ARCHIVES OF ACOUSTICS
9, 3, 305-316 (1984)

THE ACOUSTIC FIELD ON THE AXIS OF A CIRCULAR CONE

HENRYK TYGIELSKI, WITOLD RDZANEK

Department of Theoretical Physics, Higher Pedagogical School
(65-562 Zielona Góra, pl. Słowiański 6)

It was assumed in this paper that the sound source was placed on the surface of an ideal rigid circular cone. The vibration velocity amplitude at the source was constant. Solution of the wave equation in a system of spherical coordinates, by using the Kontorovich-Lebedev transformation, gave the acoustic potential. Expressions for the acoustic pressure on the axis of a circular cone were derived, and these calculations were represented graphically.

Notation

a, b – radial coordinates of the sound source
c – sound velocity
$H_\mu^{(2)}$ – cylindrical Hankel function of the μth order, of the second kind
J_μ – cylindrical Bessel function of the μth order
k – wave number
p – acoustic pressure
P_μ – Legendre function
r – coordinate in a spherical system
t – time
z – coordinate in a Cartesian system
v_0 – normal component of the vibration velocity amplitude on the surface of the source
β – conical angle (measured from the axis z to the surface of the cone)
θ – angular coordinate in a spherical system
μ – variable occurring in the Kontorovich-Lebedev transformation
ν_n – nth root of equation (20)
ϱ – density of the medium
Φ – acoustic potential
ω – angular frequency

1. Introduction

Vibrating planar, cylindrical or spherical surfaces are among the most frequent practical surface acoustic sources and the deeply investigated fields radiated by these sources.

There is less knowledge on the acoustic field distribution radiated by sources with more complex geometry, e.g. vibrating spheroidal or conical surfaces.

In paper [1] CARLISE considered a vibrating source element on a cone as a system of pairs of point sources. On the basis of the results obtained, he analysed the radiation conditions of a conical loudspeaker and gave experimental results.

The problems of the acoustic field of a source placed on a cone, the latter being in an ideal rigid and planar baffle, were considered in the papers of SLUSARENKO and DOBRUCKI [4, 14]. These authors, using the Rayleigh-Huygens integral, derived an expression for the acoustic pressure distribution. However, these results were approximate and can only be used in calculating pressures at a large distance from the source for some conical angles.

In his paper [15] TYGIELSKI considered the problem of the acoustic field of a source situated on the surface of an infinitely long, ideal rigid cone with circular termination. He solved the inhomogeneous equation for a Green function in a system of spherical coordinates. Integrating the Green function over the surface of the source, he obtained the acoustic potential. He also considered the case of the acoustic field at a large distance from the top of the cone.

The acoustic field of a point source close to an ideal rigid or an ideal compliant cone was analysed in the papers by CARSLAW [3], FELSEN [5, 6] and VAYSLEYB [16].

The present paper considered the problem of the acoustic field of a source situated on the surface of an ideal rigid, infinitely long circular cone. It was assumed that the normal component of the vibration velocity at the source was constant. Solution of the wave equation in a system of spherical coordinates, using the Kontorovich-Lebedev transformation, gave the acoustic potential. An expression was given for the acoustic pressure on the axis of the cone. Assumption that the conical angle was $\pi/2$ led to formulae representing the pressure on the axis of a source situated in an ideal rigid, planar baffle, which are known from the literature. These calculations were represented graphically.

The expressions derived in this paper can be used to calculate the acoustic pressure at any distance from the source, with conical angles from 0 to π.

2. Acoustic potential of the cone

On an ideal rigid, infinitely long conical baffle with a divergence angle β there is a surface sound source ($a \leqslant r \leqslant b$, $0 \leqslant \varphi < 2\pi$) with a uniform vibration velocity amplitude distribution (Fig. 1). The top of the cone is at the origin of the coordinate system. The radiation area is defined as follows: $0 \leqslant r < \infty$, $0 \leqslant \theta \leqslant \beta < \pi$, $0 \leqslant \varphi < 2\pi$.

