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INVESTIGATION OF THE WAVEGUIDE PROPERTIES OF A BOREHOLE AND THEIR USE FOR ACOUSTIC MEASUREMENTS IN SITU

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A method of measurement of the velocity of a shear wave travelling in the vicinity of a borehole, using its waveguide properties, is both analyzed and verified. In particular, by this method, the shear wave velocity is determined from the velocity of the so-called tube wave which corresponds to the zero frequency limit of the lowest radial mode of propagation in the fluid filling the borehole. Investigations of the necessary conditions to obtain a tube wave and of its propagation were performed on a laboratory model of a borehole and in boreholes in situ. The results obtained show the possibility of a practical use of the method investigated for the determination of the mean velocities of shear waves in the rock mass surrounding the borehole.

1. Introduction

Geoacoustical investigations of boreholes in situ are performed in order to obtain the fullest possible information concerning the physical properties of the rock mass surrounding a borehole. One of the commonly used methods is sonic logging [17], which essentially provides data from the continuous measurement of the compressional wave propagation parameters along a borehole wall, i.e. as a function of depth (Fig. 1). Usually only one transmitter-receiver probe is used in sonic logging. The borehole is filled with a fluid, which couples acoustically the transducers of the logging tool with the formation. However, at present, the possible interpretation of the data obtained from sonic logging does not, in general, deliver all the information required about the formations. Analysis of the acoustical pulse travelling in the fluid-filled borehole and in its walls indicates that various elastic waves occur, depending on the geometry of the borehole and the physical properties of the two media involved. Detailed knowledge of the conditions necessary to excite the different specific

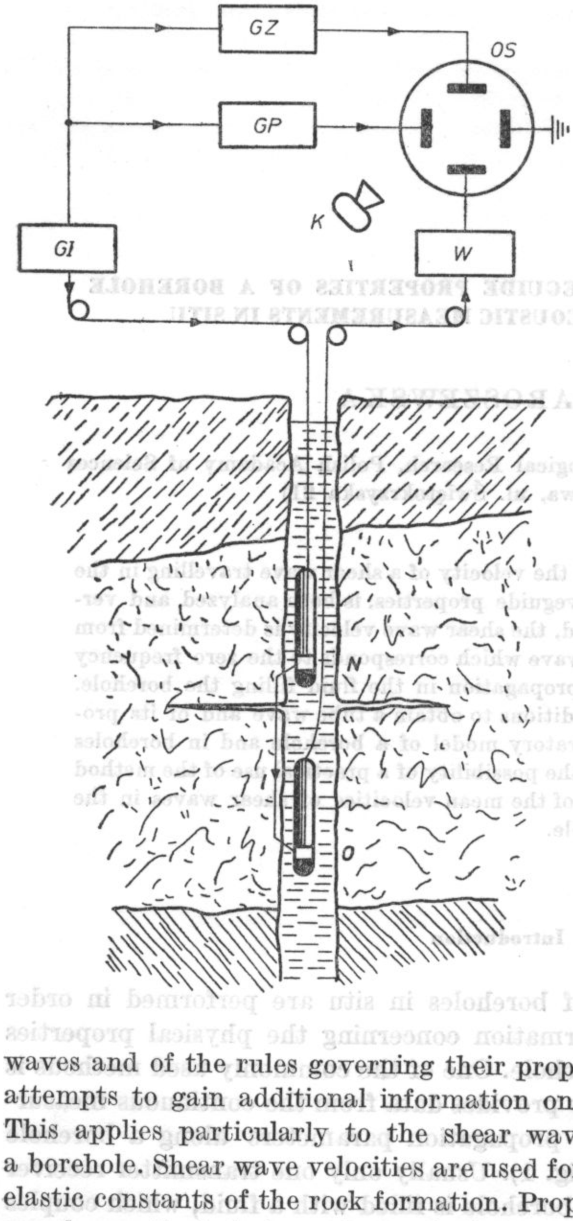

Fig. 1. The principle of geoacoustic measurement in the borehole

GI – pulse generator, *GZ* – time marker generator, *GP* – time base generator, *OS* – oscilloscope, *W* – amplifier, *K* – photographic camera, *N* – transmitter, *O* – receiver, *l* – the tested section of rock formation

waves and of the rules governing their propagation is potentially useful in the attempts to gain additional information on the properties of rock formation. This applies particularly to the shear waves propagating in the vicinity of a borehole. Shear wave velocities are used for the determination of the dynamic elastic constants of the rock formation. Propagation parameters of these waves are also particularly important as indicators of fracture and rock porosity. Read-out of the shear wave propagation data from the recorded response obtained from sonic logging is not accurate and may be ambiguous, as a result of the masking of their first times of arrival by refracted compressional waves and their multiple reflections. Direct measurement of the shear wave propagation data in the vicinity of a borehole, on the other hand, is very difficult, since access to a given location in a borehole is difficult.

In this paper, an analysis is presented of a method of determination of the velocities of shear wave travelling in the walls of a borehole, from the velocity of the so-called tube wave [10, 12, 14, 15, 20]. This wave corresponds to the zero-frequency limit of the lowest radial mode and travels in the borehole fluid. The tube wave velocity is a function of the elastic properties of this fluid and of the rock formation and in particular depends on the shear wave velocity in the formation surrounding the borehole. Investigations of the conditions of excitation and propagation of tube waves were carried out on a laboratory model of a borehole and in the field. The method for the determination of the tube wave velocities was developed from theoretical results describing elastic wave propagation in a fluid-filled cylindrical borehole in an infinite elastic solid. Such a model can be regarded as an approximation of a real borehole.

2. Propagation of acoustic waves in fluid-filled cylindrical boreholes surrounded by an infinite elastic solid

The propagation of acoustic waves in a fluid-filled cylindrical borehole surrounded by an infinite elastic solid was investigated both theoretically and experimentally and described in numerous papers (Biot [1], Gratsinskij [3 - 6], Peterson [10], Riggs [13], Somers [16], White *et al.* [18, 20 - 22]). However, the formal representation of the propagation of acoustic waves excited in a borehole by an impulsive pressure point source was given by Roever *et al.* [14]. In this report, ordinary asymptotic results are also obtained on the basis of an expansion in terms of rays and on the basis of an analysis in terms of propagation modes. Solution of the wave equation in terms of the characteristic modes of propagation of the fluid-filled borehole is particularly representative for describing dispersive wavetrains at large axial distances from the source or long-time oscillations in the vicinity of the source. Ray theory, in contrast, has remarkable practical value in the analysis of the refracted compressional and shear waves, known also as head waves [7, 8, 14, 15], travelling in the solid medium surrounding a borehole.

The propagation of acoustic waves excited in a fluid-filled cylindrical borehole by an impulsive pressure point source was described by Roever *et al.* [14] for a homogeneous and isotropic solid medium, with idealized borehole geometry and neglecting the influence of the logging instrument on the configuration of the acoustic field.

