematical model of determining elements of the transmission matrix of the described above type of filter. The impedance of the perforated surface is described in the form of a discrete parameter model and other elements of wave-guides are described with the application of a distributed parameter model. Because of several editorial errors in mathematical models presented in literature, it seems advisable to present the full model in this paper. Also relationships which enable the determination of generally applied attenuation measures – TL and IL – are described.

2. Outline of physical model

The diagram of the physical model is presented in Fig. 1, on the assumption that conditions of plane wave propagation are satisfied in channels. This model was developed on the basis of Sullivan's propositions [14]. Two channels with cross-sections S_1 and S_2, are connected on a segment with length L by several rows of perforations. Acoustic impedances are arbitrarily determined at the beginning of the segment $Z_{1,N+1}$, $Z_{2,N+2}$ and at its end – $Z_{1,1}$ and $Z_{2,1}$. An individual j-row of the perforation has a joint surface of S_{0j} and ratio of perforations related to this

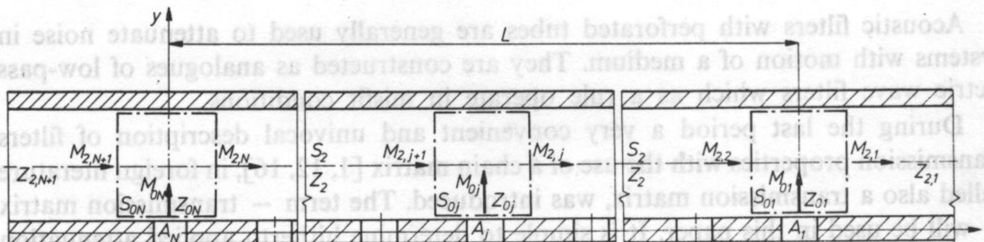


Fig. 1. Diagram of an acoustic filter with a perforated tube

surface σ_j ratio of the joint surface of orifices and the perforated surface. The flow of the medium in channels is determined by Mach numbers $M_{1,j}$, $M_{1,j+1}$ with relation to wave-guide 1, M_{2+j}; $M_{2,\ j+1}$ with relation to wave-guide 2, and M_{0j} with relation to the flow between channels in terms of surface S_{0j}. It is assumed that the value of acoustic admittance of perforations A_j is known.

The acoustic element located accordingly in wave-guide 1 and 2, connected together by orifices of the perforation can be isolated for every row of perforations. Every particle is connected with neighbouring acoustic element in the wave-guide. Segments of wave-guides 1 and 2 with lengths l_{j-1} and l_j are such connections. Taking into consideration mentioned above assumptions an analogue diagram of the filter under consideration (Fig. 1) is presented in Fig. 2. Acoustic parameters of individual elements of the model are described.

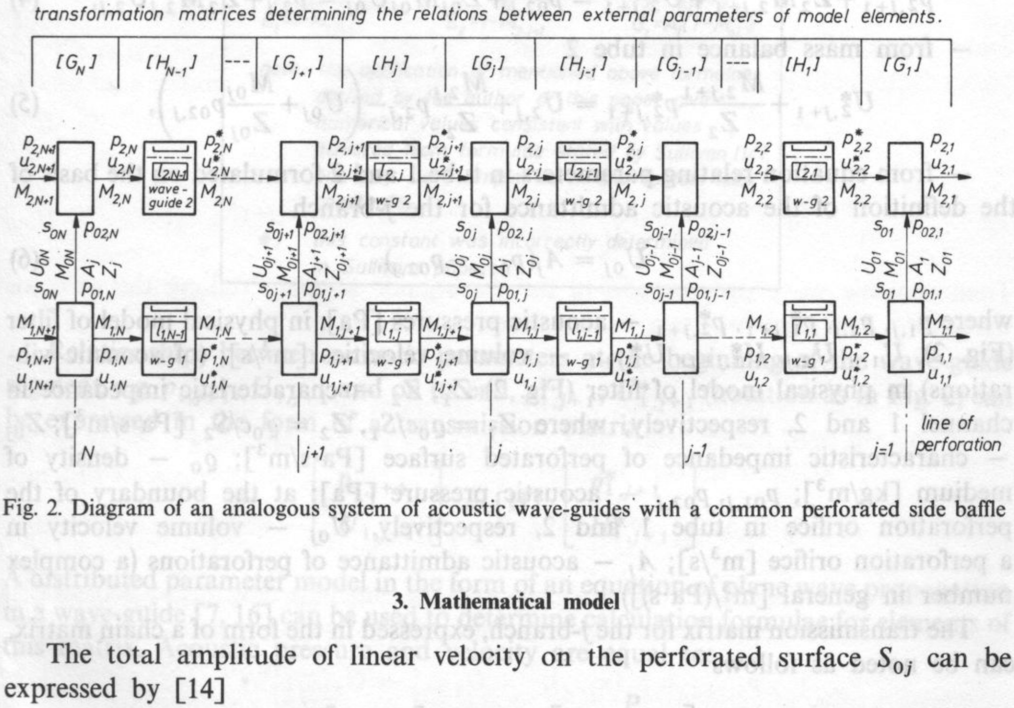


Fig. 2. Diagram of an analogous system of acoustic wave-guides with a common perforated side baffle

3. Mathematical model

The total amplitude of linear velocity on the perforated surface S_{0j} can be expressed by [14]

$$u_c = M_{0j}\, c + U_{0j}/S_{0j}, \tag{1}$$

where U_{0j} – volume acoustic velocity in the perforation orifice [m^3/s], c – sound velocity in the medium [m/s]. It is assumed that Mach numbers for the flow in channels and flow between channels in rows of perforations, M_{0j}, are known from separate hydrodynamic calculations of these channels.