The acoustic potential for the time dependence of the $\exp(i\omega t)$ type satisfies the wave equation

$$\Delta\Phi(\mathbf{r}) + k^2\Phi(\mathbf{r}) = 0, \tag{1}$$

where $\mathbf{r}$ – the tracing vector of the observation point, $k = \omega/c$ – the wave number. This equation is solved with the Neumann boundary condition. In

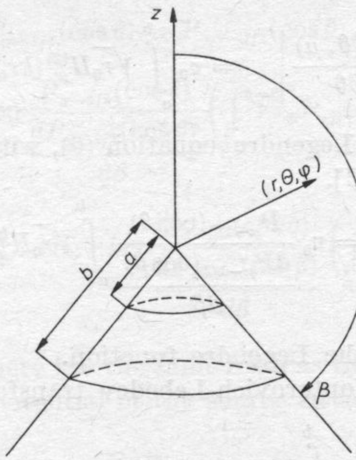

Fig. 1. The sound source on the surface of a revolution cone

view of the axial symmetry of the sound source, equation (1) can be written in a system of spherical coordinates

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi(r,\theta)}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi(r,\theta)}{\partial\theta}\right) + k^2\Phi(r,\theta) = 0. \tag{2}$$

It is considered in the above equation that the acoustic potential does not depend on the angle variable φ. The Neumann boundary condition becomes

$$\frac{1}{r}\frac{\partial\Phi(r,\theta)}{\partial\theta}\bigg|_{\theta=\beta} = \begin{cases} v_0 & \text{for the source,} \\ 0 & \text{beyond the source.} \end{cases} \tag{3}$$

In order to eliminate the variable r from equation (2), the following substitution can be used:

$$\Phi(r,\theta) = \frac{\Phi_0(r,\theta)}{\sqrt{r}}. \tag{4}$$

This substitution gives an equation in which the radial part of the Laplace operator occurs in a cylindrical coordinate system. Use of the Kontorovich-Lebedev transformation [9]

$$\Psi(\theta,\mu) = \int_0^\infty \Phi_0(r,\theta)\,\frac{H_\mu^{(2)}(kr)}{r}\,dr \tag{5}$$

and consideration that the Hankel function $H_\mu^{(2)}(kr)$ satisfies the Bessel equation [8, 12] lead to the equation with one independent variable θ

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Psi(\theta,\mu)}{\partial\theta}\right)+\left(\mu-\frac{1}{2}\right)\left(\mu+\frac{1}{2}\right)\Psi(\theta,\mu)=0. \tag{6}$$

The boundary condition (3), when considering transformation (5), becomes

$$\left.\frac{\partial\Psi(\theta,\mu)}{\partial\theta}\right|_{\theta=\beta}=v_0\int_a^b\sqrt{r_0}H_\mu^{(2)}(kr_0)\,dr_0. \tag{7}$$

The solution of the Legendre equation (6), with the boundary condition (7), is the function [7, 12].

$$\Psi(\theta,\mu)=v_0\frac{P_{\mu-1/2}(\cos\theta)}{\dfrac{dP_{\mu-1/2}(\cos\beta)}{d\beta}}\int_a^b\sqrt{r_0}H_\mu^{(2)}(kr_0)\,dr_0, \tag{8}$$

where $P_{\mu-1/2}$ represents the Legendre function.

Using the inverse Kontorovich-Lebedev transformation [9]

$$\Phi_0(r,\theta)=\frac{1}{4i}\int_a^b\mu\exp(-i\mu\pi)\sin\mu\pi\Psi(\theta,\mu)H_\mu^{(2)}(kr)\,d\mu \tag{9}$$

and from formula (4),

$$\Phi(r,\theta)=\frac{v_0}{4i\sqrt{r}}\int_{-i\infty}^{+i\infty}\mu\exp(-i\mu\pi)\sin\mu\pi H_\mu^{(2)}(kr)\frac{P_{\mu-1/2}(\cos\theta)}{\dfrac{dP_{\mu-1(2}(\cos\beta)}{d\beta}}\times$$

