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A PRACTICAL PROBABILISTIC PREDICTION OF ROAD TRAFFIC NOISE FROM A FILTERED POISSON PROCESS MODEL WITH A SIMPLIFIED ELEMENTARY TIME PATTERN OF TRIANGULAR TYPE

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To date, many theoretical methods for predicting the statistics of road traffic noise have been proposed involving the introduction of vehicle distribution models, such as an equally spaced model, an exponentially distributed model, or an Erlang distribution type model. In such cases, very often, the sound propagation characteristic was first restricted to an idealized case like a free sound field, and then sometimes extended to an arbitrary sound propagation environment. Needless to say, however, it was too difficult to predict systematically the probability distribution of road traffic noise fluctuation at an observation point under the actual sound propagation environment with the complex diffraction and/or attenuation effects.

Thus, this paper is devoted above all to consideration of a practical probabilistic method of prediction of the statistics of road traffic noise by use of a filtered Poisson process model with a simplified elementary time pattern.

The effectiveness of the proposed simple method is experimentally verified too, by applying it to the actual road traffic noise data observed in a large city.

1. Introduction

When the problem of statistical prediction of road traffic noise is theoretically considered, it is essentially important to pay our attention to the actual situation of road traffic flow and its surrounding sound propagation characteristic. The former characteristic can be grasped rather easily as several types of statistical traffic flow models, such as an exponentially distributed vehicle model, an Erlang or a gamma headway distribution type model, etc. [2, 4, 6, 7, 11]. On the other hand, it is very difficult to identify the actual system characteristic of the surrounding sound propagation environment with the sound diffraction and/or attenuation effects. In fact, in most of the previous papers, the sound propagation characteristic was restricted to an idealized case like a free sound field.

For purpose of generalizing these theoretical studies, several kinds of prediction approaches, applicable to an arbitrary sound propagation environment, have been proposed by introducing the well-known filtered Poisson process model and the Stratonovich's random point system model, based on the energy composition of component elementary time patterns [8, 9, 12]. It is essentially too difficult to predict the probability distribution of the road traffic noise fluctuation under an actual sound field, owing to the difficulty of identifying the surrounding sound propagation environment.

From the above practical points of view, in this paper, a practical method of prediction of the level probability distribution form of the road traffic noise is theoretically proposed by use of the filtered Poisson process model with a simplified elementary time pattern. The effectiveness of the present prediction theory is experimentally confirmed too, by applying it to the actual data of road traffic noise level fluctuation observed in a large city. The experimental results show fair agreement with the theory, in spite the of introduction of an extremely practical approximation of the elementary time pattern.

2. Theoretical considerations

2.1. *Cumulant statistics of sound intensity fluctuation*

In the problems of evaluation and/or regulation of road traffic noise, the statistics such as L_x sound levels (like median, L_5 and L_{10}), defined as a $(100-X)$ percentage point of the level probability distribution, as well as the lower order statistics like L_{eq}, are very often used. Accordingly, it is first fundamental to find an explicit expression for the noise level distribution function. In this section, let us consider the relationship between the elementary time pattern due to one vehicle passing and the various cumulant statistics of the total sound intensity fluctuation due to many vehicles passing.

Now, let us consider the road traffic noise shown in Fig. 1, and introduce the following assumptions [7]:

1) the road considered here has J lanes of an arbitrary length of straight or curved lines;

2) the traffic on the jth lane flows with a constant speed v_j;

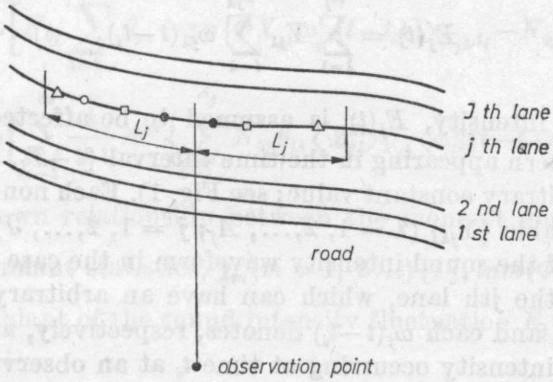

Fig. 1. Traffic flow of random point sources on a multi-lane road 0, •, □, △ – vehicle types

3) the traffic flowing on the jth lane is formed by Λ_j different types of vehicles;

4) the average number of vehicles flowing on the jth lane is N_j per unit time interval, where the number of the λth type of vehicles is $N_{j\lambda}(\lambda = 1, 2, \ldots, \Lambda_j)$;

5) n_j is a random variable expressing the number of vehicles in the arbitrary line segment $[-L_j, L_j]$ of the jth lane, and its probability function is of the usually used Poisson distribution type, i.e.

$$P(n_j) = \frac{1}{n_j!} \exp(-N_{0j}) N_{0j}^{n_j}, \quad \left(N_{0j} \triangleq N_j \frac{2L_j}{v_j}\right); \tag{1}$$

6) the probability that $n_{j\lambda}$ vehicles of the λth type appear in the above n_j vehicles is governed by the following multi-nomial distribution:

$$P(n_{j1}, n_{j2}, \ldots, n_{j\Lambda_j} | n_j) = \frac{n_j}{n_{j1}!\, n_{j2}!\, n_{j\Lambda j}!} \theta_{j1}^{n_{j1}} \theta_{j2}^{n_{j2}} \ldots \theta_{j\Lambda_j}^{n_{j\Lambda_j}}$$

$$\left(\sum_{\lambda=1}^{\Lambda_j} n_j = n_j \text{ for all } j\right), \tag{2}$$

where $\theta_j = N_{j\lambda}/N_j \left(\sum_{\lambda=1}^{j} \theta_{j\lambda} = 1 \text{ for all } j\right)$ denotes the mixture ratio of the λth type of vehicles in the jth lane of road;

7) the sound intensity fluctuation emitted from each source is statistically uncorrelated with that from every other source.