2.1. *Results of mode theory.* According to mode theory, two types of modes of propagation can be excited in the borehole discussed, i.e. circumferential modes and radial modes differing with regard to the pressure distribution along the circumference and along the diameter of a borehole. Characteristic modes are labelled with the indexes l and n, where l refers to the number of nodal planes through a borehole axis and n — to the number of pressure nodes along a borehole radius. Fig. 2 shows some of the first circumferential modes and the nodal diameters. For l even, the circumferential oscillations are symmetric with respect to the plane through the borehole axis, for l odd, the oscillations are antisymmetric with respect to this plane. The fundamental symmetric circumferential oscillations $l = 0$ propagate as a bulging and constriction of the borehole, whereas the fundamental antisymmetric $l = 1$ propagates as a bending of the borehole. Fig. 3 presents the first few radial modes in the borehole for $l = 0$. The concentric circles correspond to the pressure nodes in the borehole fluid. The lengths of successive radii of the nodal circles are proportional to the n zeros j_{ln} of the Bessel functions of the first kind, J_l. The fundamental radial mode $n = 0$ can exist for the two lowest circumferential modes, i.e. for $l = 0$ and $l = 1$. For $l > 0$, the pressure on the borehole axis is zero for all radial modes, i.e. for all values of n. For $l = 0$, in turn, the pressure is maximum on the borehole axis for all radial modes. i.e. all values of n. Hence, in the case of a source located on the borehole axis, only radial modes n associated with the fundamental symmetric circumferential mode $l = 0$ will be excited.

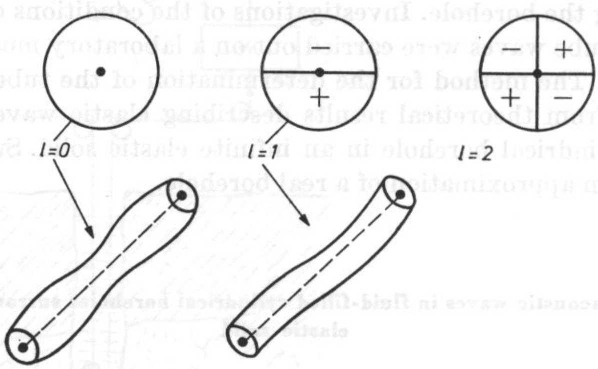

Fig. 2. Circumferential modes in the fluid filling a cylindrical borehole surrounded by an elastic solid

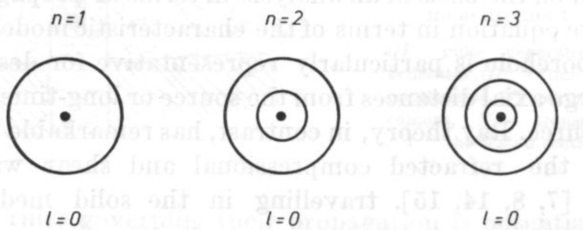

Fig. 3. Radial modes in the fluid filling a cylindrical borehole surrounded by an elastic solid

Propagation of sinusoidal, axially symmetric, radial waves in an infinitely long fluid-filled borehole in an infinite homogeneous elastic medium has been described in detail by BIOT [1]. PETERSON [10] extended this theory for the case of a point source located on the borehole axis and also for the case of an impulsive source. These works [1, 10] present a detailed theory of dispersive wavetrains propagating in the borehole fluid, whereas the report by ROEVER [14] is devoted mainly to the description of refracted arrivals. Fig. 4 shows, after BIOT [1], the group velocity c_g dispersion curves for the radial modes, and

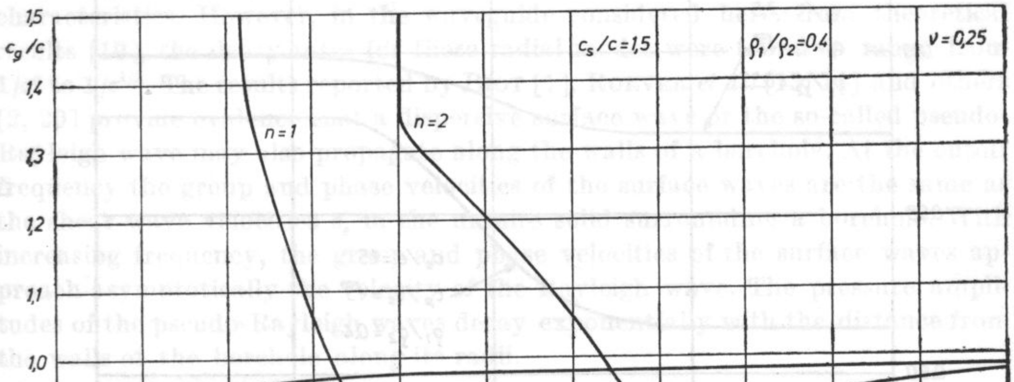

Fig. 4. Dependence of the group velocities of the first few radial modes in a fluid-filled cylindrical borehole surrounded by an elastic solid on the ratio of borehole diameter to wavelength

their dependence on the ratio of the borehole diameter D to the wavelength λ, for $c_p > c_s > c$, where c_p and c_s are the compressional and shear wave velocities, respectively, in the infinite solid surrounding the borehole, ν — Poisson's ratio for the solid medium, ϱ_1 — the density of the fluid in the borehole and ϱ_2 — the density of the solid. The group velocities are related here to the velocity of the dilatational wave c in the borehole fluid. It is easily seen from Fig. 4 that there is only one mode of the lowest order, i.e. for $l = 0$ and $n = 0$, which may propagate in the borehole over the whole range of frequencies, thus having no cutoff frequency. Both group and phase velocities of propagation for this mode are smaller than the dilatational wave velocity in the borehole fluid.

Group and phase velocities of the higher radial modes $n \geqslant 1$ reach, at the cutoff frequency, their maximum value which is equal to the shear wave velocity c_s in the solid surrounding the borehole. In the high frequency range, that is for $\lambda \ll D$, these velocities approach asymptotically the velocity c of dilatational waves in the borehole fluid. For each of these modes a very pronounced minimum in the group velocity, or the so-called Airy phase, is observed.

Plots of the group velocity dispersion curves for the lowest mode $n = 0$, $l = 0$ as a function of the ratio of the borehole diameter to the dilatational wave wavelength λ_d in the fluid, for two solids characterized by different elastic constants, and for $c_p > c_s > c$ are presented after PETERSON [10] in Fig. 5.

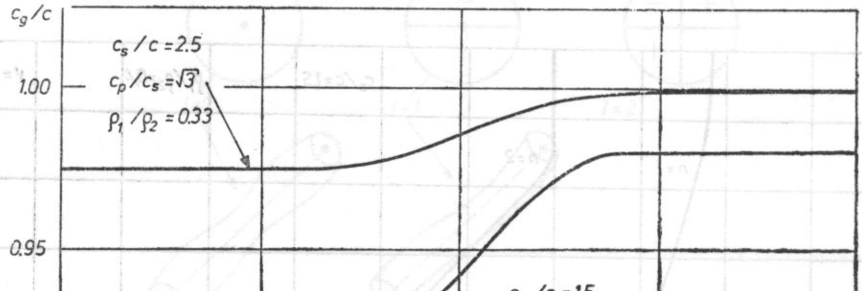

Fig. 5. Dependence of the group velocities of the fundamental mode in a fluid-filled cylindrical borehole surrounded by an elastic solid on the ratio of borehole diameter to wavelength

Both the group and the phase velocities for the lowest mode tend, for $\lambda \ll D$, to the velocity of STONELEY wave with increasing frequency, whilst for the sufficiently low frequencies ($\lambda \geqslant 5D$) they tend to the velocity of tube wave [10, 14, 15, 20], which coresponds to the so-called "waterhammer" phenomenon in a pipe. Minima of the group and phase velocity, in the case of fundamental radial mode, exist in the zero-frequency limit and thus correspond to the velocity of the tube wave c_T. The asymptotic value of this velocity is expressed by the following formula [1, 14, 20], which is valid for $\lambda \geqslant 5D$ and for $c_s > c$,

$$c_T = \frac{c}{\left[1+\frac{\varrho_1 c^2}{\varrho_2 c_s^3}\right]^{1/2}}, \tag{1}$$

where c_T — the tube wave velocity, ϱ_1 and ϱ_2 — the densities of the fluid and solid, respectively, c — the dilatational wave velocity in the fluid, c_s — the shear waves velocity in the solid.