Relations describing dependencies in such a model of a j-branch can be determined from an energy and mass balance in volumes of acoustic elements in channels 1 and 2, if we assume that there are no internal acoustic energy sources in these volumes and that acoustic pressure and velocity undergoe isentropic processes. On the basis of SULLIVAN'S considerations [14] and for the above mentioned assumptions the mathematical model of a j-branch of orifices can be presented in the following form:

— from energy balance in tube 1

$$p^*_{1,j+1} + Z_1 M_{1,j+1} U^*_{1,j+1} = p_{01,j} + Z_{0j} M_{0j} U_{0j} = p_{1,j} + Z_1 M_{1,j} U_{1,j} \tag{2}$$

— from mass balance in tube 1

$$U^*_{1,j+1} + \frac{M_{1,j+1}}{Z_1} p^*_{1,j+1} = U_{1,j} + \frac{M_{1,j}}{Z_1} p_{1,j} + \left(U_{0j} + \frac{M_{0j}}{Z_{0j}} p_{01,j} \right), \tag{3}$$

— from energy balance in tube 2

$$p^*_{2,j+1} + Z_2 M_{2,j+1} + U^*_{2,j+1} = p_{02,j} + Z_{0j} M_{0j} U_{0j} = p_{2,j} + Z_2 M_{2,j} U_{2,j}, \tag{4}$$

— from mass balance in tube 2

$$U^*_{2,j+1} + \frac{M_{2,j+1}}{Z_2} p^*_{2,j+1} = U_{2,j} + \frac{M_{2,j}}{Z_2} p_{2,j} - \left(U_{0j} + \frac{M_{0j}}{Z_{0j}} p_{02,j} \right), \tag{5}$$

— from equation relating parameters in tube 1 and 2 formulated on the basis of the definition of the acoustic admittance for the j-branch

$$U_{0j} = A_j (p_{01,j} - p_{02,j}), \tag{6}$$

where $p_{1,j}$, $p_{2,j}$, $p^*_{1,j+1}$, $p^*_{2,j+1}$ — acoustic pressures [Pa]; in physical model of filter (Fig. 2), $U_{1,j}$, $U_{2,j}$, $U^*_{1,j+1}$, $U^*_{2,j+1}$ — volume velocities [m^3/s]; (of acoustic vibrations) in physical model of filter (Fig. 2) Z_1, Z_2 — characteristic impedance in channel 1 and 2, respectively, where $Z_1 = \varrho_0 c/S_1$, $Z_2 = \varrho_0 c/S_2$ [$Pa \cdot s/m^3$]; Z_{0j} — characteristic impedance of perforated surface [$Pa \cdot s/m^3$]; ϱ_0 — density of medium [kg/m^3]; $p_{01,j}$, $p_{02,j}$ — acoustic pressure [Pa]; at the boundary of the perforation orifice in tube 1 and 2, respectively, U_{0j} — volume velocity in a perforation orifice [m^3/s]; A_j — acoustic admittance of perforations (a complex number in general [$m^3/(Pa \cdot s)$]).

The transmission matrix for the j-branch, expressed in the form of a chain matrix, can be noted as follows

$$\begin{bmatrix} p^*_{1,j+1} \\ U^*_{1,j+1} \\ p^*_{2,j+1} \\ U^*_{2,j+1} \end{bmatrix} = [G_j] \begin{bmatrix} p_{1,j} \\ U_{1,j} \\ p_{2,j} \\ U_{2,j} \end{bmatrix} \tag{7}$$

and elements of this (4×4) matrix can be determined from equations (2)–(6). Equations from which elements of this matrix can be determined are presented in Table 1.

Table 1

element denotation	calculation formula	element denotation	calculation formula
G_{11}	$1-B_3G_1/E_1$	G_{21}	G_1/E_1
G_{12}	$B_1-B_3[1-B_1(B_2-G_1)]/E_1$	G_{22}	$[1-B_1(B_2-G_1)]/E_1$
G_{13}	B_3G_1/E_1	G_{23}	$-G_1/E_1$
G_{14}	$B_3C_1G_1/E_1$	G_{24}	$-C_1G_1/E_1$
G_{31}	C_3G_1/E_2	G_{41}	$-G_1/E_2$
G_{32}	$B_1C_3G_1/E_2$	G_{42}	$-B_1G_1/E_2$
G_{33}	$1-C_3G_1/E_2$	G_{43}	G_1/E_2
G_{34}	$C_1-C_3[1-C_1(C_2-G_1)]/E_2$	G_{44}	$[1-C_1(C_2-G_1)]/E_2$
$B_1=M_{1,j}Z_1$ *)	$B_2=M_{1,j}/Z_1$	$B_3=M_{1,j+1}Z_1$	
$C_1=M_{2,j}Z_2$	$C_2=M_{2,j}/Z_2$	$C_3=M_{2,j+1}Z_2$	
$E_1=1-M^2_{1,j+1}$	$E_2=1-M_{2,j+1}$	$G_1=A_j(1-M^2_{oj})$	

note: the application of mentioned above formulae derived by the author of this paper give numerical values consistent with values achiered from formulae stated by Sullivan [14], but they have simpler form and contain less constants.

*) this constant was incorrectly determined in Sullivans paper

Relationships between acoustic parameters at the beginning of the wave-guide segment, e.g. $p_{1,j+1}$, $U_{1,j+1}$ and on its end, $p^*_{1,j+1}$, $U^*_{1,j+1}$ (notation as in Fig. 2) can be expressed in the form of a transmission matrix

$$\begin{bmatrix} p_{1,j+1} \\ U_{1,j+1} \end{bmatrix} = [\mathrm{H}] \begin{bmatrix} p^*_{1,j+1} \\ U^*_{1,j+1} \end{bmatrix}.$$

A distributed parameter model in the form of an equation of plane wave propagation in a wave-guide [7, 16] can be used to determine calculation formulae for elements of this matrix. Acoustic pressure and velocity are equal to:

$$p_1(x) = P_1 e^{-ikx}, \quad U_1(x) = \frac{P_1}{Z_1} e^{-ikx},$$

where: P_1 — pressure amplitude of acoustic wave in wave-guide 1, [Pa], Z_1 — characteristic impedance of wave-guide, [Pa·s/m^3], k — wave number, [m^{-1}]; $k = \omega/c$, ω — angular velocity (pulsation), [rad/s]; $\omega = 2\pi f$, f — frequency, [Hz], c — velocity of sound propagation in the medium at rest, [m/s], x — coordinate of the reference system along the wave-guide axis (as in Fig. 1).

