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SOUND TRANSMISSION THROUGH SLITS AND CIRCULAR APERTURES

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In this paper the sound transmission through slits and circular apertures is examined. Approximate theoretical expressions for the transmission loss are derived using simple energy balance considerations. Both resonant and nonresonant transmissions are taken into account. Theoretical values of the transmission loss are compared with measurements.

1. Introduction

The transmission of sound through small apertures (circular or narrow slit-shaped) is of interest in many noise control problems and in particular in architectural acoustics. Slits and apertures are found in most typical partitions decreasing their sound insulation.

Theories for the transmission of sound through slits and circular apertures have been advanced by several authors [1, 2, 3, 4]. All these theories in spite of their exact mathematical formulation are rather complicated to be used in cases of practical interest.

The aim of this paper is to show that using simple energy balance equations well known from *S.E.A.* (statistical energy analysis) approximate expressions for the transmission loss of small apertures can be obtained. Comparison between measurements and theoretical predictions by means of the obtained formulae is found to be in good agreement for all partical cases.

2. Theoretical transmission loss

We consider the transmission suite shown in Fig. 1. The coupling element between the two rooms is a small aperture (slit or circular opening). The whole system may be considered to consist of three coupled subsystems as shown schematically in Fig. 2. Subsystems 1, 2 and 3 represent source room, aperture and receiving room respectively.

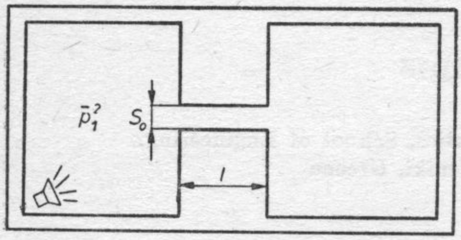

Fig. 1. Transmission suite

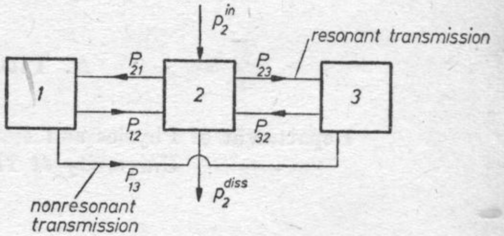

Fig. 2. Block diagram of power flow between the three coupled systems

In case of acoustical excitation of the source room, the power transmitted to the receiving room is made up of the contribution of nonresonant and resonant transmission. The nonresonant transmission, represented by the power flow P_{13} directly from system 1 to 3, is due to modes that are resonant outside of the frequency band under consideration. In this situation, system 2 acts only as a coupling element for the sound transmission between systems 1 and 3. This forced power flow is responsible for the "mass law" transmission at low frequencies. The resonant transmission, represented by the power flow P_{12}, i.e. P_{23}, is due to resonant modes and is dominant in higher frequencies as the wavelength of sound becomes comparable to the depth of the aperture.

a) Nonresonant transmission

Referring to Fig. 1 and considering that the dimensions of the opening are small compared to the wavelength, there will be a sound pressure doubling at the face S_0 of the opening due to sound reflection. If $Z_0 = R_0 + j\omega M_0$ is the acoustic impedance of the opening and M the radiation mass loading due to its finite depth, the sound power transmitted through the opening can be expressed as

$$P_{\mathrm{tr}} = \frac{4\bar{p}^2 S_0^2}{|2Z_0 + j\omega M|^2} R_0, \tag{1}$$

where $\bar{p}^2$ is the mean square pressure in the source room. For low frequencies the radiation resistance R_0 in the denominator can be neglected so that equation (1) simplifies into:

$$P_{\text{tr}} = \frac{4\overline{p}^2 S_0^2}{\omega^2(2M_0+M)^2} R_0. \tag{2}$$

In case of a circular aperture of a radius a the radiation resistance at low frequencies is [5]:

$$R_a = \varrho c \frac{a^2 k^2 S_a}{2}, \tag{3}$$

where $k = \omega/c$ the corresponding wave number.

The total effective mass $2M+M$ can be expressed as [6]:

$$2M_0+M = \frac{\varrho S_a^2}{G_a}, \tag{4}$$

where $G_a = \dfrac{S_a}{l_{\text{eff}}}$, with $l_{\text{eff}} = l+1,7a$ [5], is the effective length of the opening.

