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# DEVIATIONS FROM EQUAL TEMPERAMENT IN TUNING ISOLATED MUSICAL INTERVALS

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Four music students tuned the frequency of a variable-tone oscillator to set it at given musical intervals up and down from the reference tone of 500 Hz. Pure tones and complex tones were used. Despite the dispersion of the results a general tendency appeared: small intervals were mostly diminished and large intervals mostly enlarged in comparison with their equally-tempered values. No distinct dependence of the size of musical intervals on the sound spectrum could be observed.

## 1. Introduction

The traditional theory of music attached high significance to mathematical calculations of numerical proportions representing musical intervals. Sets of these proportions, making up theoretical interval systems, had been discussed in numerous papers and treatises in the period preceding the introduction of the equally tempered scale in music. However, still after the introduction of equal temperament, the belief in the superiority of "pure" systems, applying simple proportions of integers, was shared by most musicians. This bielef, which still generally exists, applied not only to harmonic intervals in chords but also to intervals in melodic sequences. Most people speaking of perfect intonation believe that perfection attainable in this respect consists in accurate intonation of intervals corresponding to the frequency ratios resulting from the Pythagorean or just musical scale.

An important stage in the process of studying intonation of musical intervals were investigations by GARBUZOV [1, 2], who found that intonation in solo violin play was performed within certain zones of tolerance. The intonation zones of some intervals exceeded half the semitone. A slightly different approach to this problem was presented by those authors who, by means of electronically generated tones, investigated the intonation of isolated melodic intervals. Some of these investigations were concerned with octaves [11–15]. Within these studies, a tendency was found to tune octaves in a physical interval exceeding the frequency ratio 2 : 1. Investigations of the intonation of the other within-octave intervals [7, 8] indicated that these intervals were characterized by high intonation instability. The ranges within which the intervals tuned by using electronic oscillators were considered correct, appeared repeatedly to be wider than those resulting from the discrepancies between the just, Pythagorean and equally tempered scales.

The discovery of considerable differences in the size of the same interval tuned by the same musician on various occasions led to a revision of the previous views on the problem of melodic intonation and the function of theoretical interval systems in practice. However, in order to generalize these conclusions, some doubts had to be removed. The above-mentioned experiments [7, 8] were carried out by using pure tones as stimuli. It could be suspected that the use of such stimuli, not typical in music practice, had significantly disturbed the results. To clarify the matter, it was decided that experiments on tuning musical intervals would be performed once more, this time with the use of complex tones, in order to apply stimuli more similar to musical sounds.

## 2. Experiment

The experiment was carried out in individual sessions with four music students (women) who served as subjects. Two of them were pianists and two were violinists. Their task was to tune the frequency of an oscillator to set it at given musical intervals up and down from the preceding reference tone. The time paradigm of the stimuli was the following: 0,5-s reference tone, 0.5-s break, 0.5-s variable tone, 1.5-s break. The tones were presented binaurally through headphones at 50 dB SL. The frequency of the reference tone was 500 Hz, i.e. 79 cents below $C_5$. The reference tone frequency not belonging to normal musical keys was used to prevent the listeners from performing absolute pitch evaluation. (The basic means of prevention was the fact that none of the subjects had absolute pitch.) Each individual experimental session lasted about one hour with several short breaks. The order of tuning various intervals was quasi-random, indicated by the operator.

At the first stage of the experiment, sine-waves, triangle-waves, and square—waves were employed. After a few training tasks, each subject performed 10 series of experimental tunings for each kind of stimuli (altogether $3 \times 10$ series of 12 intervals up and 12 intervals down from the reference tone). Each subject could listen to the repeated sequence of stimuli and correct the frequency

