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REFLECTION AND TRANSMISION OF A BLEUSTEIN-GULAYEV SURFACE WAVE BY THE EDGE OF A PIEZOELECTRIC MATERIAL

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The paper presents an analysis of the effect of reflection and transmission of a BLEUSTEIN-GULAYEV transverse surface wave by an edge of a piezoelectric area formed by two mechanically free and electrically shorted planes intersecting at an angle θ. The fundamental properties of the *B-G* waves have been discussed, as well as the possibilities of their application. A method of calculating the coefficients of reflection, A_R, and transmission, A_T, for a *B-G* wave by a metallized piezoelectric edge of a 6 mm symmetry, has been presented schematically. The values of coefficients $|A_R|^2$ and $|A_T|^2$ were measured in $LiJO_3$ crystal samples. Suggestions concerning further research on the analyzed effect have been inducted in the conclusions.

Introduction

The former research has shown, that transverse surface waves do not exist in homogeneous elastic materials [1]. BLEUSTEIN'S discovery in 1968 [2] and independantly GULAYEV's discovery of transverse surface waves in homogeneous piezoelectric materials was a certain surprise to the scientists. BLEUSTEIN-GULAYEV type surface waves can propagate on a mechanically free surface of a piezoelectric, which has a two-fold axis of symmetry [4]. LOVE type surface transverse waves have one non-vanishing component of the mechanical displacement and can propagate in elastic materials having a non-homogeneous subsurface layer [5]. As opposed to them BLEUSTEIN-GULAYEV waves, except for the transverse component of the mechanical displacement U_2 (Fig. 1), have an electric potential φ, induced by the piezoelectric effect of the foundation [9]. For this reason BLEUSTEIN-GULAYEV waves are called acoustoelectric waves. In agreement with the state of research, the BLEUSTEIN-GULAYEV waves do not exhibit dispersion, what has a significance in the measurements of their velocities with the application of the PAPADAKIS' reflection method [6]. In the last years a hypothesis was put forward, stating that the position of a metallic layer on the surface of a piezoelectric causes the formation of a subsurface intermediate layer with decreased piezo-

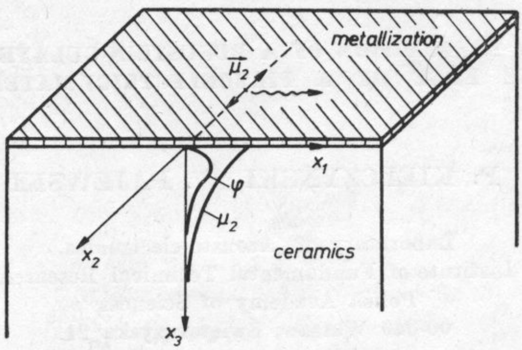

Fig. 1. Mechanical displacement U_2 and electric potential of a BLEUSTEIN-GULAYEV wave propagating on a metallized surface of a piezoelectric with a diad axis $\parallel x_2$

electric properties [7]. If this hypothesis would prove itself true, this would radically change the understanding of the essence of the BLEUSTEIN-GULAYEV waves, which in this case should be dispersive and exhibit a multimodal structure. Hitherto existing research results do not solve this problem finally [7], [8].

The research of effect of reflection and transmission of a BLEUSTEIN-GULAYEV wave by an edge is of great theoretical significance, because up to now there an accurate method of solving this problem does not exist [10], [11]. Furthermore the knowledge of the coefficient of reflection and transmission of a *B-G* wave by an edge is of fundamental significance in the construction of acoustoelectric devices for analogue processing of telecommunication signals: delay lines [12] and broad-band resonators [13]. Taking advantage of the invertibility of the diffraction effects on the investigaled edge, a new method of generating *B-G* waves with the aid of volume transverse SH waves falling from the inside of the medium onto the studied edge, was given in paper [14].

