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PROCESSING OF THE ACOUSTIC WAVE IMAGE

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This paper presents investigations of a system for processing the image of acoustic wave generated in transparent media. It also gives an analysis of the system and discusses the influence of parameters of spatial filtre apertures and of the acoustic field on the quality of representation of the field intensity distribution.

1. Introduction

Image processing is one of the basic processes of optical information elaboration and involves analysis, transfer and synthesis of an image. A mathematical model of this process is provided by simple and inverse Fourier transforms and by optical multiplication of two functions. In this range, both the analysis of the processing and its physical interpretation have been long known and frequently described [1, 2].

It is different in the case of the processing of a dynamic image, i.e. the image of a progressive or standing acoustic wave (oscillating in time) (Fig. 1b). Interest in the processing of the image of the acoustic wave results mainly from the two respects:

1.1. Visualization of the acoustic field. The processing of the image of acoustic wave allows visualization, analysis and registration of the acoustic field distribution in transparent media (bulk waves) and in opaque media (surface waves). The visual processing and the estimation of the performance of transducers of acoustic waves are highly significant practically, i.e. they permit the analysis and estimation of the performance of piezoelectric and acoustooptic equipment without disturbing the conditions of their work and without damage.

1.2. Optical processing of signals. Most equipment for optical processing of signals uses spectral and correlation signal analysis [3]. The spectral analysis is a component part of the processing system (Fig. 1a), while in the correlation process the image of the signal $S_1(t)$ from the first modulator being processed correlates with the signal $S_2(t)$ in the second acoustooptic modulator (Fig. 1c).

In both cases, what is essential is the problem of exact representation

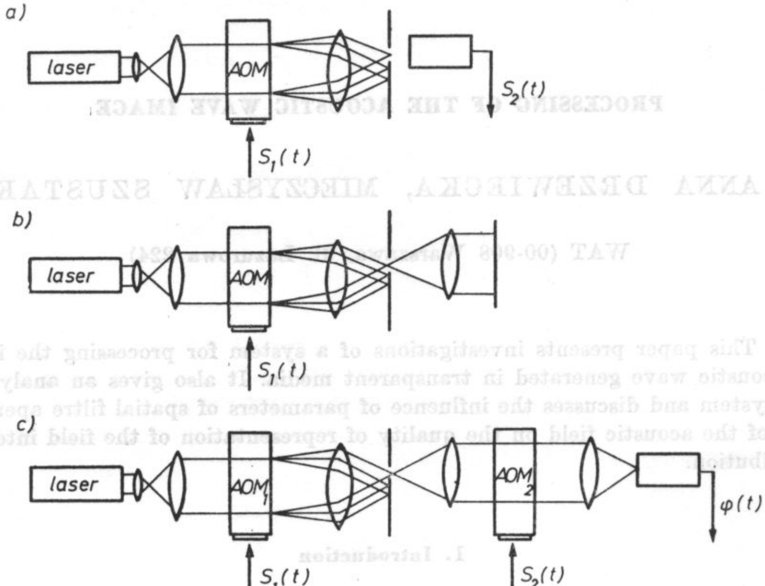


Fig. 1. Image processing in the process of visualization of the acoustic field and acoustooptical processing of signals: a) spectral analysis, b) image processing (visualization of the acoustic field), c) correlation analysis

in the image of both the field intensity distribution and of its shape, particularly in the pulsive performance. In addition, derivation of the signal correlation function [3]

$$\varphi(t) = \int r\left(v_1 t - \frac{M}{n}\, x\right) S_1(Nt)\, S_2(v_1 t - nx)\, dx,$$

where $n = v_1/v_2$ is the velocity ratio of acoustic waves in the first and second light modulators, N — the variation degree of the time scale, M — the image magnification; requires the image of the signal $S_1(t)$ to be inverted in time and also the satisfaction of the conditions

$$1 - \frac{M}{n} = -1, \quad N = \frac{M}{n}, \quad \left(v_1 t - \frac{M}{n}\, x\right) = 1.$$

The inversion of the image, the change of its spatial and time scale, and the representation quality are all implemented in the processing. Therefore, the investigation of the processing of the image of acoustic wave, the determination of the dependence of the parameters of the image on the parameters of the processing system and of the signal of the acoustic wave constitute an essential investigation problem. Investigation of the dependencies mentioned above has also a high practical significance, related to the designing and implementation of both systems for visualization of acoustic fields and acousto-optical processing of signals.