$$\times\int_a^b\sqrt{r_0}H_\mu^{(2)}(kr_0)\,dr_0\,d\mu. \tag{10}$$

In order to calculate the integral over the variable μ, formula (10), representing the acoustic potential, can be changed to another form. From the expression [8, 12]

$$H_\mu^{(2)}(kr_0)=\frac{\exp(i\mu\pi)J_\mu(kr_0)-J_{-\mu}(kr_0)}{i\sin\mu\pi}, \tag{11}$$

$$\Phi(r,\theta)=-\frac{v_0}{4\sqrt{r}}\int_{-i\infty}^{+i\infty}\mu H_\mu^{(2)}(kr)\frac{P_{\mu-1/2}(\cos\theta)}{\dfrac{dP_{\mu-1/2}(\cos\beta)}{d\beta}}\left(\int_a^b\sqrt{r_0}J_\mu(kr_0)\,dr_0\right)d\mu+$$

$$+\frac{v_0}{4\sqrt{r}}\int_{-i\infty}^{+i\infty}\mu\exp(-i\mu\pi)H_\mu^{(2)}\frac{P_{\mu-1/2}(\cos\theta)}{\dfrac{dP_{\mu-1/2}(\cos\beta)}{d\beta}}\left(\int_a^b\sqrt{r_0}J_{-\mu}(kr_0)dr_0\right)d\mu, \tag{12}$$

where J_μ is a Bessel function of the nth order. A new variable, $\mu = -\nu$, is introduced to the other of these integrals. From the relations for the Hankel function [12]

$$H_\nu^{(2)}(kr) = \exp(i\nu\pi) H_{-\nu}^{(2)}(kr) \tag{13}$$

and the Legendre function [7, 12]

$$P_{\nu-1/2}(\cos\theta) = P_{-\nu-1/2}(\cos\theta), \tag{14}$$

$$\int_{-i\infty}^{+i\infty} \mu \exp(-i\mu\pi) H_\mu^{(2)}(kr) \frac{P_{\mu-1/2}(\cos\theta)}{\dfrac{dP_{\mu-1/2}(\cos\beta)}{d\beta}} \left(\int_a^b \sqrt{r_0} J_{-\mu}(kr_0)\, dr_0 \right) d\mu$$

$$= -\int_{-i\infty}^{+i\infty} \nu H_\nu^{(2)}(kr) \frac{P_{\nu-1/2}(\cos\theta)}{\dfrac{dP_{\nu-1/2}(\cos\beta)}{d\beta}} \left(\int_a^b \sqrt{r_0} J_\nu(kr_0)\, dr_0 \right) d\nu. \tag{15}$$

The integral derived here has the same form as the first integral in formula (12). Hence the acoustic potential of the source situated on the cone becomes

$$\Phi(r, \theta) = -\frac{v_0}{2\sqrt{r}} \int_{-i\infty}^{+i\infty} \mu H_\mu^{(2)}(kr) \frac{P_{\mu-1/2}(\cos\theta)}{\dfrac{dP_{\mu-1/2}(\cos\beta)}{d\beta}} \left(\int_a^b \sqrt{r_0} J_\mu(kr_0)\, dr_0 \right) d\mu. \tag{16}$$

The integral from $-i\infty$ to $+i\infty$ can be calculated by using the method for calculating contour integrals. To this end, the integration contour is complemented with a semicircle with an infinitely long radius, situated to the right of the imaginary axis. The integral over the semicircle is zero for $r \geqslant b$. This can be shown by using the asymptotic representations for the Bessel [9] and Legendre functions [3, 7]. For high values of μ

$$J_\mu(kr) \int_a^b \sqrt{r_0} J_\mu(kr_0)\, dr_0 \sim \frac{1}{2\pi} \left(\frac{kr}{2}\right)^\mu \frac{\exp(2\mu)}{\mu^{2(\mu+1)}} \left[b^{3/2} \left(\frac{kb}{2}\right)^\mu - a^{3/2} \left(\frac{ka}{2}\right)^\mu \right], \tag{17}$$