This paper is concerned with the theoretical and experimental study of the effect of reflection and transmission of a *B-G* type transverse surface wave by a metallized egde of a piezoelectric with a 6 mm, 4 mm symmetry, or of piezoelectric ceramics. This effect was also investigated experimentally for piezoelectric crystals with a 6 symmetry [15].

Theory

The piezoelectric material covers an area limited by two planes: $x_3 = 0$ and $\xi_3 = 0$, intersecting under an angle Θ (Fig. 2). The two intersecting planes are unbounded mechanically free and electrictrically shorted (infinitely thin layer of an perfect conductor). A *B-G* wave propagating on surface $x_3 = 0$ in the $+x_1$ direction encounters a strong geometrical discontinuity of the surface – the

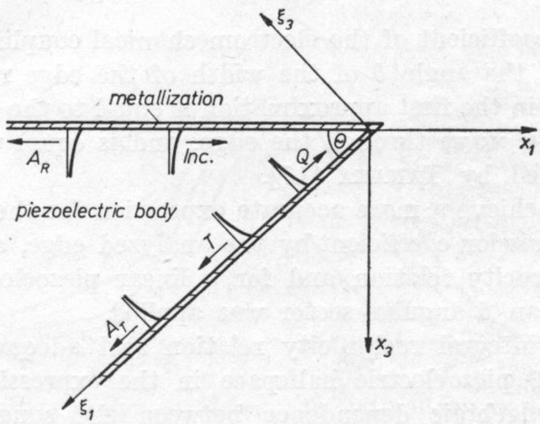

Fig. 2. Schematic diagram of an incident (i), reflected (r), transmitted (t) and produced by auxiliany sources (T, Q) Bleustein-Gulayev wave in an edge area of an θ angle of flare

edge area of an angle θ. Therefore, two *B-G* waves appear: reflected, propagating in direction $-x_1$, and passing through, propagating in direction $+\xi_1$, on the surfaces limiting the edge region. The rest of the incident wave energy is changed into a volume transverse wave radiated into the material in the angular sector Θ.

Further on the method will be presented of calculating the coefficient of reflection and transmission of a *B-G* type plane wave by the edge of a piezoelectric. This method is presented in detail in paper [15].

A *B-G* wave, incident on the investigated region edge, fulfills the (zero) boundary conditions for the stress tensor component τ_{23} and electric potential φ, only on the guiding surface $x_3 = 0$. Components τ_{23} and φ of the incident wave are not zeroed on the reflecting surface $\xi_3 = 0$. In order to fulfil the zero boundary conditions on surface $\xi_3 = 0$, an assumption was done, that on this surface auxiliary sources of stress and electric potential act, which together with τ_{23} and φ of the incident wave satisfy the zero boundary conditions on this surface. Introduced auxiliary sources generate on surface $\xi_3 = 0$ two *B-G* type plane waves with amplitudes T and Q, propagating in directions $+\xi_1$ and $-\xi_1$, respectively. Amplitudes T and Q (Lamb problem for a piezoelectric halfspace $\xi_3 \geqslant 0$), calculated with the application of the methods of the FOURIER analytical functions and transformations [26], [16], are expressed by the following formulae:

$$T = \frac{k_{15}^2}{k_{15}^2 + j\,\mathrm{ctg}(\theta/2)}, \tag{1}$$

$$Q = j\frac{k_{15}^2}{1+k_{15}^2}\,\mathrm{ctg}(\theta/2), \tag{2}$$

where k_{15} is the coefficient of the electromechanical coupling of the piezoelectric material and the angle θ of the width of the edge region.

Amplitude T in the first approximation is equal to the of coefficient transmission of the B-G wave through the edge, and is equal to the transmission coefficient obtained by TANAKA [17].

In order to achieve a more accurate expression for the B-G wave for reflection and transmission coefficient by the analyzed edge, a double integrated form of the reciprocity relation and for a linear piezoelectric material [18] in the region of an θ angular sector was applied.