2. Analysis of the system for processing the image of acoustic wave

The processing of the image of acoustic wave was analyzed on the basis of the Raman-Nath diffraction theory. A schematic diagram of the image processing system is shown in Fig. 2.

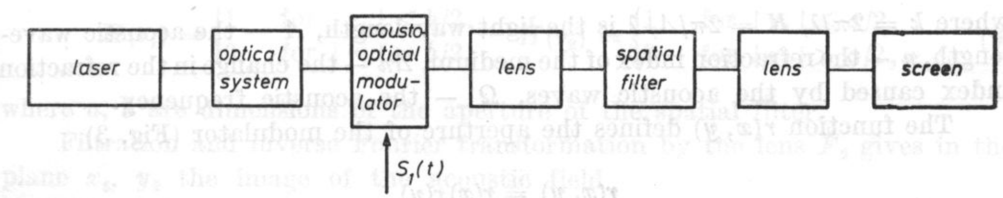


Fig. 2. A schematic diagram of the image processing system

The plane light wave

$$E(x, y, z, t) = E_0\{\exp[i(\omega t - kz)]\} \tag{1}$$

in the plane $z = 0$ falls onto an acoustooptic light modulator with a progressive plane acoustic wave [4]. The propagation direction of the light wave is perpendicular to the direction of the acoustic wave (Fig. 3). In the plane $z = L$

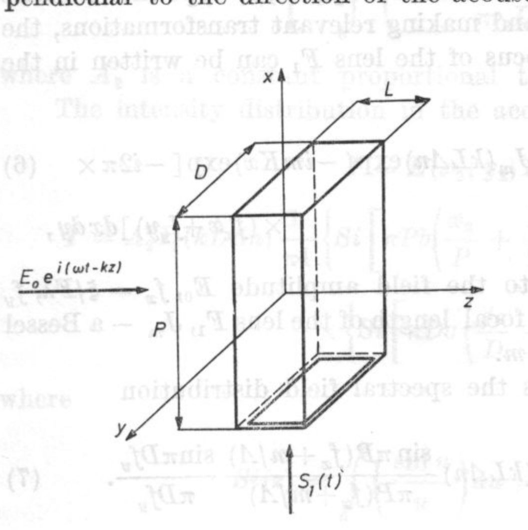


Fig. 3. Acoustooptical light modulator

the light wave has the form

$$E(x, y, L, t) = E(x, y, 0, t)T(x, y), \tag{2}$$

where $T(x, y)$ is the transfer function of the modulator,

$$T(x, y) = T_0(x, y)\exp[i\varphi(x, y)]r(x, y). \tag{3}$$

Under the assumption that the light wave crosses the acoustic field in a rectilinear manner, the optical nonuniformity related to a change in the refraction index caused by the acoustic wave influences only the phase of the light wave which crosses the acoustic wave stream. Thus, the light propagating in the plane $z = L$ undergoes only phase modulation, and therefore $T_0(x, y) = 1$,

$$\varphi(x, y) = kLn + kL\Delta n\cos(\Omega t - Kx), \tag{4}$$

where $k = 2\pi/\lambda$, $K = 2\pi/\Lambda$, λ is the light wavelength, Λ — the acoustic wavelength, n — the refraction index of the medium, Δn — the change in the refraction index caused by the acoustic waves, Ω — the acoustic frequency.

The function $r(x, y)$ defines the aperture of the modulator (Fig. 3)

$$r(x, y) = r(x)r(y),$$

$$r(x) = \begin{cases} 1 & \text{for } |x| \leqslant P/2, \\ 0 & \text{for } |x| > P/2, \end{cases} \quad r(y) = \begin{cases} 1 & \text{for } |y| \leqslant D/2, \\ 0 & \text{for } |y| > D/2. \end{cases}$$

Thus, the light distribution in the plane $z = L$ can be represented by the relation

$$E(x, y, L, t) = E_0[\exp(i\omega t)]\exp\{ikL[n + \Delta n\cos(\Omega t - Kx)]\}. \tag{5}$$

