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INVESTIGATION OF THE ULTRASONIC WAVE PROPAGATION ALONG THE BOUNDARY OF TWO HALF-SPACES: THE ELASTIC ONE OF A SOLID AND THE VISCOELASTIC ONE OF A LIQUID

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By considering the problem of the travelling wave propagation along two half-spaces: the ideally elastic (solid) and the viscoelastic (liquid), with bulk viscosity, as a result of solving the wave equations and taking into account the boundary conditions, the complex characteristic equation was obtained.

The characteristic equation was solved numerically for a frequency of 2.5 MHz for two different viscoelastic bodies, for biological tissue and the acetic acid CH_3COOH, bordering on an elastic medium, steel.

The wave velocity was sought close to the longitudinal wave velocity characteristic of the given media. It was shown that the wave could propagate at a velocity only slightly less than that of longitudinal waves, but with attenuation being slightly larger than that in an unbounded medium.

It follows from the representations obtained of the displacement potentials that, apart from the wave propagation along the boundary of the media, there is also wave propagation towards the liquid damping medium. This phenomenon did not occur in considering the ideal liquid medium.

In both cases, the distributions of the normal and tangential stresses and of the partial displacements were obtained. The wave decays exponentially as the distance from the boundary increases (on both sides).

The distributions are close in character to those of stresses and displacements obtained in the previous paper of the author, where a similar, but a lossless, model was considered.

The acoustic impedance in a viscoelastic medium was also found for the wave type propagating along and across the boundary.

1. Introduction

The investigations of the ultrasonic wave propagation along the boundary of two half-spaces: a viscoelastic liquid and an ideally elastic solid, involve a phenomenon observed during biopsy controlled by ultrasound. In specific physical conditions, there emerges a wave, which propagates along the needle, reaches its end and returns, giving an image of the needle end on the oscilloscope screen. This problem was already considered theoretically with specific physical restrictions in papers [1–3]. These papers showed that the velocity of the propagating wave is close to that in the biological medium surrounding the needle. The previous investigations have dealt with the wave propagation in ideally elastic media.

At present, it assumed that the biological medium surrounding the needle is a viscoelastic liquid, as the biological structures, such as muscle, kidney, liver, on which biopsy is performed, show viscous properties. The needle used in the puncture of a given biological structure is reduced to the infinitely long half-space of an ideally elastic solid. Thus, the wave reflected from needle end will not be considered; the wave analysed will be a travelling one. It is also assumed that the viscoelastic biological medium is unilaterally unbounded. These simplifications will ensure better knowledge of the phenomenon of the wave propagation itself along the boundary of the two media.

The coordinate system was chosen in the way shown in Fig. 1. The axis x coincides with the boundary of the two half-spaces and is parallel to the direction of the wave propagation. The axis z is directed vertically upwards. The ideally elastic solid medium is a homogeneous, isotropic material with the density ϱ_s and the Lamé constants λ_s and μ_s. Here, the velocities of the longitudinal and transverse waves are, respectively, c_d and c_t. The ideally elastic medium borders on a viscoelastic isotropic medium with the density ϱ_c and the viscoelastic constants λ_c and μ_c, defined as

$$\lambda_c = \lambda' + j\omega\lambda'', \quad \mu_c = \mu' + j\omega\mu'',$$

where λ' and μ' are the elastic constants, λ'' and μ'' are the viscous constants, $\omega = 2\pi f$; f = frequency; $j^2 = -1$.

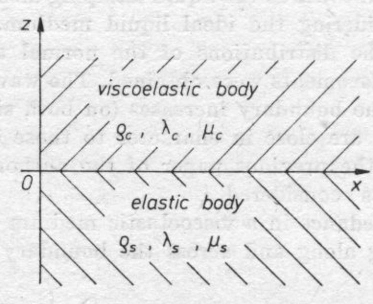

Fig. 1. The system of the media considered

The purpose of the investigations is to determine the parameters characterizing the wave motion propagating in the direction x and the approximate magnitude of the velocity and attenuation of the wave propagating along the boundary of the two half-spaces.

Further on in this paper, the biological medium will be defined as a viscoelastic liquid, where the propagating transverse wave is so rapidly, compared with the longitudinal wave propagation, attenuated that it can be neglected.

2. Initial formulae

In elasticity theory, it is assumed that the components of the stress tensor are linear functions of the components of the strain tensor. These assumptions (the Hook law) are valid only when the purely elastic forces are much stronger than those depending on the strain velocities (the viscous forces). When these forces are comparable and the stress components are also linear functions of the strain velocities, it is said that a given body also has viscous properties (a Voigt body). This body model will be assumed as an approximation of the biological medium. When isotropic, such a body is characterized by four material constants: λ', μ', λ'' and μ'', where λ' and μ' define the elastic properties and λ'' and μ'' the viscous properties of the body.

