#### ARCHIVES OF ACOUSTICS 9, 4, 291-304 (1984)

#### A VIBROACOUSTIC MODEL OF A REDUCTION LINE OF A GAS INSTALLATION

the delivative of a Hankel function of the will of an unit of a philipping balling

two ridiation by the radial direction from the product

### ZBIGNIEW ENGEL, RYSZARD PANUSZKA

Institute of Mechanics and Vibroacoustic of the Academy of Mining and Metallurgy (30 - 059 Cracow, al. Mickiewicza 30)

#### MIECZYSŁAW MENŻYŃSKI

#### Carpathian District Gas Plant (33 - 100 Tarnów, ul. Nowy Świat 78)

This paper proposes a method for predicting the sound power level of a reduction line of a gas installation, based on a vibroacoustic cylindrical equivalent model. This model was verified experimentally on a real object. It permits approximate prediction of the sound power level of a reduction line, depending on its technological and structural parameters, particularly for medium and high frequencies.

# biateo biel est at vienetal content of the barrow barrow of the learness of the bield of the bie

Prediction of the value of sound power radiated by sound sources on machinery and industrial facilities is one of the essential problems in machinery vibroacoustics. Attempts to develop methods for predicting the values of the power level of sound sources were undertaken for plates in the case of flexural resonance vibration [2] and for pipes involving turbulent water flow [1, 6].

The present authors are concerned with the problem of predicting values of the sound power radiated by sources with complex geometry, such as occur in gas pipelines. The considerations apply only to radiation of sound power by surface sources [8].

The principal object of this paper is to build and verify experimentally a model of radiation by a system of sound sources with complex geometry, where acoustic energy radiation is caused by gas flow in the process of pressure reduction. This purpose was carried out by forming an equivalent cylindrical radiation model. It is assumed that the velocity of radial vibration of the walls of the line is a function of the gas flow intensity. This assumption was verified experimentally.

#### 2. Acoustic model of the line

In building the acoustic equivalent model of the line, elements with com plex geometry were replaced by cylindrical ones with equivalent surface area and length equal to the linear dimension of a given element, measured along the axis of the line. A system of cylindrical sources thus formed was replaced by one cylindrical source with the length of the real line and an equivalent diameter (Fig. 1). The assumption of the equivalent model of the line in the form of a cylindrical source was based on the acoustic field distribution as published in papers [3, 4].

In modelling the following assumptions were made:

- the emission of acoustic energy outside a cylinder with infinite length, vibrating stochastically and stationarily over a broad frequency range, is considered;

- the surface of the cylinder vibrates in the radial mode;

- the frequency range  $\Delta f = 500-8000$  Hz;

- there is critical flow in the reductor value ( $\beta = p_2/p_1 < 0.542$ , where  $p_1$  and  $p_2$  are the values of the pressures occurring before and behind the reductor, respectively).

The cylindrical acoustic model of the line is replaced by a system of rectangular pistons with dimensions  $2z_0$  and 2a (Fig. 2), separated on its side surface and vibrating with equal amplitude and phase. It is assumed that the motion of the pistons is not correlated. Hence, the mean square values of the acoustic pressures and the sound power radiation intensity in the field outside the side surface of the cylinder are approximately proportional to the sum of the mean square values of the pressures generated successively by each of the pistons vibrating in an infinite cylindrical baffle.

LAIRD and COHEN [7] derived the expression for the acoustic pressure in a point in the far field with the coordinates R,  $\Phi$ ,  $\theta$  (Fig. 3), in the form

$$p = 2\varrho cv_r P \exp\left[j\left(kR - 2\pi ft\right)\right],\tag{1}$$

where  $\rho$  is the density of the medium, *c*-the acoustic velocity,  $v_r$  — the amplitude of the radial component of the velocity of the piston, k — the wave number, and P is expressed by the relation

$$P = \frac{1}{\pi kR} \frac{\sin(kz_0 \cos\theta)}{\cos\theta \sin\theta} \left\{ \left( \frac{a}{\pi} \right) \frac{1}{H_0^{(1)'}(ka_{eq}\sin\theta)} + \sum_{m=1}^{\infty} \left[ \frac{2\sin ma}{m\pi} \frac{\exp\left(-jm\pi/2\right)}{H_m^{(1)'}(ka_{eq}\sin\theta)} \right] \times \cos m\Phi \right\}.$$
(2)