Using equations (3) and (4), the transmitted sound power can be written as:

$$P_{\text{tr}} = \frac{2\overline{p}^2 S_a}{\varrho c}\left(\frac{a}{l_{\text{eff}}}\right)^2. \tag{5}$$

Thus, the nonresonant transmission loss of a circular aperture, referring to the total wall area, will be:

$$10\log\frac{P_{\text{in}}}{P_{\text{tr}}} = 10\log\frac{\left(\dfrac{\overline{p}^2 S_w}{\varrho c}\right)}{\left(\dfrac{2\overline{p}^2 S_a}{\varrho c}\right)}\left(\frac{l_{\text{eff}}}{a}\right)^2 = 10\log\frac{1}{2}\left(\frac{S_w}{S_a}\right) + 20\log\left(\frac{l_{\text{eff}}}{a}\right), \tag{6}$$

where $P_{\text{in}} = \dfrac{\overline{p}^2 S_w}{\varrho c}$ the incident sound power on the wall area S_w.

In case of a long narrow slit the sound power transmitted can be similarly expressed as:

$$P_{\text{tr}} = \frac{4\overline{p}^2 S_s^2}{|2Z_s+j\omega M|^2} R_s, \tag{7}$$

where R_s is the radiation resistance, Z_s the impedance and S_s the surface of the slit.

The radiation resistance of a long narrow slit at low frequencies is [6]:

$$R_s = \frac{\varrho c k S_s}{4}, \tag{8}$$

where b is the breadth of the slit and S_s its cross section.

The effective mass can be expressed as [6]:

$$2M_s + M = \frac{\varrho S_s^2}{G_s} \tag{9}$$

with $1/G_s = (l/b) + 0.7 + \frac{2}{\pi} 1\eta\left(\frac{2c}{\omega b}\right)$ per unit length, where l is the depth and b the breadth of the slit. Substituting the values of R_s and effective mass in equation (7) we obtain for the transmitted power

$$P_{\text{tr}} = \frac{\overline{p}^2 L}{\omega \varrho} \frac{1}{\left[l/b + 0.7 + \frac{2}{\pi} \ln \frac{2c}{\omega b}\right]^2}, \tag{10}$$

where L is the length of the slit. The corresponding nonresonant transmission loss of the slit can be written in the form:

$$TL_s = \frac{P_{\text{in}}}{P_{\text{tr}}} = 10\log \frac{\omega S_w}{cL} G_s^2 = 10\log \frac{\omega S_w}{cL} + 20\log\left[(l/b) + 0.7 + \frac{2}{\pi} \ln \frac{c^2}{\omega b}\right]. \tag{11}$$

b. Resonant transmission

At higher frequencies, as the wavelength becomes comparable to the depth of the opening ($\lambda \sim l$), system 2 becomes resonant and modes are excited and the sound power is transmitted to the receiving room.

In this case, the power balance equation expressing the principle of energy conservation for the opening as a system (neglecting the power flow from the receiving room back to the source room) can be written as

$$p_2^{\text{in}} = p_2^d + P_{23} + P_{21}, \tag{12}$$

where p_2^{in} denotes the power supplied to the opening, p_2^d the power dissipation in the opening and P_{23}, P_{21}' the power flow from the opening (system 2) to the rooms (systems 1 and 3).

The dissipation of stored energy in system 2 can be expressed as:

$$p_2^d = \omega n_2 E_2, \tag{13}$$

where n_2 is the loss factor of the opening and E_2 the stored energy. The power flow P_{23}, P_{21} to the rooms following *S.E.A.* [7] can be respectively expressed as:

$$P_{23} = P_{21} = \omega n_{23} E_2, \tag{14}$$

where n_{23} is the parameter called coupling loss factor, representing the losses of system 2 due to its coupling to the rooms. The input power supplied to the opening is:

$$p_{\text{in}}^2 = \omega n_{12} E_1, \tag{15}$$

with n_{12} the coupling loss factor between source room and opening and E_1 the energy stored in source room.