Using the expansion of the functions of the $\exp(i\cos x)$ type into a series of Bessel functions of the first kind and making relevant transformations, the light amplitude distribution in the focus of the lens F_1 can be written in the form

$$E(\xi, \eta) = A_1 \int_{-P/2}^{P/2} \int_{-D/2}^{D/2} \sum_{m=-\infty}^{\infty} (-i)^m J_m(kL\Delta n)\exp(-imKx)\exp[-i2\pi \times (f_x x + f_y y)]dx\,dy, \tag{6}$$

where A_1 is a constant proportional to the field amplitude E_0, $f_x = \xi/F\lambda$, $f_y = \eta/F\lambda$ spatial frequencies, F — the focal length of the lens F_1, J_m — a Bessel function of the first kind of order m.

Integration of equation (6) gives the spectral field distribution

$$E(\xi, \eta) = A_1PD \sum_{m=-\infty}^{\infty} (-i)^m J_m(kL\Delta n)\frac{\sin\pi P(f_x + m/\Lambda)}{\pi P(f_x + m/\Lambda)}\frac{\sin\pi Df_y}{\pi Df_y}. \tag{7}$$

For the sake of the simplicity of notation, the new variable spatial frequencies can be introduced

$$\nu_1 = f_x + m/\Lambda, \quad \nu_2 = f_y. \tag{8}$$

In the domain of spatial frequencies the spectrum undergoes filtration, by multiplication of the spectral function of the acoustic field (7) by the spectral function of the filter $H(\nu_1, \nu_2)$

$$E(\nu_1, \nu_2) = E(\xi, \eta) H(\nu_1, \nu_2),$$

where only the first diffraction order $m = 1$ is filtered.

The filter function has the form

$$H(\nu_1, \nu_2) = H(\nu_1) H(\nu_2),$$

$$H(\nu_1) = \begin{cases} 1 & \text{for } |\nu_1| \leqslant b/2, \\ 0 & \text{for } |\nu_1| > b/2, \end{cases} \quad H(\nu_2) = \begin{cases} 1 & \text{for } |\nu_2| \leqslant a/2, \\ 0 & \text{for } |\nu_2| > a/2, \end{cases} \tag{9}$$

where a, b are dimensions of the aperture of the spatial filter.

Filtration and inverse Fourier transformation by the lens F_2 gives in the plane x_2, y_2 the image of the acoustic field

$$E(x_2, y_2) = \int_{-b/2}^{b/2} \int_{-a/2}^{a/2} (\xi, \eta) H(\nu_1, \nu_2) \exp[i2\pi(\nu_1 x_2 + \nu_2 y_2)] d\nu_1 d\nu_2$$

$$= A_2 J_1(kL\Delta n) \left[iP \int_0^{b/2} \left(\frac{\sin 2\pi\nu_1(x_2 + P/2)}{\pi\nu_1 P} - \frac{\sin 2\pi\nu_1(x_2 - P/2)}{\pi\nu_1 P} \right) d\nu_1 \right] \times \tag{10}$$

$$\times \left[iD \int_0^{a/2} \left(\frac{\sin 2\pi\nu_2(y_2 + D/2)}{\pi\nu_2 D} - \frac{\sin 2\pi\nu_2(y_2 - D/2)}{\pi\nu_2 D} \right) d\nu_2 \right],$$

where A_2 is a constant proportional to E_0.

The intensity distribution in the acoustic wave image has the form

$$I = E(x_2, y_2) E^*(x_2, y_2),$$

$$I = A_2^2 J_1^2(kL\Delta n) \frac{1}{\pi 4} \left\{ Si \left[\pi Pb \left(\frac{x_2}{P} + \frac{1}{2} \right) \right] - Si \left[\pi Pb \left(\frac{x_2}{P} - \frac{1}{2} \right) \right] \right\}^2 \times$$

$$\times \left\{ Si \left[\pi Da \left(\frac{y_2}{D} + \frac{1}{2} \right) \right] - Si \left[\pi Da \left(\frac{y_2}{D} - \frac{1}{2} \right) \right] \right\}^2, \tag{11}$$

where

$$Si(z) = \int_0^z \left(\frac{\sin u}{u} \right) du = \frac{1}{2} \int_{-z}^{z} \left(\frac{\sin u}{u} \right) du.$$