Taking the motion of the medium into consideration, the velocity of sound propagation in terms of coordinate x will change by a value equal to the velocity of the motion of the medium – v (convection of the acoustic wave in the medium). Thus, sound velocity of a progressive wave c^+ and reflected wave c^- can be determined from relationships:

$$c^+ = c+v = c(1+v/c) = c(1+M_{1,j+1}),$$
$$c^- = c-v = c(1-v/c) = c(1-M_{1,j+1}).$$

In consequence wave numbers and wave impedances will change also:

– for a progressive wave

$$k^+ = \frac{\omega}{c^+} = \frac{\omega}{c(1+M_{1,j+1})} = \frac{k}{1+M_{1,j+1}},$$
$$Z_1^+ = \frac{\varrho_0 c^+}{S_1} = \frac{\varrho_0 c}{S_1}(1+M_{1,j+1}) = Z_1(1+M_{1,j+1}),$$

– for a reflected wave

$$k^- = \frac{\omega}{c^-} = \frac{\omega}{c(1-M_{1,j+1})} = \frac{k}{1-M_{1,j+1}},$$
$$Z_1^- = \frac{\varrho_0 c_1}{S_1} = \frac{\varrho_0 c}{S_1}(1-M_{1,j+1}) = Z_1(1-M_{1,j+1}).$$

Then acoustic wave propagation in the wave-guide segment (taking the reflected wave into consideration) can be expressed as follows:

$$p_1(x) = P_1^+ \cdot e^{-ik^+x} + P_1^- \cdot e^{ik^-x},$$
$$U_1(x) = \frac{1}{Z_1(1-M_{1,j+1}^2)}[(1-M_{1,j+1})P_1^+ \cdot e^{-ik^+x} - (1+M_{1,j+1})P_1^- \cdot e^{ik^-x}],$$

where: P_1^+ – amplitude of progressive wave, [Pa], P_1^- – amplitude of reflected wave, [Pa].

Including parameters at the begining and the end of a considered wave-guide segment in given above formulae, we achieve the following system of equations:

$$p_{1,j+1} = p_1(x_{j+1}), \qquad U_{1,j+1} = U_1(x_{j+1}),$$
$$p_{1\,j+1}^* = p_1(x_j), \qquad U_{1,j+1}^* = U_1(x_j).$$

When constants P_1^+ and P_1^- are eliminated and several simple algebraic conversions are performed, then formulae for elements of the transmission matrix [H] are obtained. According to the author's considerations [6] these formulae have the following form:

$$H_{11} = F_1(\cos\alpha_1 + iM_{1,j+1}\sin\alpha_1)$$
$$H_{12} = iF_1 z_1(1-M_{1,j+1}^2)\sin\alpha_1$$

$$H_{21} = i\frac{F_1}{Z_1}\sin\alpha_1$$

$$H_{22} = F_1(\cos\alpha_1 - iM_{1,j+1}\sin\alpha_1)$$

where: α_1 – phase shift, [rad],

$$\alpha_1 = \frac{k \cdot l_{1,j}}{1 - M_{1,j+1}^2},$$

k – wave number of the medium at rest ($v = 0$), [m^{-1}], $l_{1,j}$ – length of considered wave-guide segment, [m], F_1 – auxiliary function related to the motion of the medium

$$F_1 = \cos(M_{1,j+1} \cdot \alpha_1) - i\sin(M_{1,j+1} \cdot \alpha_1).$$

There is a necessity of introducing a notation in the form of a (4×4) matrix which would take parameters in both wave-guides into consideration in the muffler model (Fig. 2):

$$\begin{bmatrix} p_{1,j+1} \\ U_{1,j+1} \\ p_{2,j+1} \\ U_{2,j+1} \end{bmatrix} = [\mathbf{H}_j] \begin{bmatrix} p^*_{1,j+1} \\ U^*_{1,j+1} \\ p^*_{1,j+1} \\ U^*_{2,j+1} \end{bmatrix}. \tag{8}$$

considered segments of wave-guides 1 and 2 are not connected (this does not concern connections between acoustic elements in rows of perforations), acoustic parameters at the beginning of wave-guide 1: $p_{1,j+1}$, $U_{1,j+1}$ are independent from parameters on the end of wave-guide 2: $p^*_{2,j+1}$, $U^*_{2,j+1}$. Thus, it can be noted:

$$H_{13} = H_{14} = H_{23} = H_{24} = 0.$$

Other elements of the $[\mathbf{H}_j]$ matrix are obtained by assigning derived relationships to parameters describing acoustic wave propagation in wave-guide 2. Table 2 presents these formulae.

The transmission matrix for the whole filter (which describes the relationship between acoustic parameters in the last, $N+1$, and first row of orifices) can be noted as:

$$\begin{bmatrix} p_{1,N+1} \\ U_{1,N+1} \\ p_{2,N+1} \\ U_{2,N+1} \end{bmatrix} = [\mathbf{T}] \begin{bmatrix} p_{1,1} \\ U_{1,1} \\ p_{2,1} \\ U_{2,1} \end{bmatrix}. \tag{9}$$

Taking advantage of properties of a chain matrix, the transmission matrix for the filter can be determined from successive multiplications of transmission matrices

Table 2

element denotation	calculation formula	element denotation	calculation formula
H_{11}	$F_1(\cos\alpha_1 + iM_{1,j+1}\sin\alpha_1)$	H_{21}	$i\frac{F_1}{Z_1}\sin\alpha_1$
H_{12}	$iF_1 Z_1(1-M_{1,j+1}^2)\sin\alpha_1$	H_{22}	$F_1(\cos\alpha_1 - iM_{1,j+1}\sin\alpha_1)$
H_{13}	0	H_{23}	0
H_{14}	0	H_{24}	0
H_{31}	0	H_{41}	0
H_{32}	0	H_{42}	0
H_{33}	$F_2(\cos\alpha_2 + iM_{2,j+1}\sin\alpha_2)$	H_{43}	$i\frac{F_2}{Z_2}\sin\alpha_2$ *)
H_{34}	$iF_2 Z_2(1-M_{2,j+1}^2)\sin\alpha_2$	H_{44}	$F_2(\cos\alpha_2 - iM_{2,j+1}\sin\alpha_2)$

where:

α_1, α_2 – *phase shifts,* [rad]

$$\alpha_1 = \frac{k \cdot l_{1,j}}{1-M_{1,j+1}^2} \qquad \alpha_2 = \frac{k \cdot l_{2,j}}{1-M_{2,j+1}^2}$$

k – *wave number,* [m^{-1}]; $k = \omega/c$; $\omega = 2\pi f$

ω – *angular velocity,* [rad/s]; f – *frequency,* [Hz],

$l_{1,j}, l_{2,j}$ – *length of wave-guide segments between rows of perforations in wave-guide 1 and 2, respectively,* [m],

Z_1, Z_2 – *wave impedance in wave-guide 1 and 2, respectively,* [$Pa \cdot s/m^3$],

$M_{1,j+1}, M_{2,j+2}$ – *Mach number behind the j-row of perforations /Fig. 1 and 2/ in wave-guides 1 and 2, respectively,*

F_1, F_2 – *auxiliary functions,*

$$F_1 = \cos(M_{1,j+1}\alpha_1) - i\sin(M_{1,j+1}\alpha_1) = e^{-iM_{1,j+1}\alpha_1}$$

$$F_2 = \cos(M_{2,j+1}\alpha_2) - i\sin(M_{2,j+1}\alpha_2) = e^{-iM_{2,j+1}\alpha_2}$$

*note: Sullivan [14] introduced simplifications in his formulae which do not influence significantly calculation results for low Mach numbers (less than 0.1); besides, the formula for element H_{43} marked */ is given incorrectly.*

$[G_j]$ and $[H_j]$

$$[T] = \prod_{j=1}^{N} [H_j][G_j], \tag{10}$$

where:

$$[H_N] = [I],$$

$[I]$ – unit matrix.

It is necessary to know the value of the acoustic admittance of perforations in order to calculate dependencies given in Table 1. As it was done by SULLIVAN [15] it can be calculated from formula (13) on the basis of specific acoustic resistance Θ_0

and specific acoustic admittance χ_0 of perforations determined from investigations. If we do not have mentioned above experimental data, then these quantities be estimated from known theoretical models.

4. The determination of the admittance of a perforated surface for a discrete parameter model

The acoustic impedance of a single orifice in a perforated baffle can be determined from formulae describing the discrete parameter model of a perforated plate [18]

$$Z_d = \frac{4\varrho_0}{\pi d^2}\left\{2\sqrt{2\omega\nu}\left[\frac{h}{d}+\left(1-\frac{\pi}{4}\cdot\frac{d^2}{b^2}\right)\right]+i\omega h_{ef}\right\},^{(1)} \tag{11}$$

where $d < 0.06\cdot\lambda$; ω – angular velocity (pulsation) [rad/s]; $\omega = 2\pi f$; f – frequency, [Hz]; λ – length of acoustic wave, [m], $\lambda = c/f$; ϱ_0 – density of medium [kg/m^3]; ν – kinematic viscosity of medium, [m^2/s], in order to consider losses due to heat exchange between condensed and rarefied places in the medium it is suggested that this coefficient should be increased by 114% [9]; d – diameter of orifices [m], b – scale of perforation orifices, [m], h – thickness of perforated plate (baffle), [m]; h_{ef} – effective length of orifices of the perforation, including the mass of the medium adjoining the orifice; according to source material [18]

$$h_{ef} = h+0.85\left(1-\frac{d}{2b}\right)\cdot d.$$

Applying the above formula to the j-row of perforations and considering dimensionless specific resistance and reactance of perforations, we have

$$Z_j = Z_{0j}(\Theta_{0j}+i\chi_{0j})/\sigma_j,$$

where: Θ_{0j} – specific acoustic resistance of perforation, χ_{0j} – specific acoustic reactance of perforations.

Substituting formulae for characteristic impedance of the perforated surface Z_{0j} and for the ratio of perforation σ_j

$$Z_{0j} = \frac{\varrho_0 c}{S_{0j}} \qquad \sigma_j = \left(\frac{\pi d^2}{4}\right)/b^2,$$

in above relationships, comparing both sides of the relationships and after several algebraic conversions we reach the following notation of the specific acoustic

(1) The denotation of the coefficient of dynamic viscosity which does not satisfy dimensional relationships was given mistakenly in source materials in [18].

resistance and reactance of perforations

$$\Theta_{0j} = \frac{4\sqrt{\pi f \nu}}{c}\left[\frac{h}{d}+(1-\sigma_j^2)\right],$$

$$\chi_{0j} = \frac{2\pi f}{c} h_{ef,j}. \tag{12}$$

On the basis of considerations presented in paper [2] the author suggests that the formula including the Fok function for effective length of an orifice of the perforation should be applied

$$h_{ef,j} = h + \frac{\pi}{4\varphi(d/b_j)} \cdot d$$

where: $\varphi(d/b_j)$ — Fok function which according to the author's approximation can be expressed by the following formula

$$\varphi(d/b_j) = \frac{1.5\, d/b_j + 0.48}{0.9 - d/b_j} \quad \text{for} \quad d/b_j < 0.9,$$

$$\varphi(d/b_j) = 0 \quad \text{for} \quad d/b_j \geqslant 0.9.$$

According to author's own computational verifications, given above formulae for specific acoustic resistance and reactance of the perforation (12) present good consistence with results of empirical research [10, 13, 15] which were within author's means, within the range of applicability of the theory of propagation of acoustic waves with infinitely small amplitudes, i.e. for acoustic pressure levels practically below 120 dB.

The acoustic admittance of the j-row of perforations, A_j, can be determined from the definition of admittance as the inverse of impedance:

$$A_j = \frac{1}{Z_j} = \frac{\sigma_j}{Z_{0j}} \cdot \frac{\Theta_{0j} - i\chi_{0j}}{\Theta_{0j}^2 + \chi_{0j}^2}. \tag{13}$$

5. The influence of flow on the specific acoustic resistance of perforation orifices

On the basis of INGARD'S and ISING'S papers [10] GARRISON et al. [15] formulated a mathematical model of the specific acoustic resistance of perforation orifices at flow conditions of the medium

$$\Theta_0 = K_0 \frac{\bar{v}_0}{c},$$

where: $\bar{v}_0$ — average velocity at laminar flow of medium in the orifice, [m/s],

K_0 – numerical constant.