Placing the integral reciprocity relation and adequate Green function [19] for a $x_3 \geqslant 0$ piezoelectric halfspace in the expression, we obtain the following linear algebraic dependence between the sought reflection, A_R, and transmission, A_T, coefficients:

$$A_R = R + Q(T + A_T), \tag{3}$$

where R is the reflection coefficient in the first approximation and is expressed by the following formula:

$$R = \frac{1}{2}\,\frac{k_{15}^2}{k_{15}^2 + j\,\mathrm{ctg}\,\theta}\,\frac{\sin\theta + jk_{15}^2\cos\theta}{\sin\theta - jk_{15}^2\cos\theta}. \tag{4}$$

The application of the reciprocity relation and the GREEN function for the $\xi_3 \geqslant 0$ piezoelectric halfspace, leads to another linear algebraic dependence between coefficients A_R and A_T

$$A_T = T + Q(R + A_R). \tag{5}$$

Solving a system of two algebraic equations, (4) and (5), in relation to A_R and A_T, we reach the final form of the expressions of the coefficients of reflection and transmission of a B-G wave by a metallized edge of a piezoelectric:

$$A_R = \frac{1+Q^2}{1-Q^2}R + \frac{2Q}{1-Q^2}T, \tag{6}$$

$$A_T = \frac{1+Q^2}{1-Q^2}T + \frac{2Q}{1-Q^2}R, \tag{7}$$

where amplitudes T, Q and R are expressed by formulas (1), (2) and (4), respectively.

The functional discussion of the dependence of the reflection A_R, and transmission, A_T, coefficients on the electromechanical coupling coefficient k_{15} and the θ angle, on the basis of expressions (6) and (7) is rather inconvenient. To this

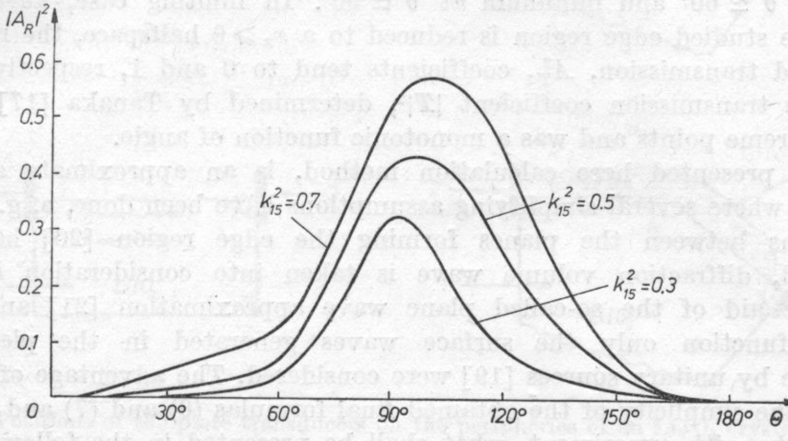

Fig. 3. Calculated reflection energy coefficient $|A_R|^2$ of a Bleustein-Gulayev wave as a function of angle θ. Electromechanical coupling coefficient k_{15} is a parameter

