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CALCULATION OF ULTRASONIC FIELD DISTRIBUTION RADIATED BY PLANAR TRANSDUCERS*

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This paper presents a method for numerical calculation of amplitude and phase distribution of ultrasonic fields radiated by a planar transducer in the case of the different boundary conditions. Some theoretical consideration are reported that yield to the numerical approach for the investigated problems. Limited number of calculations are presented to illustrate facilities of the programme invented for this purpose.

1. Introduction

Amplitude and phase distribution of ultrasonic fields transmitted as well as received is of the great importance to many investigators. The assumption of the plane-wave is commonly used, however, it is not fulfiled in most cases. When the real distribution is considered, especially in such applications as calibration of the transducers, velocity and attenuation measurements, ultrasonic propagation in complex media, it helps to obtain the correct results [2-4].

2. Theory

An ultrasonic field radiated by a planar transducer can be regarded as a diffraction problem of an arbitrary incident wave $u(x, y, z)$ through an aperture A in an infinite plane screen S of vanishing thickness and can be solved by means of integral equations. Any solution u of the wave equation that is regular inside a close surface Σ is expressible in terms of the boundary values on Σ of either u or $\partial u/\partial n$, at least if the corresponding Green's functions of

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the first and second kinds are known [1]. (Σ consists of an infinite plane $S+A$ and a half-sphere at infinity). Accordingly, if u is any wave function that is regular for $z \geqslant 0$ and satisfies appropriate conditions at infinity [1] the Rayleigh formulae express u:

$$u = -\frac{1}{2\pi}\int \frac{\partial u}{\partial n}\frac{\exp(ikr)}{r}d\Sigma, \tag{1}$$

or

$$u = \frac{1}{2\pi}\int u\frac{\partial}{\partial n}\left(\frac{\exp(ikr)}{r}\right)d\Sigma, \tag{2}$$

where the integration is over the plane $z' = 0$ (a planar diffraction problem) and r denotes the distance between the field point (x, y, z) and the source point (x', y', z').

If two different boundary condition are considered

$$u|_S = 0 \quad \text{soft screen}, \tag{3}$$

$$\left.\frac{\partial u}{\partial n}\right|_S = 0 \quad \text{– rigid screen (or baffled transducer)}, \tag{4}$$

and applied to (1) and (2) the solutions of the problems are obtained

$$u = \frac{1}{2\pi}\int\limits_A u\frac{\partial}{\partial n}\left(\frac{\exp(ikr)}{r}\right)d\Sigma \quad \text{soft screen}, \tag{5}$$

$$u = -\frac{1}{2\pi}\iint\limits_A \left(\frac{\exp(ikr)}{r}\right)\frac{\partial u}{\partial n}d\Sigma \quad \text{rigid screen}. \tag{6}$$

The other well known approximation to this problems is Kirchhoff's approximation

$$u = \frac{1}{4\pi}\int\limits_A\left\{\frac{\exp(ikr)}{r}\frac{\partial u}{\partial n} - u\frac{\partial}{\partial n}\left(\frac{\exp(ikr)}{r}\right)\right\}dA. \tag{7}$$

There is an inconsistency of Kirchhoff's theory because u and $\partial u/\partial n$ cannot be simultaneously prescribed on A, and that is why there exist various modification of Kirchhoff's theory. Kirchhoff's solution can be interpreted as the average of two Rayleigh's solutions. These three formulae give different results only within the near field of the aperture, and at a distance from it integrals (5), (6) and (7) tend to the same value.

3. Numerical approach

The programme was invented to calculate pressure distribution of the ultrasonic field in the plane parallel to the transducer face. The source was assumed to be planar and in that case Rayleigh's formulae can be treated as the transform relation

$$p(x, y, z_{\text{const}}) = f_1(x, y, 0) ** f_2(x, y, z), \tag{8}$$

where $**$ denotes twodimensional convolution with respect to x and y and

$$f_1(x, y, 0) = \begin{cases} \dfrac{\partial p}{\partial n} & \text{rigid boundary condition} \\ p & \text{soft boundary condition} \end{cases} \tag{9}$$

$$f_2(x, y, z) = \begin{cases} \dfrac{1}{2\pi} \dfrac{\exp(-ikr)}{r} & \text{rigid boundary condition} \\ -\dfrac{1}{2\pi} \dfrac{\partial}{\partial n} \left(\dfrac{\exp(-ikr)}{r} \right) & \text{soft boundary condition} \end{cases} \tag{10}$$

The computations were performed by a convolution approach utilizing a Fast Fourier Transform routine. The programme was run on a PRIME computer, however, in order to optimize it, some parts were implemented by the Array Proccessor (*AP*).