$$J_{-\mu}(kr) \int_a^b \sqrt{r_0} J_\mu(kr_0)\, dr_0 \sim \frac{\sin\mu\pi}{\pi\mu^2} \left[b^{3/2} \left(\frac{b}{r}\right)^\mu - a^{3/2} \left(\frac{a}{r}\right)^\mu \right], \tag{18}$$

$$\frac{P_{\mu-1/2}(\cos\theta)}{\dfrac{dP_{\mu-1/2}(\cos\beta)}{d\beta}} \sim \frac{1}{\mu} \exp[\pm i\mu(\beta - \theta)], \tag{19}$$

where the sign $(+)$ refers to this part of the right half-plane where $\operatorname{Im}(\mu) > 0$; the sign $(-)$, to that with $\operatorname{Im}(\mu) < 0$.

The subintegral function occurring in formula (16) has its poles at points where

$$\frac{dP_{\mu-1/2}(\cos\beta)}{d\beta} = 0. \tag{20}$$

The poles are single and occur on the real axis [2, 3, 17]. Application of residua theory to the integral over a closed contour [10] gives the acoustic potential of the source situated on a cone, in the form of an infinite series,

$$\Phi(r,\theta) = \frac{iv_0\pi}{2\sqrt{r}} \sum_{n=1}^{\infty} (2\nu_n+1) H_{\nu_n+1/2}(kr) \frac{P_{\nu_n}(\cos\theta)}{\left.\dfrac{\partial^2 P_\nu(\cos\beta)}{\partial\nu\partial\beta}\right|_{\nu=\nu_n}} \times$$
$$\times \int_a^b \sqrt{r_0} J_{\nu_n+1/2}(kr_0)\, dr_0 \tag{21}$$

for $r \geqslant b$. In this formula summation is carried out over all the positive roots $\nu_n = \mu_n - 1/2$ of equation (20).

In order to obtain the potential for the region $a \geqslant r$, it is possible to use formula (10), in which the Hankel function $H_\mu^{(2)}(kr)$ can be represented by formula (11). Proceeding in an analogous way to the previous case, we obtain

$$\Phi(r,\theta) = -\frac{v_0}{2\sqrt{r}} \int_{-i\infty}^{+i\infty} \mu J_\mu(kr) \frac{P_{\mu-1/2}(\cos\theta)}{\dfrac{dP_{\mu-1/2}(\cos\beta)}{d\beta}} \left(\int_a^b \sqrt{r_0} H_\mu^{(2)}(kr_0)\, dr_0 \right) d\mu$$

$$= \frac{iv_0\pi}{2\sqrt{r}} \sum_{n=0}^{\infty} (2\nu_n+1) J_{\nu_n+1/2}(kr) \frac{P_{\nu_n}(\cos\theta)}{\left.\dfrac{\partial^2 P_\nu(\cos\beta)}{\partial\nu\partial\beta}\right|_{\nu=\nu_n}} \int_a^b \sqrt{r_0} H_{\nu_n+1/2}(kr_0)\, dr_0. \tag{22}$$

From formulae (21) and (22), the acoustic potential can be obtained for the region $a < r < b$,

$$\Phi(r,\theta) = \frac{iv_0\pi}{2\sqrt{r}} \sum_{n=0}^{\infty} (2\nu_n+1) \frac{P_{\nu_a}(\cos\theta)}{\left.\dfrac{\partial^2 P_\nu(\cos\beta)}{\partial\nu\partial\beta}\right|_{\nu=\nu_n}} \times$$
$$\times \left[H_{\nu_n+1/2}^{(2)}(kr) \int_a^r \sqrt{r_0} J_{\nu_n+1/2}(kr_0)\, dr_0 + J_{\nu_n+1/2}(kr) \int_r^b \sqrt{r_0} H_{\nu_n+1/2}^{(2)}(kr_0)\, dr_0 \right]. \tag{23}$$

In formulae (22) and (23) summation is also carried out over all the positive roots $\nu_n = \mu_n - 1/2$ of equation (20).