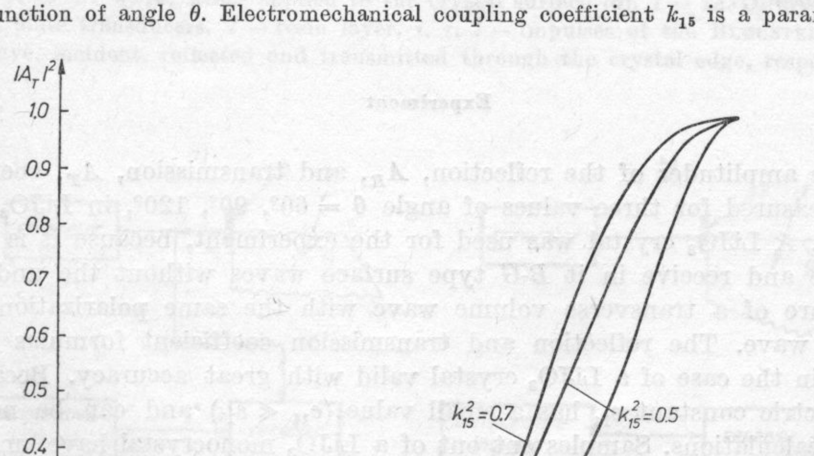

Fig. 4. Calculated transmission energy coefficient $|A_T|^2$ of a Bleustein-Gulayev wave as a function of angle θ. Electromechanical coupling coefficient k_{15} is a parameter

end, expressions (6) and (7) were tabularized with the aid of a computer and the calculation results are presented in Fig. 3 and 4. These figures show, that with the increase of the electromechanical coupling coefficient k_{15}, the values of the energetic coefficients of reflection, $|A_R|^2$, and transmission, $|A_T|^2$, increase for the whole θ angle range. Furthermore, coefficient $|A_T|^2$ has a maximum at $\theta \simeq 90°$, while coefficient $|A_T|^2$ has two extreme values, i. e. maximum at $\theta \simeq 60°$ and minimum at $\theta \simeq 90°$. In limiting case, at $\theta \simeq 180°$, when the studied edge region is reduced to a $x_3 \geqslant 0$ halfspace, the reflection, $|A_R|^2$ and transmission, A_T^2, coefficients tend to 0 and 1, respectively. The energetic transmission coefficient $|T|^2$, determined by Tanaka [17] did not have extreme points and was a monotonic function of angle.

The presented here calculation method, is an approximate analytical method, where several simplifying assumptions have been done, e. g. multiple reflections between the planes forming the edge region [20] have been neglected, diffraction volume wave is taken into consideration indirectly with the aid of the so-called plane wave approximation [21] and in the GREEN function only the surface waves generated in the piezoelectric halfspace by unitary sources [19] were considered. The adventage of this method is the simplicity of the obtained final formulas (6) and (7) and the good conformity with experiment, what shall be presented in the following parts of the paper.

Experiment

The amplitudes of the reflection, A_R, and transmission, A_T, coefficients were measured for three values of angle $\theta = 60°, 90°, 120°$, in $LiJO_3$ crystal samples. A $LiJO_3$ crystal was used for the experiment, because it is easy to generate and receive in it *B-G* type surface waves without the undesirable admixture of a transverse volume wave with the same polarization as the surface wave. The reflection and transmission coefficient formulas (6) and (7) are in the case of a $LiJO_3$ crystal valid with great accuracy. Because its piezoelectric constant e_{14} has a small value ($e_{14} \ll e_{15}$) and can be neglected in the calculations. Samples cut out of a $LiJO_3$ monocrystal have undergone mechanical treatment (grinding) and were vaccum evaporated with an Al ~ 0.5 μm layer on the side surfaces. The radius of curvature of the edge region was observed to equal ~ 70 μm, so it was many times smaller than the length of the surface wave used in the experiment: ~ 600 μm for $f \simeq 4$ MHz. To generate and defect the *B-G* waves plate ceramic transducers were used, bounded to the crystal with epoxy resin. Fig. 5 presents the position of the plate transducers on the pheripheries of the $LiJO_3$ crystal during the measurements of the pulse amplitudes of the incident (i), reflected (r) and

