ARCHIVES OF ACOUSTICS
10, 4, 335-344 (1985)

ANALYSIS OF DIRECTIONAL RADIATION PATTERNS OF A SYSTEM OF FLAT PLANE SOUND SOURCES

ZBIGNIEW ENGEL, LESŁAW STRYCZNIEWICZ

Institute of Mechanics and Vibroacoustics of the Academy of Mining and Metallurgy in Cracow, Poland

This paper presents a method of determining directional radiation patterns of a system of arbitrary flat surface sound sources, source vibration velocities phase shifted by an arbitrary angle. Also the results of verification studies of this method for a system of two circular pistons are presented.

1. Introduction

One of the main causes for noise emission by machines and industrial installations are the material vibrations of their elements.

During the past few years several methods, allowing quantitative evaluation of noise emitted in industrial conditions, have been developed [4]. Research has been also conducted on predicting the vibroacoustical power emitted by a sound source in certain conditions [2]. The second problem becomes quite complicated in the case of simultaneous sound emission by several sound sources. This is due to the occurrence of interactions between source and air acoustical waves, which cause changes in the acoustical power system in relation to the power emitted by these sources independently (sources mutually isolated). Conducting quantitative research on acoustical energy emission requires the development of an effective method of determining acoustical self-impedance and mutual impedance of the sources. One of the acoustical impedance determination methods is based on the integration of the square root of the source directional radiation pattern modulus.

This paper presents a method of determining directional radiation patterns of a system of two arbitrary plane sound sources, vibrating out of phase with equal frequencies.

2. Directional radiation pattern of a system of flat plane sound sources

Mathematical analysis of an acoustical field produced by a source is a complex problem. An effective solution to this problem is possible in a case of flat plane sources vibrating in an infinite acoustic baffle in a free field. It is a theoretical case, rarely met in practice, but its solution can be applied with small corrections in the analysis of existing sound sources. Assuming an ideal acoustic baffle, the acoustic pressure in the Fraunhofer zone for a monoharmonic wave can be determined from the Fraunhofer integration formula

$$p(r,\theta,\varphi)=\frac{ik\varrho_0 c}{2\pi}\frac{e^{-ikr}}{r}\int_S v(\vec{r}_0)\exp[ikr_0\sin\theta\cos(\varphi-\varphi_0)]dS_0 \quad [\mathrm{Pa}], \tag{1}$$

where $i=\sqrt{-1}$, k – wave number [m^{-1}], $\varrho_0 c$ – specific acoustical impedance of the medium [$\mathrm{kg\,m^{-2}s^{-1}}$], r, θ, φ – space polar coordinates of the observation point, r_0, φ_0 – polar coordinates of a point laying on the source surface, S – source surface [m^2], $v(\vec{r}_0)$ – normal component of the vibration velocity on the source surface in a point determined by vector $\vec{r}_0$[$\mathrm{m\,s^{-1}}$].

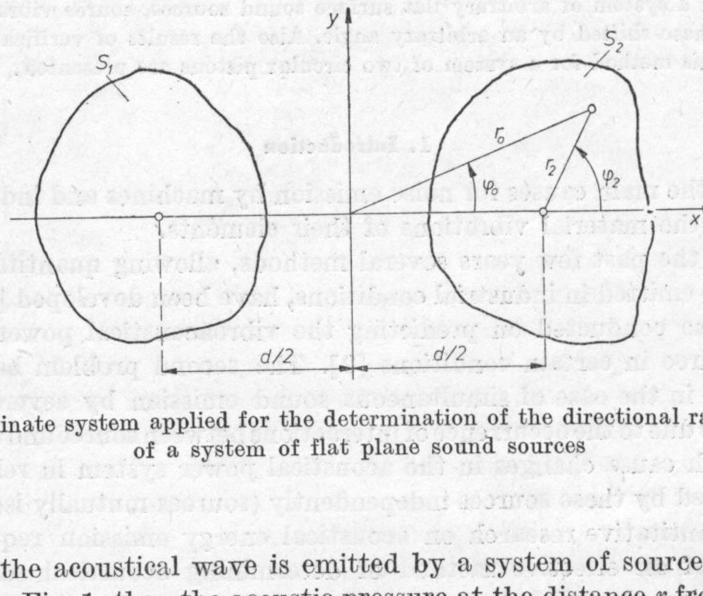


Fig. 1. Coordinate system applied for the determination of the directional radiation system of a system of flat plane sound sources