Then, the constitutive equation of the viscoelastic body becomes

$$\tau_{ij} = \left(2\mu' + 2\mu'' \frac{\partial}{\partial t}\right) \varepsilon_{ij} + \left(\lambda' + \lambda'' \frac{\partial}{\partial t}\right) \varepsilon_{ij} \delta_{ij}, \qquad (i, j = x, y, z), \tag{1}$$

where τ and ε are, respectively the stress and the strain. Substitution of (1) into the motion equations

$$\varrho_c \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \tau_{ij}}{\partial x_j} \tag{2}$$

gives the equation of the displacements of the isotropic viscoelastic body

$$\varrho_c \frac{\partial^2 \overline{u}}{\partial t^2} = \left[(\lambda' + \mu') + (\lambda'' + \mu'') \frac{\partial}{\partial t}\right] \operatorname{grad} \operatorname{div} \overline{u} + \left(\mu' + \mu'' \frac{\partial}{\partial t}\right) \nabla^2 \overline{u}. \tag{3}$$

In the case of harmonic motion with the frequency $f = \omega/2\pi$, which is assumed in this paper,

$$\overline{u}(x, y, z, t) = \overline{u}(x, y, z) \exp(j\omega t)$$

and, after using equation (3),

$$-\varrho_c \omega^2 \overline{u} = (\lambda_c + \mu_c) \operatorname{grad} \operatorname{div} \overline{u} + \mu_c \nabla^2 \overline{u}, \tag{4}$$

where

$$\left.\begin{aligned} \lambda_c &= \lambda' + j\omega\lambda'' \\ \omega &= 2\pi f \\ \mu_c &= \mu' + j\omega\mu'' \end{aligned}\right\}. \tag{5}$$

The displacement equation (4) has formally the same shape as that in elasticity theory. The only difference involves the coefficients λ_c and μ_c which are at present complex and depend on the frequency, according to dependence (5), whereas in the case of the wave propagation in an ideally elastic medium the coefficients were real quantities. Thus, the ideally elastic isotropic body is characterized by two constants, the Lame constants; the viscoelastic isotropic body, by four constants. λ' and μ' define, respectively, the bulk and structural elasticity, whereas λ'' and μ'' are, respectively, the coefficients of the bulk and tangential viscosity. Equation (4) is solved in the same way as in elasticity theory [7].

For the viscoelastic medium, in order to solve equation (4), the displacement vector $\bar{u}$ is respresented in the form

$$\left.\begin{aligned} \bar{u}^c &= (\bar{v}^c + w^c)\exp(j\omega t) \\ \text{with the condition}\ \operatorname{rot}\bar{v}^c &= 0 \text{ and } \operatorname{div} w^c = 0 \end{aligned}\right\}. \tag{6}$$

Here, the vector is respresented in the form of the sum of the scalar potential gradient ϕ^c and the rotation of the vector potential ψ^c with the coordinates ψ^c_x, ψ^c_y and ψ^c_z.

The vector rot $\bar{\psi}^c$ has components of the form

$$\operatorname{rot}\bar{\psi}^c = \left[\frac{\partial\psi^c_z}{\partial y} - \frac{\partial\psi^c_y}{\partial z}, -\left(\frac{\partial\psi^c_z}{\partial x} - \frac{\partial\psi^c_x}{\partial z}\right), \frac{\partial\psi^c_y}{\partial x} - \frac{\partial\psi^c_x}{\partial y}\right]. \tag{7}$$

In view of the twodimensional character of the problem in expression (7), only one component of the vector $\bar{\psi}^c$ occurs, i.e. ψ^c_y, while the vector rot $\bar{\psi}^c$ has the components

$$\operatorname{rot}\bar{\psi}^c = \left[-\frac{\partial\psi^c_y}{\partial z}, 0, \frac{\partial\psi^c_y}{\partial x}\right]. \tag{7a}$$

Therefore, the displacement vector $\bar{u}^c$ has the form

$$\bar{u}^c(x, z, t) = [\operatorname{grad}\phi^c(x, z) + \operatorname{rot}\psi^c_y(x, z)]\exp(j\omega t), \tag{8}$$

and, after breaking it into the components u^c and w^c, they have the form

$$\left.\begin{aligned} u^c &= \left(\frac{\partial\phi^c}{\partial x} - \frac{\partial\psi^c_y}{\partial z}\right)\exp(j\omega t) \\ w^c &= \left(\frac{\partial\phi^c}{\partial z} + \frac{\partial\psi^c_y}{\partial x}\right)\exp(j\omega t) \\ \bar{u} &= (u, w). \end{aligned}\right\} \tag{8a}$$

The potentials ϕ^c and ψ_y^c satisfy the equations

$$\left.\begin{aligned}\nabla^2\phi^c &= \frac{-\varrho_c\omega^2}{\lambda_c+2\mu_c}\,\phi^c\\ \nabla^2\psi_y^c &= \frac{-\varrho_c\omega^2}{\mu_c}\,\psi_y^c\end{aligned}\right\}, \tag{9}$$

where $\lambda_c = \lambda'+j\omega\lambda''$, $\mu_c = \mu'+j\omega\mu''$ and $\omega = 2\pi f$.

Equations (9) result from the application of dependence (8) in the displacement equation (4), i.e.

$$(\lambda_c+\mu_c)\operatorname{grad}\operatorname{div}(\operatorname{grad}\phi^c+\operatorname{rot}\psi_y^c)+\mu_c\nabla^2(\operatorname{grad}\phi^c+\operatorname{rot}\psi_y^c) = -\varrho_c\omega^2(\operatorname{grad}\phi^c+\operatorname{rot}\psi_y^c),$$

but

$$\operatorname{div}\operatorname{grad}\phi^c = \nabla^2\phi^c,$$

$$\operatorname{div}\operatorname{rot}\psi_y^c = 0,$$

$$(\lambda_c+\mu_c)\operatorname{grad}\nabla^2\phi^c+\mu_c\nabla^2\phi^c+\mu_c\nabla^2\operatorname{rot}\psi_y^c = -\varrho_c\omega^2(\operatorname{grad}\phi^c+\operatorname{rot}\psi_y^c),$$

$$\operatorname{grad}[(\lambda_c+2\mu_c)\nabla^2\phi^c+\varrho_c\omega^2\phi^c]+\operatorname{rot}[\mu_c\nabla^2\psi_y^c+\varrho_c\omega^2\psi_y^c] = 0,$$

hence, (9) follows.