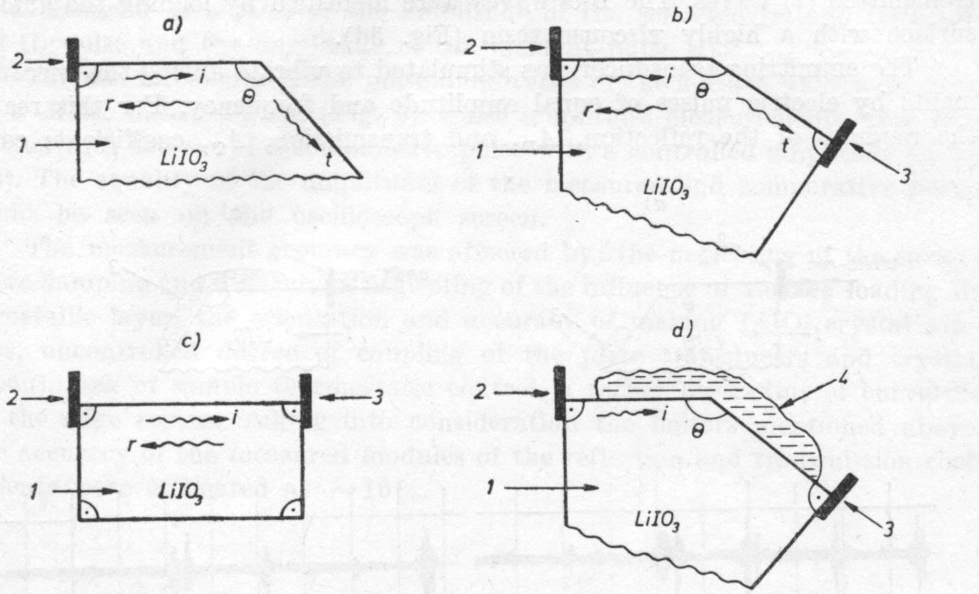

Fig. 5. Positions of the plate transducers on the peripheries of an $LiJO_3$ crystal during the measurements of the pulse amplitude of the *a*) reflected, *b*) transmitted, *c*) incident BLEUSTEIN-GULAYEV wave. Resin applied to the crystal surface (*d*). *1* — $LiJO_3$ crystal sample, *2*, *3* — plate transducers, *4* — resin layer, *i*, *r*, *t* — impulses of the BLEUSTEIN-GULAYEV wave, incident, reflected and transmitted through the crystal edge, respectively

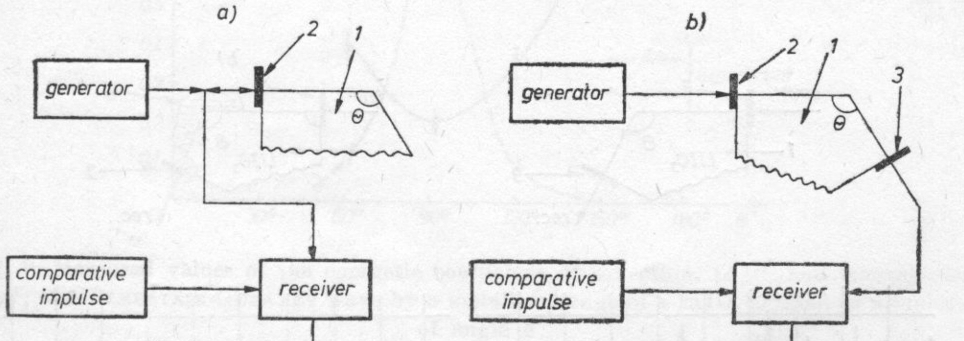

Fig. 6. Measuring set for determining the pulse amplitude of an reflected (*a*) transmitted (*b*) and incident (*c*) on the edge of the crystal BLEUSTEIN-GULAYEV wave. *1* — crystal sample, *2* — emmitting-receinving transducer, *3* — receiving transducer

transmitted (t) waves. The *B-G* waves were identified by loading the guiding surface with a highly viscouse resin (Fig. 5d).