When the acoustical wave is emitted by a system of sources, S_1 and S_2, presented in Fig. 1, then the acoustic pressure at the distance r from the centre of the coordinate system in the Fraunhofer zone is expressed by the following formula:

$$p(r,\theta,\varphi)=\frac{ik\varrho_0 c}{2\pi}\frac{e^{-ikr}}{r}\Big[\int_{S_1} v(\vec{r}_0)\exp\big(ikr_0\sin\theta\cos(\varphi-\varphi_0)\big)dS_0+$$

$$+\int_{S_2} v(r_0)\exp\big(ikr_0\sin\theta\cos(\varphi-\varphi_0)\big)dS_0\Big] \quad [\mathrm{Pa}]. \tag{2}$$

Acoustical pressure on the direction normal to the source surface $\theta=0$, determined from (2), is

$$p_0(r) = \frac{ik\varrho_0 c}{2\pi}\frac{e^{-ikr}}{r}\left[\int_{S_1} v(\vec{r_0})\,dS_0 + \int_{S_2} v(\vec{r_0})\,dS_0\right] \quad [\mathrm{Pa}]. \tag{3}$$

Defining the directional pattern as a ratio of the acoustical pressure on the given direction to the pressure on the direction normal to the source surface, can be troublesome, because the expression in brackets in formula (3) can in certain cases equal 0. For this reason, the directional radiation pattern in the further part of this paper will be understood as the ratio of the acoustical pressure on the given direction to the acoustical pressure on the direction normal to the surface of an imaginary source, where the velocity distribution is expressed as

$$\hat{v}(\vec{r_0}) = |v(\vec{r_0})| \quad [\mathrm{m\,s^{-1}}]. \tag{4}$$

In a case of a system of sources, the acoustical pressure on the direction normal to the surface of an imaginary source is equal to

$$\hat{p}_0(r) = \frac{ik\varrho_0 c}{2\pi}\frac{e^{-ikr}}{r}\left[\int_{S_1} |v(\vec{r_0})|\,dS_0 + \int_{S_2} |v(\vec{r_0})|\,dS_0\right] \quad [\mathrm{Pa}]. \tag{5}$$

Substituting in formula (5) the following expressions

$$[v_1] = \frac{1}{S_1}\int_{S_1} |v(\vec{r_0})|\,dS_0 \quad [\mathrm{m\,s^{-1}}], \qquad [v_2] = \frac{1}{S_2}\int_{S_2} |v(\vec{r_0})|\,dS_0 \tag{6}$$

we obtain

$$\hat{p}_0(r) = \frac{ik\varrho_0 c}{2\pi}\frac{e^{-ikr}}{r}\left([v_1]S_1 + [v_2]S_2\right) \quad [\mathrm{Pa}]. \tag{7}$$

As relation

$$r_0\cos(\varphi-\varphi_0) = \frac{d}{2}\cos\varphi + r_2\cos(\varphi-\varphi_2) \tag{8}$$

is valid for source 2, and for source 1,

$$r_0\cos(\varphi-\varphi_0) = -\frac{d}{2}\cos\varphi + r_1\cos(\varphi-\varphi_1) \tag{9}$$

where d — distance between the centres of the sources [m],
r_1, φ_1 — local polar coordinates of source 1,
r_2, φ_2 — local polar coordinates of source 2, then substituting expressions (8) and (9) in formula (2) and dividing it by expression (7), an expression for the

directional radiation pattern for a system of sources is obtained

$$D(\theta, \varphi) = \frac{\exp\left(-ik\frac{d}{2}\cos\varphi\sin\theta\right)S_1[v_1]D_1(\theta, \varphi) + \exp\left(ik\frac{d}{2}\cos\varphi\sin\theta\right)S_2[v_2]D_2(\theta, \varphi)}{S_1[v_1]+S_2[v_2]}. \tag{10}$$