Now, let us introduce the filtered Poisson process as one of the stochastic time series models for the sound intensity wave, $E_j(t)$, at an observation point, due to the traffic flowing only on the jth lane, as follows:

$$E_j(t) = \sum_{\lambda=1}^{\Lambda_j} Y_{j\lambda} \sum_{i=1}^{n_{j\lambda}} \omega_{j\lambda}(t-t_i), \tag{3}$$

where the sound intensity, $E_j(t)$, is assumed to be affected only by the elementary time pattern appearing in the time interval $[t-T_j,\, t+T_j]$ ($T_j \overset{\Delta}{=} L_j/v_j$, where L_j is an arbitrary constant value; see Fig. 1). Each non-negative independent random variable $\{Y_{j\lambda}\}$ ($\lambda = 1, 2,\ldots, \Lambda_j$; $j = 1, 2,\ldots, J$), reflects the random peak value of the sound intensity waveform in the case when the λth type vehicle flows on the jth lane, which can have an arbitrary type probability distribution form, and each $\omega_j(t-t_i)$ denotes, respectively, an elementary time pattern of sound intensity occurring at time t_i at an observation point, which is standardized so that the maximum value of the elementary time pattern is equal to one. Furthermore, $\{t_i\}$ are mutually independent random time points with the uniform distribution function $P(t_i) = 1/2T_j$ (for all i).

The moment generating function of $E_j(t)$ is given as follows:

$$\begin{aligned} m_j(\Phi) \overset{\Delta}{=} \langle \exp[\Phi E_j(t)] \rangle &= \Big\langle\Big\langle\Big\langle \exp\Big\{\Phi \sum_{\lambda=1}^{\Lambda_j} Y_{j\lambda} \sum_{i=1}^{n_{j\lambda}} \omega_{j\lambda}(t-t_i)\Big\}\Big\rangle_{Y_{j\lambda},t_i|n_{j\lambda},n_j}\Big\rangle_{n_{j\lambda}|n_j}\Big\rangle_{n_j} \\ &= \Big\langle\Big\langle \prod_{\lambda=1}^{\Lambda_j} \langle \exp\{\Phi Y_{j\lambda}\, \omega_{j\lambda}(t-t_i)\}\rangle^{n_{j\lambda}}_{Y_{j\lambda},t_i|n_{j\lambda},n_j}\Big\rangle_{n_{j\lambda}|n_j}\Big\rangle_{n_j}, \end{aligned} \tag{4}$$

where $\langle\cdot\rangle_u$ denotes an averaging operation with respect to its subindex, u, and $\langle\cdot\rangle_{u|v}$ a conditional averaging operation with respect to u when v is set at a constant value.

By using equation (2), the following expression can be easily derived:

$$\begin{aligned} &\Big\langle\Big\langle \prod_{\lambda=1}^{\Lambda_j} \exp\{\Phi Y_{j\lambda}\, \omega_{j\lambda}(t-t_i)\}\Big\rangle^{n_{j\lambda}}_{Y_{j\lambda},t_i|n_{j\lambda},n_j}\Big\rangle_{n_{j\lambda}|n_j} \\ &= \sum_{n_{j1},n_{j2},\ldots,n_{j\Lambda_j}} \frac{n_{j1}}{n_{j1}!\, n_{j2}!\ldots n_{j\Lambda_j}!} [\theta_{j1}\langle \exp\{\Phi Y_{j1}\, \omega_j(t-t_i)\}\rangle_{Y_{j1},t_i|n_j}]^{n_{j1}} \times \\ &\quad \times [\theta_{j\Lambda_j}\langle \exp\{\Phi Y_{j\Lambda_j}\, \omega_{j\Lambda_j}(t-t_i)\}\rangle_{Y_{j\Lambda|j},t_i|n_j}]_{n_{j\Lambda_j}} \\ &= \Big[\sum_{\lambda=1}^{\Lambda_j} \theta_{j\lambda}\langle \exp\{\Phi Y_{j\lambda}\, \omega_{j\lambda}(t-t_i)\}\rangle_{Y_{j\lambda},t_i}\Big]^{n_j}. \end{aligned} \tag{5}$$

Thus, using equation (1), the moment generating function can be explicitly derived as follows:

$$m_j(\Phi) = \sum_{n_j=0} \frac{1}{n_j!} \exp(-N_{0j}) \Big[N_{0j} \sum_{\lambda=1}^{\Lambda_j} \theta_{j\lambda} \langle \exp\{\Phi Y_{j\lambda} \omega_{j\lambda}(t-t_i)\}\rangle_{Y_{j\lambda},t_i} \Big]^{n_j}$$

$$= \exp\Big[N_{0j} \sum_{\lambda=1}^{\Lambda_j} \theta_{j\lambda} \langle \exp\{\Phi Y_{j\lambda} \omega_{j\lambda}(t-t_i)\}\rangle_{Y_{j\lambda},t_i} - N_{0j} \Big]$$

$$= \exp\Big[\sum_{m=1}^{\infty} \frac{1}{m!} \Phi^m \Big\{ \sum_{\lambda=1}^{\Lambda_j} N_{0j} \theta_{j\lambda} \langle Y_{j\lambda}^m \rangle_{Y_{j\lambda}} \langle \omega_{j\lambda}^m (t-t_i)\rangle_{t_i} \Big\} \Big]. \tag{6}$$