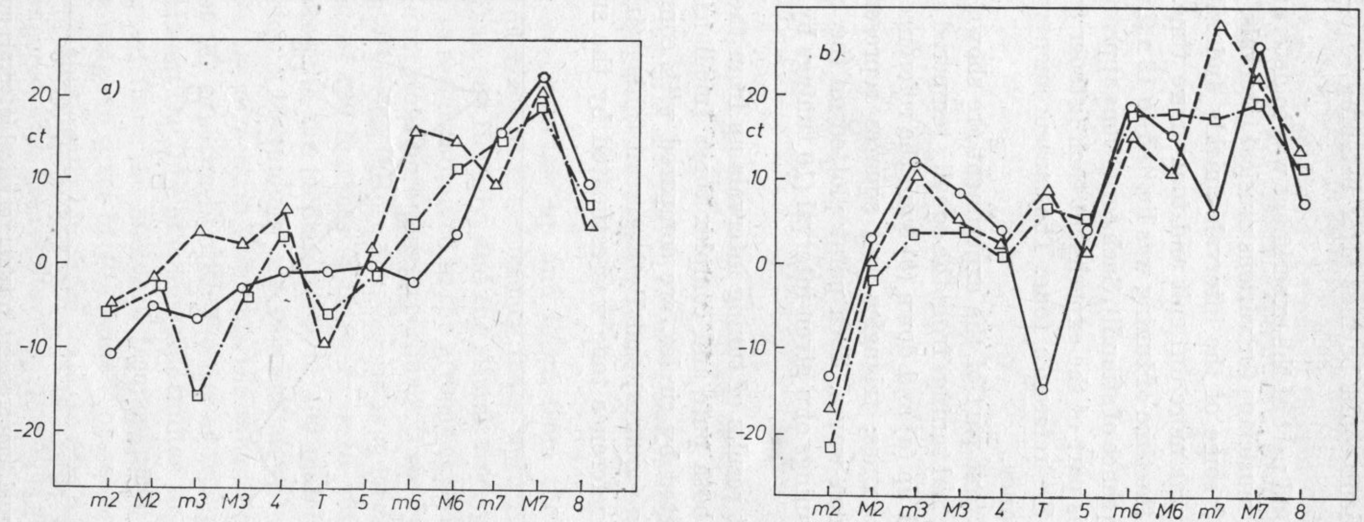


**Fig. 1.** Deviations from equal temperament in tuning musical intervals up (*a*) and down (*b*) from the reference tone. ○ – sine-wave, △ – trangle-wave, □ – square-wave. Intervals marked on the abscissa: minor and major second, minor and major third, pure fourth, tritone, pure fifth, minor and major sixth, minor and major seventh, octave

of the variable tone, until the given interval appeared to him as satisfactory. In practice, this requirement could be met after listening to 6–20 3-second sequencies.

The aim of the second part of the experiment was to determine the strength of the memory trace in musicians for various musical intervals. It was observed that the listeners tuned some of the intervals quickly and without difficulty. Tuning other intervals was more difficult and took more time. In the second part of the experiment the time of tuning was limited to 12 s (i.e. four presentations of the 3-second sequence of stimuli). Such conditions appeared to be closer to musical practice. In this part of the experiment each listener tuned 10 series of 12 intervals up from the reference tone. The measurements were performed with triangle-wave tones.

The results of the first part of the experiment are shown in Fig. 1 in the form of deviations of actual tunings from the equally tempered scale. The results concern intervals tuned up (*a*) and down (*b*) from the reference tone of 500 Hz. Circles represent pure tones; triangles and squares represent, respectively, triangle-waves and square-waves. Each point plotted on the curve represents a median value of 40 tunings of a given interval (10 tunings by each of the four subjects).

Fig. 2. shows joint results of tuning intervals up from the reference tone, comprising the use of both pure and complex tones (medians of 120 values). These results, represented by circles, are compared with computed values of intervals in just tuning (squares) and Pythagorean tuning (triangles). Tuning intervals up from the reference tone was estimated by the subjects as easier