In this expression,  $a_{eq}$  is the equivalent radius of the cylinder,  $H_0^{(1)'}$  is the derivative of a Hankel function of the zeroth order, of the first kind,  $H_m^{(1)'}$  — the derivative of a Hankel function of the *m*th order, of the 1st kind.

Subsequently, KENNEDY and YOUNG [6] determined the intensity of sound power radiation in the radial direction from the product

 $\frac{1}{2} \left( 2 \varrho c v_r P \right) \left( 2 v_r \sin \theta P^* \right),$ 

where  $P^*$  is a complex conjugate of P.



Fig. 1. Procedure of the stage transition from the real object to the equivalent cylindrical model of the line. The values are given in m

Taking the imaginable cylindrical surface with the radius  $r = R\sin\theta$ , parallel and concentric with the cylindrical baffle, the total sound power radiation intensity in the radial direction, corresponding to unit length, is

$$\overline{w} = 2 \int_{0}^{\pi} \frac{1}{2} (2\varrho c v_r P) (2 v_r \sin \theta P^*) R \sin \theta d\Phi =$$

$$=4\varrho cv_r^2 R \int_0^n PP^* \sin\theta d\Phi [Wm^{-2}].$$
(3)



Fig. 2. Piston acoustic model of the line

Equation (3), when derived for discrete frequency, also determines approximately the power radiation intensity over a narrow frequency range (the value of P for any frequency over that range does not deviate greatly from that of P for the centre frequency of the range).  $v_r^2$  is then the mean square value of the velocity of the piston, averaged over that range.



Fig. 3. Piston model of the line in a cylindrical coordinate system

#### REDUCTION LINE OF GAS INSTALLATION

The total power radiation intensity is equal to the sum of the radiation power densities of particular vibrating pistons. The system of the pistons is defined by an infinite series of rings with the axial length 2  $z_0$ . Each of the rings contains about  $\pi/a$  pistons, hence the sound power radiated by all the pistons in the first ring is

$$\overline{w} = \left(\frac{\pi}{a}\right) w = 4 \varrho c v_r^2 \ \widetilde{W}(r, \, k, \, z_0, \, \alpha, \, \theta, \, a_{eq}) \quad [W], \tag{4}$$

where

$$\tilde{W} = \frac{\pi}{a} R \int_{0}^{\pi} PP^* \sin\theta d\Phi = \frac{\sin\theta}{\pi a k^2 r} \left( \frac{\sin^2(kz^0 \cos\theta)}{\cos^2\theta} \right) \left\{ \frac{a^2}{\pi} \left| \frac{1}{H_0^{(1)'}(ka_{eq} \sin\theta)} \right|^2 + \sum_{m=1}^{\infty} \left[ \frac{2\sin^2 ma}{m^2 \pi} \left| \frac{1}{H_m^{(1)'}(ka_{eq} \sin\theta)} \right|^2 \right] \right\}, \quad (5)$$

and  $r = R\sin\theta$ . In the limits when  $\theta \to 1/2\pi$ 

$$\begin{split} \tilde{W}\left(r,\,k,\,z_{0},\,a,\,a_{\mathrm{eq}},\,\theta\,=\,\frac{\pi}{2}\right) &= \lim_{\theta \to \frac{\pi}{2}} \tilde{W} \,=\, \frac{z_{0}^{2}}{ra\pi^{2}} \left[ \left| \frac{a}{H_{0}^{(1)'}(ka_{\mathrm{eq}}\sin\theta)} \right|^{2} + \right. \\ &\left. + \sum_{m=1}^{\infty} 2 \left| \frac{\sin ma}{mH_{m}^{(1)'}(ka_{\mathrm{eq}}\sin\theta)} \right|^{2} \right], \end{split}$$
(6)

when x tends to zero

$$\frac{1}{H_m^{(1)'}(x)} - \{2\pi i/m \Gamma(m)\} \left(\frac{1}{2} x\right)^{m+1} \quad \text{for } m \neq 0$$
and