Using the well-known relationship between the moment generating function $m(\Phi)$, and the cumulant statistics, $\chi_m (m = 1, 2 \ldots)$ [1]; $ln m(\Phi) = \sum_{m=1}^{\infty} \Phi^m \chi_m / m!$, the mth order cumulant of the sound intensity fluctuation $E_j(t)$ can be directly derived as follows:

$$\chi_{mE_j} = \sum_{\lambda=1}^{\Lambda_j} N_{0j} \theta_{j\lambda} \langle Y_{j\lambda}^m \rangle Y_{j\lambda} \langle \omega_{j\lambda}^m (t-t_j)\rangle_{t_i}$$

$$= \sum_{\lambda=1}^{\Lambda_j} N_{0j} \theta_{j\lambda} \langle Y_{j\lambda}^m \rangle Y_{j\lambda} \frac{1}{2T_j} \int_{-T_j}^{T_j} \omega_{j\lambda}^m (\tau)\, d\tau. \quad (7$$

Thus, the mth cumulant of the total sound intensity $E(t) = \sum_{j=i}^{J} E_j(t)$, due to the traffic flowing on a road having J lanes of infinite length ($L_j \to \infty$; for all j), can be easily obtained as:

$$\chi_{mE} = \sum_{j=1}^{J} \chi m E_j = \sum_{j=1}^{J} \sum_{\lambda=1}^{\Lambda_j} N_{j\lambda} \langle Y_{j\lambda}^m \rangle \int_{-\infty}^{\infty} \omega_{j\lambda}^m (\tau)\, d\tau, \tag{8}$$

where

$$N_{j\lambda} \overset{\Delta}{=} \lim_{T_j \to \infty} \frac{1}{2T_j} N_{0j} \theta_{j\lambda} = N_j \theta_{j\lambda} \tag{9}$$

is the average number of the λth type vehicles flowing on the jth lane per unit time interval (see assumption (4) and equation (1)).

2.2 Cumulant statistics of sound level fluctuation

The moment generating function of the sound level fluctuation defined by

$$L = M \ln(E/E_0) \qquad \Big(M \overset{\Delta}{=} \frac{10}{\ln 10}, \quad E_0 = 10^{-12} W/\mathrm{m}^2 \Big), \tag{10}$$

can be obtained as follows:

$$m_L(\Phi) \overset{\Delta}{=} \langle \exp(\Phi L) \rangle_L = \left\langle \left(\frac{E}{E_0} \right)^{\Phi M} \right\rangle_E . \tag{11}$$

Using the previous relationship between the moment generating function and the cumulant statistics, and letting $\Phi M = m$ in equation (11), the following equality can be obtained:

$$\sum_{i=1}^{\infty} \frac{1}{i!} \left(\frac{m}{M} \right)^i \chi_{iL} = \ln \langle E^m \rangle - m \ln E_0 \quad (m = 1, 2 \ldots), \tag{12}$$

here, χ_{iL} denotes the ith order cumulant of the sound level L.

Equation (12) can be concretely written as

$$\begin{aligned}
\ln\langle E\rangle - \ln E_0 &= \frac{1}{M}\chi_{1L} + \frac{1}{2!}\left(\frac{1}{M}\right)^2 \chi_{2L} + \frac{1}{3!}\left(\frac{1}{M}\right)^3 \chi_{3L} + \frac{1}{4!}\left(\frac{1}{M}\right)^4 \chi_{4L} + \ldots, \\
\ln\langle E^2\rangle - 2\ln E_0 &= \frac{2}{M}\chi_{1L} + \frac{1}{2!}\left(\frac{2}{M}\right)^2 \chi_{2L} + \frac{1}{3!}\left(\frac{2}{M}\right)^3 \chi_{3L} + \frac{1}{4!}\left(\frac{2}{M}\right)^4 \chi_{4L} + \ldots, \\
\ln\langle E^3\rangle - 3\ln E_0 &= \frac{3}{M}\chi_{1L} + \frac{1}{2!}\left(\frac{3}{M}\right)^2 \chi_{2L} + \frac{1}{3!}\left(\frac{3}{M}\right)^3 \chi_{3L} + \frac{1}{4!}\left(\frac{3}{M}\right)^4 \chi_{4L} + \ldots, \\
\ln\langle E^4\rangle - 4\ln E_0 &= \frac{4}{M}\chi_{1L} + \frac{1}{2!}\left(\frac{4}{M}\right)^2 \chi_{2L} + \frac{1}{3!}\left(\frac{4}{M}\right)^3 \chi_{3L} + \frac{1}{4!}\left(\frac{4}{M}\right)^4 \chi_{4L} + \ldots.
\end{aligned} \tag{13}$$

By solving these linear simultaneous equations, the cumulant statistics $\chi_{nL}(n = 1, 2, \ldots)$ of sound level can be obtained from the statistical information $\langle E^n \rangle$ on the moment statistics of sound intensity. Furthermore, as is well-known, these moment statistics $\langle E^n \rangle$ of sound intensity can be easily calculated from the cumulant statistics χ_{nE} of sound intensity as follows:

$$\begin{aligned}
&\langle E \rangle = \chi_{1E}, \quad \langle E^2 \rangle = \chi_{2E} + \langle E \rangle^2, \quad \langle E^3 \rangle = \chi_{3E} + 3\langle E \rangle \langle E^2 \rangle - 2\langle E \rangle^3, \\
&\langle E^4 \rangle = \chi_{4E} + 4\langle E \rangle \langle E^3 \rangle + 3\langle E^2 \rangle - 12\langle E \rangle^2 \langle E^2 \rangle + 6\langle E \rangle^4, \ldots.
\end{aligned} \tag{14}$$