4. Results

All calculations presented were performed for a circular transducer of a 5.7 mm radius and with a frequency of 0.948 MHz radiating into water. The uniform distribution on the transducer surface was assumed.

In Fig. 1 the amplitude and phase distributions are shown at a distance of 10 mm away from the transducer for three boundary conditions: rigid, soft boundary condition, and Kirchhoff's approximation. For quantitative comparison several cross-sections for previous results are shown in Fig. 2.

Computer simulated formation of ultrasonic field in the case of the rigid boundary condition (baffled transducer) is shown in Fig. 3.

5. Conclusions

Using the approach presented in this paper one can calculate the real field distribution, both amplitude and phase, radiated by a planar transducer for different boundary conditions, different shape of the transducers and different distribution across the transducer surface. The latter can be assumed theoretically [6] or obtained experimentally [7].

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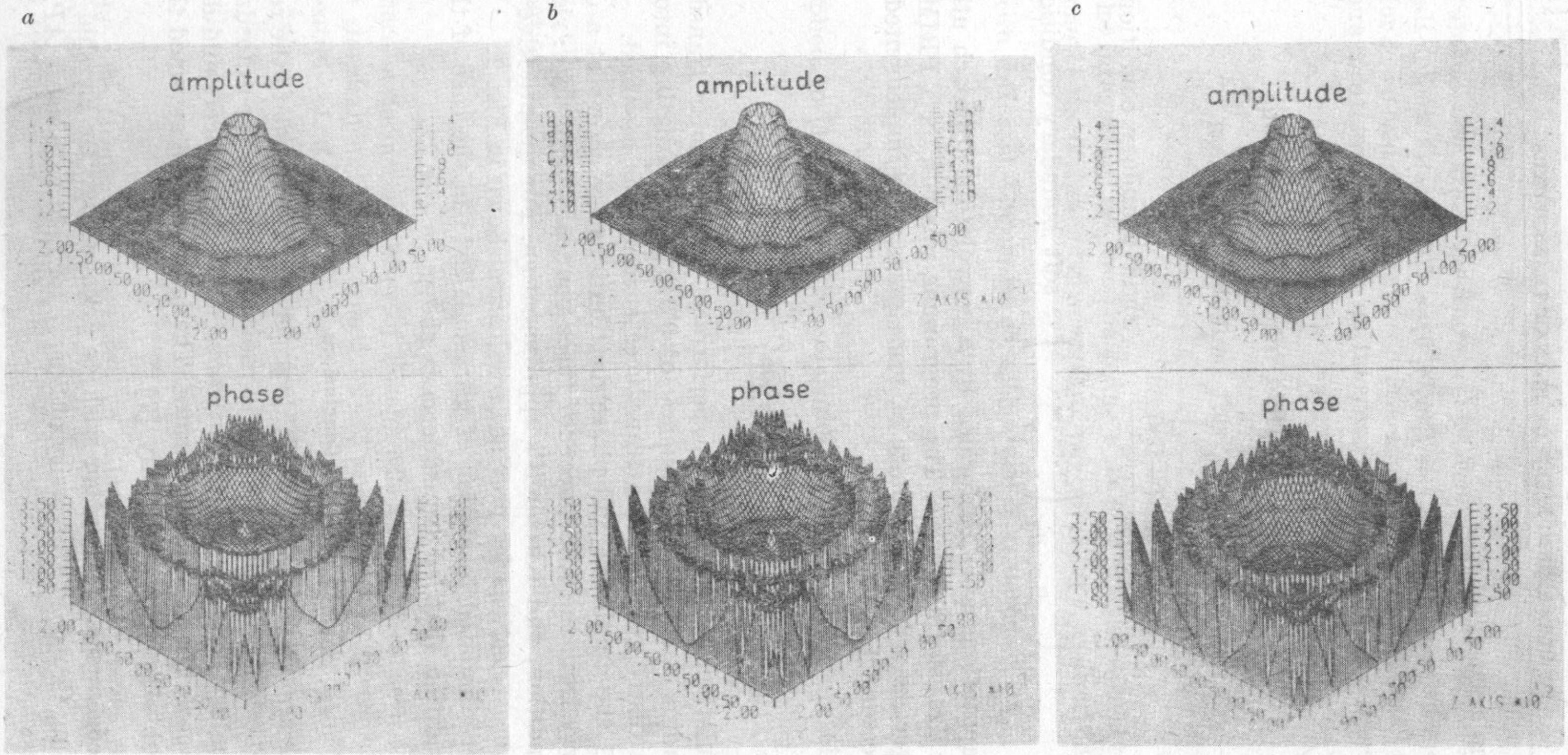
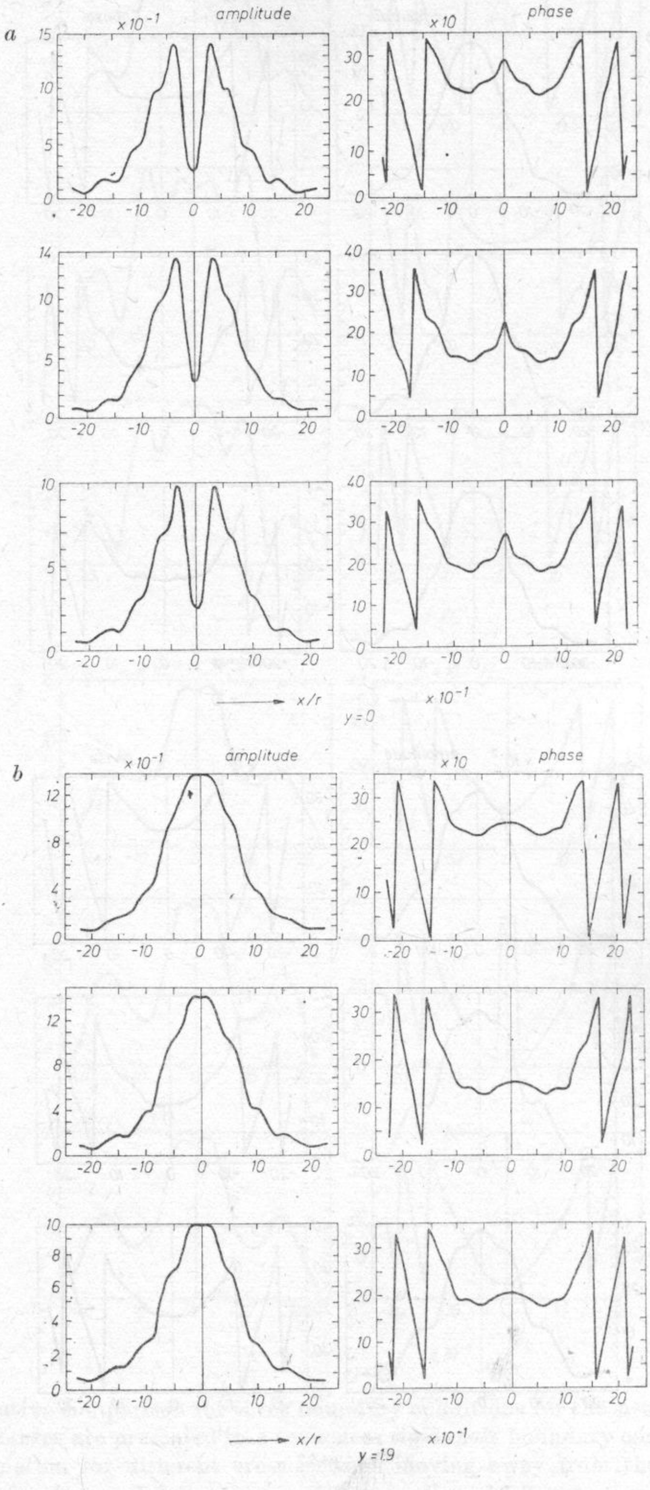