The emmitting transducer was stimulated to vibrate during the measurements by electric pulses of equal amplitude and frequency. For this reason the modules of the reflection, A_R, and transmission, A_T, coefficients could

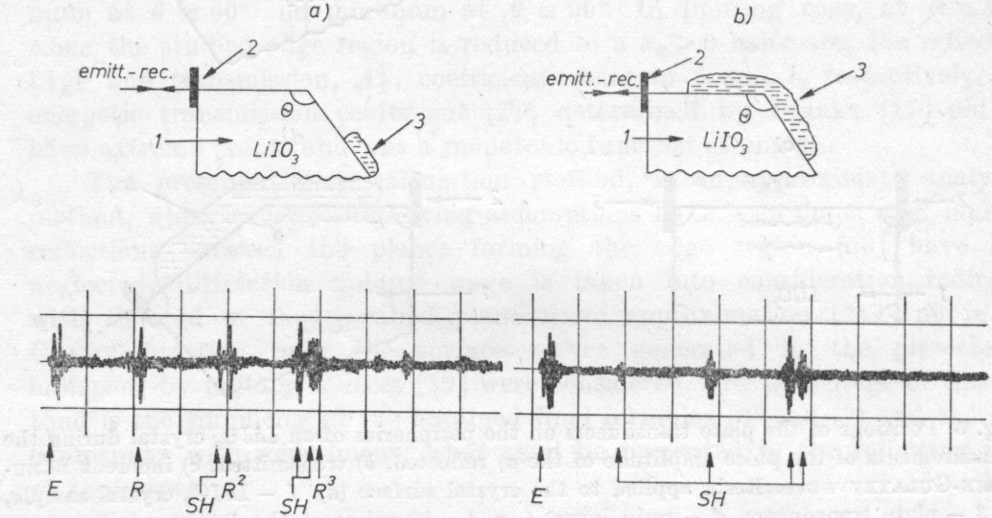

Fig. 7. Oscillograms of impulses of a BLEUSTEIN-GULYEV wave reflected from the edge of a $LiJO_3$ crystal, for an angle $\theta = 90°$. The crystal surface with resin (b) and without resin (a). *1* – $LiJO_3$ crystal sample, *2* – emmitting-receiving transducer, *3* – resin layer

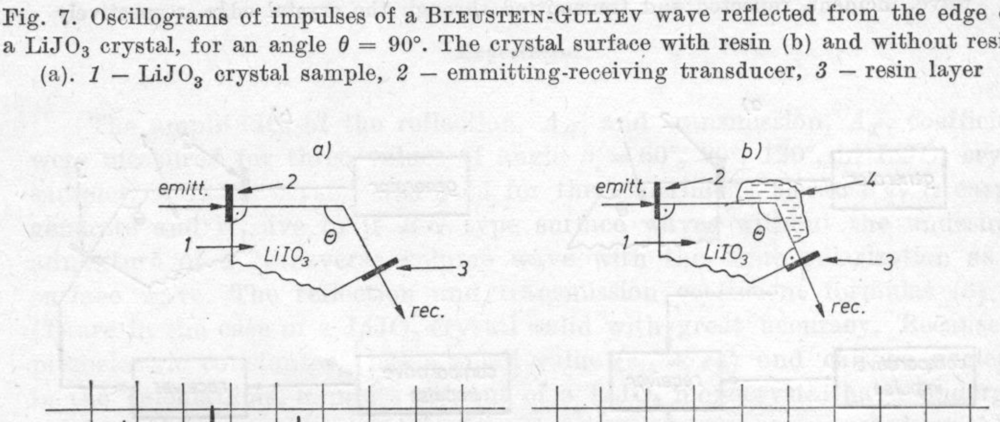

Fig. 8. Oscillograms of impulses of a BLEUSTEIN-GULAYEV wave transmitted by a $LiJO_3$ crystal edge. The crystal surface with out resin (a) and with resin (b). $\theta = 90°$

be determined as a ratio of the amplitude of the reflected (r) and transmitted (t) pulse and the amplitude of the incident pulse (i).

The measurements of the pulse amplitudes of the surface wave were done on a *Matec* measuring set (Fig. 6). Pulse amplitude measurements were conducted with the aid of a comparative pulse with a controlled amplitude (± 0.2 dB). The equality of the amplitudes of the measured and comparative pulses could be seen on the oscilloscope screen.