The solution of system (9) (by the classical method of separation of variables) leads to the form

$$\begin{aligned}\phi^c(x,z) &= \big(D\exp(-jl_dz)+D_1\exp(jl_dz)\big)\exp(-jpx),\\ \psi_y^c(x,z) &= \big(B\exp(-jl_tz)+B_1\exp(jl_tz)\big)\exp(-jpx),\end{aligned} \tag{10}$$

where

$$\left.\begin{aligned}l_d^2 &= \frac{\omega^2\varrho_c}{\lambda_c+2\mu_c}-p^2\\ l_t^2 &= \frac{\omega^2\varrho_c}{\mu_c}-p^2\end{aligned}\right\}, \tag{11}$$

p is the sought propagation constant,

$$\left.\begin{aligned}c &= \omega/\mathrm{Re}(p), \quad p = \big(\mathrm{Re}(p),\mathrm{Im}(p)\big)\\ \alpha &= -\mathrm{Im}(p)\end{aligned}\right\}, \tag{12}$$

c is the phase velocity of the wave and α is the attenuation coefficient of the wave.

A damped planar harmonic wave travelling in the direction x is characterized by the factor $\exp(j\omega t)\exp(-jpx)$.

Since the biological medium is bounded only on one side (by the elastic half-space), the wave radiated in the direction z will not be reflected and the term containing the factor $\exp(jl_d z)$ can be neglected (i.e. $D_1 \equiv 0$). Analogously, $B_1 \equiv 0$.

Finally, formulae (10) become

$$\begin{aligned} \phi^c(x, z) &= D\exp(-jl_d z)\exp(-jpx), \\ \psi_y^c(x, z) &= B\exp(-jl_t z)\exp(-jpx). \end{aligned} \tag{13}$$

Then, the normal and tangential stresses, expressed by the displacement components, u^c and w^c, according to the formulae

$$\begin{aligned} \tau_{zz}^c &= \lambda_c\left(\frac{\partial u^c}{\partial x} + \frac{\partial w^c}{\partial z}\right) + 2\mu_c\frac{\partial w^c}{\partial z}, \\ \tau_{xz}^c &= \mu_c\left(\frac{\partial w^c}{\partial x} + \frac{\partial u^c}{\partial z}\right), \end{aligned} \tag{14}$$

where

$$\begin{aligned} u^c &= \big(-pjD\exp(-jl_d z) + jl_t B\exp(-jl_t z)\big)\exp(-jpx), \\ w^c &= \big(-jl_d D\exp(-jl_d z) + jl_t B\exp(-jl_t z)\big)\exp(-jpx), \end{aligned} \tag{15}$$

and defined by representations (8) and (13), become

$$\begin{aligned} \tau_{zz}^c = [-D\exp(-jl_d z)(\lambda_c p^2 + \lambda_c l_d^2 + 2\mu_c l_d^2) + B\exp(-jl_t z) \times \\ \times 2\mu_c p l_t]\exp(-jpx + j\omega t), \\ \tau_{xz}^c = \mu_c[-2pl_d D\exp(-jl_d z) + (l_t^2 - p^2)B\exp(-jl_t z)] \times \\ \times \exp(-jpx + j\omega t). \end{aligned} \tag{16}$$

The constants l_d, l_t and p are complex conjugate quantities, related by formulae (11). The propagation constant p is related to the wave number k and the wavelength λ by the formula

$$\mathrm{Re}(p) = k = \frac{\omega}{c} = 2\pi/\lambda. \tag{17}$$

In general, the propagation constant has the form $p = \omega/c - ja$, where c denotes the phase velocity of the wave propagation in the system and a is the coefficient of the wave attenuation.

3. Viscoelastic liquid

Formulae (13) describe the displacement potentials in the viscoelastic body, where longitudinal and transverse waves can propagate. This body is characterized by the density ϱ_c, the Lamé elastic constants λ' and μ' and by the coefficients of the bulk and structural viscosity, λ'' and μ'', respectively.

The determination of the specific values of the material constants λ', λ'', μ' and μ'' requires the solution of the following equations:

$$\left.\begin{aligned} c_d &= \omega/\mathrm{Re}(h) \\ a_d &= -\mathrm{Im}(h) \end{aligned} \quad \text{where } h = \left[\frac{\varrho_c\omega^2}{\lambda_c+2\mu_c}\right]^{1/2} \atop \begin{aligned} c_t &= \omega/\mathrm{Re}(l) \\ a_t &= -\mathrm{Im}(l) \end{aligned} \quad \text{where } l = \left[\frac{\varrho_c\omega^2}{\mu_c}\right]^{1/2} \right\}. \tag{18}$$