Assuming that the sources are identical, they vibrate in the same phase with equal velocities, we obtain a known formula [6]

$$D(\theta, \varphi) = D_1(\theta, \varphi)\cos\left(\frac{kd}{2}\sin\theta\cos\varphi\right). \tag{11}$$

If sources are identical and vibrate with equal amplitudes of velocity, but the vibration velocity of the second source is out of phase at an angle of φ in relation to the vibration velocity of the first source, then after some simple conversions we achieve

$$D_2(\theta, \varphi) = e^{i\psi}D_1(\theta, \varphi). \tag{12}$$

Therefore the directional pattern of the system will be

$$D(\theta, \varphi) = \frac{1}{2}D_1(\theta, \varphi)\left[\exp\left(-ik\frac{d}{2}\sin\theta\cos\varphi\right) + \exp\left(i\psi + \frac{d}{2}\sin\theta\cos\varphi\right)\right] \tag{13}$$

while the pattern modulus will be expressed by

$$|D(\theta, \varphi)| = |D_1(\theta, \varphi)|\cos\left(\frac{kd}{2}\sin\theta\cos\varphi + \frac{\psi}{2}\right). \tag{14}$$

In a case of a system of rigid, circular vibrating pistons, the pattern modulus will equal

$$|D(\theta, \varphi)| = \frac{2J_1(ka\sin\theta)}{ka\sin\theta}\cos\left(\frac{kd}{2}\sin\theta\cos\varphi + \frac{\psi}{2}\right), \tag{15}$$

where a – piston radius [m], J_1 – first type Bessel function of the first order.

3. Physical model of a system of two piston sources

A system of two circular pistons vibrating in a rigid acoustic baffle has been analysed in the course of experimental investigations. Pistons with smooth and flat surfaces, mounted to two backing plates of electromagnetic heads of vibration inductors (RTF 11075) were the main element of the emission source excitation system. The bases of the inductors have been mounted to a soundproof casing (closed, wooden box), placed on a platform of an anechoic chamber.

Piston plates vibrated in circular openings of the acoustic baffle which separated the faces of the inductor heads and the bases of the inductors from the half-space above the pistons' surface. The vibrations of the pistons were forced kinematically by the head of the inductor.

Two identical pistons, of $2a = 0.14$ m diameter, forced to vibrate with a constant amplitude of vibration velocity in specified frequency ranges, were applied in investigations:

$v = 0.02\,[\mathrm{ms}^{-1}]$ for frequency range 200–500 [Hz];

$v = 0.004\,[\mathrm{ms}^{-1}]$ for frequency range 500–750 [Hz];

$v = 0.002\,[\mathrm{ms}^{-1}]$ for frequency range 850–2000 [Hz].

Frequency band 750–850 [Hz] was not taken into account, because of the occurrence of the effect of self-resonance of the inductor with the mounted piston. A piston of $b = 0.005\,[\mathrm{m}]$ thickness was placed in a square acoustic baffle with side length $l = 2\,[\mathrm{m}]$ and wall thickness $g = 0.07\,[\mathrm{m}]$, attached to the soundproof casing of the inductors.

Fulfiling the condition $l > \lambda$, the lower measuring frequency was fixed at 200 Hz.

The condition, $ka = 2$, for the analysed piston corresponds to a frequency of 775 Hz.

With the above system parameters, there should be a conformity between the calculation and measurement results in the far field in the frequency range 200–775 Hz.

In order to check quantitatively the discrepancies in the higher frequency range, the measuring range was widened to 2000 Hz. Preliminary tests of the distribution of piston vibrations have been conducted in order to determine to what extent the physical model complies with the theoretical model of two rigid pistons.

To this end following measurements were performed:

— measurement of the attenuation diagram of the vibrations acceleration, velocity and displacement, for the midpoint of a single piston,

— measurement of the characteristic of phase shifts between the vibration velocities of two points on the piston surface.