Fig. 1. Calculation of ultrasonic field distribution for the transmitter of 0.948 MHz frequency and 5.7 mm radius in the plane parallel to the transducer face at a distance of 10 mm away from it for different boundary conditions: *a*) rigid boundary, *b*) soft boundary condition, *c*) Kirchhoff's approximation. x and y axes are normalized to the radius of the transducer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amplitude
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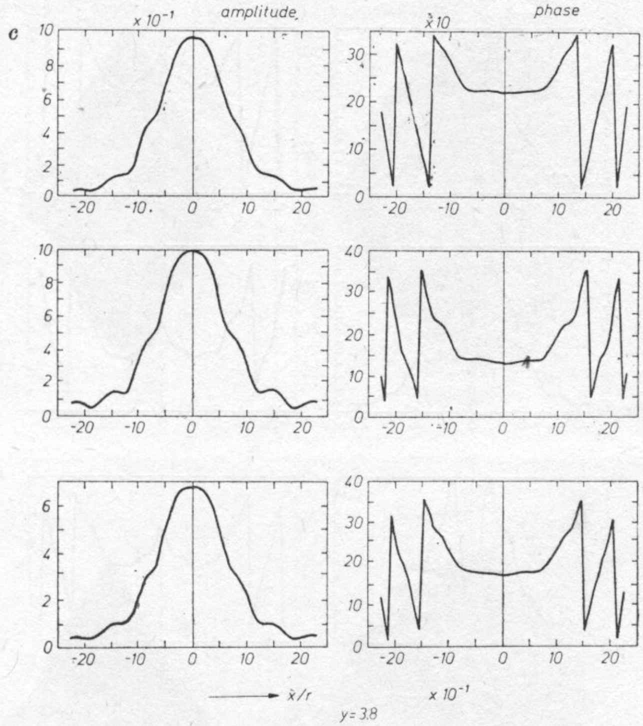
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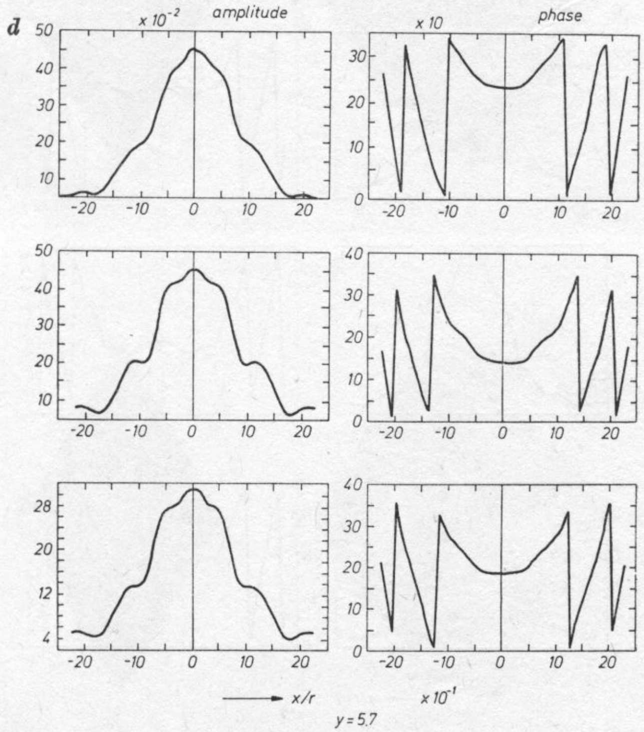
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x 10^-2
phase
x 10
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y = 5.7