The above formula is valid for average flow velocities in the perforation orifice which exceed half the value of the vibration velocity amplitude of an acoustic particle in this orifice

$$\bar{v}_0 > \frac{1}{2}\cdot\hat{u}_0,$$

where $\hat{u}_0$ is the vibration velocity amplitude of an acoustic element in the perforation.

Sullivan [15] proposed the value of the numerical constant to be equal $K_0 = 2.57$. Hence, the acoustic resistance of perforations determined by the flow of the medium can be noted as

$$\Theta_0 = 2.57\cdot M_{00} = 2.57\cdot M_0/\sigma, \tag{14}$$

where: M_{00} – Mach number with the consideration of the average flow velocity in the orifice; $M_{00} = \bar{v}_0/c$, M_0 – Mach number with the consideration of the average flow velocity related to the perforated surface S_0 (as it is accepted in equations of energy and mass balances and as it is denoted in Figs. 1 and 2).

Given above expressions for the specific acoustic resistance of perforations, (12) and (14), do not include flows with small velocities in the interval

$$\bar{v}_0 \in (0, \tfrac{1}{2}\cdot\hat{u}_0).$$

The author of this paper suggests that the following approximation of the specific acoustic resistance of the perforation related to the *j*-row of perforations should be used in practical calculations:

$$\Theta_{0j} = \frac{4\sqrt{\pi f \nu}}{c}\left[\frac{h}{d}+(1-\sigma_j)\right]+2.57\frac{M_{0j}}{\sigma_j}. \tag{15}$$

Formula (15) exhibits good conformity with calculation results obtained from formula (12) for a medium at rest ($\bar{v}_0 = 0$) and from formula (14) for flow velocities in perforation orifices: $v_0 > 5\,\mathrm{m/s}$. In this last case the first term in formula (15) is by an order of magnitude smaller than the second term. The mentioned above formula gives good conformity of calculation and measurement results of insertion loss at flow velocities of the medium in perforation orifices $v_0 < 5\,\mathrm{m/s}$ (discrepancies did not exceed 4 dB).

It should be noted that presented above calculation formulae for a case of a medium in motion ($\bar{v}_0 > 0$) are valid for small orifices which satisfy the laminar flow condition

$$\mathrm{Re} = \frac{\bar{v}_0\cdot d}{\nu} < 10^3.$$

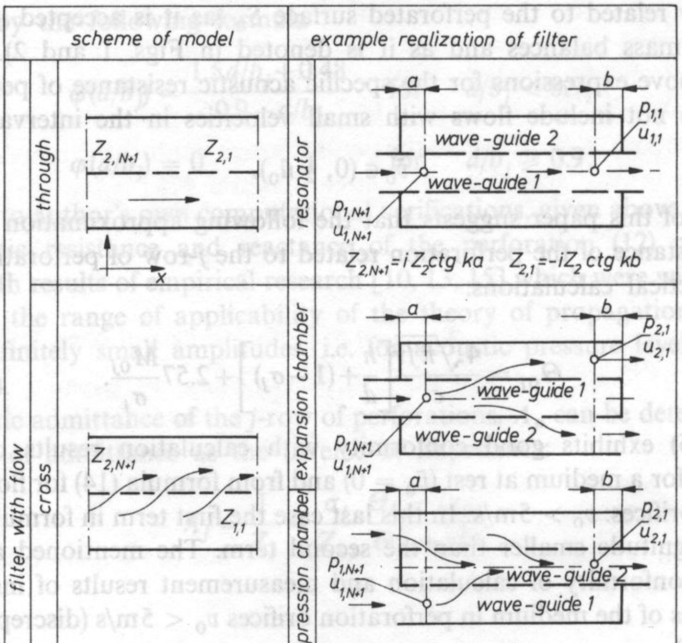
Additional experimental verification is necessary for the turbulent flow range. As for a medium at rest or for small flow velocities in perforation orifices $\bar{v}_0 < 0.5\,\mathrm{m/s}$ the condition stated for formula (11) is valid. It can be expressed as the limitation of the frequency range in terms of the diameter of the perforation orifices

$$f < 0.06c/d.$$

6. The determination of the transmission matrix for detailed solutions of filters

Table 3 presents three basic possibility of flows in the studied filter and corresponding to them special solutions of filters with a part of their channels closed with rigid acoustically impermeable baffles. In every case considered systems have only one inlet channel and one outlet channel. The impedance of a segment from the

Table 3. Transformation matrices determining the relations between external (extrinsic) parameters of model elements



baffle to the axis of the nearest row of perforation orifices can be determined in segments of closed channels.

The relationship between acoustic parameters at the inlet of the acoustic wave to the investigated muffler model and the outlet from this model can be presented in the form of a four-element transmission matrix. The conversion of the general form of the transmission matrix [T] for extreme rows of perforations in the channel, into the mentioned four-element matrix is also interesting. This problem will be considered separately for individual cases of filter solutions presented in Table 3.

6.1. *Filter with through flow*

This case can be characterised by the following form of the transmission matrix:

$$\begin{bmatrix} p_{1,N+1} \\ U_{1,N+1} \end{bmatrix} = [\mathbf{T}'] \begin{bmatrix} p_{1,1} \\ U_{1,1} \end{bmatrix}.$$

Besides, relationships

$$p_{2,N+1} = Z_{2,N+1} \cdot U_{2,N+1}, \qquad p_{2,1} = Z_{2,1} \cdot U_{2,1} \tag{17}$$

are valid. When these formulae are included in expressions for elements of the (4×4) transmission matrix [T] and parameters $p_{2,1}$ and $U_{2,1}$ are eliminated from equations, then we reach the following formulae which describe elements of a (2×2) transmission matrix [T']

$$T'_{11} = T_{11} + \frac{a \cdot b}{e}, \quad T'_{12} = T_{12} + \frac{a \cdot c}{e}, \qquad T'_{21} = T_{21} + \frac{b \cdot d}{e}, \quad T'_{22} = T_{22} + \frac{c \cdot d}{e}, \tag{18}$$

where:

$$a = T_{14} + Z_{2,1} \cdot T_{13}, \quad b = T_{31} - Z_{2,N} \cdot T_{41}, \qquad c = T_{32} - Z_{2,N} \cdot T_{42}, \quad d = T_{24} + Z_{2,1} \cdot T_{23},^{(2)} \qquad e = (T_{44} + Z_{2,1} \cdot T_{43}) Z_{2,N} - (T_{34} + Z_{2,1}\, T_{33}). \tag{19}$$