Equation (1) shows that the tube wave velocity depends only on the elastic properties of the borehole fluid (ϱ_i, c) and on the elastic properties of the surrounding solid; namely, on its shear modulus ($\varrho_2 c_s^2$).

According to PETERSON and ROEVER *et al.* [10, 14], the pressure amplitudes for each of the radial modes, attenuation in both media being neglected, decay with the distance z from the impulsive point source, as $1/z^{1/2}$ along the borehole axis. The pressure amplitudes which correspond to the minima of the group velocities (the Airy phase) decay as $1/z^{1/3}$ [10, 14], whilst the tube wave propagates, according to PETERSON [10], without decaying along the borehole axis. It should be noted that in general the actual decay rates of the pressure amplitudes for all the normal radial modes depend on: the mode number, the frequency, the group velocity, the distance to the source and the source signal characteristics. However, in the waveguide considered here, from theoretical results [10], the decay rates for these radial modes were found to range from $1/z^0$ to $1/z^{1/2}$. The results reported by BIOT [1], ROEVER *et al.* [13, 14] and others [2, 20] provide evidence that a dispersive surface wave or the so-called pseudo-Rayleigh wave may also propagate along the walls of a borehole. At the cutoff frequency the group and phase velocities of the surface waves are the same as the shear wave velocities c_s in the infinite solid surrounding a borehole. With increasing frequency, the group and phase velocities of the surface waves approach asymptotically the velocity of the Rayleigh wave. The pressure amplitudes of the pseudo-Rayleigh waves decay exponentially with the distance from the walls of the borehole along its radii.

2.2. *The results of ray theory*. Ray theory applied to the analysis of the propagation of acoustic waves in a cylindrical fluid-filled borehole surrounded by an infinite elastic solid [14] gives, in effect, the refracted compressional and shear waves [2, 4, 7 - 15]. In geoacoustics these waves are usually designated as $P_1 P_2 P_1$ and $P_1 S P_1$. They are excited when the spherical waves are incident on the boundary of two media at an angle larger or equal to the angle of total internal reflection.

Refracted waves propagate along the paths characterized by the minimum travel time relative to the other waves, i.e. along the walls of a borehole. In the case of the propagation of acoustic pulses, the refracted waves thus arrive first at the points of observation in the borehole. The geometry of the borehole causes the rays of the acoustic waves propagating in it to be focused on the borehole axis, which leads to a caustic phenomenon and in effect they are totally reflected from the axis with a 90° phase shift. In the case of the propagation of acoustic pulses, the reflected pulse changes its shape according to a Hilbert transformation [14].

The results from the asymptotic ray theory reported by ROEVER [14] for a source on the borehole axis, and with attenuation in the medium being neglected, indicate that the amplitude of the first refracted compressional arrival decays with the distance z from the source as $1/z\log^2 z$ (i.e. approximately as $1/z$), whilst the amplitude of the first shear arrival decays as $1/z^2$. These results are also supported by empirical data [14] and by the analysis of the complex-valued sections of dispersion curves.

As shown by PETERSON [10], the decrease of the peak amplitude of the first refracted compressional arrival, with distance, z, from the source along the borehole axis, follows the formula $1/z^2 \log z$, which is not in agreement with the formula given by ROEVER [14]. On the other hand, TSANG [18], in his work pertaining to the propagation of compressional refracted waves in a borehole, obtained the same results as those of ROEVER [14], i.e. the decay rate of the amplitude for the first arrival is proportional to $1/z \log^2 z$. TSANG [18] has also shown that the expression for the decay rate of the first amplitude for these wave components along the borehole axis given by PETERSON [10] is erroneous. With reference to the decay rate of the amplitude for the first arrival of shear waves, however, the results obtained by these three authors are in complete agreement ($1/z^2$).

If the source is located in the borehole off its axis, the refracted waves travel in borehole walls along spiral pathways [2, 4, 5, 14]. The decay rate of the amplitude for the first arrival of these "spiral" refracted waves changes, according to ROEVER, with the distance between the source and the borehole walls as well as that between the receiver and the borehole walls and with their spacing along the borehole axis within the limits from $1/z$ to $1/z^2$, when attenuation is not taken into account.

The results by BULATOVA *et al.* [2] and by GRATSINSKIJ [6], obtained on the grounds of an approximate analysis using ray theory and empirical data, seem to indicate that the amplitudes of the first reflected arrivals of both compressional and shear waves travelling along the borehole walls fall off as $1/z^{3/2}$ for the case of a point source located on the axis. For a source off the axis they found that the first amplitudes of both compressional and shear arrivals fall off as $1/z^2$. These results were obtained for the conditions analogous to those assumed by ROEVER [14]. It seems, however, that in these two cases a significant difference should be pointed out, namely that ROEVER [14] measured the amplitude on the borehole axis and not close to its walls [2, 6]. If the attenuation in the surrounding rock-formation is taken into account, the total decrease of the amplitudes for an axial location of the source in the borehole follows the formula [2, 6]

$$A = \frac{A_0 \exp(-\alpha z)}{z^{3/2}}, \tag{2}$$

and for an off-axis location of the source

$$A = \frac{A_0 \exp(-\alpha z)}{z^2}, \tag{3}$$

where z — the cylindrical axial coordinate of the borehole, A_0 — the wave amplitude for $z = 0$, A — the wave amplitude at the distance z from the source, α — the attenuation coefficient in the rock formation.

In summary, the results presented in the reports cited indicate, in spite of differences, that the amplitudes of the refracted waves decrease more rapidly as a function of the distance from the source of acoustic waves in the borehole than the amplitudes of the dispersive wavetrains travelling in the borehole fluid. These conclusions hold, however, for the case where energy losses in both media are neglected.

2.3. *Acoustic response obtained from the borehole.* On the basis of the above description of acoustic wave propagation in a fluid-filled borehole surrounded by an elastic solid, the theoretical acoustic response from a borehole for the case of a short pulsed disturbance [9, 11] can be predicted. Fig. 6 illustrates schematically the acoustic waves propagating in a borehole from an impulsive source

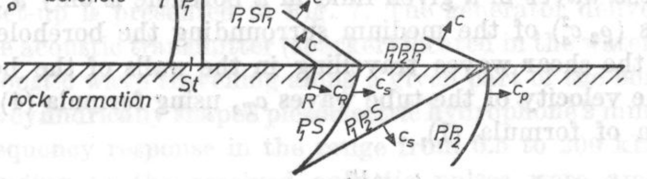

Fig. 6. Theoretical acoustic response in a borehole

$P_1P_2P_1$ – refracted compressional wave, $P_1 S P_1$ – refracted shear wave, R – pseudo-Rayleigh wave, P_1 – direct wave, St – Stoneley wave, T – tube wave, c – velocity of dilatational waves in the fluid filling the borehole, c_p and c_s – velocity of compressional and shear waves in rock formation, c_T – tube wave velocity, O – acoustic source

o, situated in the figure on its left-hand side. For simplification, only one boundary of the borehole is shown. In the case of a pulse-like disturbance, at the point of observation located some distance from the source, refracted waves: compressional, $P_1 P_2 P_1$, and shear, $P_1 S P_1$, arrive first, being characterized by minimum travel times, and propagating along the borehole walls with velocities c_p and c_s, respectively, and with velocity c in the borehole fluid. Next, with a short delay relative to the shear wave, a dispersive surface wave R and dispersive wavetrains in the borehole fluid arrive. Among the latter waves, the so-called direct wave P_1, characterized by high frequency components and travelling with velocity of a dilatational wave c in the fluid and the Stoneley wave can be distinguished. The tube wave T characterized by a relatively low frequency and travelling with velocity $c_T < c$ arrives last. In the tail of the pulse, oscillations, corresponding to the frequencies of the Airy phase of the first few modes, may also be observed.