2.9. A simplified expression of sound level distribution for practical use

In this section, in view of the establishment of a practical evaluation method, a simplified expression of the probability distribution is first introduced, as follows (its theoretical background is shown in [10]):

$$P(L) = \frac{1}{\sqrt{2\pi\chi_{2L}}} \exp\left\{ -\frac{(L-\chi_{1L})^2}{2\chi_{2L}} \right\} \text{(Gaussian distribution)}. \tag{15}$$

Accordingly, it is sufficient to solve only two lower order cumulants χ_{1L} and χ_{2L} in equation (13).

2.4. *A simplification of the elementary time pattern*

In the practical evaluation procedure, the following points must be noticed:

1) In order to calculate concretely $\chi_{mE}(m = 1, 2, \ldots)$ by use of equation (8), it is first necessary to obtain the statistical or deterministic information on $N_{j\lambda}$, $Y_{j\lambda}$ and $\omega_{j\lambda}$ $(\lambda = 1, 2, \ldots, \Lambda_j;\ j = 1, 2, \ldots, J)$. As is mentioned in the introduction, the statistical properties of $N_{j\lambda}$ and $Y_{j\lambda}$ can be grasped rather easily from the usual theoretical and/or experimental considerations. On the contrary, it is not so easy to systematically evaluate $\omega_{j\lambda}(\tau)$ under an actual sound propagation environment.

2) It is not necessary to estimate $\omega_{j\lambda}(\tau)$ itself accurately, but it is quite enough to obtain only the whole value of the definite integral $\int_{-\infty}^{\infty} \omega_{j\lambda}^{m}(\tau)\,d\tau$, for the purpose of determining $\chi_{mE}(m = 1, 2, \ldots)$. Moreover, only the total sum for each cumulant statistics in each lane and vehicle type case is useful for the above probabilistic evaluation. That is, the values of the cumulant statistics of sound intensity, χ_{mE}, are not influenced sensitively by the instantaneous waveform itself of $\omega_{j\lambda}(\tau)$, owing to the above smoothing operation effect.

Thus, the probabilistic evaluation index like L_x for the sound level fluctuation, is not influenced sensitively by the instantaneous waveform of the individual time pattern $\omega_{j\lambda}(\tau)$. Based on this fundamental viewpoint, it is possible to find the reason why the elementary time pattern can be extremely simplified.

First, let us consider the standard case when the same type vehicles flow on a single lane road $(J = \Lambda = 1)$ under a free sound field. At this time, χ_{mE} can be exactly expressed as follows (see equation (8)):

$$\chi_{mE} = N_{11}\langle Y_{11}^{m}\rangle \int_{-\infty}^{\infty} \omega_{11}^{mm}(\tau)\,d\tau = N_{11}\langle Y_{11}^{m}\rangle \int_{-\infty}^{\infty} \frac{1}{(1+\beta\tau^2)^m}\,d\tau$$

$$= N_{11}\langle Y_{11}^{m}\rangle \frac{\pi}{\sqrt{\beta}} A_m, \tag{16}$$

with

$$A_m = \frac{(2m-3)!!}{(2m-2)!!}, \tag{17}$$

where β is a certain constant value and $(*)!!$ is defined as

$$(2m)!! = 2m(2m-2)\ldots 4\cdot 2,$$
$$(2m-1)!! = (2m-1)(2m-3)\ldots 5\cdot 3\cdot 1,$$
$$0!! = (-1)!! = 1. \tag{18}$$

Hereupon, let us consider an extremely simplified case when this original elementary waveform on dB scale is approximated by a triangular waveform, as shown in Fig. 2, under the condition of equivalence of the average sound intensity. That is, the mth order cumulant $\chi_{mE}(\Delta)$ of a triangular wave-

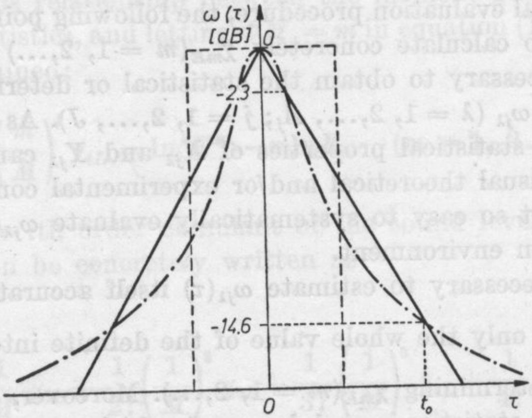

Fig. 2. Relationship between an original elementary waveform and its simplified triangular time pattern for sound level. Original waveform: ——·——, simplified triangular time pattern: ——

form corresponding to χ_{mE}, can be easily derived under the condition $\chi_{1E}(\Delta) = \chi_{1E}$, as follows:

$$\chi_{mE}(\Delta) = N_{11}\langle Y_{11}^m\rangle \int_{-\infty}^{\infty} \exp\left[-m\frac{2}{\pi}\sqrt{\beta}\,|\tau|\right]d\tau$$

$$= N_{11}\langle Y_{11}^m\rangle \frac{\pi}{\sqrt{\beta}} B_m = \frac{B_m}{A_m}\chi_{mE}, \tag{19}$$

with

$$B_m = \frac{1}{m}. \tag{20}$$

If the original elementary waveform is approximated in trial by a square waveform, the corresponding $\chi_{mE}(\square)$ can be directly derived as follows:

$$\chi_{mE}(\square) = N_{11}\langle Y_{11}^m\rangle \int_{-\pi/2\sqrt{\beta}}^{\pi/2\sqrt{\beta}} 1d\tau = N_{11}\langle Y_{11}^m\rangle \frac{\pi}{\sqrt{\beta}} C_m, \quad C_m = 1. \tag{21}$$