The dimensionless product of the field apertures and the *Pb* filter is an analogue of the product $T\Delta f$ in the analysis of time signals. The value of *Pb* or *Da* defines the shape of the field image and affects essentially the quality of representation of the intensity and shape of the input distribution of the acoustic field [5]. Explanation of the effect of the product *Pb* or *Da* on the

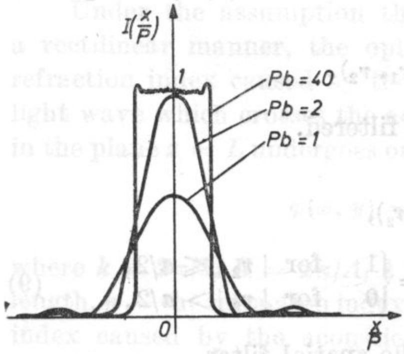


Fig. 4. Light intensity distribution in the image for different values of the product *Pb*

image of the acoustic field is essential for the interpretation of the results of visualization and acoustooptical processing of signals. The present analysis is valid for linear systems, and accordingly the intensity of acoustic fields should be low, thus satisfying the condition of linearity of the system $J_1(kL\Delta n) \approx kL\Delta n/2$. For low intensity of acoustic fields the light intensity distribution in the image is proportional to the acoustic wave intensity. Fig. 4 shows a theoretical field distribution, illustrating the effect of the product *Pb* calculated from expression (11).

3. Experimental investigations

The processing of the acoustic wave image was investigated in the system shown in the diagram (Fig. 5). The acoustic field image was visualized on a TV monitor [6, 7]. The image analysis was performed by the method of electronic image line selection [8]. The system used permitted the field intensity distribution in the acoustic wave image to be registered on the oscilloscope display and recorded on the *XY* plotter.

In the measurements of the acoustic field distribution there occurs a systematic error which results from the processing characteristics of the equipment used and is 6 percent. A more detailed discussion of errors was given in the previous paper [8].

3.1. Processing of the image of a progressive acoustic wave. The investigations of the processing of the acoustic wave image were performed on an acoustooptic modulator with a progressive wave, made of SF6. Transducers of lithium iodate generated a uniform acoustic field of the frequency $f = 50$ MHz. The uniformity of the acoustic field was checked by the method of optical probing in parallel directions.

Fig. 6 shows images of the acoustic wave with the index of the line being selected and theoretical and experimental light intensity distributions in the image

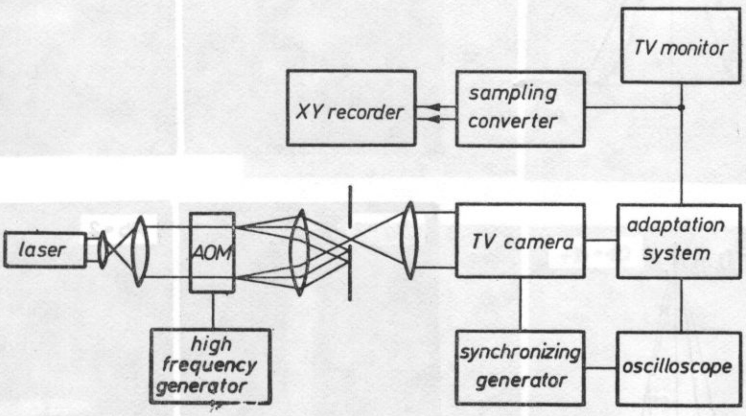


Fig. 5. A block diagram of image processing and analysis

for different values of the aperture product. The values of Pb and Da were varied by changing the width of the spatial filter; $Pb = 1, 2, 3, 5, 40, 50$ correspond to the widths of the filter aperture 0.01, 0.02, 0.03, 0.05, 0.4, 0.5 mm. For $Da = 1, 2, 3, 5, 40, 50$ the filter widths were, respectively, 0.005, 0.010, 0.015, 0.025, 0.20, 0.25 mm. It follows from the investigations performed that the image of the field is well represented at $Pb > 40$ and $Da > 40$, which corresponds to a filter width comparable with the diameter of the diffracted light stream. At lower values of Pb and Da the image of the field is not complete and distorted by diffraction effects on the edges of the spatial filter.