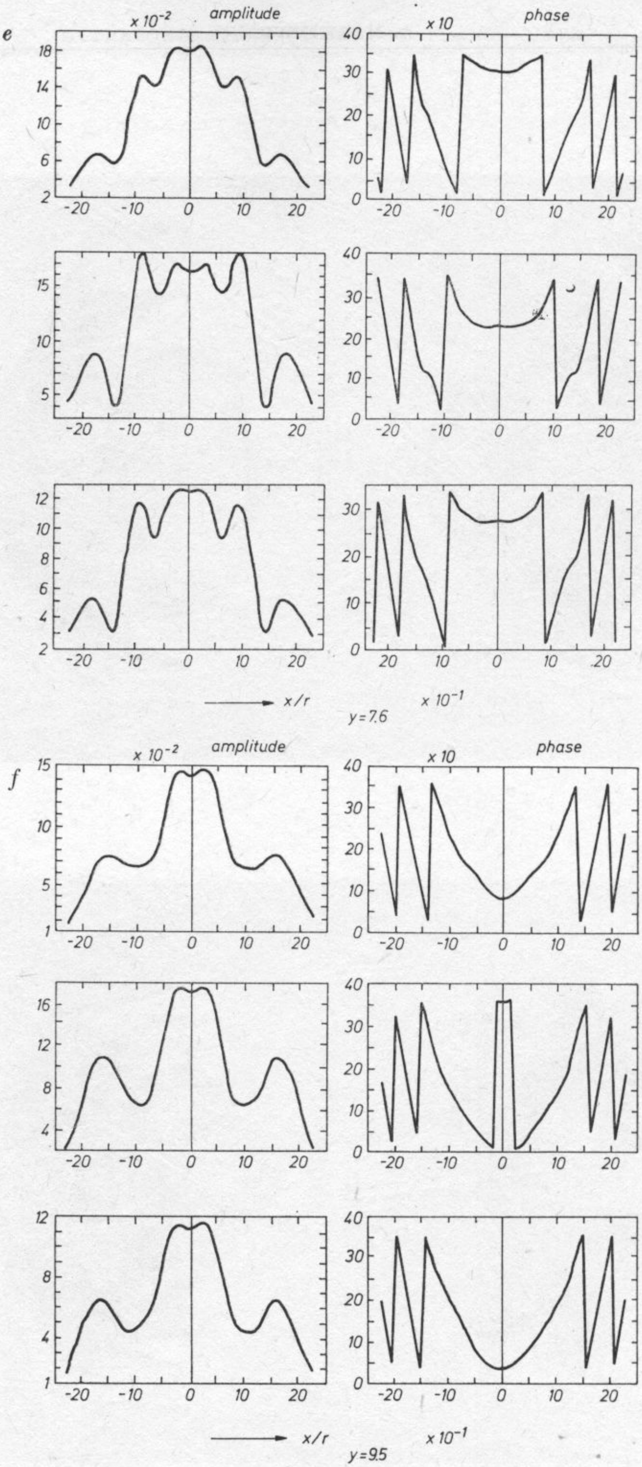
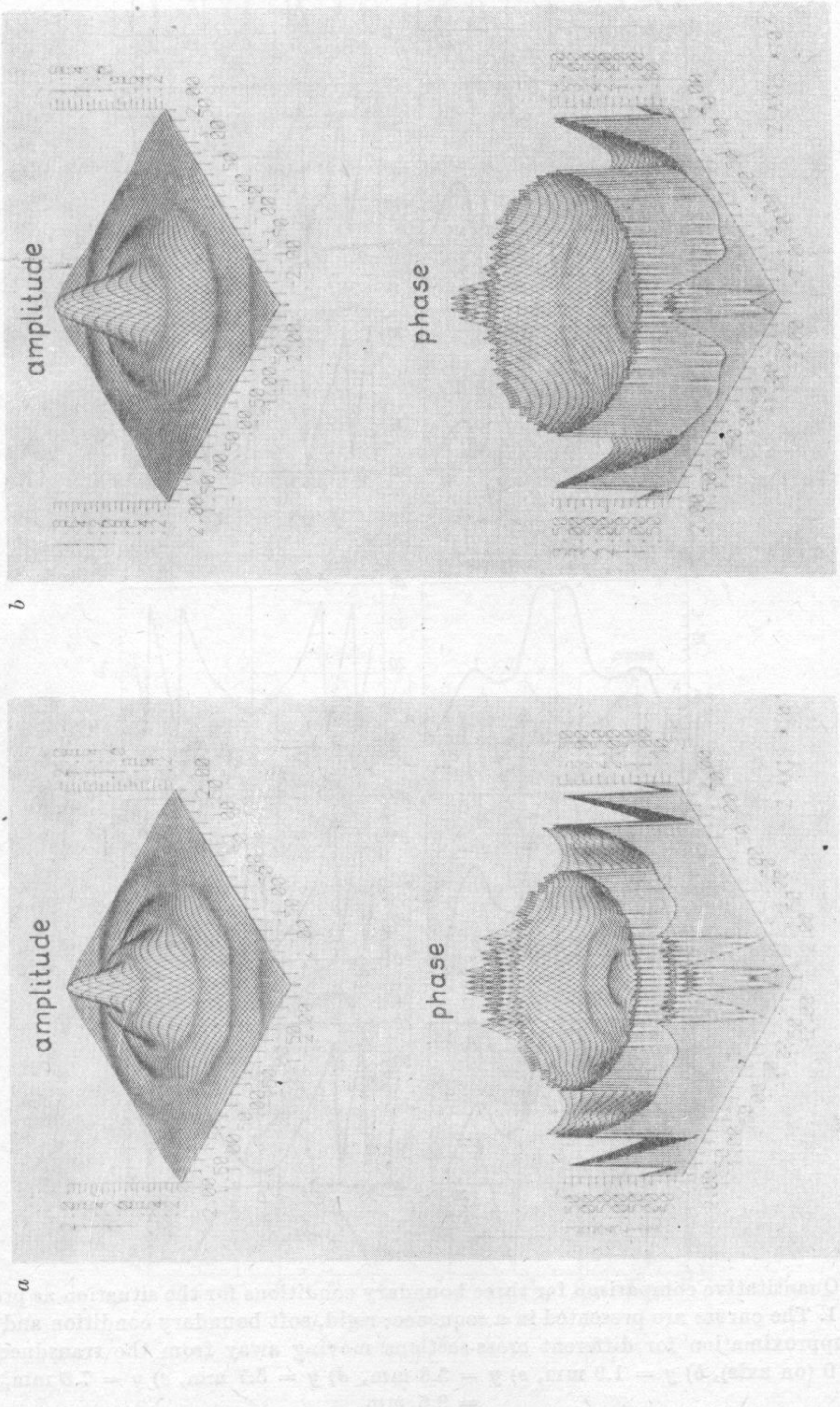