Table 1 shows a comparison between A_m and B_m for several values of m (of course, $\chi_{mE}(\Delta) \simeq_{mE}$ if $B_m \simeq A_m$). It is very interesting to note that the value

Table 1. A comparison between A_m and B_m for several values of m

m	A_m	B_m	C_m
1	1.000	1.000	1.0
2	0.500 (= 1/2)	0.500 (= 1/2)	1.0
3	0.375 (= 3/8)	0.333 (= 3/9)	1.0
4	0.313 (= 5/16)	0.250 (= 4/16)	1.0
5	0.273 (= 35/128)	0.200 (= 25/125)	1.0

of χ_{2E} (i.e. the variance of sound intensity) is equal to that of $\chi_{2E}(\Delta)$. From this table, it is obvious that $\chi_{mE}(m = 1\text{–}5)$ for an original waveform can be successfully approximated by $\chi_{mE}(\Delta)$ for a simplified triangular waveform and that it cannot be approximated by $\chi_{mE}(\square)$ for a simplified square waveform.

3. Experimental considerations

3.1 Outline of experiment

The actual location of the road, bank and two observation points is shown in Fig. 3. The road considered here has two lanes. The received random noise fluctuation waves at the two observation points O_1 and O_2 were coincidently put on record by use of a data recorder. Table 2 gives the observed values of road traffic flow in every observation time interval.

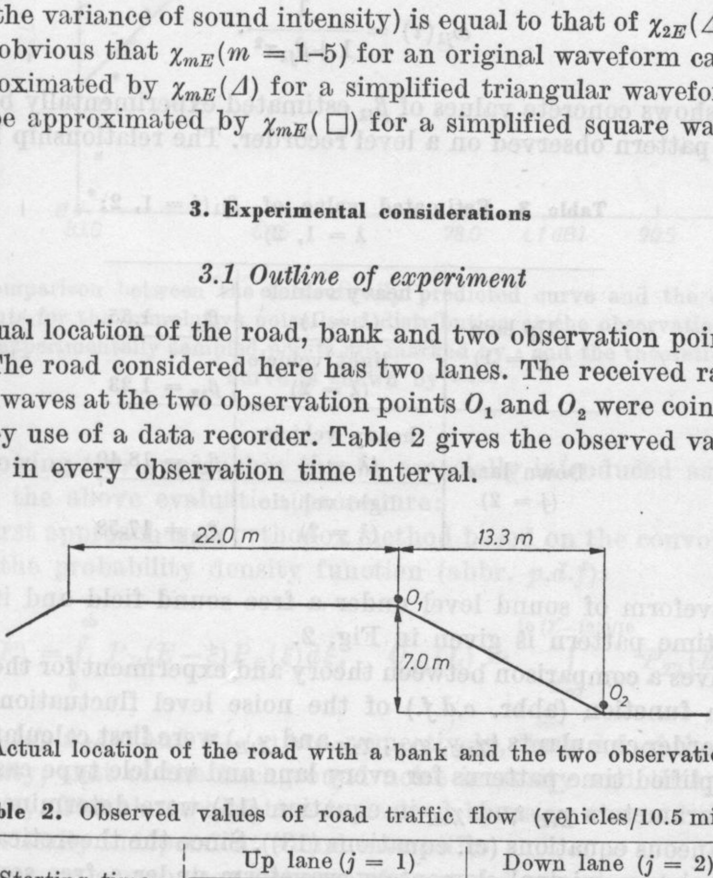

Fig. 3. Actual location of the road with a bank and the two observation points

Table 2. Observed values of road traffic flow (vehicles/10.5 min.)

Starting time point of observation	Up lane ($j = 1$)		Down lane ($j = 2$)	
	have vehicle ($\lambda = 1$)	light vehicle ($\lambda = 2$)	heavy vehicle ($\lambda = 1$)	light vehicle ($\lambda = 2$)
13.10 *PM*	16	21	12	9
13.30	13	13	13	11
14.00	7	13	12	8
14.30	8	16	12	7
15.00	12	8	8	12
Average	11.2	14.2	11.4	9.4

3.2. *Results of experiments*

a) *Noise level probability distribution at the observation point* O_1

The sound propagation environment between an individual vehicle on the road and the observation point O_1 can be regarded as a free sound field. At this time, the time pattern of sound intensity, $\omega_{j\lambda}(\tau)$, in equation (8) can be given as follows ($\lambda=1, 2$; $j=1, 2$; see Table 2):

$$\omega_{j\lambda}(\tau)=\frac{1}{1+\beta_{j\lambda}\tau^2}. \tag{22}$$

Table 3 shows concrete values of $\beta_{j\lambda}$ estimated experimentally by use of the actual time pattern observed on a level recorder. The relationship between the original waveform of sound level under a free sound field and its simplified triangular time pattern is given in Fig. 2.