These dependencies relate the material constants to the longitudinal and transverse wave velocities in tissue, c_d and c_t, and to the attenuation coefficients a_d and a_t, corresponding to the propagation of these waves for given frequencies. It is assumed, after [4], that

$$c_d = 1.5\times 10^5\ \mathrm{cm/s}, \qquad c_t = 3\times 10^3\ \mathrm{cm/s},$$

$$a_d = 0.37\ \mathrm{cm}^{-1}, \qquad a_t = 2\times 10^3\ \mathrm{cm}^{-1},$$

where a_d is the attenuation coefficient of the longitudinal wave propagating along the muscle. With these assumptions and the frequency $f = 2.5$ MHz, the values of the material constants of biological tissue, determined from relations (18), are

$$\lambda' = 2.27\times 10^{10}\ \mathrm{dyne/cm^2} = 2.27\times 10^9\ \mathrm{N/m^2},$$

$$\mu' = 5.9\times 10^6\ \mathrm{dyne/cm^2} = 5.9\times 10^5\ \mathrm{N/m^2}, \tag{19}$$

$$\lambda'' = 10.23\ \mathrm{dyne\ s/cm^2} = 1.023\ \mathrm{Ns/m^2}, \qquad \mu'' = 0.33\ \mathrm{dyne\ s/cm^2} = 0.033\ \mathrm{Ns/m^2}.$$

It is seen that the coefficient of the bulk viscosity λ'' is larger by almost 2 orders of magnitude than μ''. In paper [12] O'Brien showed that μ'' has the same value for tissue as that for water (soft tissue contains 70 % of water) and, subsequently, made the assumption $\mu'' < \lambda''$ for water. The same assumption can be made here for tissue. Subsequently, comparison of the elastic coefficients λ' and μ' in (9) shows that μ' is lower by 4 orders of magnitude than λ'. Therefore, it is assumed that $\mu' \ll \lambda'$.

In summing up the above assumptions, tissue is defined as a liquid with the coefficients of bulk viscosity $\lambda'' = 10.23$ dyne s/cm² and of bulk viscosity $\lambda' = 2.25\times 10^{10}$ dyne/cm². In this liquid only the longitudinal wave propagates, since, as Frizzell showed in paper [4], the attenuation of the transverse wave in tissue is 1000 times as large as that of the longitudinal wave.

Thus, the displacement potential has the form

$$\phi^c(x, z) = D\exp(-jl_d z)\exp(-jpx), \quad l_d^c = \frac{\omega^2 \varrho_c}{\lambda_c} - p^2. \tag{20}$$

The displacement $\bar{u}$ is, when broken into components,

$$\left.\begin{aligned} u^c &= -jpD\exp(-jl_d z)\exp(-jpx)\exp(j\omega t) \\ w^c &= -jl_d D\exp(-jl_d z)\exp(-jpx)\exp(j\omega t) \end{aligned}\right\}. \tag{21}$$

The stresses have the form

$$\left.\begin{aligned} \tau_{zz}^c = \tau_{xx}^c &= -D\exp(-jl_d z)\lambda_c(p^2+l_d^2)\exp(-jpx)\exp(j\omega t) \\ \tau_{xz} &= 0. \end{aligned}\right\}. \tag{22}$$

The propagation constant p is common to the potential ϕ^c and the potentials ϕ^s and ψ_y^s, defined below, in the ideally elastic half-space of the solid. These (displacement) potentials can be given in the following form (considering, analogously to formulae (13), only the possibility of the wave propagation away from the boundary, i.e. in the direction $-z$):

$$\left.\begin{aligned} \phi^s(x, z, t) &= A\exp(jk_d z)\exp(-jpx)\exp(j\omega t) \\ \psi_y^s(x, z, t) &= E\exp(jk_t z)\exp(-jpx)\exp(j\omega t) \end{aligned}\right\}, \tag{23}$$

where

$$k_d^2 = \frac{\omega^2}{c_d^2} - p^2, \quad k_t^2 = \frac{\omega^2}{c_t^2} - p^2. \tag{24}$$

The components of the displacement vector, u^s and w^s, and the normal and tangential stresses, calculated from formulae (16) and expressed by potentials (23), have the form

$$\left.\begin{aligned} u^s &= [-Ajp\exp(jk_d z) - jk_t\exp(jk_t z)E]\exp(-jpx+j\omega t) \\ w^s &= [jk_d A\exp(jk_d z) - jpE\exp(jk_t z)]\exp(-jpx+j\omega t) \end{aligned}\right\}; \tag{25}$$

$$\left.\begin{aligned} \tau_{zz}^s &= [-(\lambda_s p^2 + \lambda_s k_d^2 + 2\mu_s k_d^2)A\exp(jk_t z) + 2\mu_s p k_t E\exp(jk_t z)]\exp(-jpx+j\omega t) \\ \tau_{xz}^s &= [2A\exp(jk_d z)k_d p + E\exp jk_t z(k_t^2 - p^2)]\mu_s\exp(-jpx+j\omega t)] \end{aligned}\right\}. \tag{26}$$

4. Boundary conditions

On the boundary of the half-space $z = 0$, the normal and tangential stresses, and the normal components of the displacement vector, must be continuous, i.e. the following conditions should be statisfied:

$$\left.\begin{aligned} \tau_{zz}^{s} &= \tau_{zz}^{c} \\ \tau_{xz}^{s} &= 0 \\ w^{s} &= w^{c} \end{aligned}\right\}. \tag{27}$$

Substitution of dependences (8a), (21), (22), (25) and (26) in the boundary conditions (27) gives a system of three homogeneous equations with the unknons A, E and D and the coefficients a_{ij}; $i, j = 1, 2, 3$. The coefficients include the material constants of the elastic half-space and the viscoelastic biological medium, and the wave numbers k_d, k_t and l_d and the sought propagation constant $p = (\omega/c)-ja$. The determinant formed from conditions (27), after substituting the previously determined displacements (8a) and (25), and stresses (22) and (26), has the form

$$|a_{ij}| = \begin{vmatrix} 2\mu_s p^2 - \omega^2 \varrho_s & 2\mu_s p k_t & \omega^2 \varrho_c \\ 2\mu_s p k_d & \mu_s(k_t^2 - p^2) & 0 \\ jk_d & -jp & -jl_d \end{vmatrix}. \tag{28}$$