Hence, when  $\theta \to 0$ ,  $\tilde{W} \to 0$ . In the acoustic field, for the *n*th ring, the following relation is valid:

$$\theta = \tan^{-1} \left( \frac{r}{2nz_0} \right).$$

Hence, the mean total sound power per unit length of the imaginable surface surrounding the cylinder is

$$\overline{w}_{\text{total}} = 4\varrho c v_r^2 \Big( \tilde{W}(r, k, z_0, a, a_{\text{eq}}, \theta = \frac{1}{2} \pi \Big) + 2 \sum_{n+1}^{\infty} \tilde{W} \Big[ r, k, z_0, a, a_{\text{eq}}, \theta = \tan^{-1} \left( \frac{r}{2nz_0} \right) \Big] \Big\}.$$
(7)

295

 $\frac{1}{H_0^{(1)'}(x)} \rightarrow \frac{1}{2} \pi i x.$ 

The expression  $2 \sum_{n=1}^{\infty} \tilde{W}[r, k, z_0, a, a_{eq}, \theta = \tan^{-1}(r/2nz_0)]$  can be approximated, when  $r \to \infty$ , by the relation [6]

$$\int_{0}^{\pi/2} \frac{r}{z_0 \sin^2 \beta} \tilde{W}(r, k, z_0, \alpha, a_{eq}, \beta) d\beta.$$
(8)

It follows from equations (5) and (6) that the first term of equation (7) is inversely proportional to r. The remainder of the expression is given by equation (8) and does not depend on r within the interval  $0 \le \beta \le 1/2\pi$ . Because the first term of equation (7) is inversely proportional to r and the summary term does not depend on r, for sufficiently large values of r the first term can be neglected.

With  $Z = z_0/\lambda$  and  $A = -a_{eq}/\lambda$ , where  $\lambda$  is the wave length, the following expression for sound power can be derived from equations (7) and (8),

7

$$\overline{v}_{\text{total}} = \varrho c v_r^2 \lambda I(\alpha, Z, A) \quad [W], \tag{9}$$

where

$$I = \frac{1}{\pi^4 a z_0} \int_0^{\pi/2} \frac{\sin^2(2\pi Z \cos\beta)}{\cos^2\beta \sin\beta} \left\{ \left| \frac{a}{H_0^{(1)'}(2\pi A \sin\beta)} \right|^2 + \sum_{m=1}^{\infty} \left[ \frac{2\sin^2 ma}{m^2} \left| \frac{1}{H_m^{(1)'}(2\pi A \sin\beta)} \right|^2 \right] \right\} d\beta.$$
(10)

Fig. 4 shows the characteristic of log I as a function of the quantity A, as determined by KENNEDY and YOUNG [6].



Equation (9) was derived for an infinitely long system of pistons vibrating in a cylindrical baffle. In the case of a long pipe with finite length  $l[m](l/a_{eq} \ge 1)$ , for which the diffraction phenomena at its ends can be neglected, the theoretical sound power level  $L_{Wl}$  radiated for  $\varrho c = 428.6 \text{ Nsm}^{-3}$  can be expressed approximately by the relation

$$L_{Wt} = 10\log\frac{\overline{w}_{\text{total}}}{W_0}l = 146 + 20\log\overline{v}_r + 10\log\lambda + 10\log I \quad [\text{dB}], \quad (11)$$

where  $W_0 = 10^{-12} [W]$  is the reference power level,  $\lambda$  - the wave length [m], log *I* is the value determined from the curve in Fig. 4,  $v_r [ms^{-1})$  - the mean value of the velocity of radial vibration.