Table 3. Estimated value of $\beta_{j\lambda}$ ($j=1, 2$; $\lambda=1, 2$)

Up lane ($j=1$)	heavy vehicle ($\lambda=1$)	$\beta_{11}=1.55$
	light vehicle ($\lambda=2$)	$\beta_{12}=1.23$
Down lane ($j=2$)	heavy vehicle ($\lambda=1$)	$\beta_{21}=18.40$
	light vehicle ($\lambda=2$)	$\beta_{22}=17.58$

Fig. 4 gives a comparison between theory and experiment for the cumulative distribution function (abbr. *c.d.f.*) of the noise level fluctuation. Hereupon, dour lower order cumulants (χ_{1E}, χ_{2E}, χ_{3E} and χ_{4E}) were first calculated by introfucing simplified time patterns for every lane and vehicle type case, and then, only two parameters χ_{1L} and χ_{2L} in equation (15) were determined by solving the simultaneous equations (cf. equations (13)). Since the theoretical *c.d.f.* curve predicted from an original elementary waveform under a free sound field (cf. equation (22)) agrees with that from a simplified triangular waveform, this curve was omitted here. As is directly found in this figure, the effect of the background noise on the resultant traffic noise level distribution form cannot be neglected in a specific case with a light traffic flow (cf. Table 2). In order to increase the accuracy of prediction of the noise level probability distribution, the above background noise generated independently by the other different noise sources should be taken into consideration for the above noise evaluation method.

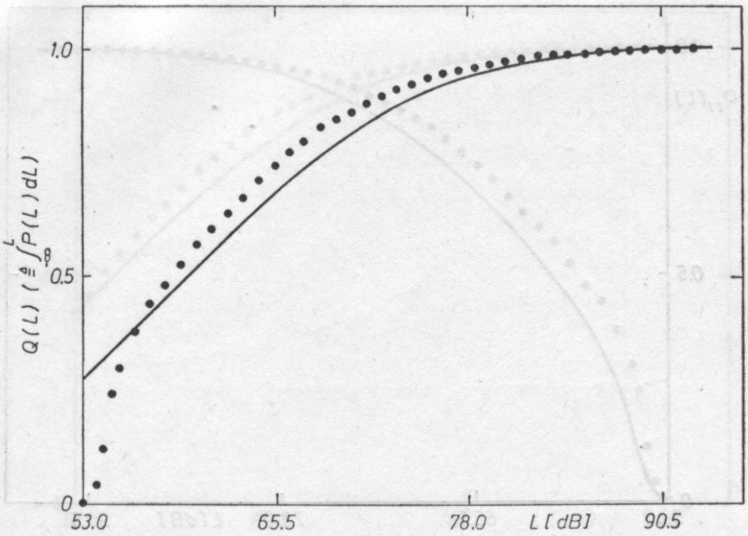

Fig. 4. A comparison between the theoretically predicted curve and the experimentally sampled points for the cumulative noise level distribution at the observation point O_1 (cf. Fig. 3). The experimentally sampled points are marked by · and the theoretically predicted curve is shown by ——

The following two approaches can be especially introduced as an attempt to improve the above evaluation procedure:

1) the first approach is an orthodox method based on the convolution of the ntegral of the probability density function (abbr. *p.d.f*):

$$P_{T1}(E) = \int_0^\infty P_R(E-\xi)P_B(\xi)\,d\xi, \quad Q_{T1}(L) = \int_{-\infty}^{10^{(L-120)/10}} P_{T1}(E)\,dE. \tag{23}$$

Here, $P_R(*)$, $P_B(*)$ and $P_{T1}(*)$ are, respectively, the *p.d.f.* of the road traffic noise intensity, that of the background noise intensity and that of the total noise intensity. The above equation is derived by using the additive property of two statistically independent sound intensities.

Fig. 5 shows a comparison between theory and experiment in the form of the sound level distribution. The theoretically predicted curve was calculated by using equation (23) (the lognormal distribution was employed as $P_R(*)$ in equation (23); see equation (15)). It is fairly troublesome to obtain the finite integral of equation (23), even if this method is quite orthodox.

2) the second approach is a practical method to avoid the above trouble of calculating the finite integral.

If the noise level probability distribution restricted only in the level range $[L_0,\ \infty]$ (L_0 — an arbitrary constant value) is considered, the following expres-

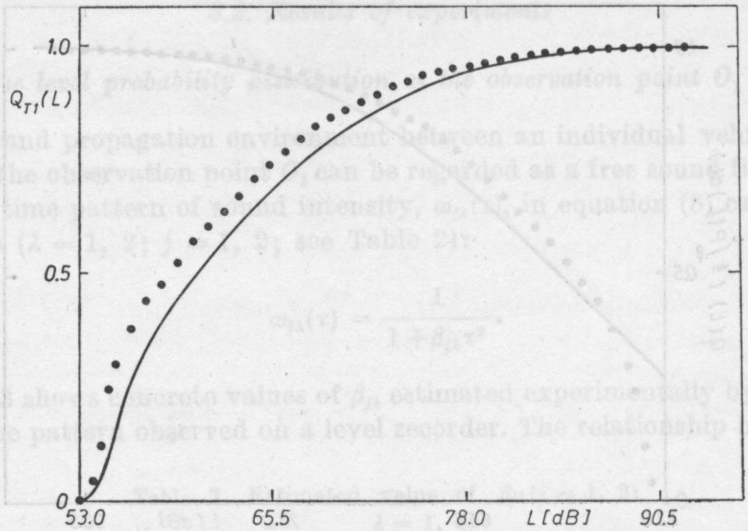

Fig. 5. A comparison between the theoretically predicted curve and the experimentally sampled points for the cumulative noise level distribution at the observation point O_1. The experimentally sampled points are marked by · and the theoretical curve predicted by use of equation (23) is shown by ——

sion derived on the basis of the fundamental property of a conditional probability must be employed:

$$Q_{T2}(L) = Q_{\mathrm{EX}}(L_0) + [1 - Q_{\mathrm{EX}}(L_0)]\frac{Q_T(L) - Q_T(L_0)}{1 - Q_T(L_0)} \quad (L \geqslant L_0). \tag{24}$$