The curves of the intensity distribution of the acoustic fields obtained by the method of electronic image analysis are close in shape to the theoretical curves. The differences observed are caused by the additional light scattering on the faults of the modulator crystal (scratches, dust). Nonuniformities of the image background, which are strongest at the edges of the image, are caused by the inertia of the vidicon of the camera and by the presence of the so-called false signals [8].

3.2. Processing of the image of the crossed acoustic fields. In the system described above investigations of the modulator were performed, in which the streams of two acoustic waves of the same frequency and running at the right angle to one another (Fig. 7) were crossed.

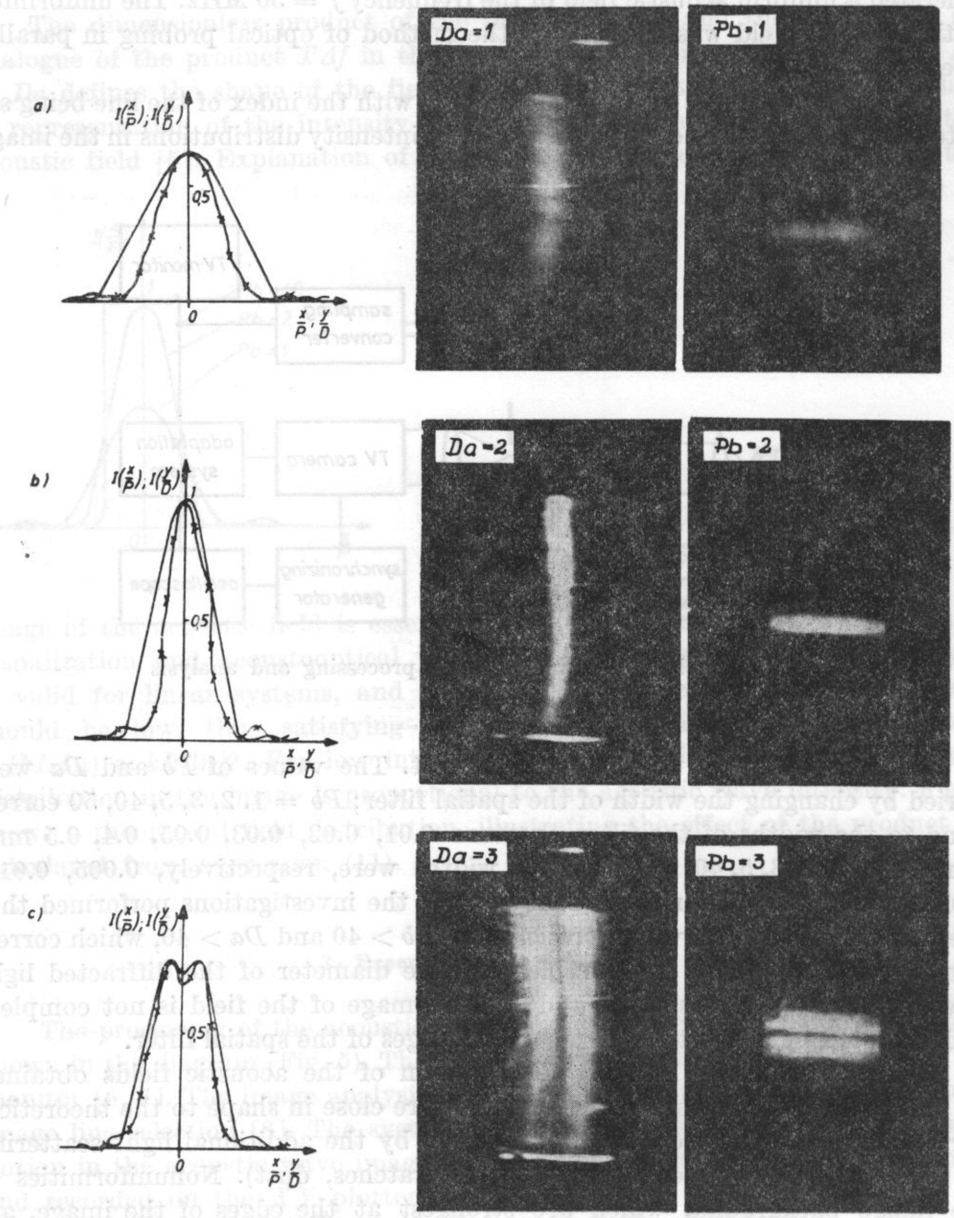
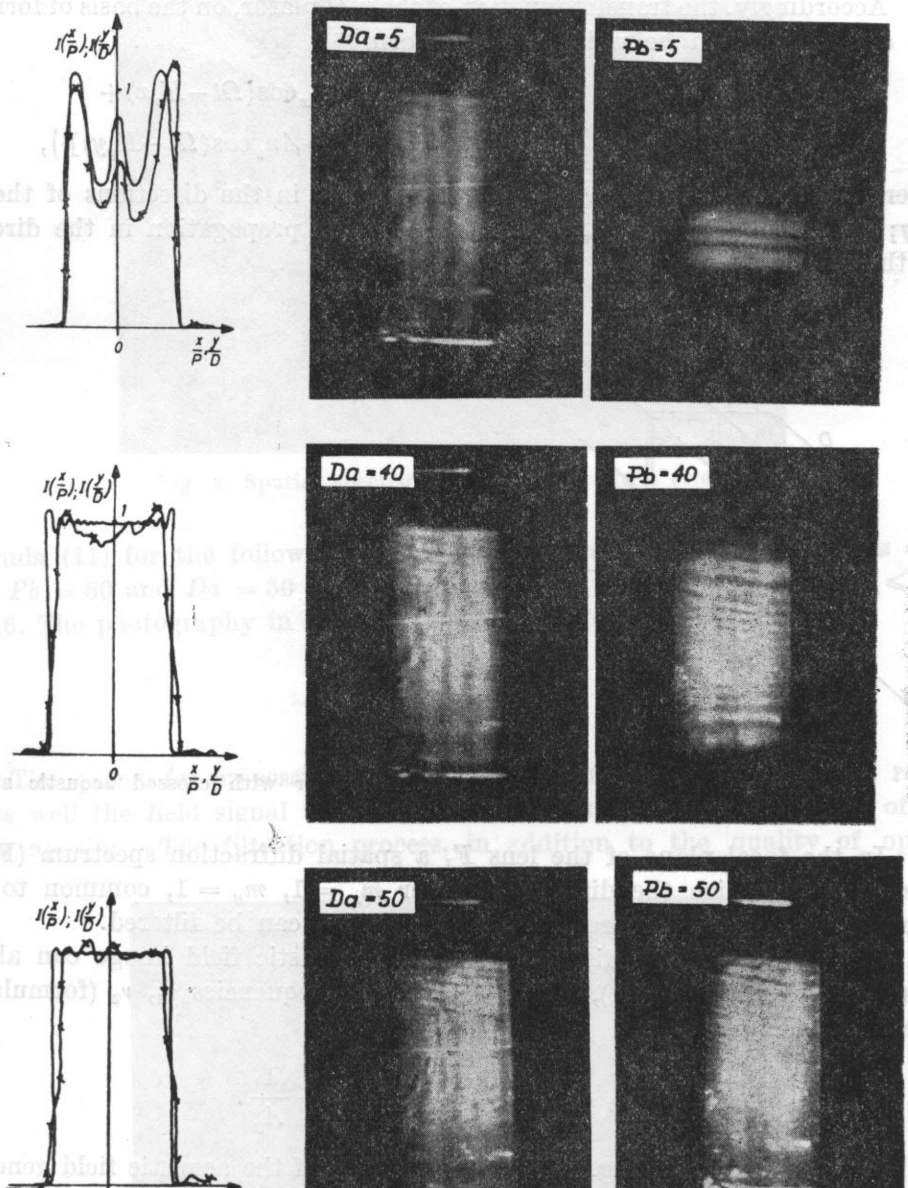