#### 3. Vibrating model of the line

The acoustic model makes the mean sound power  $\overline{W}$  radiated dependent on the value of the mean radial vibration velocity of the walls of the line,  $\overline{v}_r$ , according to the function

$$\overline{W} = \varphi_1(\overline{v}_r, f, a_{eq}, l) \quad [W].$$
(12)

In the vibration model, the mean value of the radial vibration velocity of the walls of the reduction line,  $\bar{v}_r$ , depends on the gas flow intensity  $Q[\mathbf{m}^3\mathbf{h}^{-1}]$ , the vibration frequency  $f[\mathrm{Hz}]$ , the equivalent radius  $a_{\mathrm{eq}}[\mathbf{m}]$  and the length  $l[\mathbf{m}]$ , according to the function

$$\overline{v}_r = \varphi_2(Q, f, a_{eq}, l) \quad [m \cdot s^{-1}].$$
 (13)

The vibration model and the acoustic one, which form together a vibroacoustic model, are shown in Fig. 5.



Fig. 5. Schematic diagram of the vibroacoustic equivalent model of the line

The function given by relation (13) can be determined by investigations performed on real lines. In the case of a set of K lines, it is possible to derive the functions

$$v_{rK} = \varphi_k(Q, f), \tag{14}$$

where  $a_{eq}$ , l = const.

Hence, the mean function is sought:

$$\overline{v}_r = \overline{\varphi}(Q, f), \tag{15}$$

where

$$Q[Q_{\min}, Q_{\max}], \varphi[f_{\min}, f_{\max}].$$
(16)

The flow intensity Q varies in time. Assuming that the discrete values of the flow intensity,  $Q_{K_i}$ , are known for particular lines, the function  $Q_K$  can be written as

$$Q_{K} = Q_{\text{VAR}}[Q_{K_{1}}, Q_{K_{2}}, \dots, Q_{K_{i}}, \dots, Q_{K_{m}}],$$
(17)

where m = 1, 2... are discrete values of the gas flow intensity  $Q_{K}$ .

For each of the lines, the mean value of the function is determined from (13) by averaging

$$\bar{v}_{r_{K}} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} v_{r_{i}}^{2}},$$
(18)

where n is the number of points for the measurement of the radial vibration velocity on the line. Hence, the mean value of the function of the radial vibration velocity can be determined in the form

$$\overline{v}_r(Q,f) = \frac{1}{K} \sum_{i=1}^K v_{r_K} \quad [\mathrm{ms}^{-1}].$$
(19)

The function given by relation (13) was determined experimentally on 3 lines with the first degree of pressure reduction, in the same technological and structural system, as shown in Fig. 1.

The investigations were carried out for gas flow intensities contained within the interval (2315-6480)  $m^{3}h^{-1}$  under normal conditions. The distribu-



Fig. 6. Distribution of the measurement points on the surface of the line

tion of the measurement points on the surfaces of the particular elements is shown in Fig. 6. In turn, Fig. 7 gives the mean values of the amplitudes of the radial vibration velocity of the line, as obtained from relation (19). They are the basis for determination of the sound power level radiated by the reduction line.





#### 4. Experimental verification of the vibroacoustical model

The aim of the verification was to estimate agreement between the values of the sound power levels calculated theoretically from relation (11) and the values of the sound power levels obtained by measurements on a real object. The investigations were based on the sound pressure level distributions in the near and far fields which were given in papers [3] and [4].

In the theoretical calculations, four variant ways of averaging the radius of the equivalent cylinder,  $a_{eq}$  were employed:

- from the total real surface area of the elements of the line

$$a_{eq}^{e} = \frac{S}{2\pi l} \quad [m]; \tag{20}$$

- from the arithmetic mean of the radii of the equivalent cylindrical ele-

ments

$$a_{eq}^{a} = \frac{\sum_{i=1}^{8} d_{i}}{2 \cdot 8}$$
 [m]; (21)

- from the geometric mean of the radii of the equivalent cylindrical elements

$$u_{cq}^{g} = \frac{1}{2} \sqrt[8]{d_1 d_2 \dots d_8} \quad [m];$$
 (22)

- from the quadratic mean of the radii of the equivalent cylindrical elements

$$a_{eq}^{k} = \frac{1}{2} \sqrt{\frac{\sum_{i=1}^{8} d_{i}^{2}}{8}} \quad [m].$$
 (23)

The dimensions of the elements of the reduction line used in the tests are listed in Table 1.