The measurement accuracy was affected by: the neglecting of the surface wave damping and diffraction, neglecting of the influence of surface loading by a metallic layer, the orientation and accuracy of making $LiJO_3$ crystal samples, uncontrolled degree of coupling of the plate transducers and crystal, (resin), lack of sample thermostatic control, a rather big radius of curvature of the edge region. Taking into consideration the factors mentioned above, the accuracy of the measured modules of the reflection and transmission coefficients were estimated at $\sim 10\%$.

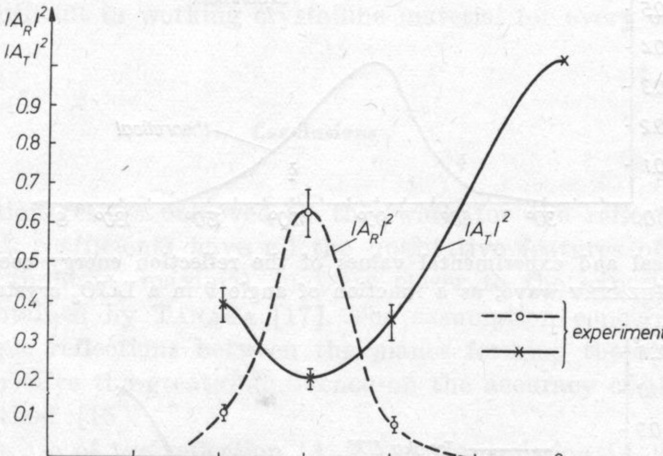

Fig. 9. Measured values of the energetic coefficient of reflection, $|A_R|^2$, and transmission, $|A_T|^2$, of a BLEUSTEIN-GULAYEV wave by a metallized edge of a $LiJO_3$ crystal, as a function of angle θ

Fig. 7 and 8 present exemplary oscillograms obtained during the amplitude measurements of the pulses of a wave reflected and transmitted by a metallized edge of a $LiJO_3$ crystal for an angle of $\theta = 90°$. Individual pulses on the oscillograms were identified as pulses of a *B-G* wave multiply passing or reflected by the crystal edge. For example, the pulse marked with symbol r^2t in Fig. 8 is a pulse which after passing through the investigated edge (t), was reflected twice from it (r^2) before reaching the receiving transducer. Symbol SH marks the existing in the studied structure pulses of a transverse volume wave.

Fig. 9 shows the values of the reflection $|A_R|^2$ and transmission $|A_T|^2$ energy coefficients, measured in the described measuring set-up.

Comparison of Theoretical and Experimental Results

The maximum of the measured reflection energy coefficient, $|A_R|^2$, was localized near the angle $\theta \simeq 90°$ (Fig. 10), what is in accordance with the theoretical expectations expressed by equation (6). It is worth noting, that at an angle of $\theta \simeq 90°$ a minimum of the reflection coefficient occurs for RAYLEIGH type surface waves [22]. This can be explained by the fact, that RAY-

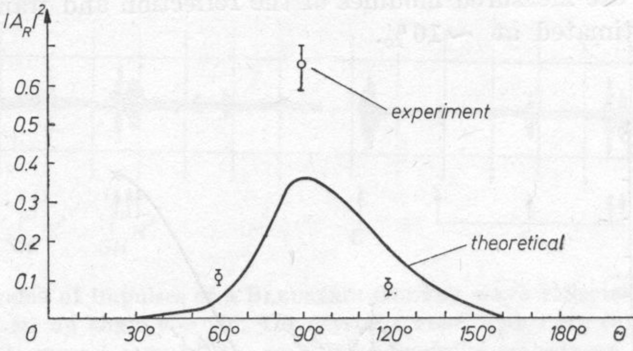

Fig. 10. Theoretical and experimental values of the reflection energy coefficient, $|A_R|^2$ of a BLEUSTEIN-GULAYEV wave, as a function of angle θ in a $LiJO_3$ crystal ($k_{15}^2 = 0.38$)