6.2. *Filter with cross flow*

The relation between parameters at the entry of an acoustic wave to the muffler model under consideration and at its exit from this model is

$$\begin{bmatrix} p_{1,N+1} \\ U_{1,N+1} \end{bmatrix} = [\mathbf{T}'] \begin{bmatrix} p_{2,1} \\ U_{2,1} \end{bmatrix}. \tag{20}$$

(2) The transmission matrix element T_{24} was mistakenly given instead of T_{23} in SULLIVAN's paper [14].

Formulae for closed segments of the channel can be written in the following form:

$$p_{2,N+1} = Z_{2,N+1} \cdot U_{2,N+1},$$
$$p_{1,1} = Z_{1,1} U_{1,1}. \tag{21}$$

The following expressions for elements of matrix [T'] are achieved as a result of transformations similar to those performed previously:

$$T'_{11} = T_{13} + \frac{ab}{e}, \quad T'_{12} = T_{14} + \frac{ac}{e},$$
$$T'_{21} = T_{23} + \frac{bd}{e}, \quad T'_{22} = T_{24} + \frac{cd}{e}, \tag{22}$$

where:

$$a = T_{12} + Z_{1,1} T_{11}, \quad b = T_{33} - Z_{2,N} T_{43},$$
$$c = T_{34} - Z_{2,N} T_{44}, \quad d = T_{22} + Z_{1,1} T_{21}, \tag{23}$$
$$e = (T_{42} + Z_{1,1} T_{41}) Z_{2,N} - (T_{32} + Z_{1,1} T_{31}).$$

6.3. Filter with reverse flow

In this case the transmission matrix has the following form:

$$\begin{bmatrix} p_{1,N+1} \\ U_{1,N+1} \end{bmatrix} = [\mathrm{T}'] \begin{bmatrix} p_{2,N+1} \\ U_{2,N+1} \end{bmatrix}, \tag{24}$$

and formulae for closed channels are:

$$p_{1,1} = Z_{1,1} U_{1,1},$$
$$p_{2,1} = Z_{2,1} U_{2,1}. \tag{25}$$

As a result of certain transformations, elements of the matrix [T'] can be noted as

$$T'_{11} = \frac{de - cf}{ad - bc}, \quad T'_{12} = \frac{af - be}{ad - bc},$$
$$T'_{21} = \frac{dg - ch}{ad - bc}, \quad T'_{22} = \frac{ah - bg}{ad - bc}, \tag{26}$$

where:

$$a = T_{32} + Z_{1,1} T_{31}, \quad b = T_{34} + Z_{2,1} T_{33},$$
$$c = T_{42} + Z_{1,1} T_{41}, \quad d = T_{44} + Z_{2,1} T_{43}, \tag{27}$$
$$e = T_{12} + Z_{1,1} T_{11}, \quad f = T_{14} + Z_{2,1} T_{13},$$
$$g = T_{22} + Z_{1,1} T_{21}, \quad h = T_{24} + Z_{2,1} T_{23}.$$

7. The determination of values of basic filter loss measures

Transmission loss TL and insertion loss IL are generally applied attenuation measures in acoustic filters. Properties, significant differences and inaccuracies of these attenuation measures are discussed in WYRZYKOWSKI's, PUCH's and SNAKOWSKI's papers [12, 16], and papers written by the author [3, 5].

7.1. *Insertion loss*

Figure 3 presents two diagrams of electric analogues which consist of substitute four-terminal networks. Their acoustic properties are described with a transmission matrix for a case before and after the insertion of the investigated filter (acoustic element or system) for which the insertion loss is to be determined. The studied filter is represented by a four-terminal network determined by a transmission matrix [**T**]. A four-terminal network with transmission matrix [**A**] substitutes the inlet system

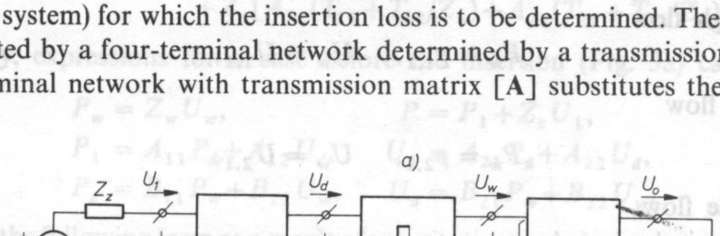


Fig. 3. Diagrams of analogue systems reflecting conditions corresponding to the definition of insertion loss a – system before insertion of filter (element of acoustic system), b – system after insertion of filter

through which the acoustic wave is supplied to the studied filter and a four-terminal network with transmission matrix [**C**] and outlet load impedance Z_0 represent the outlet system which carried away the acoustic wave. According to the definition of insertion loss, in both considered cases the inlet system and outlet system are identical. They are denoted by the same matrices, [**A**] and [**C**], and by impedance Z_0 in both diagrams (Fig. 3a and 3b).

Also a frequently encountered case of diversified cross-sections of the inlet and outlet wave-guide is included in the diagram. It is modelled in Fig. 3a by a four-terminal network. Its transmission matrix [**B**] describes the relationship for a sudden change of the cross-section, typical for expansion chamber mufflers [4].

Acoustic pressure P is the equivalent of voltage and volume acoustic velocity U is the equivalent of current in mentioned diagrams. It was also accepted that in both cases under consideration an acoustic wave with amplitude P is supplied to the inlet system and the impedance of the source of this wave is equal to Z_z. Except for special cases, it can be expected that values of other acoustic parameters on terminals of individual four-terminal networks in both diagrams will vary. Parameters of a system with an inserted filter are denoted by a star (Fig. 3b). Furthermore, parameters on input terminals of the four-terminal network which represents the studied muffler are denoted P_d, U_d, and on the outlet terminals – P_w, U_w.