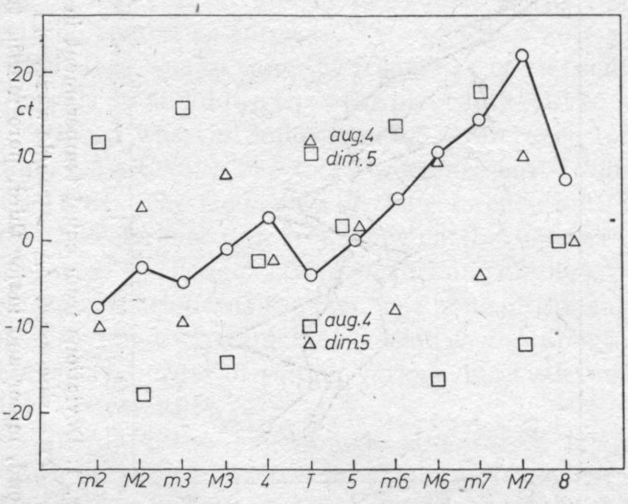


Fig. 2. Deviations from equal temperament in tuning musical intervals up from the reference tone of 500 Hz. Cumulative results ○, comparison with just □ and Pythagorean tuning △; aug. 4 – augmented fourth, dim. 5 – diminished fifth

and more natural than tuning down from the reference tone, therefore only the results of tuning upwards were presented in the figure.

The data in Fig. 2 were next compared with the results obtained in the investigations carried out previously by one of the authors [8]. In these previous investigations, like in the present experiment, four experienced musicians

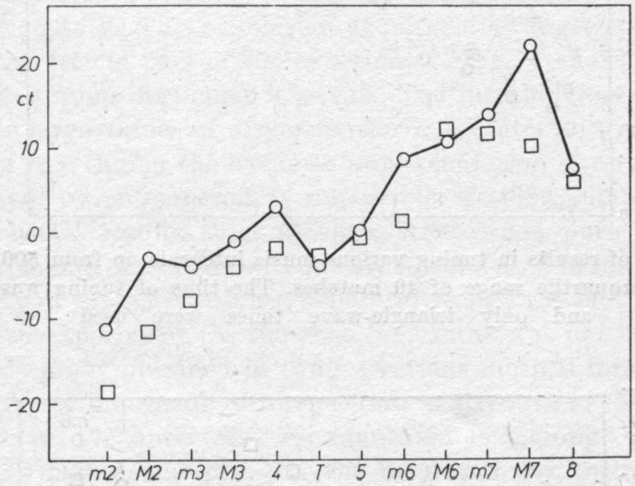


Fig. 3. Cumulative results as in Fig. 2 (○) compared with the results of the previous experiment (□) performed with pure-tone stimuli over a wide frequency range

tuned twelve-octave intervals up and down from the reference tones using headphones. Frequencies of the reference tones were 125, 250, 500, 1000 and 2000 Hz rather than 500 Hz only, and the stimuli were pure tones. The comparison of cumulative results obtained in the present experiment with the cumulative results of the previous pure-tone experiment is shown in Fig. 3. The present results are shown as circles, the previous results as squares. Each circle represents the median value of 120 matches (4 listeners $\times 10$ tunings $\times 3$ waveforms). Each square represents the median value of 400 matches (4 listeners $\times 5$ reference frequencies $\times 10$ tunings $\times 2$ (up and down)).

Fig. 4. shows the dispersion (interquartile ranges) of the results, obtained in second part of the experiment with the use of triangle-wave tones. In this part of the experiment the time of tuning was limited to 12 s and the intervals were tuned only up from the standard frequency 500 Hz. Each circle shows the interquartile range of a set of 40 results representing the deviation of the given interval from the equally tempered scale. In Fig. 5. the results from Fig. 4 (circles) were compared with the results of the previous study (squares). Squares represent interquartile ranges of cumulative data obtained with the use of pure-tones at the reference frequencies of 125, 250, 500, 1000 and 2000 Hz [8].