Substituting equations (13), (14) and (15) in (12) we obtain the following equation:

$$\omega n_{12} E_1 = \omega n_2 E_2 + 2\omega n_{21} E_2. \tag{16}$$

The energy stored in the source room can be written as

$$E_1 = \frac{V_1 \bar{p}_1^2}{\varrho c^2}, \tag{17}$$

where $\bar{p}_1^2$ is the mean square pressure and V_1 the volume of the source room, while the energy stored in the opening

$$E_2 = m_2 \bar{v}_2^2, \tag{18}$$

where m_2 – the air mass and $\bar{v}_2^2$ – the mean square velocity of the opening. Combining equations (16), (17) and (18) as follows:

$$n_{12} \frac{V_1}{\varrho c^2} \bar{p}_1^2 = n_2 m_2 \bar{v}_2^2 + 2 n_{21} m_2 \bar{v}_2^2 \tag{19}$$

and solving for the mean square velocity of the opening we obtain:

$$\bar{v}_2^2 = \bar{p}_1^2 \frac{V_1}{V_2} \frac{1}{\varrho^2 c^2} \frac{n_{12}}{n_{21}} \frac{1}{2 + \frac{n_2}{n_{21}}}. \tag{20}$$

The power radiated from the opening to the receiving room P_{23}, i.e. P_{21}, can be expressed as:

$$P_{23} = P_{21} = \omega n_{21} m_2 \bar{v}_2^2 = \varrho c S_2 \sigma_2 \bar{v}_2^2, \tag{21}$$

where σ_2 is the radiation efficiency of the opening.

It follows for the coupling loss factor n_{21}:

$$n_{21} = \frac{\sigma_2}{kl}. \tag{22}$$

Furthermore, from the statistical energy analysis it is well known [7] that the ratio of the coupling loss factor in two opposite directions is inversely proportional to the corresponding model densities, i.e.

$$\frac{n_{12}}{n_{21}} = \frac{N_2}{N_1}, \tag{23}$$

where N_1, N_2 are the model densities of the source room and the opening respectively.

Using equations (22) and (23) we obtain for the mean square velocity of the opening:

$$\overline{v}_2^2 = \overline{p}_1^2 \frac{V_1}{V_2} \frac{1}{\varrho^2 c^2} \frac{N_2}{N_1} \frac{1}{2\left(1+\frac{n_2 kl}{\sigma_2}\right)}, \tag{24}$$

and for the power transmitted through the opening:

$$P_{\mathrm{tr}} = \varrho c S_2 \sigma_2 \overline{v}_2^2 = \overline{p}_1^2 \frac{V_1}{2} S_2 \frac{\sigma_2}{\varrho c} \frac{N_2}{N_1} \frac{1}{2\left(1+\frac{n_2 kl}{\sigma_2}\right)}. \tag{25}$$

In case of a circular aperture [8]

$$N_2 = \frac{1}{c\pi}, \quad N_1 = \frac{V_1 \omega^2}{2\pi^2 c^3}. \tag{26}$$

So that (25) can be written in the form:

$$P_{\mathrm{tr}} = \overline{p}_1^2 \frac{c}{\varrho c} \sigma_a \frac{1}{\left(1+\frac{n_2 kl}{\sigma_a}\right)}. \tag{27}$$

In case of a long narrow slit [8]:

$$N_2 = \frac{l S_s \omega}{4\pi c^2 b}, \tag{28}$$

so that from equation (25) it can be obtained for the transmitted power:

$$P_{\mathrm{tr}} = \overline{p}_1^2 \frac{S_s \sigma_s}{4 \varrho b \omega} \frac{\pi}{\left(1+\frac{n_2 kl}{\sigma_s}\right)}. \tag{29}$$

WILSON and SOROKA [9] have shown that the change from a circular piston to a square one produces only a slight change in radiation resistance and hence, it is quite appropriate to apply impedance functions for circular pistons to the case of a square piston by substituting the value of the radius a with the equivalent "radius" $(S_s/\pi)^{1/2}$. Exact expressions for the radiation resistance of a circular or an equivalent rectangular piston have been given earlier [9, 10] in the form:

$$R\left[2k\left(\frac{S}{\pi}\right)^{1/2}\right] = \frac{2}{\pi} \sum_{n=0}^{\infty} \frac{B_{2n}\left[k\left(\frac{S}{\pi}\right)^{1/2}\right]^{2n+2}}{(2n+3)!} \pi^{n+1}, \tag{30}$$

where B_{2n} are coefficients depending on the geometry of the opening.

Using the values of the radiation resistance given by equation (30) the resonant power transmission can be obtained.

c. Overall transmission

The sound power transmitted to the receiving room through the opening is made up of the contribution of nonresonant and resonant transmission. Combining equations (5), (10), (27) and (29) a composite transmission loss for all frequencies can be obtained.