Measurement results have led to the following conclusions:

— the resonance frequency of the system equals 1100 Hz,

— the phase shift angle between the vibration acceleration of two points on the piston surface is constant and equals 0° in frequency range 200–750 Hz. Only for several narrow frequency bands it achieves the values between 10°–15°. At a frequency of 600 Hz a clear local change of the phase shift angle, achieving 90°, occurs,

— quick, transient changes of the phase shift angle in frequency range 750–850 Hz,

— the phase shift is constant and equals 180° in the range 850–2000 Hz; only in the range 1070–1120 Hz sudden changes of the phase shift occur, caused by resonance,

— the phase shift between the accelerations of vibrations of piston midpoints is constant in frequency ranges 200–750 Hz and 1250–2000 Hz,

— transient changes of the phase shift angle occur in the range 750–850 Hz.

Research has resulted in accepting the following measuring ranges: 200–750 Hz and 850–2000 Hz.

4. Theoretical and experimental investigations of the acoustical field distribution

The aim of the research was to determine the true and theoretical directional radiation patterns for a given model of a system of two pistons vibrating in an acoustic baffle. The measurements of the true patterns were conducted in an anechoic chamber. Sources were excited to vibrate monoharmonically with an equal amplitude by a system consisting of a sinusoidal signal generator which allows simultaneous generation of two signals shifted by an arbitrary angle, and two power amplifiers. Piston vibrations were controlled by a system of two accelerators and charge amplifiers, and a two-channel oscilloscope. Additionally, the phase shift angle between the vibration velocities of the pistons was measured

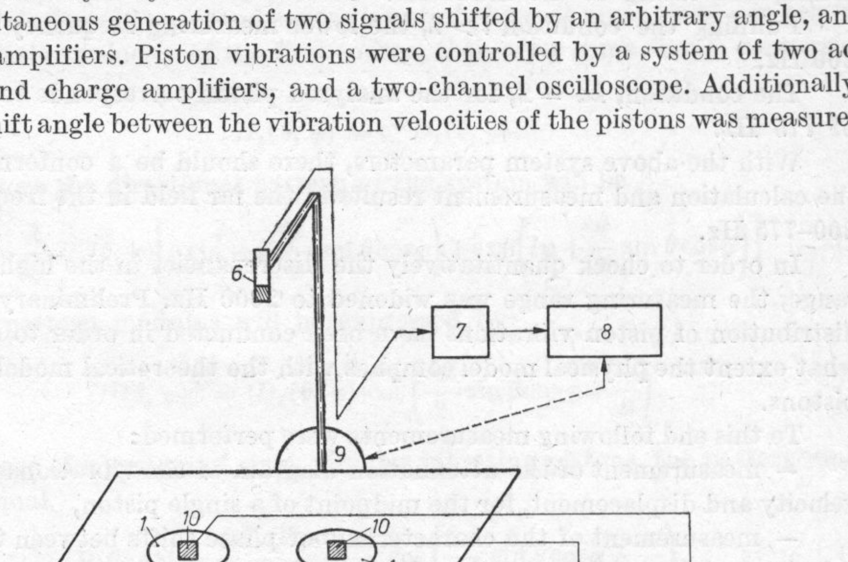
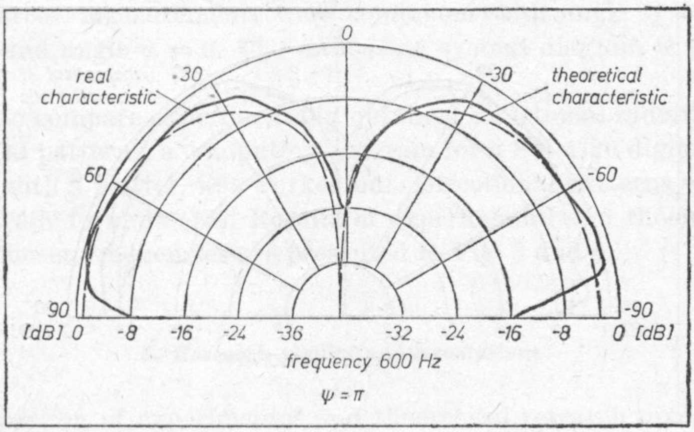
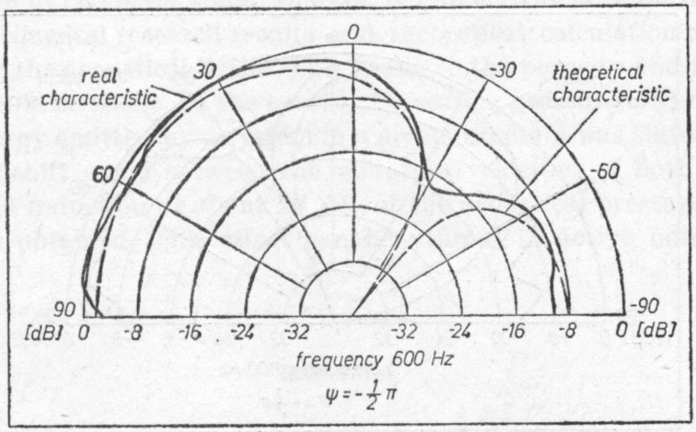