Here, $Q_{\mathrm{EX}}(L_0)$ denotes the experimental *c.d.f* at a level point L_0 and $Q_T(*)$ is defined as

$$Q_T(*) = \int_{-\infty}^{*} P(L)\,dL \quad (* = L \text{ or } L_0). \tag{25}$$

The *c.d.f.* curve predicted theoretically by use of equations (24), (15) and (25) and the experimentally sampled points are compared in Fig. 6.

From Figs. 5 and 6, it is obvious that the above two evaluation methods show a fairly good agreement with the experimental results.

b) Noise level probability distribution at the observation point O_2

The elementary time pattern of sound intensity, $\omega_{j\lambda}(\tau)$, at the observation point O_2 can be approximated as follows:

$$\omega_{j\lambda}(\tau) = \frac{1}{1+\beta_{j\lambda}\tau^2}\,\frac{a_j^2 + (v_j\tau)^2}{b_j^2 + (v_j\tau)^2}\,10^{-\Delta L_j(\tau;f_0)}, \tag{26}$$

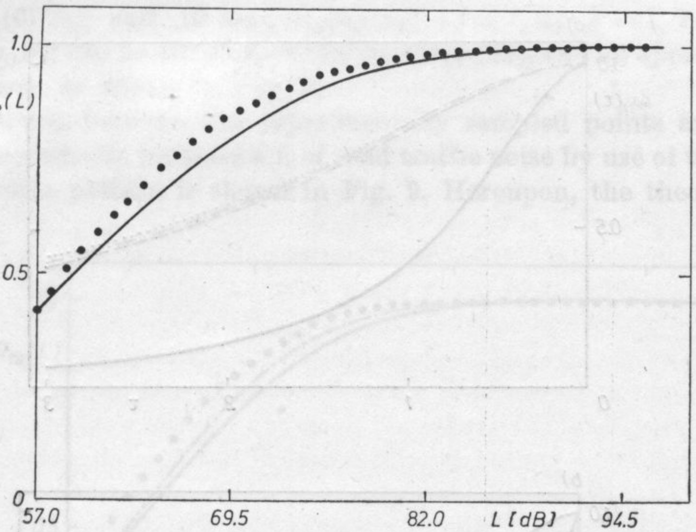

Fig. 6. A comparison between the theoretically predicted curve and the experimentally sampled points for the cumulative noise level distribution at the observation point O_1. The experimentally sampled points are marked by · and the theoretical curve predicted by use of equations (24) and (15) is shown by ——

where the values of $\beta_{j\lambda}(j = 1, 2; \lambda = 1, 2)$ are given in Table 3. Furthermore, a_j and b_j are the shortest distances between the jth lane in the road and the observation points O_1 and O_2 respectively (see Fig. 7), and $\Delta L_j(\tau; f_0)$ denotes the sound attenuation which can be easily calculated by use of an acoustical evaluation chart of the barrier, based on a value of the Fresnel number $N_j(\tau; f_0)$ [3, 5]:

$$N_j(\tau; f_0) = 2\delta_j(\tau) f_0/c. \tag{27}$$

Here, f_0 denotes the representative frequency in the power spectrum of an actual road traffic noise and c is the speed of sound. Furthermore, $\delta_j(\tau)$ is the difference $((\overline{SP} + \overline{PO_2}) - \overline{SO_2})$ of the sound propagation path length which can be determined by using the location of a vehicle on the jth lane at the time τ and the observation point O_2. Fig. 8 shows the normalized elementary time