It is obvious that the wider the spectrum of the pulse emitted in the borehole the more complex the received response will be, i.e. more of the various waves will be excited in the borehole. At large distances of the observation point from the source, relative to the wavelength, along the borehole axis, the waveform differs significantly from that for small distances, i.e. the vicinity of the source.

This results from the significantly different decay rates of the amplitude for various types of acoustic waves with the distance to the source discussed above, as well as their different attenuation in the medium. The present experimental data and the results of theoretical works thus indicate that, at sufficiently large distances from the transmitter, only dispersive wavetrains in which the amplitudes fall off relatively slowly with distance arrive at the point of observation. In particular, this pertains to the tube wave which is least attenuated, since its frequency is the lowest.

3. Propagation of acoustic waves investigated on the model of a borehole

3.1. *Technique*. The conditions necessary to excite a tube wave and its propagation data were investigated on a model of the borehole. The propagation velocity of these waves in a given fluid in a borehole is only a function of the shear modulus ($\varrho_2 c_s^2$) of the medium surrounding the borehole (1). Thus the velocity c_s of the shear waves travelling in the walls of the borehole can be found from the velocity of the tube waves c_T, using formula (4) obtained from transformation of formula (1),

$$c_s = \left(\frac{\varrho_1}{\varrho_2}\right)^{1/2} \frac{c}{\left[1-\left(\frac{c_T}{c}\right)^2\right]^{1/2}}. \tag{4}$$

The present idea of recording and measuring the velocity of the tube waves originated from the theoretical and experimental works discussed which indi-

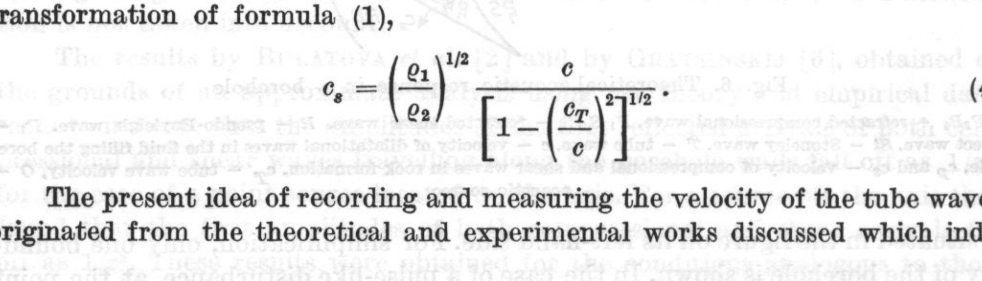

Fig. 7. Block diagram of the laboratory set-up for investigation of the propagation of acoustic waves in a model borehole

cate that the amplitudes of these waves fall off comparatively less relative to the amplitudes of the other waves as a function of distance from the source. To check the truth of this statement, the amplitudes of the first arrivals of the tube waves were measured on a laboratory model along the borehole axis as a function of distance from the source. To the present writer's knowledge such measurements have never previously been carried out.

A borehole model 12 mm in diameter was made along the axis of symmetry of a concrete block of $116 \times 30 \times 30$ cm. The compressional and shear elastic wave velocities in the block determined using conventional techniques were $c_{p1} = 4562$ m/s and $c_{s1} = 2660$ m/s, respectively. The compressional wave velocity was measured using a transmission technique and the shear wave velocity using piezoelectric transducers with suitable electrical polarization.

Investigations of the tube waves were carried out on the model borehole with the block immersed in a special water-filled vessel. A block diagram of the laboratory set-up is presented in Fig. 7. The generator delivering electric pulses excited the acoustic transmitter (sparker) located in the water-filled model borehole. The acoustic wave travelling along a given part of the borehole length was received by a cylindrically shaped piezoelectric hydrophone 2 mm in diameter, having a flat frequency response in the range from 0.5 to 200 kHz. Electrical signals corresponding to the received acoustic pulses were amplified using a suitable preamplifier and fed to the synchroscope input. The generator delivering pulses to the transmitter and the synchroscope were triggered by a quartz-stabilized clock. To excite the wave in the model borehole, the sparker used emitted acoustic waves of a sufficiently low frequency spectrum, i.e. $\lambda \geqslant 5D$. The construction of the sparker used is presented in Fig. 8. It consists of a teflon-insulated copper wire 0.7 m min diameter and of a copper tube 1.5 mm in diameter pressed tight over the wire insulator. The inner wire is one, and the outer tube the other electrode of the sparker, which, due to its dimensions, was regarded as a point source. The frequency spectrum emitted by such a transmitter depends on its geometry and on the duration of the electric pulse leading to the spark discharge. In the present experiments, 4.5 kV rectangular *dc* pulses were used. Their duration was adjustable in the range from 5 to 100 μs and the repetition frequency in the range from 1 to 50 pulses per second.

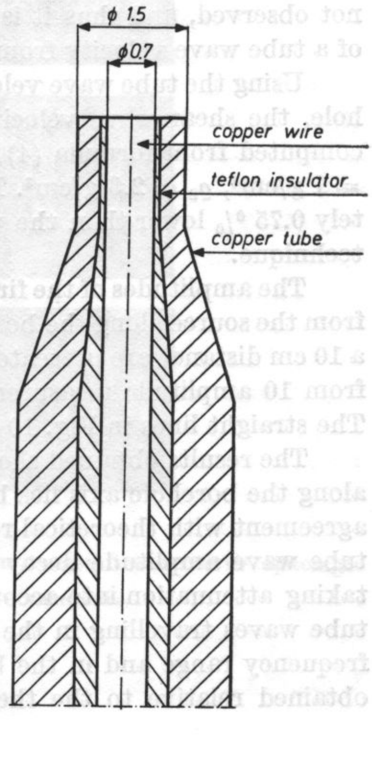

Fig. 8. Sparker cross-section

The wave forms were photographed from a synchroscope screen as a function of the transmitter-receiver distance along the borehole axis. This distance was changed from 10 to 90 cm in 10 cm steps. Both transducers used in the measurement of the tube wave amplitudes were constrained to be on the borehole model axis by mechanical centralizers.

3.2. *Results of the laboratory measurements*. Fig. 9 shows the oscillograms of the acoustic signals obtained on the borehole model at constant amplification and sweep frequency for various transmitter-receiver spacings. As can be easily observed from these oscillograms, the most pronounced frequency component in the received pulses amounts to about 25 kHz.

At front of the received pulses, a very weak wave component is observed. The travel-times of this wave indicate that it is a refracted shear arrival. The subsequent segment of the received pulses contains a large amplitude wave of approximately the same frequency (25 kHz) travelling at a velocity of about 1390 m/s, i.e. lower than the velocity of dilatational waves in water. This indicates the dispersive character of this wave. For this wave and the borehole diameter $D = 12$ mm, the relation $\lambda \geqslant 5D$ is valid. In the range of wavelengths thus determined, only the fundamental radial mode, at the tube wave velocity may propagate according to the mode theory. As can be readily seen from Fig. 9, a very substantial part of the pulse energy travels at the tube wave velocity. With large transmitter-receiver spacings other waves than tube waves are not observed, and thus it is possible to achieve a very accurate determination of a tube wave velocity from its travel time.