Fig. 2. Quantitative comparison for three boundary conditions for the situation as presented in Fig. 1. The curves are presented in a sequence: rigid, soft boundary condition and Kirchhoff's approximation for different cross-sections moving away from the transducer axis: *a*) $y = 0$ (on axis), *b*) $y = 1.9$ mm, *c*) $y = 3.8$ mm, *d*) $y = 5.7$ mm, *e*) $y = 7.6$ mm, *f*) $y = 9.5$ mm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amplitude
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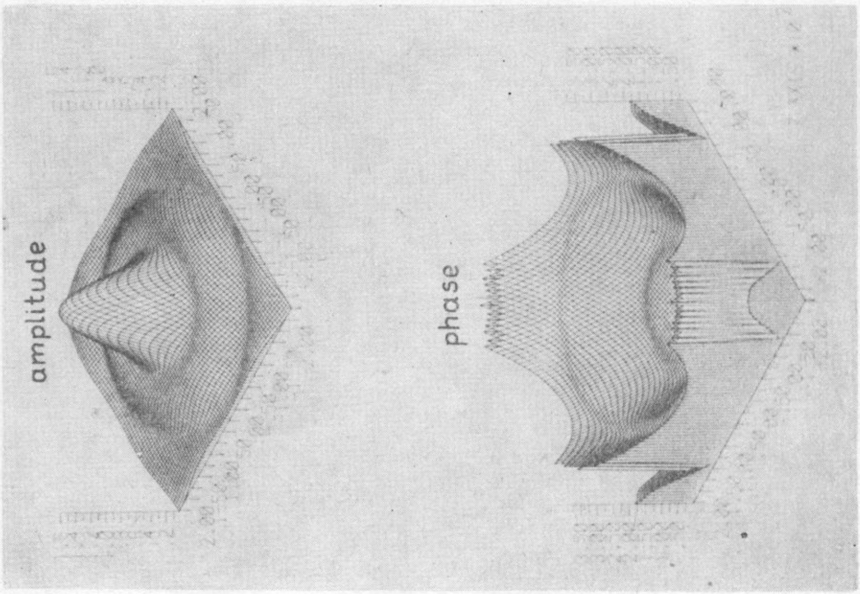