The characteristic equation

$$|a_{ij}| = 0 \quad (i, j = 1, 2, 3) \tag{29}$$

signifies that there exists a nontrivial solution of system (27). The characteristic equation is an algebraic equation with complex conjugate terms and the complex conjugate unknown p. The propagation constant p occurs explicitly and is contained in the terms k_d, k_t and l_d, defined by relationships (11) and (24). It is impossible to transform equation (29) to achieve an analytical solution. Therefore, equation (29) will be solved numerically for the characteristic parameters of the biological medium when biopsy is performed. Subsequently, the results obtained will be used in another model, providing clues as to in what ranges the propagation constant p and other parameters, characteristic of the wave propagating along a hollow elastic cylinder immersed in a viscoelastic liquid, should be sought. This problem will be considered in another paper.

The characteristic equation (29) was solved numerically by the method of successive approximations. The aim was to find a complex conjugate solution whose real part lies close to the longitudinal wave velocity in tissue (close to 1.5×10^5 cm/s) and whose imaginary part is close to the value of the longitudinal wave attenuation in tissue (close to 0.370 cm^{-1}). The signs of k_d, k_t and l_d from formulae (13a) and (24) were chosen in such a way that the wave decayed with increasing distance from the boundary of the two media. The solution of the complex conjugate characteristic equation gave zero for the velocity of the wave sought $c_x = 1.499593 \times 10^5$ cm/s, i.e. slightly lower than the longitudinal wave velocity assumed in the unbounded viscoelastic liquid $c = 1.5 \times 10^5$ cm/s. The value of the attenuation coefficient obtained was $\alpha_x = 0.3734$ cm^{-1}, i.e. higher than the attenuation coefficient of the longitudinal wave in an unbounded viscoelastic medium assumed as $\alpha = 0.370$ cm^{-1}.

As a result of solving the equation, the following values of the wave numbers k_d, k_t and l_d were obtained:

$$
\begin{aligned}
k_d &= -0.39 - j101,\\
k_t &= -0.42 - j92.8, \qquad (30)\\
l_d &= 0.013 - j2.59,
\end{aligned}
$$

and the following values of the displacement potential amplitudes:

$$
\begin{aligned}
E &= 0.00067 - j1.08,\\
D &= 4.85 - j0.047, \qquad (31)
\end{aligned}
$$

for the assumed value of the amplitude $A = 1 + j0$.

Subsequently, in order to verify whether the phenomenon is similar for a highly damping liquid, the acetic acid CH_3COOH was assumed as the viscoelastic liquid. This is a medium which shows 10 times as much attenuation as that in biological tissue. Namely, according to [9] the measured attenuation coefficient is

$$\frac{\alpha_d}{f^2} \times 10^{17} \frac{s^2}{\text{cm}} = 90{,}000. \qquad (32)$$

Then for the frequency $f = 2$ MHz the attenuation $\alpha_d = 5.625$ cm^{-1}. The other characteristic parameters of the acetic acid are: the longitudinal wave velocity $c_d = 1.15 \times 10^5$ cm/s, the density $\varrho = 1.049$ g/cm^3, the coefficients of elasticity and bulk viscosity λ' and λ'', respectively (from formulae(18)): $\lambda' = 1.38 \times 10^{10}$ dyne/cm^2 $= 1.38 \times 10^9$ N/m^2, $\lambda'' = 72.5$ dyne s/cm^2 $= 7.25$ Ns/m^2. Then, the solution of the characteristic equation (29) by the method of successive approximations gives the sought velocity of the wave guided along the boundary, $c_x = 1.14913 \times 10^5$ m/s and the attenuation of this wave $\alpha_x = 5.635$ 1/cm.

Analogously to the first case considered (biological tissue), the velocity of the sought wave was found to be slightly lower than the longitudinal wave velocity in the medium and the attenuation slightly higher than the wave attenuation in an unbounded medium. By assuming the displacement amplitude $A = 1 + j0$, the remaining displacements amplitudes D and E and the wave displacements and stresses in the system considered were calculated. For the velocity c_x and the attenuation α_x determined numerically, the calculated numbers k_d, k_t and l_d and the displacement amplitudes E and D are

$$\left.\begin{aligned} k_d &= -5.74 - j134 \\ k_t &= -6.03 - j128 \\ l_d &= 0.0937 - j1.86 \end{aligned}\right\}, \tag{33}$$

$$\left.\begin{aligned} E &= 0.00411 - j1.047 \\ D &= 4.863 - j\,0.0256 \end{aligned}\right\}. \tag{34}$$