Using the tube wave velocity $c_T = 1390$ m/s as measured on the model borehole, the shear wave velocity $c_{s2} = 2640$ m/s in the concrete block itself was computed from formula (4) for the following parameters: $c = 1482$ m/s, $\varrho_1 = 1$ g/cm³, $\varrho_2 = 2.3$ g/cm³. The obtained velocity $c_{s2} = 2640$ m/s is approximately 0.75 % lower than the velocity $c_{s1} = 2660$ m/s obtained using the classical technique.

The amplitudes of the first arrivals of the tube wave as a function of distance from the source along the borehole axis normalized relative to the amplitude at a 10 cm distance are presented in Fig. 10. The data points in the graph are means from 10 amplitude measurements. Standard deviation never exceeded 0.3 dB. The straight lines in Fig. 10 have slopes of $1/z^{1/2}$, $1/z^{2/3}$ and $1/z$.

The results obtained show that the decay rate for the tube wave amplitude along the borehole axis lies between $1/z^{2/3}$ and $1/z$. These results are not in full agreement with theoretical results which determine the maximum slopes of the tube wave amplitude decay rate, due to the geometrical factors and without taking attenuation into account, as not exceeding $1/z^{1/2}$. The attenuation of the tube waves travelling in the water medium in the borehole is negligible in the frequency range and in the borehole segments examined [8]. The discrepancy obtained relative to the theoretical works may thus have resulted from the

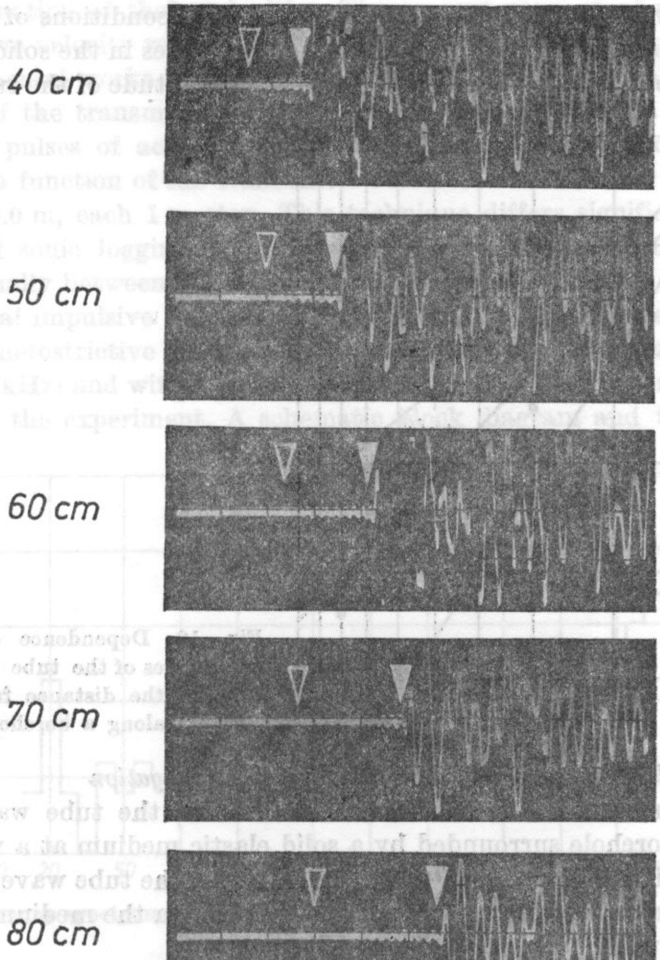

Fig. 9. Oscillograms of acoustic model responses for various transmitter-receiver spacings

time base — 100 μs/cm; ▽ — shear wave, ▼ — tube wave assumptions which simplify and idealize the actual conditions of measurement and mainly from the assumption that the energy losses in the solid medium surrounding the borehole have no influence on the amplitude of the tube wave.

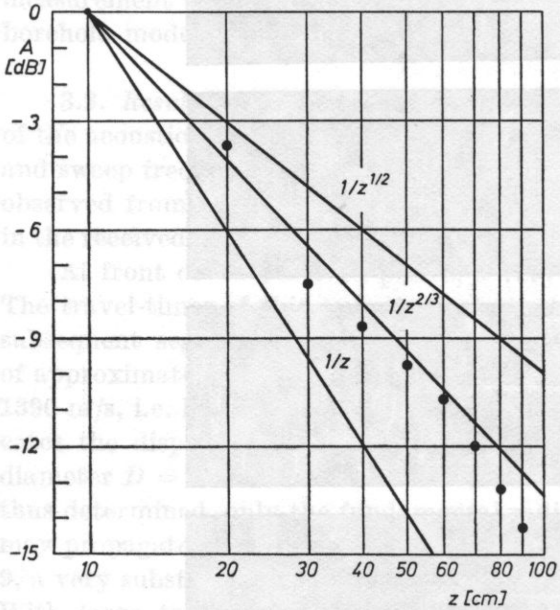

Fig. 10. Dependence of the relative amplitudes of the tube wave frist arrivals on the distance from transmitter along a borehole axis

3.3. Concluding remarks on the laboratory investigation

(a) In agreement with the theoretical results, the tube wave travels in a fluid-filled borehole surrounded by a solid elastic medium at a velocity lower than the velocity of dilatational waves in the fluid. The tube wave velocity may be used for computation of the shear wave velocity in the medium surrounding the borehole, using formula (4).

(b) The amplitude of the tube wave decreases more slowly as a function of axial transmitter-receiver spacing than the amplitudes of the refracted waves.

(c) At sufficiently large distances from the transmitter only dispersive wavetrains are observed in the model response recorded. If the acoustic pulses emitted have a frequency spectrum of sufficient density at frequencies which are low enough ($\lambda \geqslant 5D$), then the tube wave is observed.

(d) The decay rate of the tube wave amplitude observed actually as a function of transmitter-receiver spacing is larger than that predicted theoretically. This discrepancy probably results from theoretical assumptions which idealize and simplify the actual conditions of the measurement.

4. Field measurements

4.1. *Method.* Investigations in boreholes in situ pertaining to the conditions of excitation and propagation data of tube waves were aimed chiefly at the determination of their velocities. The present concept of measurement of the tube wave velocity results, as it was already discussed, from the theoretical and experimental works which point out that it is only possible to obtain the tube wave if the transmitter-receiver spacing is large enough. In the present project the pulses of acoustic waves travelling in the borehole were photographed as a function of the transmitter-receiver distance over the range from 0.5 m to 10.0 m, each 1 m step. This technique differs significantly from the technique of sonic logging in which, as a rule, a constant transmitter-receiver spacing (usually between 0.5 m and 1.8 m) is used.