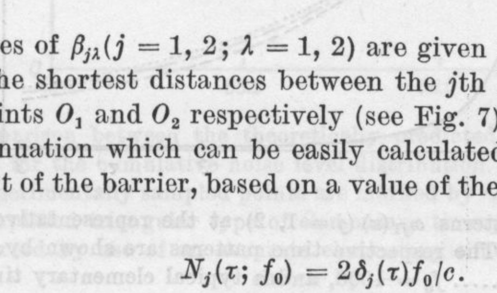

Fig. 7. Location of the jth lane and the two observation points O_1 and O_2

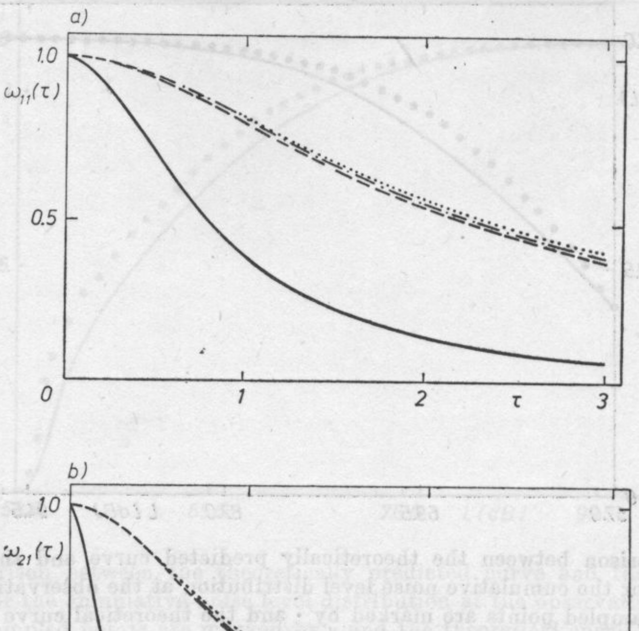

Fig. 8. Normalized time patterns $\omega_{j1}(\tau)$ $(j = 1,\ 2)$ at the representative frequency point $f_0 = 500$, 700 and 1000 Hz. The respective time patterns are shown by: — — — — $f_0 = 500$, ——·—— $f_0 = 700$, ... $f_0 = 1000$, and a typical elementary time pattern in the idealized case with a free sound field is simultaneously shown by ——. *a)* $\omega_{11}(\tau)$ $(j = 1,\ \lambda = 1)$; *b)* $\omega_{21}(\tau)$ $(j = 2,\ \lambda = 1)$

patterns $\omega_{j1}(\tau)$ ($j = 1, 2$; i.e. heavy vehicle) at $f_0 = 500$, 700 and 1000 Hz, calculated from equation (26). The normalized elementary time pattern in an idealized case with free sound field (cf. equation (22)) is simultaneously shown in this figure.

Let us predict theoretically the traffic noise level probability distribution form by using these elementary time patterns. It is not so easy, however, to determine the above elementary time pattern by use of equation (26). Accordingly, it is an effective way to introduce an extremely simplified type of elementary time pattern. That is, a triangular type elementary waveform was introduced as a simplified time pattern for the sound level fluctuation at the observation point O_2, which was determined especially by using two values;

$10 \log_{10}[\omega_{j\lambda}(0)/E_0]$ and $10 \log_{10}[\omega_{j\lambda}(t_0)/E_0]$ ($j=1, 2$; $\lambda=1, 2$). Hereupon, $\omega_{j\lambda}(0)$ and $\omega_{j\lambda}(t_0)$ can be calculated from equation (26), and an appropriate value of t_0 is chosen, as shown in Fig. 2.

A comparison between the experimentally sampled points and the curve predicted theoretically for the *c.d.f.* of road traffic noise by use of this simplified elementary time pattern is shown in Fig. 9. Hereupon, the theoretical curve

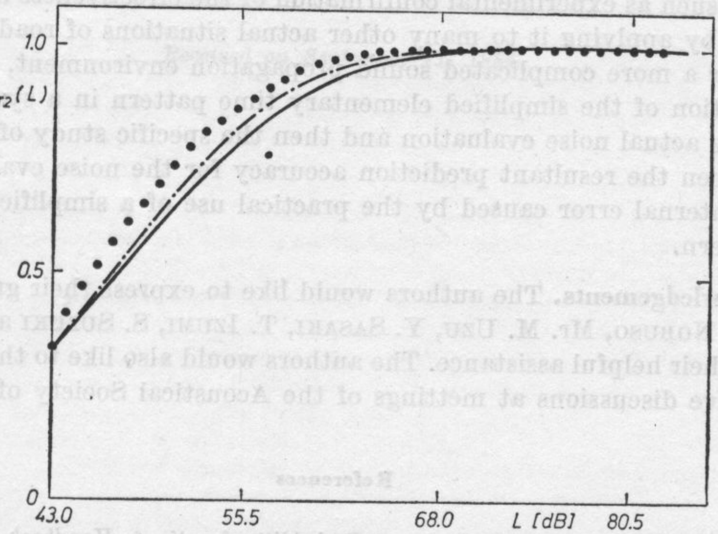

Fig. 9. A comparison between the theoretically predicted curve and the experimentally sampled points for the cumulative noise level distribution at the observation point O_2 (cf. Fig. 3). The experimentally sampled points are marked by ·. The theoretical curve predicted by use of a simplified triangular type of elementary time pattern is shown by —·— and that predicted by use of an accurate elementary time pattern (see equation (26)) is shown by ——

was calculated from equations (15) and (24) ($L_0 = 43$ dB). The theoretical curve calculated by using equation (26) is simultaneously shown in this figure. From this figure, it is obvious that its prediction accuracy can be hopefully increased by improving the degree of the approximation of the elementary time pattern. But the prediction method based on a simplified triangular type elementary time pattern, considered here, is still an effective way for the practical usage.

4. Conclusion

In this paper, especially from the practical point of view of evaluation of the actual sound propagation environment, a method of prediction of the level *p.d.f.* for the actual road trafficno isefluctuation was first proposed in a hybrid form of theory and experiment by use of a filtered Poisson process model with a simplified elementary time pattern of triangular type. Then, the effectiveness of the proposed evaluation method was experimentally confirmed by applying it to the actual road traffic noise data observed in a large city. Such a practical prediction method based on a simplified triangular type elementary time pattern is still at an early stage of study. Therefore, the present study was mainly focussed on its methodological viewpoint. There still remain many types of future problems, such as experimental confirmation of the effectiveness of the method proposed, by applying it to many other actual situations of road traffic noise data under a more complicated sound propagation environment, the optimum determination of the simplified elementary time pattern in a systematic relation to the actual noise evaluation and then the specific study of the relationship between the resultant prediction accuracy for the noise evaluation index and the internal error caused by the practical use of a simplified elementary time pattern.

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