Consideration of the numerical results of (30), (31), (33) and (34) gives the displacement potentials in the form:

a) biological tissue/steel:

$$\phi^c = D\exp(-jz0.013)\exp(-2.59z)\exp(-jx104.75 - 0.3734x) \quad \text{tissue}$$

$$\left.\begin{aligned} \phi^s &= A\exp(-jz0.39)\exp(101z)\exp(-jx104.75 - 0.3734x) \\ & \qquad \text{steel} \\ \psi_y^s &= E\exp(-jz0.42)\exp(93z)\exp(-jx104.75 - 0.3734x) \end{aligned}\right\} \tag{35}$$

b) acetic acid/steel:

$$\phi^c = D\exp(-jz0.009)\exp(-1.86z)\exp(-jx136.69 - 5.635x) \quad \text{acetic acid}$$

$$\left\{\begin{aligned} \phi^s &= A\exp(-jz5.74)\exp(134z)\exp(-jx136.69 - 5.635x) \\ & \qquad \text{steel} \\ \psi_y^s &= E\exp(-jz6.03)\exp(128z)\exp(-jx136.69 - 5.635x) \end{aligned}\right. \tag{36}$$

It follows from the form of ϕ^c, ϕ^s and ψ_y^v that the wave propagates and is attenuated in the direction x and z; from the solid to the viscoelastic liquid. The first exponential factor did not occur when the liquid medium was considered without attenuation [1].

Moreover, the wave is strongly attenuated with increasing distances from the boundary $z = 0$ in both directions.

5. Acoustic impedance in the viscoelastic liquid

Let us now calculate the impedance in the unbounded viscoelastic medium and the impedance of the half-space of the viscoelastic liquid for the wave propagating along the boundary of the two half-spaces.

The impedance in the unbounded medium, Z_{bulk}, of the viscoelastic liquid, for the planar wave propagating in the direction x, is given by the formula

$$Z_{\text{bulk}} = \frac{p'}{v} = \varrho_c c = \frac{\lambda_c + 2\mu_c}{c} = \frac{\lambda_c}{c} = \lambda_c \operatorname{Re}\left(\sqrt{\frac{\varrho_c}{\lambda}}\right), \tag{37}$$

where p' is the pressure of the wave and c is its velocity. In the case of a bounded medium the components of the vector of the impedance of the medium in the direction i ($i = x, y, z$) are defined, according to [8, 10], as

$$Z_i^c = \frac{\tau_i^c}{v_i^c}, \tag{38}$$

where

$$\tau_i^c = \tau_{xi}^c + \tau_{yi}^c + \tau_{zi}^c \quad (i = x, y, z),$$

$$\bar{v}^c = \left[\frac{du^c}{dt}, \frac{dv^c}{dt}, \frac{dw^c}{dt}\right] = [v_x^c, v_y^c, v_z^c].$$

Then for the half-space of the viscoelastic liquid, from formulae (14) (15) and (22),

$$Z_x^c = \frac{\tau_{xx}^c}{v_x^c} = \frac{\omega \varrho_c}{p}, \qquad Z_z^c = \frac{\tau_{zz}^c}{l_d} = \frac{\omega \varrho_c}{l_d}, \tag{39}$$

giving the following values of the components of the impedance

a) for tissue:

$$\begin{cases} |Z_x| = 1.5 \times 10^5 \text{ g/scm}^2, \text{ since } Z_x = 1.4995 \times 10^5 + 5.35 \times 10^2 j \\ |Z_z| = 6.2 \times 10^6 \text{ g/scm}^2, \text{ since } Z_z = 3.216 \times 10^4 + 6.234 \times 10^6 j \end{cases} \tag{40}$$

b) for acetic acid CH_3COOH:

$$\begin{cases} |Z_x| = 1.2 \times 10^5 \text{ g/scm}^2, \text{ since } Z_x = 1.2 \times 10^5 + 4.96 \times 10^3 j \\ |Z_z| = 8.8 \times 10^6 \text{ g/scm}^2, \text{ since } Z_z = 4.44 \times 10^5 + 8.82 \times 10^6 j \end{cases} \tag{41}$$

weheras the characteristic impedance for the wave in an unbounded viscoelastic medium is:

a) for tissue

$$|Z_{\text{bulk}}^c| = 1.5 \times 10^5 \text{ g/scm}^2, \text{ since } Z_{\text{bulk}} = 1.49 \times 10^5 + 5.7 \times 10^2 j;$$

b) for acetic acid CH_3COOH: (42)

$$|Z_{\text{bulk}}^c| = 1.2 \times 10^5 \text{ g/scm}^2, \text{ since } Z_{\text{bulk}} = 1.2 \times 10^5 + 9.88 \times 10^3 j.$$

It can be seen that the modulus of the component Z_x of the impedance vector for the liquid half-space (formulae (40) and (41)) is equal to the modulus of the characteristic impedance of the wave in the unbounded medium (formula (42)). In turn, the component Z_z of the impedance (perpendicular to the boundary of the medium) is in both cases distinctly larger (more than ten times). Its imaginary part is larger by one or two orders of magnitude than the real one, which results from the very weak attenuation of this component, propagating from the elastic medium to the damping liquid, perpendicularly across the boundary of the two media. For an undamped propagating wave the impedance is a real quantity [13].

6. Discussion and results

It has been shown that in a planar system of a viscoelastic half-space (biological tissue), bordering on an ideally elastic half-space (steel), a boundary wave can propagate with a velocity slightly lower ($c_x = 1.499593 \times 10^5$ cm/s) than that of the longitudinal wave, assumed for an unbounded viscoelastic medium ($c = 1.5 \times 10^5$ cm/s). In view of the properties of biological tissue, it has been assumed that it only exhibits elasticity and bulk viscosity (damping liquid).