3. Acoustic field on the axis of a cone

There is the following relationship between the pressure and the acoustic potential Φ for harmonic vibrations [11, 17],

$$p = i\omega\varrho\Phi, \tag{24}$$

where ω — angular frequency, ϱ — density of the medium. In order to derive expressions for the acoustic pressure of a source situated on the cone, formulae (21)–(23) should be multiplied by $i\omega\varrho$.

One of the quantities characterizing the acoustic field is the distribution of the field on the main axis of the source. It is convenient to calculate the acoustic field of a source situated on the cone on the axis z. Assuming that $\theta = 0$ and considering that

$$P_{\nu_n}(1) = 1, \tag{25}$$

the following expressions are derived for the acoustic pressure on the axis of the cone,

$$p(z) = -\frac{\omega\varrho v_0\pi}{2\sqrt{z}} \sum_{n=0}^{\infty} \frac{2\nu_n+1}{\left.\dfrac{\partial^2 P_\nu(\cos\beta)}{\partial\nu\partial\beta}\right|_{\nu=\nu_n}} H^{(2)}_{\nu_n+1/2}(kz) \int_a^b \sqrt{r_0} J_{\nu_n+1/2}(kr_0)\,dr_0 \tag{26}$$

for $z \geqslant b$,

$$p(z) = -\frac{\omega\varrho v_0\pi}{2\sqrt{z}} \sum_{n=0}^{\infty} \frac{2\nu_n+1}{\left.\dfrac{\partial^2 P_\nu(\cos\beta)}{\partial\nu\partial\beta}\right|_{\nu=\nu_n}} \left[H^{(2)}_{\nu_n+1/2}(kz) \int_a^z \sqrt{r_0} J_{\nu_n+1/2}(kr_0)\,dr_0 + \right.$$

$$\left. + J_{\nu_n+1/2}(kz) \int_z^b \sqrt{r_0} H^{(2)}_{\nu_n+1/2}(kr_0)\,dr_0\right]. \tag{27}$$

for $a < z < b$,

$$p(z) = -\frac{\omega\varrho v_0\pi}{2\sqrt{z}} \sum_{n=0}^{\infty} \frac{2\nu_n+1}{\left.\dfrac{\partial^2 P(\cos\beta)}{\partial\nu\partial\beta}\right|_{\nu=\nu_n}} J_{\nu_n+1/2}(kz) \int_a^b \sqrt{r_0} H^{(2)}_{\nu_n+1/2}(kr_0)\,dr_0 \tag{28}$$

for $a \geqslant z$.

Assumption in the above formulae that $\beta = 90^\circ$ permits the acoustic pressure on the axis of a circular ring situated in a planar, rigid baffle to be obtained. For $\beta = 90^\circ$ the roots of equation (20) are [2, 7, 12]:

$$\nu_n = 2n, \quad n = 0, 1, 2, \ldots \tag{29}$$

and

$$\left.\frac{\partial^2 P(\cos\beta)}{\partial\nu\partial\beta}\right|_{\substack{\nu=2n\\ \beta=90^\circ}} = (-1)^{n+1}\frac{2^{2n}(n!)^2}{(2n)!}. \tag{30}$$

Hence

$$p(z) = \frac{\omega\varrho v_0 \pi}{2\sqrt{z}} \sum_{n=0}^{\infty} (-1)^n \frac{(4n+1)(2n)!}{2^{2n}(n!)^2} H^{(2)}_{2n+1/2}(kz) \int_a^b \sqrt{r_0} J_{2n+1/2}(kr_0) dr_0 \tag{31}$$

for $z \geqslant b$,

$$p(z) = \frac{\omega\varrho v_0 \pi}{2\sqrt{z}} \sum_{n=0}^{\infty} (-1)^n \frac{(4n+1)(2n)!}{2^{2n}(n!)^2} \left[H^{(2)}_{2n+1/2}(kz) \int_a^z \sqrt{r_0} J_{2n+1/2}(kr_0) dr_0 + J_{2n+1/2}(kz) \int_z^b \sqrt{r_0} H^{(2)}_{2n+1/2}(kr_0) dr_0 \right] \tag{32}$$

for $a < z < b$,

$$p(z) = \frac{\omega\varrho v_0 \pi}{2\sqrt{z}} \sum_{n=0}^{\infty} (-1)^n \frac{(4n+1)(2n)!}{2^{2n}(n!)^2} J_{2n+1/2}(kz) \int_a^b \sqrt{r_0} H^{(2)}_{2n+1/2}(kr_0) dr_0 \tag{33}$$

for $a \geqslant z$.