A typical impulsive geoacoustic Petroscope PT-13 equipment with a ring-shaped magnetostrictive transmitter probe (radial resonant frequency 13.9 or 25.4 or 31.5 kHz) and with a piezoelectric ring-shaped transducer receiver probe was used in the experiment. A schematic block diagram and the principle of

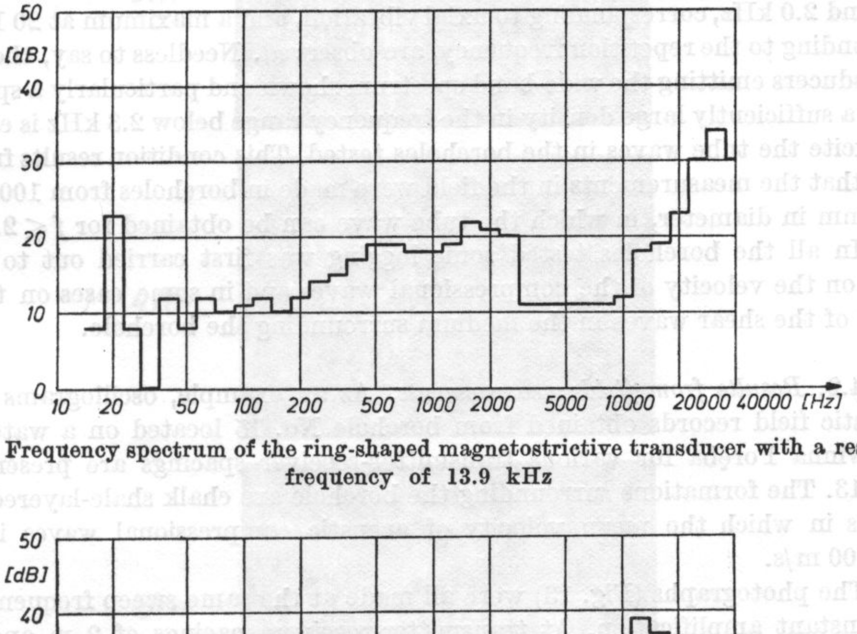

Fig. 11. Frequency spectrum of the ring-shaped magnetostrictive transducer with a resonant frequency of 13.9 kHz

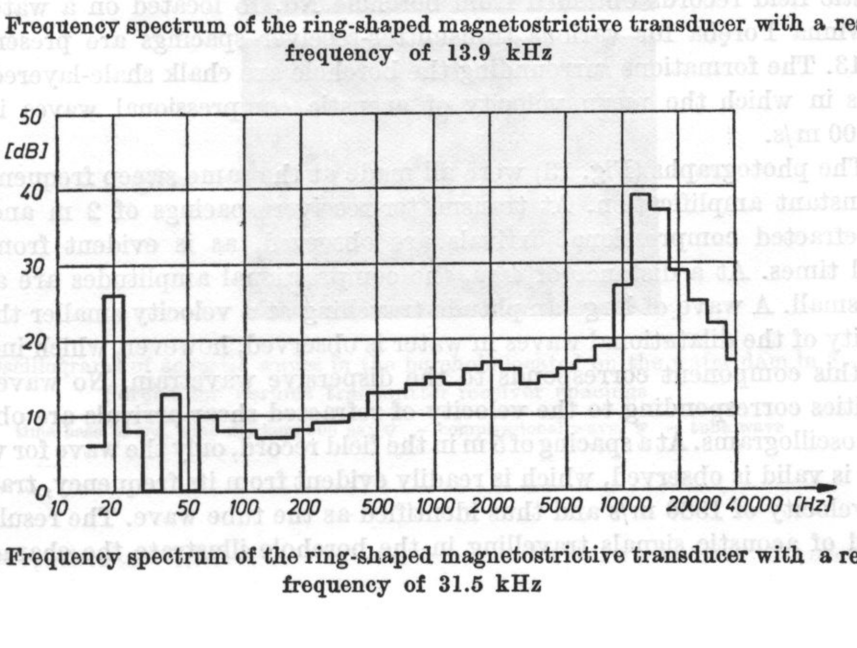

Fig. 12. Frequency spectrum of the ring-shaped magnetostrictive transducer with a resonant frequency of 31.5 kHz

operation of the equipment are given in Fig. 1. Transmitting transducers were set into vibration using *dc* pulses at a repetition frequency of 20 Hz.

The frequency spectrum of the transmitter transducers used was measured using a sphere-shaped piezoelectric hydrophone 10 mm in diameter and with a frequency response flat over the range from 2 Hz to 40 kHz. Acoustic waves were also received using the probe of a typical Petroscope PT-13 equipment. The frequency spectra of magnetostrictive transducers operating at resonant frequencies of 13.9 and 31.5 kHz, obtained from a 1/3 octave spectral analyzer (corrected for the frequency response of the PT-13 receiver channel) as being most representative are presented in Figs. 11 and 12. The measurements of these frequency spectra were carried out in a large water basin (lake) and with a transmitter-receiver spacing of 1 m. The spectra obtained provide evidence that the transducers used emitted wide-band signals. Except for the maximum at 13.9 or 31.5 kHz, which corresponds to the radial resonance, a maximum between 1.5 and 2.0 kHz, corresponding to axial vibration, and a maximum at 20 Hz, corresponding to the repetition frequency, are observed. Needless to say, the use of transducers emitting the wide-band spectrum shown and particularly a spectrum with a sufficiently large density in the frequency range below 2.3 kHz is essential to excite the tube waves in the boreholes tested. This condition results from the fact that the measurements in the field were made in boreholes from 100 mm to 130 mm in diameter, in which the tube wave can be obtained for $f \leqslant 2.3$ kHz.

In all the boreholes tested sonic logging was first carried out to obtain data on the velocity of the compressional waves and in some cases on the velocity of the shear waves in the medium surrounding the borehole.

4.2. *Results from field measurements.* As an example, oscillograms of the acoustic field records obtained from borehole No. 15 located on a water dam in Swinna Poręba for various transmitter-receiver spacings are presented in Fig. 13. The formations surrounding the borehole are chalk shale-layered sandstones in which the mean velocity of acoustic compressional waves is $c_p = 3700$ m/s.

The photographs (Fig. 13) were all made at the same sweep frequency and at constant amplification. At transmitter-receiver spacings of 2 m and 3 m, the refracted compressional arrivals are observed, as is evident from their travel times. At a distance of 4 m, the compressional amplitudes are already very small. A wave of large amplitude travelling at a velocity smaller than the velocity of the dilatational waves in water is observed, however, which indicates that this component corresponds to the dispersive wavetrain. No waves with velocities corresponding to the velocity of refracted shear arrivals are observed in the oscillograms. At a spacing of 5 m in the field record, only the wave for which $\lambda \geqslant 5D$ is valid is observed, which is readily evident from its frequency, travelling at a velocity of 1380 m/s and thus identified as the tube wave. The results presented of acoustic signals travelling in the borehole illustrate the character of

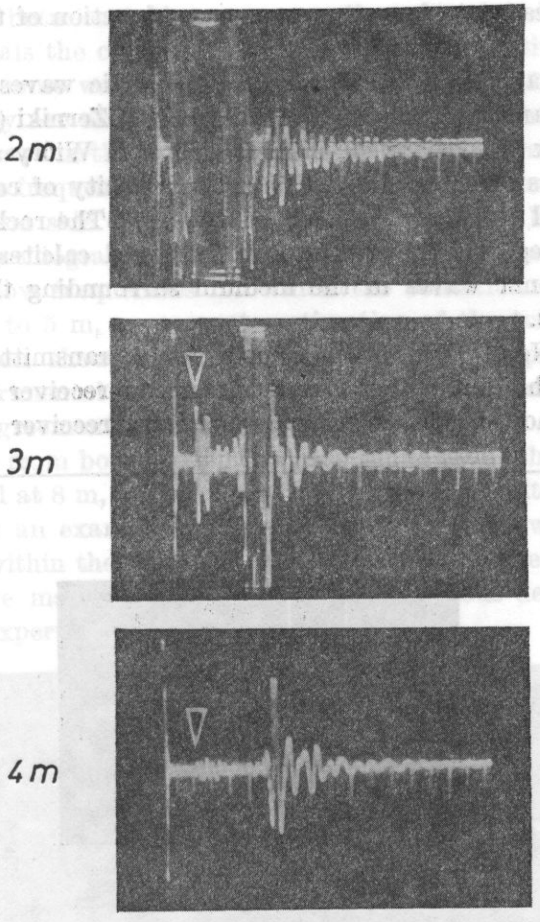

Fig. 13. Oscillograms of acoustic waves in the borehole located on the water dam in Swinna Poręba for various transmitter-receiver spacings

time base 10 ms, time markers 500 μs; ▽ – compressional wave, ▼ – tube wave

the amplitude decay rate for some waves as a function of transmitter-receiver distance.