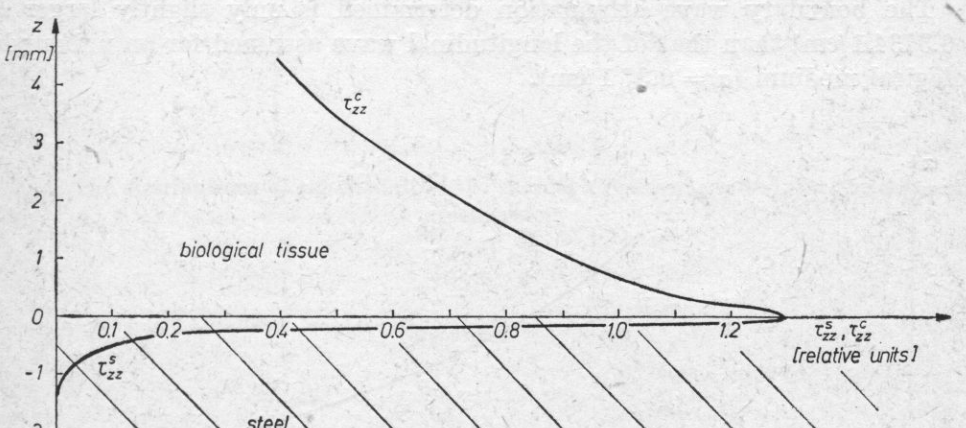

Fig. 2. The distribution of the moduli of the normal stresses τ^s_{zz} and τ^c_{zz} for the tissue-steel system

The boundary wave analysed propagates only in the boundary conditions: the thickness of this layer in steel is smaller by an order of magnitude than that in biological tissue. In turn, the thickness of the boundary layer in biological tissue is of the order of 10 wavelengths. This results from the displacement distributions (Fig. 4) determined for a frequency of 2.5 MHz. The stress distributions (Figs. 2, 5, 7), determined in biological tissue and acetic acid also confirm this character of bilateral decay of the wave across the boundary.

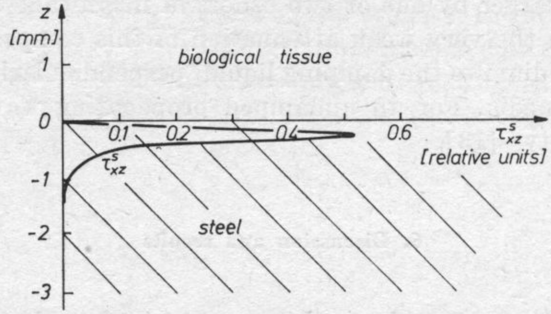

Fig. 3. The distributions of the modulus of the tangential stress τ^s_{xz} in steel for the tissue-stell system

The diagrams enclosed confirm the equality of the stresses and displacements on the boundary of the media, in keeping with the boundary conditions (27). However, the displacements u^s and u^c are different (Fig. 8), for they are directed along the boundary and not determined by any boundary condition.

The boundary wave attenuation determined is only slightly larger ($\alpha_x = 0.3734$ 1/cm) than that of the longitudinal wave assumed for an unbounded biological medium ($\alpha = 0.37$ 1/cm).

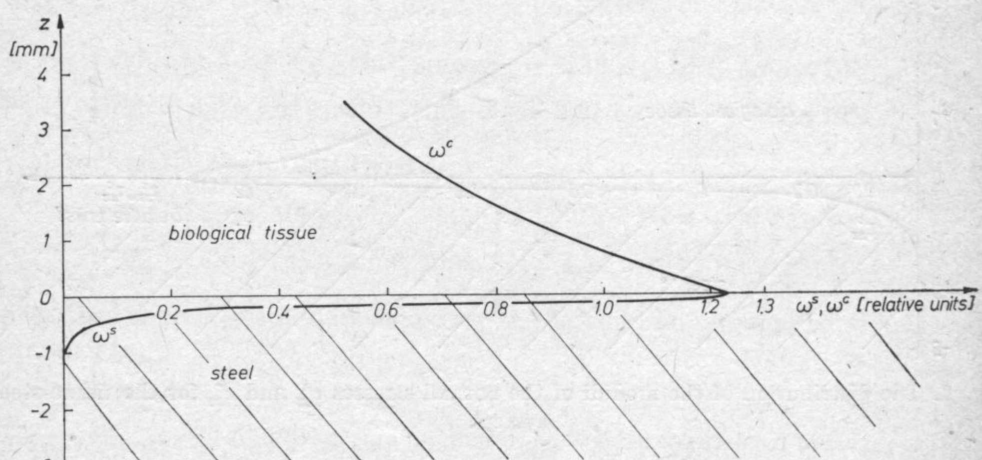

Fig. 4. The distributions of the moduli of the displacement components w^c and w^s for the tissue-steel system

It has been observed that the wave propagates in the directions x and z; from the boundary layer of the solid medium to the boundary layer of the viscoelastic liquid (see formulae (35) and (36)). In a previous paper of the author, where two elastic media were considered, the boundary wave was not observed to propagate across the boundary.