In order to calculate the sums of series occurring in formulae (31)–(33), it is possible to use an expansion of the function $\exp(-ikR)/R$ into a series of Legendre polynomials and cylindrical functions [12],

$$\frac{\exp(-ikR)}{R} = -\frac{i\pi}{2\sqrt{rr_0}} \sum_{n=0}^{\infty} (2n+1) J_{2n+1/2}(kr_0) H^{(2)}_{n+1/2}(kr) P_n(\cos\theta) \tag{34}$$

for $r > r_0$, where $R = \sqrt{r^2 + r_0^2 - 2rr_0\cos\theta}$. Assuming in the above formula that $\theta = \pi/2$ and taking into account the value of Legendre polynomials at this point [7, 12],

$$\frac{\exp(-ikR)}{R} = -\frac{i\pi}{2\sqrt{rr_0}} \sum_{n=0}^{\infty} (4n+1) \frac{(2n)!}{2^{2n}(n!)^2} J_{2n+1/2}(kr_0) H^{(2)}_{2n+1/2}(kr), \tag{35}$$

where $R = \sqrt{r^2 + r_0^2}$.

Considering formula (35), expressions (31)–(33) can be written in the form

$$p(z) = i\omega\varrho v_0 \int_a^b \frac{\exp(-ikR)}{R} r_0 dr_0. \tag{36}$$

After integration we obtain

$$p(z) = 2i\varrho c v_0 \sin\frac{k}{2}(r_b - r_a) \exp\left[-\frac{i}{2} k(r_b + r_a)\right], \tag{37}$$

where $\omega = kc$, $r_a = \sqrt{a^2 + z^2}$, $r_b = \sqrt{b^2 + z^2}$.

Expression (37) represents the acoustic pressure on the axis of a circular ring with radii a and b. Assumption that $a = 0$ gives a formula which defines the pressure distribution on the axis of the circular piston situated in a planar, rigid baffle. An analogous formula was obtained in papers [11, 13, 17] by using other methods.

When the observation point is at a large distance from the top of the cone ($kz \gg 1$), for very low values of ka and kb ($ka < kb \ll 1$), from formula (26),

$$p(z) = \frac{i\varrho cv_0}{2}\cot\frac{\beta}{2}\left[(kb)^2-(ka)^2\right]\frac{\exp(-ikz)}{kz}. \tag{38}$$

It follows therefrom that as the divergence angle of a cone, β, increases and as the distance of the observation point from the source increases, the pressure amplitude decreases. With the above assumptions formula (38) is valid for the whole variability interval of the angle θ. This source thus lacks directionality.

Assumption that $z = 0$ in formula (28) gives the expression

$$p(z) = 2i\varrho cv_0\cot\frac{\beta}{2}\sin\frac{kb-ka}{2}\exp\left(-i\,\frac{ka+kb}{2}\right), \tag{39}$$

which represents the acoustic pressure on the top of the cone.

4. Conclusion

The expressions derived in the present paper for the acoustic potential and pressure are given in the form of infinite series. The series are divergent the faster, the greater the difference is between the observation point and

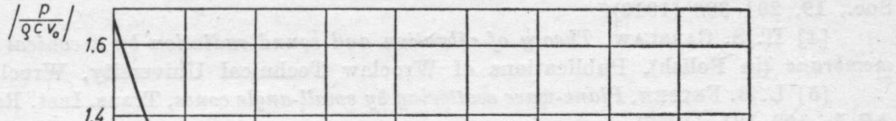

Fig. 2. The acoustic field on the axis of a circular cone. It is assumed that $a = 0$, $kb = 2$

the radial coordinates of the source (a, b). In a case when the cone divergence angle is 90°, the expression is obtained for the acoustic pressure on the axis of the circular ring, characterized by simple notation form.