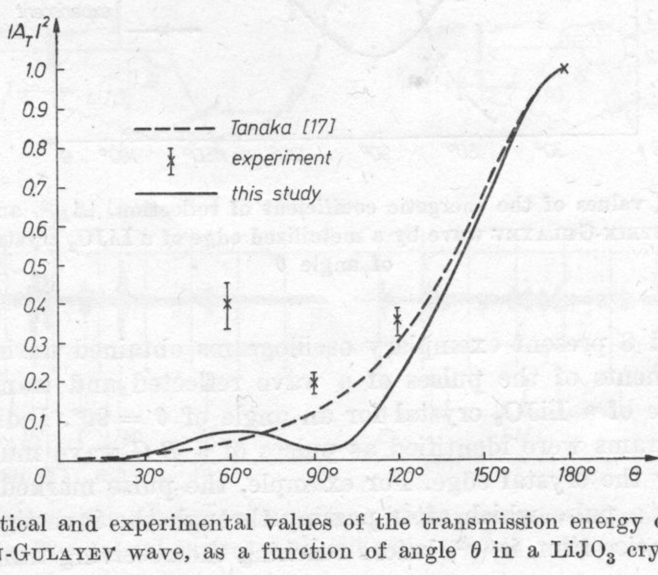

Fig. 11. Theoretical and experimental values of the transmission energy coefficient, $|A_T|^2$, of a BLEUSTEIN-GULAYEV wave, as a function of angle θ in a $LiJO_3$ crystal ($k_{15}^2 = 0.38$)

LEIGH waves are not transverse waves, but have two displacement components: SV and L.

The measured transmission energy coefficient $|A_T|^2$ has a minimum at an angle of $\theta \simeq 90°$ and a maximum at an angle of $\theta \simeq 60°$. The latter can be easily explained on the basis of simple geometrical reasoning for a wave reflected twice between the planes inclined toward each other at an angle of 60°. The profile of the measured transmission energy coefficient is in accordance with formula (7) (Fig. 11).

It is difficult to explain the discrepancies between the calculation and experiment results, due to the low accuracy of the experiment and the imperfection of the calculation method [15].

In order to fully experimentally presented in this paper verify the calculation method, the measurements of the reflection and transmission coefficients should be done for a greater amount of θ angle values, especially in its low value range ($\leqslant 60°$). But carrying out measurements for a large number of the θ angle values is very labour-consuming, because a mechanically seperate sample of considerably big dimensions (~ 3 cm) would have to be made from a difficult in working crystalline material for every case.

Conclusions

1. Calculation results achieved in this work for the reflection A_R^2 and transmission A_T^2 coefficients have all the qualitative features of the measurement results (minima, maxima), and are closer to the experiment results than results obtained by TANAKA [17]. The assumption concerning the absence of multiple reflections between the planes forming the analyzed edge region seems to have the greatest influence on the accuracy of the presented calculation method [15].

2. The increase of the reflection $|A_R|^2$ and transmission $|A_T|^2$ coefficients occuring with the increase of the electromechanical coupling coefficient, found in the work, can be an argument for the existance of a transient layer near the metallized piezoelectric surface.

3. Considering the only qualitative conformity of the calculation results, obtained with the application of existing methods, with the experiment results, a method of integral equations [23] and nummerical methods (finite element [24], boundary element [25]) can be applied in future work on the analyzed effect. These methods allow us to achieve a solution of essentially arbitrary accuracy.

4. A relatively high value of the B-G wave of coefficient reflection, $|A_R|^2$, from a rectangular edge region, enables the construction of a Fabry-Perrot resonator of a quality factor of about 5000, working in a widefrequency band.

Summarizing we have to state, that in order to sget to know fully the complicated reflection and transmission effect of a *B-G* wave by an edge, further theoretical and experimental studies have to be done.

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