The other examples of oscillograms of acoustic waves in boreholes were taken on water dams in Wióry (No. 1) and Brzegi-Żerniki (Nos. 2 and 3). The characteristic rock formations on the water dam in Wióry are chiefly medium grained sandstones of lower Trias. The mean velocity of compressional waves in these geological formations was $c_p = 2700$ m/s. The rock formations at the water dam in Brzegi-Zerniki are Jurassic fractured calcites. The mean velocity of compressional waves in the medium surrounding the borehole tested was $c_p = 3150$ m/s.

Photograph No. 1 (Fig. 14) was made for a transmitter-receiver spacing of 4 m, photographs Nos. 2 and 3 for a transmitter-receiver spacing of 5 m but with various borehole depths. For these transmitter-receiver spacings it is poss-

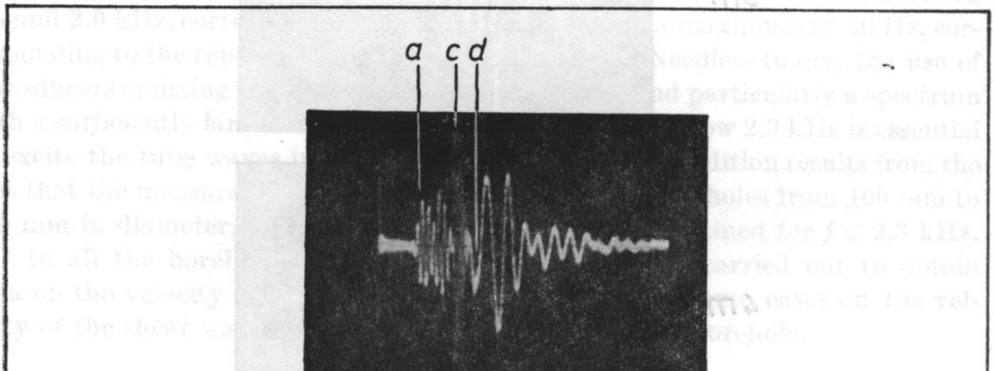

Fig. 14. Oscillograms of acoustic waves in the borehole located on the water dam in Brzegi--Żerniki and Wióry

time base 10 ms, time markers 500 μs; *a* – compressional wave, *b* – shear wave, *c* – direct wave, *d* – tube wave ible to distinguish between various waves on the basis of their travel time. At the fronts of the signals the compressional waves of comparatively high frequency are observed; in some waveforms, i.e. photographs Nos. 2 and 3 shear waves of lower frequency are also present. Direct waves travelling at the velocity of the dilatational wave in the water filling the borehole, characterized by the components of highest frequencies contained in the spectrum emitted, in agreement with the right-hand side of the group velocity curves in Fig. 4, can be easily found in all the oscillograms. Pulse tails correspond to tube waves having large amplitudes and low frequencies for which the condition $\lambda \geqslant 5D$ is valid. At distances from 4 to 5 m, an accurate estimation of the tube wave velocity is, in the oscillograms shown, rather difficult, due to the presence of the other waves. The experimental results for acoustic waves travelling in boreholes in situ are in good agreement with theoretical predictions.

Oscillograms from boreholes in the rock foundation (shale) of a water dam in Młoty obtained at 8 m, i.e. for a relatively large transmitter-receiver spacing, are presented by an example in Fig. 15. Only the tube waves for which the wavelengths lie within the limit $\lambda \geqslant 5D$ are observed in these oscillograms. The photographs were made in borehole No. 82 at various depths but otherwise under the same experimental conditions throughout.

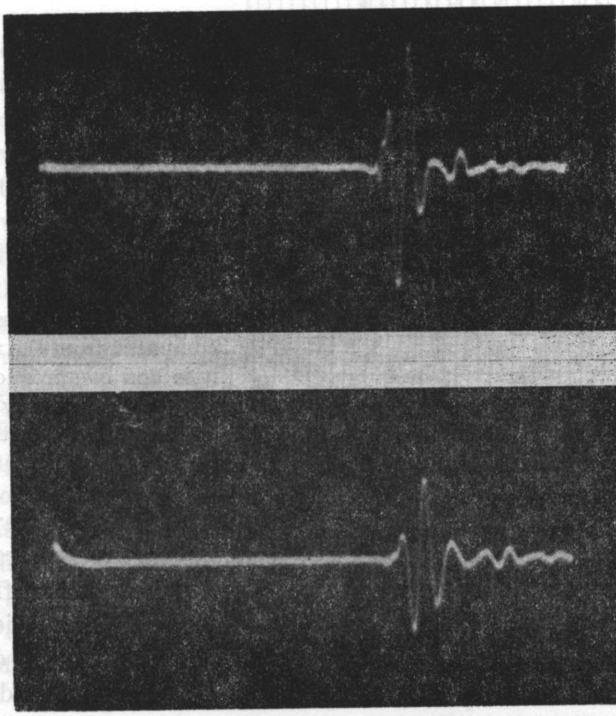

Fig. 15. Oscillograms of acoustic waves from various depths of the borehole located on the water basin in Młoty for 8 m distance to the transmitter. Time base 10 ms, time markers 500 μs

The mean values of the compressional wave velocity in the rock formation surrounding borehole No. 82 in Młoty, computed from the tube wave velocity using relation (4), as a function of depth, are presented in Fig. 16 together with the compressional wave velocities for comparison.

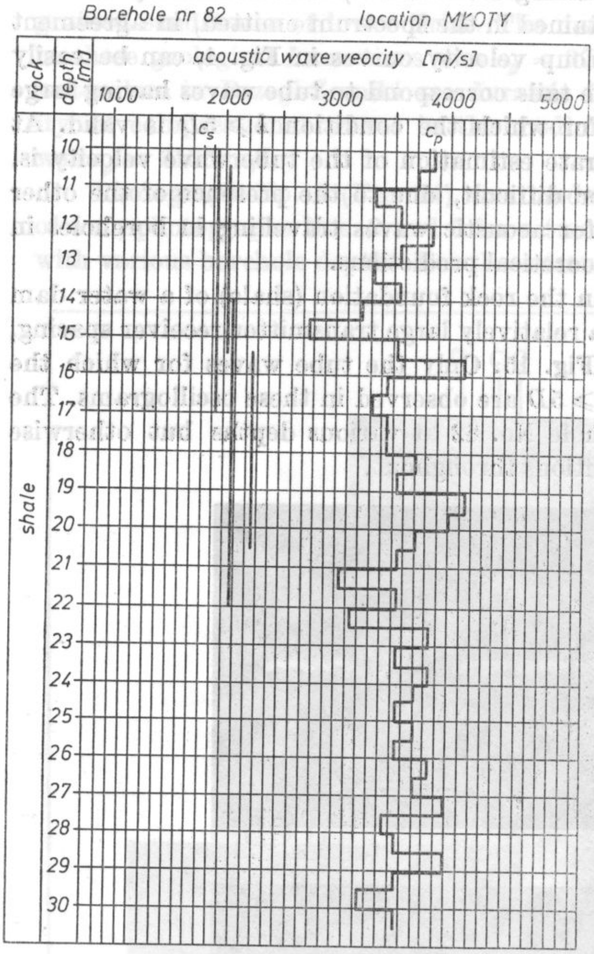

Fig. 16. Mean shear wave velocities computed from the tube wave velocities and compressional wave velocities in the vicinity of the borehole at Młoty as a function of its depth

4.3. *Concluding remarks on field measurements*

1. The records presented here illustrate the multitude of wave types which are excited in boreholes using ring-shaped magnetostrictive transducers, and also the decay rates of their amplitudes along the borehole axis in field conditions. In complete agreement with the recent data from both theoretical and experimental works, the present data also provide evidence that the decay rates for refracted waves travelling in the borehole walls as a function of distance are the largest.