Fig. 6. Acoustic wave images and theoretical (solid lines) and experimental (dash and cross lines) normalized light intensity distributions in the image, illustrating the effect of the aperture product

a) $Pb = 1$, $Da = 1$; *b)* $Pb = 2$, $Da = 2$; *c)* $Pb = 3$, $Da = 3$; *d)* $Pb = 5$, $Da = 5$; *e)* $Pb = 40$, $Da = 40$; *f)* $Pb = 50$, $Da = 50$


d)
e)
f)
Da = 5
Pb = 5
Da = 40
Pb = 40
Da = 50
Pb = 50
0
1

Accordingly, the transfer function of the modulator, on the basis of formulae (3) and (4), should be written in the form

$$T(x, y) = T_0(x, y)r(x, y)\exp\big(i\{kLn + kL[\Delta n_x \cos(\Omega t - K_x x) + \Delta n_y \cos(\Omega t - K_y y)]\}\big), \quad (12)$$

where $\Delta n_{x,y}$ are changes in the refraction index in the directions of the axis x, y; and K_x, k_y are constants of acoustic wave propagation in the direction of the axis x and y.

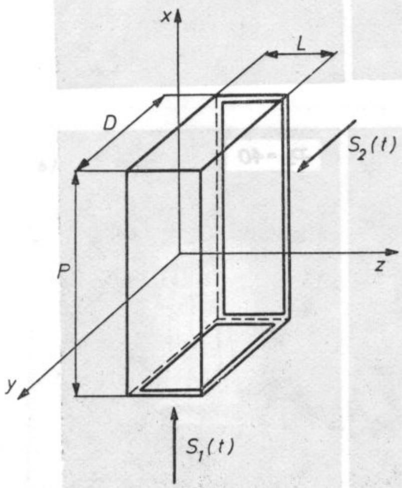


Fig. 7. Modulator with crossed acoustic streams

In the focal plane of the lens F_1 a spatial diffraction spectrum (Fig. 8) occurs, from which the diffraction order $m_x = 1$, $m_y = 1$, common to both acoustic streams and represented by an arrow, can be filtered.

The light intensity distribution in the acoustic field image can also be described by formula (11), where the spatial frequencies ν_1, ν_2 (formula (8)) are inserted in the form

$$\nu_1 = f_x + \frac{1}{\Lambda_x}, \qquad \nu_2 = f_y + \frac{1}{\Lambda_y}.$$

The photography (Fig. 9) shows the image of the acoustic field generated as a result of crossing two acoustic streams of the same intensity and the same wavelength in both directions. The difference between the acoustic streams in the two directions, which can be observed in the photograph, results from its diffraction, since the visualization was performed in the far field [9]. As previously, variation of the width of the spatial filter can affect the shape of the field image.

For comparison, Fig. 10 shows curves of the spatial distribution of light intensity in the image of crossed acoustic waves. Calculations were made from

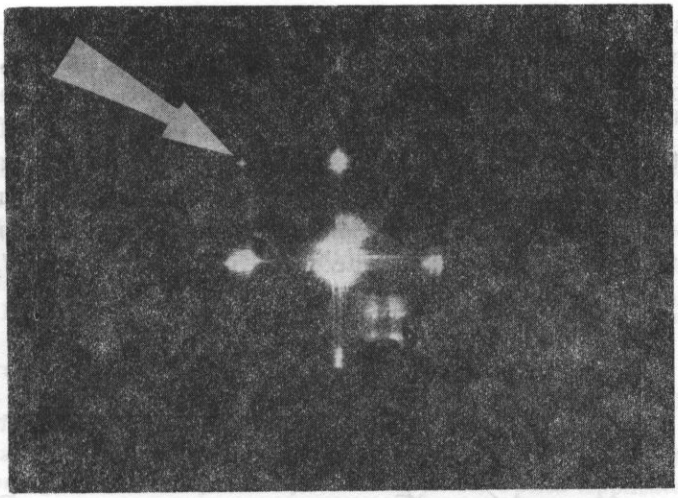


Fig. 8. Spatial spectrum of crossed acoustic streams

formula (11) for the following values of the parameters: $Pb = 40$, $Da = 40$ and $Pb = 50$ and $Da = 50$ and the coordinates $0 \leqslant x/P \leqslant 0.6$ and $0 \leqslant y/D \leqslant 0.6$. The photography in Fig. 9 was taken at $Pb = 30$ and $Da = 30$.