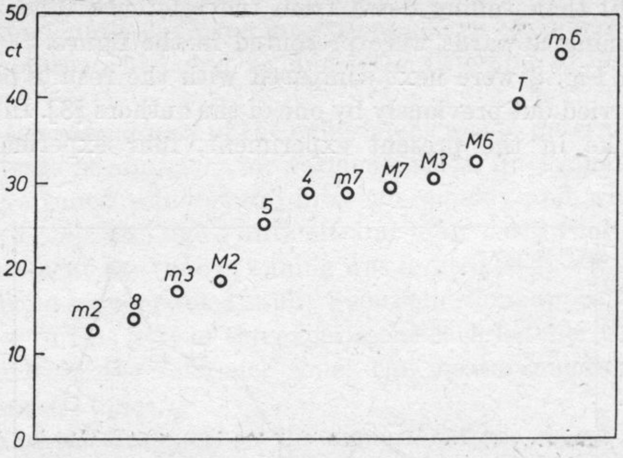


Fig. 4. Dispersion of results in tuning various music intervals up from 500 Hz. Each circle represents the interquartile range of 40 matches. The time of tuning was limited to 12 s and only triangle-wave tones were used

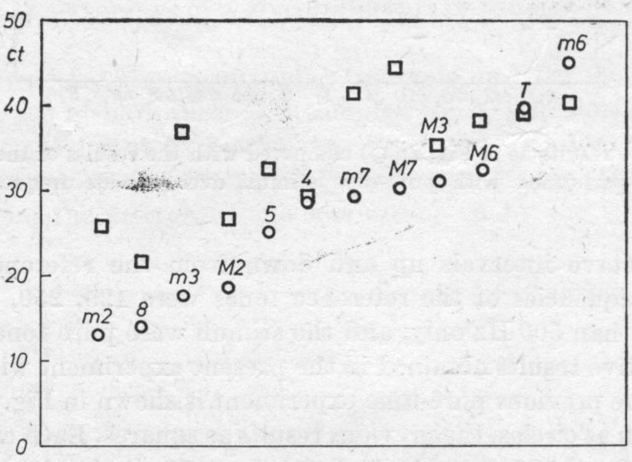


Fig. 5. Interquartile ranges of the results of tuning as in Fig. 4. (○) compared with the results of the previous study (□)

### 3. Conclusions

The results of the experiment prove that the intonation of melodic intervals is not in any distinct way dependent on the timbre of sounds. In the 1st part of the experiment, it was also confirmed, as had been noted before, that small intervals show a tendency to decrease, while large intervals tend to increase their size in comparison with the equally-tempered values. In this light, the commonly observed phenomenon of "octave enlargement" [11, 12] can be recognized as a symptom of a more general tendency.

An important conclusion follows from the observation of Fig. 2. A comparison of the mean values of freely tuned intervals with the values resulting from the assumption of just and Pythagorean tuning indicates a total lack of correlation with these scales. It can be presumed that neither just nor Pythagorean tuning play such a role in music as a large number of music theoreticians would be likely to assign to it. This conclusion acquires a stronger support following the results of the second part of the experiment. Fig. 4 shows the dispersion of the results in tuning particular intervals. The instability of the intonation, reflected by the large values of dispersion for most intervals, indicates that in the condition of free tuning the accurate implementation of any scale, whether equally tempered, or untempered, is impossible. Besides, comparison with the previously published results (Fig. 5) shows that when pure tones are used and the measurements extended to frequency regions used in music less frequently, the stability of tuning is still less accurate, despite the fact that in the previously described experiment [8] the time for tuning was not limited.

Different accuracy obtained in tuning various musical intervals was assumed to be the factor indicating differentiation in strength of the memory trace for a given interval [7]. Intervals were identified as "strong" when they were characterized by higher stability, i.e. the dispersion by their tuning results was lower. On the basis of the results of the second part of experiment (Fig. 4), the intervals of minor and major seconds, minor third and octave can be classified as "strong", whereas the intervals of minor sixth and tritone are particularly "weak". This classification coincides only partly with the one which could be assumed on the basis of the previous experiment [8] carried out in larger frequency range by using pure tones (Fig. 5), undoubtedly, the question of the criterion of interval strength requires further studies.

While drawing conclusions from the above-described experiments, one can attempt to outline a hypothesis explaining the apparently paradoxical contrast beetween the extremely high frequency sensitivity of the ear [6] and the great tolerance in tuning musical intervals. Musical intervals are discrete quality categories which function in a musical pitch system in a similar way as phonemes in natural language do. They may be considered as elements of the basic communication code in music. Their temporary structures form musical phrases and melodies which convey the largest part of musical "meaning".