2. For large transmitter-receiver spacing only dispersive wavetrains propagating in the borehole fluid were observed in the field records.

3. Using the tube wave velocity, the mean shear wave velocity in the rock mass surrounding the borehole can be computed using relation (4). The use of equation (4) for computation of the shear wave velocity, c_s, from the measured tube wave velocity, c_T, introduces an additional error σ_W determined by the general formula [19]

$$\sigma_W = \sigma_{\bar{x}} \frac{\partial W}{\partial x}, \tag{5}$$

where $\sigma_{\bar{x}}$ — the standard deviation of the mean of the measured quantity $\bar{x}$, W — a function of the measured quantity, x — the measured quantity.

Using function (4), the following dependence is obtained

$$\frac{\partial W}{\partial x} = \frac{\partial c_s}{\partial c_T} = \left(\frac{\varrho_1}{\varrho_2}\right)^{1/2} \frac{1}{\left[1-\left(\frac{c_T}{c}\right)^2\right]^{3/2}}, \tag{6}$$

and hence

$$\sigma_W = \sigma_{c_s} = \sigma_{\bar{c}_T} \left(\frac{\varrho_1}{\varrho_2}\right)^{1/2} \frac{1}{\left[1-\left(\frac{c_T}{c}\right)^2\right]^{3/2}}. \tag{7}$$

As can be seen from (7), the error $\sigma_{\bar{c}_s}$ resulting from the use of function (4), for the computation of the shear wave velocity, becomes much larger as the tube wave velocity approaches the velocity of the dilatational wave in the borehole fluid. These values are, in practice, of the same order and thus the error, σ_{c_s}, in determining the velocity of the shear wave may be significantly larger than the standard deviation ,$\sigma_{\bar{c}T}$, of the mean value of the measured tube wave velocity c_T. The greatly increased transfer of tube wave velocity measurement errors to the errors of their function (4) indicates some deficiency of the method investigated. This deficiency can be well illustrated by the following examples. The tube wave velocities determined for borehole No. 82 located on the hydro-electric power station Młoty were the means of 10 measurements with a standard deviation not exceeding ±0.1 dB. The error σ_{c_s} of the shear wave determination according to formula (7) for the tube wave velocity $c_T = 1333$ m/s was in this case $\sigma_{c_s} = 7.4\,\sigma_{\bar{c}_T}$, i.e. ± 0.74 dB, for $c_T = 1351$ m/s, correspondingly, $\sigma_{c_s} = 9.0\,\sigma_{\bar{c}_T}$, i.e. ±0.9 dB.

4. The shear wave velocity computed from the mean tube velocity for sections of the borehole some meters long can be used for the determination of the mean dynamic elastic constants of the rock mass. These constants are particularly useful and significant for engineering geology.

5. The use of two receivers that are relatively close to each other with a spacing of 0.5 m for example, at some meters' distance from the transmitter should improve, as far as can be predicted, the resolution of the method discussed.

6. Identification of the tube wave and the very accurate determination of its propagation velocity using the described method is not possible in all specific borehole conditions and is particularly difficult if the attenuation in the rock formation is small.

7. Further research in the field using the described method in lithologically different geological formation seems promising.

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References

[1] M.A. BIOT, *Propagation of elastic waves motion in a cylindrical hole containing a fluid*, J. Appl. Phys., **23**, 9, 997-1005 (1952).

[2] Z.M. BULATOVA, E.A. VOLKOWA, E.F. DUBROV, *Akusticheskij karotazh*, Izd. "Nedra" Leningrad 1970, pp. 31-45.

[3] W.G. GRATSINSKIJ, *Kinematicheskiye osobiennosti volnovey kartiny pri ultrazvukovom karotazhe skvazhin*, Izv. Ak. Nauk SSSR, ser. geofiz., **7**, 1021-1039 (1963).

[4] W.G. GRATSINSKIJ, *Kinematika volnovykh frontov pri karotazhe skvazhin a raspredelonnymi preobrazovatelami*, Izv. Ak. Nauk SSSR, ser. geofiz., **8**, 1178-1197 (1963).

[5] W.G. GRATSINSKIJ, *Issledovanie uprugich voln v modeli skvazhiny*, Izv. Ak. Nauk SSSR, ser, geofiz., **3**, 322-338 (1964).

[6] W.G. GRATSINSKIJ, *Amplitudy skolzyashchikh voln na poverkhnosti skvazhiny*, Izv. Ak. Nauk SSSR. ser. geofiz., **6**, 819-838 (1964).

[7] A. JAROSZEWSKA, *Investigation of the conditions for excitation and propagation of refracted acoustic waves in rock samples* (in Polish), Archiwum Akustyki, **15**, 4. 385-398 (1980).

[8] I. MALECKI, *Physical foundations of technical acoustics*, Pergamon Press, PWN, 1969, pp. 134, 267.

[9] R.L. MORRIS, D.R. GRINE, T.E. ARKFELD, *Using compressional and shear acoustic amplitudes for the location of fractures*, Jour. Pet. Techn., **16**, 6, 623-632 (1964).

[10] E.W. PETERSON, *Acoustic wave propagation along a fluid-filled cylinder*, J. Appl. Phys., **45**, 8, 3340-3350 (1974).

[11] G.R. PICKETT, *Acoustic character logs and their applications in formation evaluation*, Jour. Pet. Techn., **15**, 6, 659-667 (1963).

[12] S. PLEWA, *Prospecting geophysics* (in Polish), Śląsk, Katowice 1972.

[13] E.D. RIGGS, *Seismic wave types in a borehole*, Geophysics, **20**, 1, 53-67 (1955).

[14] W.L. ROEVER, J.H. ROSENBAUM, T.F. VINING, *Acoustic waves from an impulsive sources in a fluid-filled borehole*, JASA, **55**, 6, 1144-1157 (1974).

[15] W.L. ROEVER, T.F. VINING, E. STRICK, *Propagation of elastic wave motion from an impulsive source along a fluid-solid interface, I. Experimental pressure response, II. Theoretical pressure response*, Phil. Trans. R. Soc., **A 251**, 455-488 (1959).

[16] E.V. SOMERS, *Propagation of acoustic waves in a liquid-filled cylindrical hole surrounded by an elastic solid*, J. Appl. Phys., **24**, 5, 515-521 (1953).

[17] G.C. SUMMERS, R.A. BRODING, *Continuous velocity logging*, Geophysics, **18**, 3, 598 (1952).

[18] L. TSANG, J.A. KONG, *Asymptotic methods for the first compressional head wave arrival in a fluid-filled borehole*, JASA, **65**, 3, 647-654 (1979).

[19] J. Wehr, *Measurements of velocities and attenuation of ultrasonic waves* (in Polish), PWN, Warszawa 1972, p. 78.

[20] J.E. White, *Seismic waves radiation, transmission, and attenuation*, McGraw-Hill, New York 1965, p. 142-211.

[21] J.E. White, H.H. Frost, *Unexpected waves observed in fluid-filled boreholes*, JASA, **28**, 5, 924-927 (1956).

[22] J.E. White, R.E. Zechman, *Computed response of an acoustic logging tool*, Geophysics, **33**, 2, 302-310 (1968).

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