Fig. 2. Measuring system diagram: *1* — pistons, *2* — acoustical baffle, *3* — vibrations inductor, *4* — power amplifier PO–21, *5* — generator EMG 1117/6, *6* — microphone 4133 B&K, *7* — measuring amplifier 2607 B&K, *8* — level registrator 2307 B&K, *9* — rotary table 3921 B&K, *10* — accelerometer 4343 B&K, *11* — charge amplifier 2635 B&K, *12* — oscilloscope STD 501 XY, *13* — phasemeter 2971 B&K





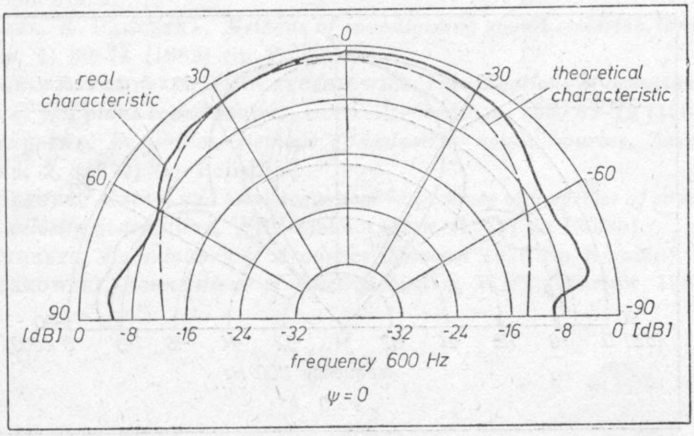


Fig. 3. Directional patterns of a piston system for a frequency forcing vibrations of 600 [Hz]