On the basis of the results presented, numerical calculations were carried out of the absolute value of the relative pressure (the ratio of the absolute value of the pressure $|p|$ and the self resistance of the medium, ϱc, and the vibration velocity amplitude at the source, v_0) on the axis of the cone, depending on the parameter kz. It was assumed that $a = 0$, $kb = 2$, $\beta = 90°$, 120° and 140°. The tables of roots of equation (20), given in paper [2] were used in the calculations. The behaviour of relative pressure changes is shown in Fig. 2.

The expressions derived for the acoustic potential and pressure can be used for calculations of the acoustic far field and acoustic impedance. These problems will be considered in another paper.

References

[1] R. W. Carlise, *Conditions for wide angle radiation from conical sound radiators*, J. Acoust. Soc. Am., **15**, 1, 44–49 (1943).

[2] F. A. Carrus, C. G. Treuenfels, *Tables of roots and incomplete integrals of associated Legendre functions of fractional orders*, J. Math. and Phys., **29**, 4, 282–299 (1951).

[3] H. S. Carslaw, *Diffraction of waves by a wedge of any angle*, Proc. London Math. Soc., **19**, 291–306 (1919).

[4] H. S. Carslaw, *Theory of vibration and sound radiation by a conical loudspeaker membrane* (in Polish), Publications of Wrocław Technical University, Wrocław 1978.

[5] L. B. Felsen, *Plane-wave scattering by small-angle cones*, Trans. Inst. Radio Engrs., **AP-5**, 109–121 (1957).

[6] L. B. Felsen, *Backscattering from wide-angle and narrow-angle cones*, J. Appl. Phys., **26**, 2, 138–149 (1955).

[7] E. W. Hopson, *Teoria sfericheskikh i elipsoidalnikh funktsij*, Izdatelstvo Innostrannoy Literatury, Moscow 1952.

[8] E. Jahne, F. Emde, F. Lösch, *Spetsyalnye funktsii, formuli, grafiki, tablitsy*, Izd. "Nauka", Moscow 1977.

[9] M. I. Kontorovich, N. N. Lebedev, *Ob adnoy metode resheniya nekatorikh zadach teorii difraktsii i rodstvennikh ey problem*, Zhurn. Eks. i Teor. Fiz., **8**, 10, 11, 1192–1206 (1938).

[10] F. Leja, *Complex functions* (in Polish), PWN, Warsaw 1976.

[11] I. Malecki, *Theory of waves and acoustic systems* (in Polish), PEW, Warsaw 1964.

[12] I. M. Ryżyk, I. S. Gradsztejn, *Tables of integrals, sums, series and products* (in Polish), PWN, Warsaw 1964.

[13] Stenzel-Brosze, *Leitfaden zur Berechnung von Schallvorgängen*, Springer-Verlag, Berlin 1958.

[14] L. B. Slusarenko, *Kharakteristiki napravlennosti kruglovo usechennogo konusa*, Akusticheskij Zhurnal, **20**, 2, 327, 328 (1974).

[15] H. Tygielski, *Acoustic far field of a cone* (in Polish), Proc. of XXVI Open Seminar on Acoustics, Wrocław – Oleśnica, 1979, 508–511.

[16] Y. V. VAYSLEYB, *Rasseyanie zvukovikh voln na konechnom konuse*, Akusticheskij Zhurnal, **17**, 1, 33–42 (1971).

[17] R. WYRZYKOWSKI, *Linear theory of the acoustic field of gaseous media* (in Polish), Higher Pedagogical School, Rzeszów 1975.

[18] M. I. ZHURINA, W. M. KARMAZINA, *Tablitsy i formuli dla sfericheskikh funktsij* $P^m_{-1/2+i\tau}(x)$, **1**, **2** Izd. AN SSSR, Moscow 1960, 1962.

Received on 8 September, 1983.