The value of the sound power level  $L_{W_t}(f)$ , which would be radiated by the equivalent cylindrical source, was determined from relation (19) (Table 2). These values were compared with those of the sound power levels obtained from measurements in the near field,  $L_{W_b}(f)$ , and in the far one,  $L_{W_d}(f)$  (Table 3).

This comparison permits an adquate variant of the radius of the equivalent model to be selected in the theoretical calculations of the power level by its experimental verification. For this purpose, for the four variants of the radius assumed, calculations were carried out of the theoretical level of the total sound power radiated over the five frequency ranges under study, according to the dependence

$$L_{W_t} = 10\log \frac{\sum_{i=1}^{5} W_{i_t}(f_i)}{W_0} \quad [dB], \qquad (24)$$

where  $W_{i_t}(f_i)$  [W] is the sound power of the line over the 1/3 octave band with the centre frequency  $f_i$ , determined theoretically.

Similar calculations of the total sound power levels were carried out for the near and far fields, according to the dependencies

$$L_{W_b} = 10\log \frac{\sum_{i=1}^{5} W_{i_b}(f_i)}{W_0} \quad [dB], \qquad (25)$$

$$L_{W_{\vec{a}}} = 10\log \frac{\sum_{i=1}^{5} W_{i_{\vec{a}}}(f_{i})}{W_{0}} \quad [dB], \qquad (26)$$

300

No	Element	Real dimen- sions [m]	Surface area [m²]	Equivalent cylinder diameter [m]
1	Inlet elbow	$d_z = 0.159$ $\mathbf{r} = 0.750$ $l_1 = 1.180$	$S_1 = 0.59$	$d_1 = 0.159$
2	Round collar valve wedge AP 5/1 25 atn	$d_n = 0.150$ $l_2 = 0.350$	$S_2 = 0.67$	$d_2 = 0.610$
3	Diffusor	$d_z = 0.159/0.219$ $l_3 = 0.500$	S <sub>3</sub> = 0.30	$d_3 = 0.190$
4	Quick-closing valve GSDS 200/25	$d_n = 0.200$ $l_4 = 0.750$	S <sub>4</sub> = 1.80	$d_4 = 0.764$
5	Pressure reducer GRDE 200/25	$d_n = 0.200$ $l_5 = 0.720$	$S_5 = 2.60$	$d_5 = 1.149$
6	Straight connection pipe	$d_z = 0.219$ $l_6 = 1.830$	$S_6 = 2.16$	$d_6 = 0.219$
7	Oval collar valve edge AP 5/1 10 atn	$d_n = 0.200$ $l_7 = 0.400$	S <sub>7</sub> = 0.90	$d_7 = 0.716$
8	Outlet elbow	$d_z = 0.219$ r = 0.600 $l_8 = 0.950$	$S_8 = 0.65$	$d_8 = 0.219$
9	Whole line	$d_n = 0.200$ l = 6.680	S = 8.77	d = 0.418

Table	1.	Dimensions	of	the	line	

where  $W_{i_b}(f_i)$  [W] is the sound power of the line in the 1/3 octave band with the centre frequency  $f_i$ , determined in the near field; and  $W_{i_d}(f_i)$  [W] is the sound power of the line in the 1/3 octave band with the centre frequency  $f_i$ determined in the far field.