With respect to the previously applied notation, the following substitution has to be taken into consideration at the entry:

$$P_d = p_{1,N+1}, \quad U_d = U_{1,N+1},$$

and on the outlet – in terms of flow direction of the acoustic wave

– through flow

$$P_w = p_{1,1}, \quad U_w = U_{1,1},$$

– cross flow

$$P_w = p_{2,1}, \quad U_w = U_{2,1},$$

– reverse flow

$$P_w = p_{2,N+1}, \quad U_w = U_{2,N+1},$$

Acoustic powers of a wave which penetrates to the outlet system correspond to active powers on input terminals of the four-terminal network characterised by transmission matrix [C]. Including notation as in Fig. 3, these powers are equal to:

– in system before insertion (Fig. 3 a)

$$N_w = \frac{1}{2}|P_w|^2 \frac{1}{\text{Re}(Z_w)},$$

– in a system after insertion of studied muffler (Fig. 3b)

$$N_w^* = \frac{1}{2}|P_w^*|^2 \frac{1}{\text{Re}(Z_w)}.$$

Outlet systems in both considered cases are identical, so their resistances are equal (real parts of impedance). Including above formulae in the definition of insertion loss, we have

$$IL = 10\lg\frac{Nw}{N_w^*} = 20\lg\left|\frac{P_w}{P_w^*}\right|. \tag{30}$$

Relationships between acoustic parameters for a case of an inserted filter (Fig. 3b) can be noted by following formulae

$$\begin{aligned} P_w^* &= Z_w U_w^*, & P &= P_1^* + Z_z U_1^*, \\ P_d^* &= T_{11} P_w^* + T_{12} U_w^*, & U_d^* &= T_{21} P_w^* + T_{22} U_w^*, \\ P_1^* &= A_{11} P_d^* + A_{12} U_d^*, & U_1^* &= A_{21} P_d^* + A_{22} U_d^*. \end{aligned} \tag{31}$$

where acoustic impedance of the outlet system is described by expression

$$Z_w = \frac{p_w}{U_w} = \frac{p_w^*}{U_w^*} = \frac{Z_0 C_{11} + C_{12}}{Z_0 C_{21} + C_{22}}. \tag{32}$$

The value of this impedance determines the value of the outlet load impedance Z_0 and the values of elements of the transmission matrix [C] of the outlet system. Substituting expressions (31) and performing several algebraic transformations, a formula for the amplitude of an acoustic wave emitted by the source after a studied filter was inserted is achieved

$$P = \{A_{11}(T_{11} + T_{12}/Z_w) + A_{12}(T_{21} + T_{22}/Z_w) + Z_z[A_{21}(T_{11} + T_{12}/Z_w) + A_{22}(T_{21} + T_{22}/Z_w)]\} P_w^*. \tag{33}$$

Similarly, expressions for a case before the insertion (Fig. 3a) can be noted

$$\begin{aligned} P_w &= Z_w U_w, & P &= P_1 + Z_z U_1, \\ P_1 &= A_{11} P_d + A_{12} U_d, & U_1 &= A_{21} P_d + A_{22} U_d, \\ P_d &= B_{11} P_w + B_{12} U_w, & U_d &= B_{21} P_w + B_{22} U_w, \end{aligned} \tag{34}$$

which have the following form as a result of substitution and algebraic transformations

$$P = \{A_{11}(B_{11} + B_{12}/Z_w) + A_{12}(B_{21} + B_{22}/Z_w) + Z_z[A_{21}(B_{11} + B_{12}/Z_w) + A_{22}(B_{21} + B_{22}/Z_w)]\} P_w. \tag{35}$$

A comparison of both sides of formulae (33) and (35), and further algebraic transformations gives an expression

$$\left|\frac{P_w}{P_w^*}\right| = \left| \frac{A_{11}\left(T_{11} + \frac{T_{12}}{Z_w}\right) + A_{12}\left(T_{21} + \frac{T_{22}}{Z_w}\right) + Z_z\left[A_{12}\left(T_{11} + \frac{T_{12}}{Z_w}\right) + A_{22}\left(T_{21} + \frac{T_{22}}{Z_w}\right)\right]}{A_{11}\left(B_{11} + \frac{B_{12}}{Z_w}\right) + A_{12}\left(B_{21} + \frac{B_{22}}{Z_w}\right) + Z_z\left[A_{12}\left(B_{11} + \frac{B_{12}}{Z_w}\right) + A_{22}\left(B_{21} + \frac{B_{22}}{Z_w}\right)\right]} \right| \tag{36}$$

According to author's derivations [4] elements of the transmission matrix for a sudden change of the cross-section, [**B**], are equal to

$$\begin{aligned} B_{11} &= 1, & B_{12} &= Z_{FC} M_C - \frac{1 - M_C^2}{1 - M_A^2} M_A Z_{FA}, \\ B_{21} &= 0, & B_{22} &= \frac{1 - M_C^2}{1 - M_A^2}, \end{aligned} \tag{37}$$

where: Z_{FA}, Z_{FC} – characteristic impedance of wave-guides at the entry and at the outlet of the investigated filter, [Pa·s/m³], M_A, M_C − Mach numbers in wave-guides at the entry and outlet of the studied filter.

As a result of the substitution of (36) and (37) in (30), an expression for insertion loss is achieved

$$IL = 20\lg\left|\frac{A_{11}\left(T_{11}+\frac{T_{12}}{Z_w}\right)+A_{12}\left(T_{21}+\frac{T_{22}}{Z_w}\right)+Z_z\left[A_{21}\left(T_{11}+\frac{T_{12}}{Z_w}\right)+A_{22}\left(T_{21}+\frac{T_{22}}{Z_w}\right)\right]}{A_{11}\left(1+\frac{B_{12}}{Z_w}\right)+A_{12}\frac{B_{22}}{Z_w}+Z_z\left[A_{21}\left(1+\frac{B_{12}}{Z_w}\right)+A_{22}\frac{B_{22}}{Z_w}\right]}\right|. \tag{38}$$

Expressions for insertion loss for chosen special cases are presented in Table 4.