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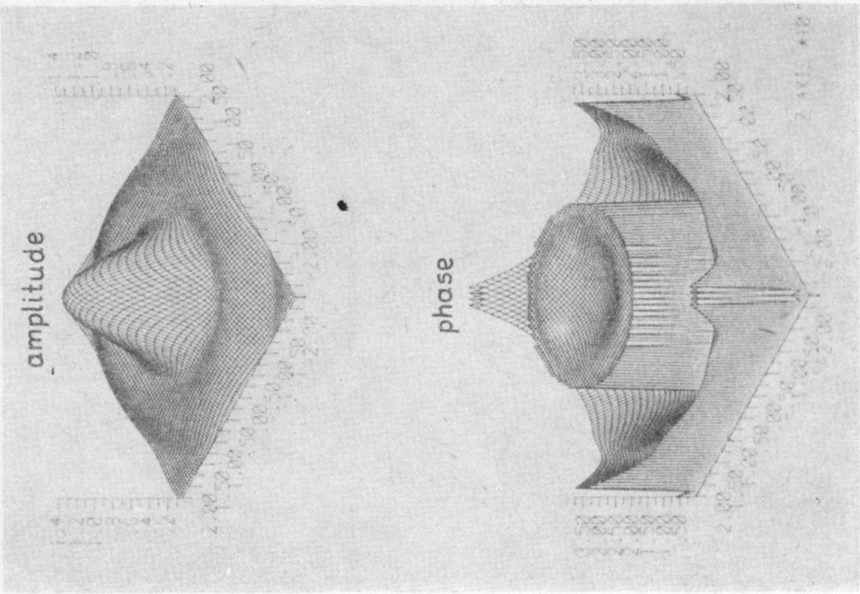

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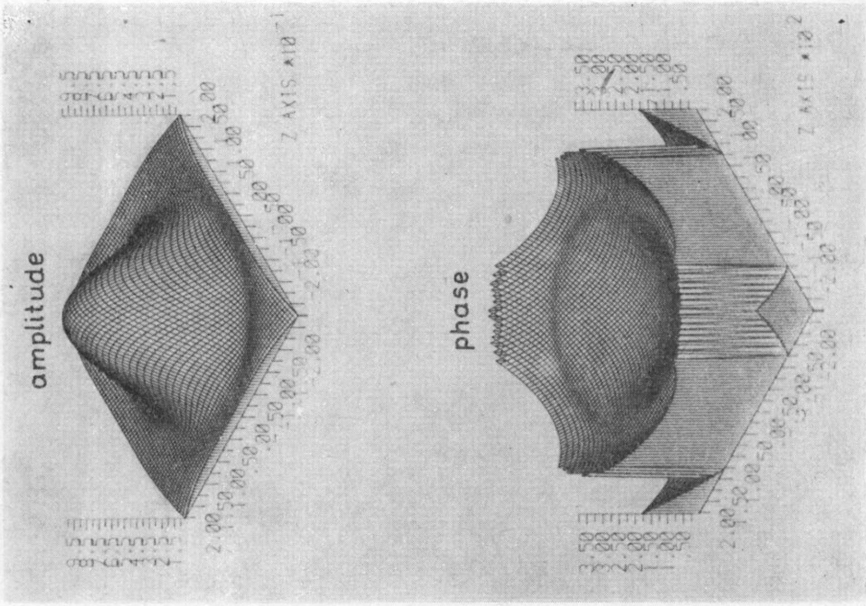