4. Conclusions

The system for processing the acoustic field image constructed here represents well the field signal under the conditions of a correct selection of the filter aperture. The filtration process, in addition to the quality of optical

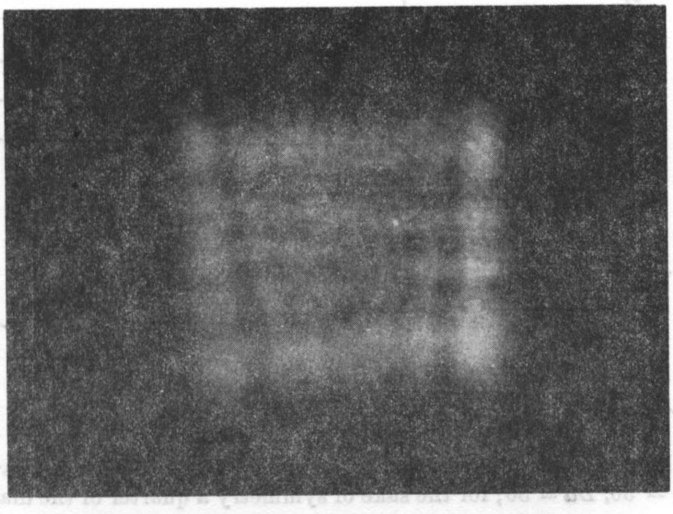


Fig. 9. Image of the acoustic field of crossed streams

elements, has an essential effect on the quality and fidelity of the image. In view to maintaining the linearity of the image processing system, the acoustic fields undergoing optical elaboration should not introduce nonlinear distortions. In employing visualization for the evaluation of the performance of piezoelectric transducers attention should be paid to such selection of a filter that the distortion of the intensity distribution in the image by the filtration process can be avoided. The filtration process is particularly important in the implementation of the correlation function, where the acoustic field of the signal $S_2(t)$ correlates with the optical image of the field of the signal $S_1(t)$.

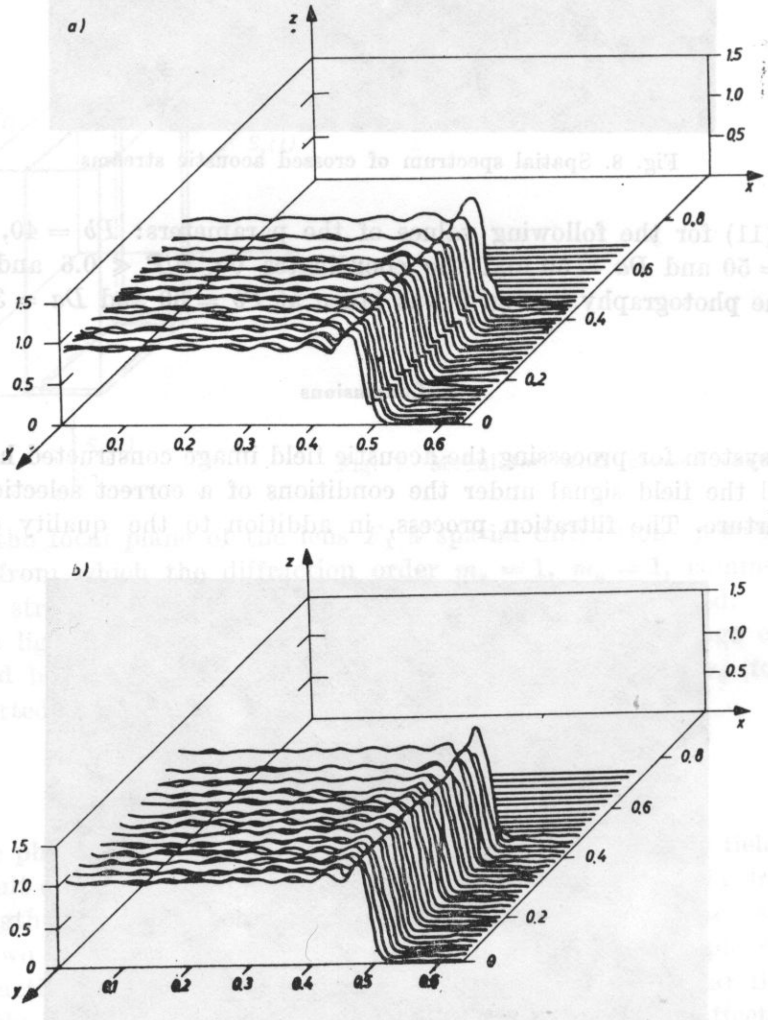


Fig. 10. Spatial light intensity distribution in the image of crossed acoustic waves; a) $Pb = 40$, $Da = 40$; b) $Pb = 50$, $Da = 50$; for the sake of symmetry a quarter of the diagram is presented here

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