Table 4

	description of model	assumptions	mathematical model of insertion loss IL
1	wave-guides identical at the entry and outlet of the muffler	$Z_{FA}=Z_{FC}$ $B_{12}=0$ $B_{22}=1$	$IL=20\lg\left\|\frac{A_{11}(T_{11}+T_{12}/Z_W)+A_{12}(T_{21}+T_{22}/Z_W)+Z_Z[A_{21}(T_{11}+T_{12}/Z_W)+A_{22}(T_{21}+T_{22}/Z_W)]}{A_{11}+A_{12}/Z_W+Z_Z(A_{21}+A_{22}/Z_W)}\right\|$
2	a reflectionless entry of an acoustic wave to the muffler	$Z_Z=Z_{FA}$ [A] - transmission matrix of wave-guide with wave impedance Z_{FA}	$IL=20\lg\left\|\frac{T_{11}+T_{12}/Z_W+Z_{FA}T_{21}+Z_{FA}T_{22}/Z_W}{1+(B_{12}+Z_{FA}B_{22})/Z_W}\right\|$
3	source with constant pressure	$Z_Z=0$	$IL=20\lg\left\|\frac{A_{11}(T_{11}+T_{12}/Z_W)+A_{12}(T_{21}+T_{22}/Z_W)}{A_{11}(1+B_{12}/Z_W)+A_{12}B_{22}/Z_W}\right\|$
4	source with constant velocity	$Z_Z\to\infty$	$IL=20\lg\left\|\frac{A_{21}(T_{11}+T_{12}/Z_W)+A_{22}(T_{21}+T_{22}/Z_W)}{A_{21}(1+B_{12}/Z_W)+A_{22}B_{22}/Z_W}\right\|$
5	reflectionless entry and outlet of an acoustic wave to and from the muffler	$Z_Z=Z_{FA}$ $Z_W=Z_{FC}$ [A], [B] - transmission matrices of wave-guides with wave impedances Z_{FA} and Z_{FC} respectively	$IL=20\lg\left\|\frac{T_{11}+T_{12}/Z_W+Z_{FA}T_{21}+Z_{FA}T_{22}/Z_{FC}}{1+(B_{12}+Z_{FA}B_{22})/Z_{FC}}\right\|$
6	reflectionless entry and outlet of anacoustic wave to and from the muffler at identical wave impedance	as in 5 and $Z_{FC}=Z_{FA}$	$IL=20\lg\left\|\frac{T_{11}+T_{12}/Z_F+Z_FT_{21}+T_{22}}{2}\right\|$

7.2. *Transmission loss*

The model presented in Fig. 3b can be applied in calculations of this loss measure on the assumption that the output of the acoustic wave from the four-terminal network representing the studied filter is reflexionless. This condition can be noted as

$$Z_w = Z_{FC}$$

Acoustic powers necessary in further calculations are derived from the following expressions:

– for a wave inciding onto studied filter

$$^{+}N_d^* = \frac{1}{2}\cdot\frac{|^{+}P_d^*|^2}{Z_{FA}}$$

– for a wave trasmitted from the filter to the outlet system (to a reflexionless outlet wave-guide)

$$N_w^* = \frac{1}{2}\frac{|P_w^*|^2}{Z_{FC}}$$

where $^{+}P_d$ denotes the pressure of an acoustic wave which incides into the studied filter.

When given above expressions are substituted in the formula for transmission loss, the following formula is obtained

$$TL = 10\lg\frac{^{+}N_d^*}{N_w^*} = 20\lg\frac{|^{+}P_d^*|}{|P_w^*|} + 10\lg\frac{Z_{FC}}{Z_{FA}}. \tag{39}$$

Using formulae (31) and following relationships between parameters at the entry to the four-terminal network representing the studied filter

$$P_d^* = {}^{+}P_d^* + {}^{-}P_d^*, \quad U_d^* = \frac{^{+}P_d^* - P_d^*}{Z_{FA}}, \tag{40}$$

where $^{+}P_d$, P_d – acoustic pressures at progressive and reflected wave, respectively, at the entry of the four-terminal network representing the studied filter, the following expression is reached

$$\frac{|^{+}P_d^*|}{|P_w^*|} = \left|\frac{T_{11} + \frac{1}{Z_{FC}}T_{12} + Z_{FA}T_{21} + \frac{Z_{FA}}{Z_{FC}}T_{22}}{2}\right|$$

as a result of substitutions and algebraic transformations. Substituting the above expression in formula (40) we achieve the following expression for transmission loss

$$TL = 20\lg\left|\frac{T_{11} + \frac{1}{Z_{FC}}T_{12} + Z_{FA}T_{21} + \frac{Z_{FA}}{Z_{FC}}T_{22}}{2}\right| + 10\lg\frac{Z_{FC}}{Z_{FA}}. \tag{41}$$

8. Conclusions

This paper presents a calculation model of a low-pass acoustic filter with a perforated baffle which separates two channels with constant cross-sections at the assumptions for the propagation conditions of a plane wave with infinitely small amplitude. The model was built on the basis of the segmentation principle for repeated segments of channels between individual rows of perforations. Acoustic properties of these segments were described with a transmission matrix on the basis of a discrete parameter model of the perforation orifice, on the assumption of a laminar flow through these orifices. A distributed parameter model describes wave-guide segments.

Properties of the whole filter can be described with a transmission matrix as a result of successive multiplications of transmission matrices of individual segments. Properties of the chain matrix were utilized as well as detailed mathematical model of the basic flow directions of an acoustic wave through the filter.

The presented above description of properties of discussed filters is very convenient for further computer processing and determination of acoustic characteristics of these filters. Also numerical values of both generally applied measures – transmission loss TL and insertion loss IL – can be calculated when entry and source impedances are known. Calculations can be performed on widely applied personal computers.

Unfortunately the application of presented calculation models is limited to small orifices of the perforation, e.g. orifices which satisfy conditions of a discrete parameter model and laminar flow of the medium through these orifices. Further studies are aimed at the elimination of these restrictions.

The condition of plane wave propagation is another limitation. Much more complicated calculation methods which would include spatial propagation of an acoustic wave (such as the method of finite elements) are necessary in order to remove this limitation.

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