Physically, this phenomenon can be explained by energy flow from the boundary layer to the viscoelastic liquid. Without this energy flow, as a result of absorption, the amplitude (and energy) of the wave in the boundary layer of

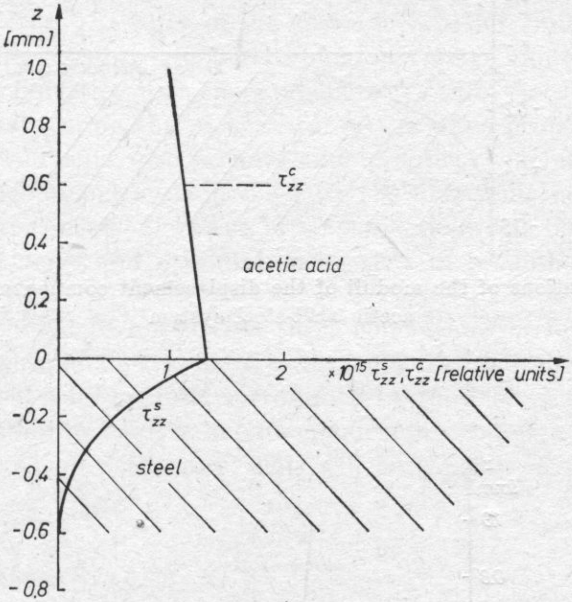

Fig. 5. The distributions of the moduli of the normal stresses τ_{zz}^{s} and τ_{zz}^{c} for the acetic acid-steel system

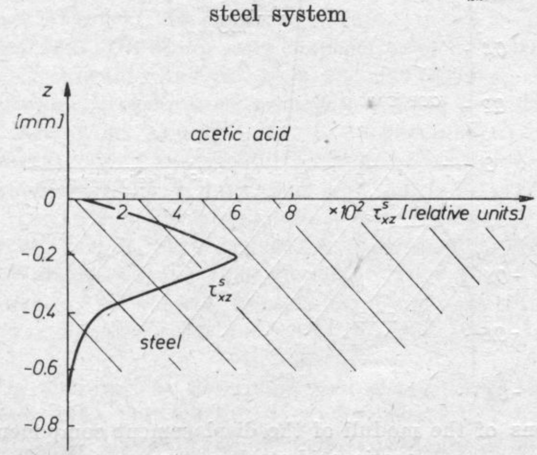

Fig. 6. The distribution of the modulus of the tangential stress τ_{xz}^{s} in steel for the acetic acid-steel system

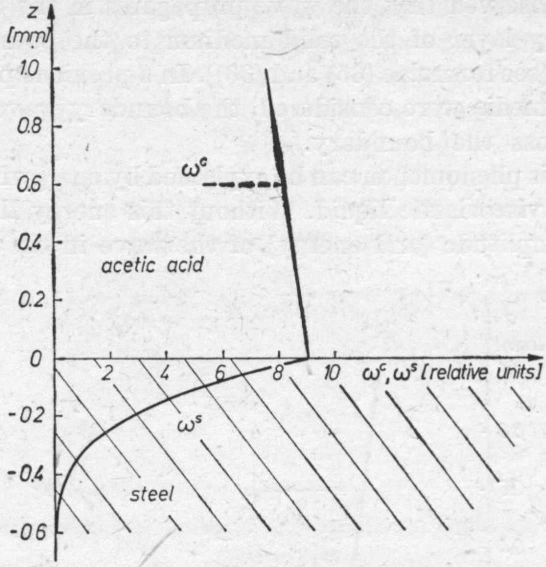

Fig. 7. The distributions of the moduli of the displacement components w^c and w^s for the acetic acid-steel system

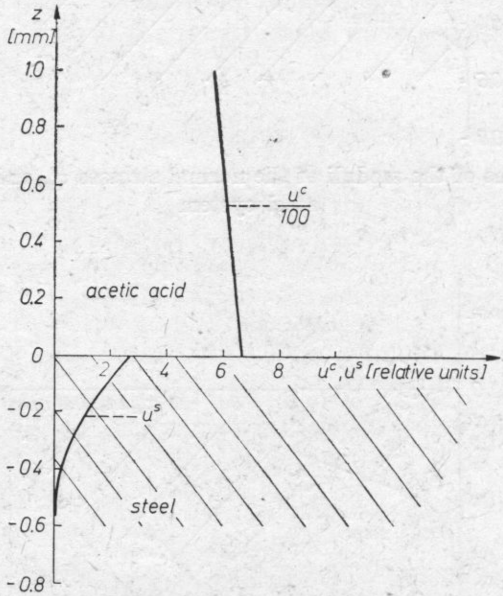

Fig. 8. The distributions of the moduli of the displacement components u^c and u^s for the acetic acid-steel system

the viscoelastic liquid would decrease as the wave would propagate in the direction x. It would, however, remain constant in the boundary layer of the ideally elastic solid medium. Therefore, after covering some distance x, it could not satisfy the boundary conditions of the equality of the stresses and displacements on both sides of the boundary. Thus, there must occur energy flow across the boundary, to increase the wave amplitude in the viscoelastic liquid and decrease it in the elastic one; and, by doing so, satisfy the boundary conditions, irrespective of the distance x.

The conclusions that there is energy flow across the boundary of the media can also be drawn from the value of the impedance Z_z determined for the viscoelastic liquid. It has a real component, which, in view of the wave propagation in the direction z, indicates that energy penetrates into the viscoelastic liquid.

The above analysis, on the mechanism of the wave propagation along the boundary of the two media, also permits some practical conclusions to be drawn for the performance of biopsy controlled by the ultrasonic beam, since it can be assumed that the ultrasonic wave propagates along the needle with practically the same velocity and attenuation as that in an unbounded biological medium.

However, it should be noted that these conclusions have been formulated for a very simplified planar system of two media. In reality, in biopsy, there is a layered cylindrical solid medium surrounded by the biological one. Therefore, it seems indispensible to analyse such a much more complex case, in order to solve the problem in an exact way. This will be considered in another paper.

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