In case of a circular aperture we easily obtain:

$$TL = 10\log\left\{\frac{1}{4}\left(\frac{S_w}{S_a}\right)\frac{1}{\left[2\left(\frac{a}{l_{\mathrm{eff}}}\right)^2+\frac{c^2\sigma_a}{S_a\omega^2}\right]}\right\} \tag{31}$$

and in case of a narrow slit respectively:

$$TL = 10\log\left\{\left(\frac{S_w}{S_s}\right)\frac{1}{\left[\frac{4cL}{\omega S_s}\frac{1}{\left[1/b+0.7+\frac{2}{\pi}\ln\frac{2c}{\omega b}\right]^2}+\frac{\pi c\sigma_s}{\omega b}\right]}\right\}. \tag{32}$$

In both cases we have assumed undamped conditions, so that the energy dissipation can be neglected.

At low frequencies the first term in the denominator dominates, so that the transmission loss follows practically the "mass law" transmission of equation (6) or (11). At higher frequencies the second term becomes predominant determining the sound transmission.

3. Comparison with experiment

The measurements were made between two small rooms. The aperture was made in the middle of a thick gypsum partition wall, while the microphone was mounted in the measuring room as near as possible to the orifice of the aperture.

Fig. 3 shows a comparison between the measured transmission loss of circular apertures of various cross section and that calculated using equation (31). The depth of the aperture, i.e. the thickness of the separation wall between the two rooms, was 5 cm.

Fig. 4 shows a similar comparison between experimental results and theoretical predictions using equation (32) for slits of various breadth. The depth of the slits, i.e. the thickness of the separation wall, was 7 cm and its length 60 cm.

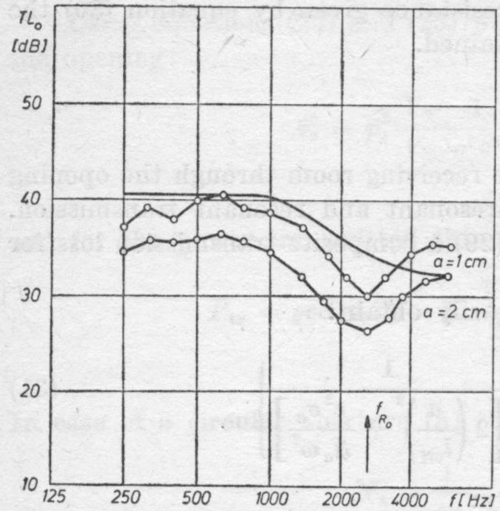

Fig. 3. Transmission loss of a 5 cm gypsum wall with a circular aperture of radius a in the middle

0 ——— 0 experiment; ——— from equation (31)

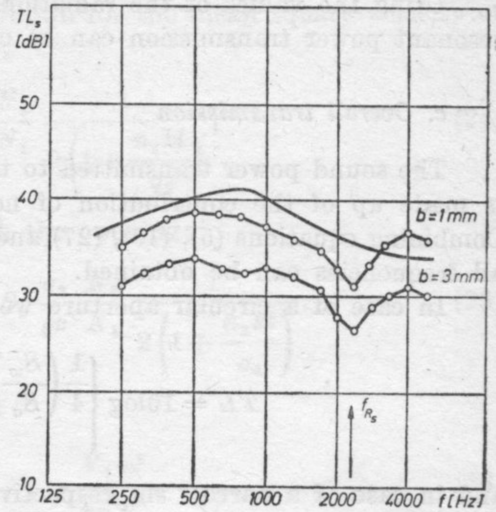

Fig. 4. Transmission loss of a 7 cm gypsum wall with a slit of length 0.60 m and variable breadth b

0 ——— 0 experiment; ——— from equation (32

4. Discussion of results

The measured and theoretical transmission loss values for slits and circular apertures appear to be in good agreement at low frequencies. In this frequency range the sound transmission is not resonant following equations (6) and (11). Within the resonance domain, where the aperture resonates as a tube open at both ends at the frequency $f_R = \frac{c}{2L}$, the agreement between measurements and theoretical predictions is not so good, even though the main trends are predicted.

It seems that due to the statistical nature of our derivation we get a mean value of the transmission loss for the higher frequencies, which can be considered as satisfactory for practical use.

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