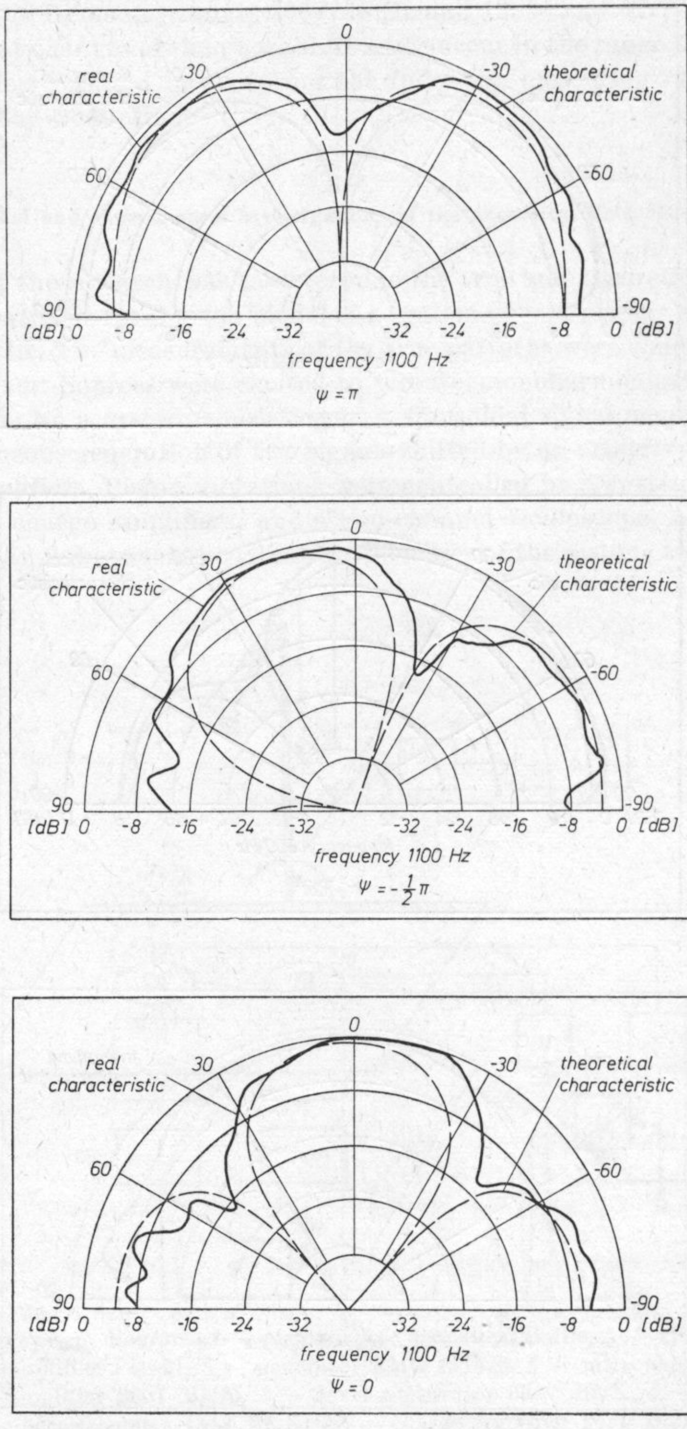


Fig. 4. Directional patterns of a piston system for a frequency forcing vibrations of 1100 [Hz]

by a phasemeter. The microphone descirbed a circle, 1 m in diameter, around the sound sources. Measurements were conducted with angle Q varying from $-\pi/2$ to $\pi/2$ and angle $\varphi = 0$. The measuring system diagram is presented in Fig. 2.

In order to compare experimentally obtained directional radiation patterns with theoretical patterns, a computing program for a SM 4/20 digital computer, collaborating with a plotter, was worked out. Directional patterns were calculated directly from formula (15). Results of experimental and theoretical investigations for chosen frequencies are presented in Fig. 3 and 4.

5. Research results and conclusions

The comparison of experimental and theoretical research gives their good qualitative conformity. Therefore we can see that the applied mathematical model of a system of flat plane sound sources, is proper. The greatest discrepancies between experimental research results and theoretical calculation results were observed near the acoustical baffle. This is due to the porosity and flexibility of the applied acoustic baffle. In the course of research, great diversification of the acoustical energy emitted by a system in a given direction was stated depending on the phase shift angle between the vibration velocities of both sources. In certain cases a reduction of about 20 dB of the acoustical pressure on a given direction was obtained. This effect can be utilized in active noise reduction methods.

References

[1] S. Czarnecki, Z. Engel, R. Panuszka, *Acoustical power measurements of a piston in a near field with the application of the correlation method,* Archives of Acoustics, **11, 3,** 261–273 (1976) (in Polish).

[2] Z. Engel, R. Panuszka, *Methods of investigating sound emission through a circular plate,* Mechanika, **1**, 59–74 (1982) (in Polish).

[3] Z. Engel, R. Panuszka, L. Stryczniewicz, *Investigations of acoustical energy emission of a system of flat plane sound sources,* CIOP Proceedings, **125**, 63–72 (1985) (in Polish).

[4] R. Panuszka, *Evaluation methods of industrial sound sources,* Zeszyty Naukowe AGH, Mechanika, **2**, (1983) (in Polish).

[5] W. Rdzanek, *Mutual and total acoustical impedance of a system of sources of variable planar vibration velocity distribution,* WSP Zielona Góra, (1979) (in Polish).

[6] E. Skudrzyk, *Foundations of Acoustics,* Moscow 1976 (in Russian).

[7] Z. Żyszkowski, *Foundations of Electroacoustics,* WNT, Warsaw 1984 (in Polish).