Both phonemes of natural language and musical intervals may be objectively described in terms of their physical parameters such as frequency ratios (musical intervals), or time durations and formant frequency (vowels). However, none of those parameters may be considered as representing a single value, a point on the scale of physical magnitude. Rather they represent some more or less broad ranges of values within which the characteristic sensational quality of a given unit of a code is still preserved. Within each category, several narrower variants can be distinguished by ear. The implementation of these variants is related to the practical functioning of a given code. In the natural language they are called phonetic variants. They are divided into two subcategories: combinatorial variants, conditioned by the position of a given phoneme in the sequence of speech, and facultative variants, related to individual pronunciation and being used as indicators carrying information about the spaeker.

The existence of the phonetic variants in language has its analogy in music in form of the so-called intonation variants of musical intervals [9]. Similarly to the phonetic variants of speech, they may be divided into two kinds whose function, however, is slightly different from that in the case of phonetic variants.

The first kind of intonation variants may be called "acoustic variants". They appear as precisely defined versions of the size of a given interval, imposed by objective, physical and psychophysiological facts related to the production and perception of music sounds. Examples of such intonation variants are those which are applied to reach a minimum of dissonance (minimum of beats between partials of simultaneously sounding tones) in the conditions of particularly precise auditory control. The intervals between simultaneous tones set at minimum acoustic beats are characterized by possibly simplest frequency ratios of the fundamental tones. The above principle is best observed in just or "natural" tuning; therefore the thirds, sixths and other intervals are often tuned according to the just scale. This effect can be observed in "barbershop singing" [4] and in tuning such instruments as mouth harmonica.

Another example of acoustic intonation variants are some intervals played on bowed instruments such as the violin. The strings of these instruments are tuned in pure fifths; it forces the performer to raise most intervals in Pythagorean tuning [3]. Other acoustic variants can be observed commonly in piano and organ music and are know as the equally tempered scale. These intonation variants, although initially opposed to by musicians, finally became rooted in the auditory memory of music listeners and even appear to be preferred to others [16].

Whereas acoustic intonation variants are to a large extent objective, the other kind of intonation variants, which can be called "expressive variants", is characterized by direct relation to the subjective sensation of the musical content of a piece. Expressive intonation variants concern changes in intonation subordinated to the expressive function of music. In the Western music, the expression of some specific harmonic structures is emphasized by intonation. The shift of pitch accompaning the harmonic tensions can be mentioned as an example of this phenomenon; e.g. the shift of the leading note towards the keynote, or that of a dissonant tone towards its resolution [5, 10].

In contrast to both acoustic variants in music and phonetic variants in speech, the expressive variants in music can carry important elements of information transferred by the performer to the listener. Intonation deviations of this kind distinctly extend the emotional expressiveness of a piece. The possibility of using them in singing and playing instruments with free intonation gives music a peculiar feature, an additional dimension as it were, impossible to achieve while playing the fixed-pitch instruments.

The intonation variants of a given interval which are implemented according to the current requirements of music in performing a music piece or in listening to it, can occur randomly in the imagination of a musician while producing intervals isolated from the music context. In tuning isolated intervals, as it was done in the experiment described, subjects could not be prevented from relying subconsciously or even consciously upon memorized melodic fragments in which the given intervals were used. This could result in using various intonation variants of the same interval by various subjects or even by the same subject at various repetitions of the same tuning. The above-described processes, entirely beyond control of the experimenter, could have been responsible for the large dispersion of results in tuning isolated melodic intervals.

In the light of the above hypothesis, it should be recognized that those intervals which have more varied intonation variants, appear to be "weak" in free-tuning experiments. Conversely, "strong" intervals are those whose intonation variants in music are less varied. The group of particularly strong intervals includes an octave, a minor second and a fourth. On the other hand, a tritone and a minor sixth belong definitely to the weak group. Classification of other intervals in terms of their "interval strength" needs support in further investigations.

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