The differences among the values of the total sound power levels calculated from (24-26) were determined according to the dependencies

$$\Delta L_{W_1} = L_{W_1} - L_{W_2} \quad [dB], \tag{27}$$

$$\Delta L_{W_0} = L_{W_0} - L_{W_0} \quad [dB]. \tag{28}$$

Table 4 lists the calculated values of the total sound power levels and their differences. It follows from this table that the least difference among the Table 2. Calculated values for determination of the theoretical sound power level of a reduction line

(28)

Level [dB]	Centre frequency of 1/3 octave band [Hz]					
[ap]	500	1000	2000	4000	8000	
LPavb	69.5	79.8	93.9	98.4	92.6	
LWb	78.9	89.2	103.3	107.8	102.0	
LPavd	62.8	78.0	86.2	94.1	87.7	
LWd	80.2	95.4	103.6	111.5	105.1	

Table 3. Levels of averaged sound pressures and sound powers of the whole reduction line in the near and far fields

results obtained, both for the far and the near fields, occurs in the case of assuming in theoretical calculations the radius  $a_{ea}^{\sigma}$ .

Fig. 8 represents graphically comparison between the theoretically determined sound power levels with those measured in the near and far fields. The behaviour of these levels indicates that at low frequencies deviations occur

Variant of equivalent				Differences in total power levels		
radius	$^{L}W_{t}[dB]$	$L_{W_b}[dB]$	$  LW_d[dB]$	$LW_1[dB]$	LW2[dB]	
aceq	110.7	109.9		0.8	0.7	
aaeq	111.7		110.0	1.8	1.7	
$a_{eq}^g$	110.2		arbusting and	0.3	0.2	
akeq	112.2		an minanis	2.3	2.2	

Table 4. Measured values of the sound power levels of the line



Fig. 8. Comparison of the determined sound power levels for the line

enil noitéel

L<sub>w</sub> [dB] (being maximum, of about 7 dB, in the band of 500 Hz). At higher frequencies, however, (2000-8000 Hz), i.e. where characteristically of the reduction line maximum level values occur, there is agreement among the results obtained, likewise in total sound power levels. This agreement confirms the validity of the method assumed for selection of the equivalent model of the reduction line.

#### 5. Conclusions

The results of the experimental investigations of the spatial distribution of the sound field of the reduction line served to build an equivalent vibroacoustic model of the line and for its experimental verification.

This equivalent vibroacoustic model permits approximate prediction of the sound power level of a reduction line, depending on its technological and structural properties, with the best results being obtained for medium and high frequencies.

The method proposed here for the construction of the equivalent model was verified for lines with the nominal diameter  $d_n = 0.2 \text{ m}$ ; it can also be used to build equivalent models of lines with different parameters.

## References

[1] J. M. CLINCH, Prediction and measurement of the vibrations induced in thin - walled pipes by the passage of internal turbulent water flow, JSV, 12 (1970).

[2] S. CZARNECKI, Z. ENGEL, R. PANUSZKA, Sound power and radiation efficiency of a circular plate, Archives of Acoustics, 1, 3, 201-213 (1976).

[3] Z. ENGEL, R. PANUSZKA, M. MENŻYŃSKI, Noise propagation by technological lines of gas installations (in Polish), Gaz, Woda i Technika Sanitarna, 11-12, 270-275 (1981).

[4] Z. ENGEL, R. PANUSZKA, M. MENŻYŃSKI, Investigations of the sound field of the reduction lines of gas installations (in Polish), Proc. Conf. "Noise Control 82", Cracow, 20-22 Sept. 1982.

[5] Z. ENGEL, Issledowaniya akusticheskoy moshchnosti promyshlennykh istochnikov shuma, Proc 21 Conf. Acoustics, Bratislava, 1982, 1, 99-104.

[6] W. C. KENNEDY, F. J. YOUNG, Chart for estimating the efficiency of the broadband sound - power radiation by randomly vibrating pipewalls, J. Acoust. Soc. Am., 5 (1977).

[7] D. T. LAIRD, H. COHEN, Directionality patterns for acoustic radiation from a source on a rigid cylinder, J. Acoust. Soc. Am., 24 (1952).

[8] M. MENŻYŃSKI, Analysis of the emission of vibroacoustic energy of the reduction lines of gas installations, for its limitation (in Polish), Doct. diss., Academy of Mining and Metallurgy, 1982.

Received on 7 March, 1983; revised version on 27 April 1984.