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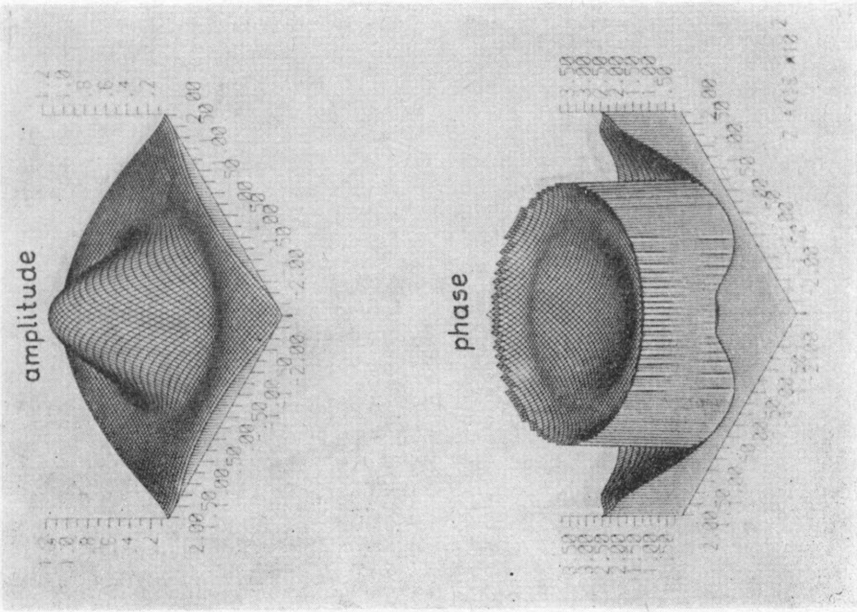
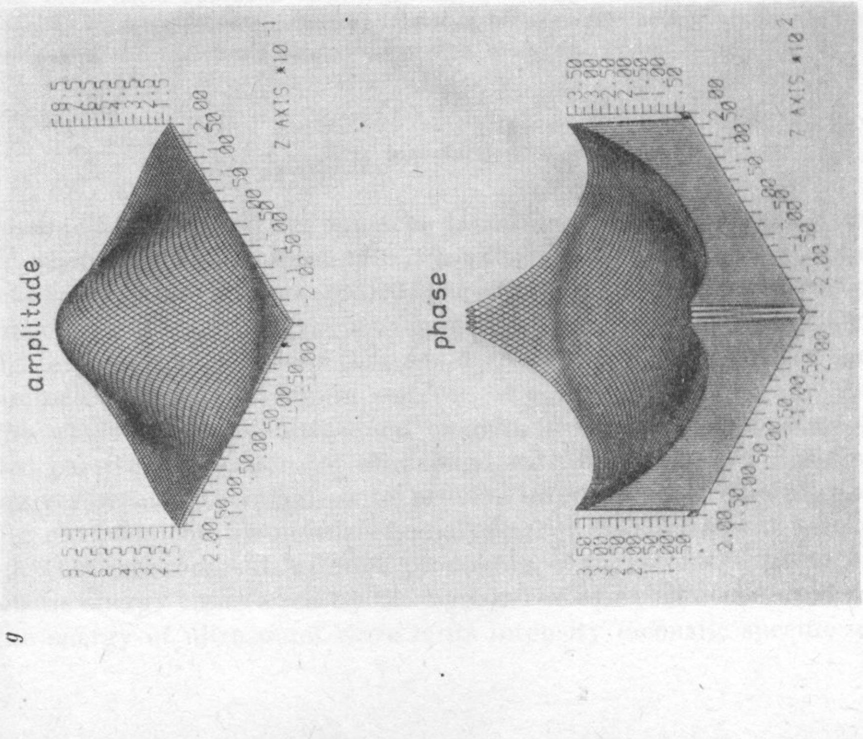


e

Fig. 3. Computer simulated formation of ultrasonic field radiated by a baffled transducer (of 0.948 MHz frequency and 5.7 mm radius) in the plane parallel to the transducer face at distances: *a)* $z = 18$ mm, *b)* $z = 27$ mm, *c)* $z = 36$ mm, *d)* $z = 45$ mm, *e)* $z = 54$ mm, *f)* $z = 63$ mm, *g)* $z = 72$ mm; x and y axes are normalized to the radius of the transducer



g

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