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DETECTABILITY OF SMALL BLOOD VESSELS AND FLAT BOUNDARIES OF SOFT TISSUES IN THE ULTRASONIC PULSE ECHO METHOD

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The detectability of a small blood vessel of a radius 0.1 mm by the pulse echo ultrasonic method using a frequency of 2.5 MHz was estimated. It was assumed that the vessel was surrounded by a homogeneous soft tissue (i.e. causing no reflection) and was in the near region of the far field radiated in a continuous manner by a plane transducer of a diameter of 2 cm. The soft tissue and the walls of the vessel were assumed to have the same elastic properties as those of a liquid.

The measurements were carried out in a plane polar coordinate system, where the incident wave, the reflected wave and the wave penetrating into the vessel were expressed in terms of Bessel and Hankel functions. The boundary conditions were assumed in the form of the equality of the acoustic pressures and the normal components of acoustic velocities on each side of the surface of the vessel. Thence the magnitude of the reflected wave was determined.

The losses of the signal due to the reflection of the wave, its divergence and absorption, are shown in the form of a graph from which it can be seen that the signal from the vessel considered is essentially detectible, although it lies near the noise level, and is critically dependent on the distance from the transducer due to attenuation in the tissues penetrated.

The detectability of the plane boundaries of soft tissues was also determined, indicating that at a distance of 20 cm from the transducer a difference in the characteristic acoustic impedance of the tissues of 0.2% is sufficient to give a detectable echo.

1. Introduction

Ultrasonography (pulse ultrasonography) gives valuable information on the human tissues investigated, permitting the detection and visualization of the boundaries of the tissues, both normal and pathological, and also visualization of the texture of these tissues. This is possible as a result of specular reflections arising from the boundaries of the tissues, and from the echoes caused by diffuse reflections, when the size of the anatomical structures is small compared to the wavelength. The present paper attempts to estimate quantitatively the detectability, using an ultrasonographic method, of a blood vessel within human soft tissues. The diameter of the vessel is assumed to be several times smaller than the ultrasonic wavelength so that the reflection of the wave is diffuse in character.

At the same time the minimum difference in the (characteristic) acoustic impedance of the tissue that is necessary to detect plane boundaries with a normally incident wave, is determined.

2. The assumptions of the analysis

It is assumed that a blood vessel with a radius $a = 0.1$ mm is in the initial ultrasonic region of the far field generated by a plane piezoelectric transducer (transmit-receive transducer) with a diameter of 2 cm and a frequency $f = 2.5$ MHz (wavelength $\lambda = 0.63$ mm).

Under the conditions assumed the boundary between the near and far fields is 16 cm. At a distance $r_0 = 20$ cm from the transducer one can assume that the acoustic pressure in this region of the field is approximately equal to the acoustic pressure averaged over the whole area of the ultrasonic beam near the transducer. The blood vessel is on the axis of the ultrasonic beam and the axis of the vessel is perpendicular to the axis of the beam. It is assumed that the ultrasonic wave falling onto the vessel is a plane, homogeneous wave, although this is essentially not satisfied in practice.

The calculations were carried out for a continuous wave despite the use of the pulse echo method in ultrasonography. However, since the diameter of the vessel is small compared to the wavelength and since the impedances of the soft tissue and blood hardly differ, the transient time becomes very short. In this case the assumption of a steady state introduces no serious error.

It was also assumed that the elastic properties of the tissues are the same as those of the liquid and also that the surrounding tissue and the walls of the vessels are sufficiently homogeneous for no reflections to occur inside them.

3. The reflection of a plane wave from a blood vessel

The calculations were made for a plane polar coordinate system where r is the radius, and θ is the azimuth. The axis of the coordinate system coincides with the axis of the blood vessel. The incident plane wave can be represented here as a series of cylindrical functions [5]

$$\varphi_i = \varphi_M \Big[J_0(k_t r) + 2 \sum_{1}^{\infty} (-j)^m J_m(k_t r) \cos(m\theta) \Big] e^{j\omega t}, \tag{1}$$

where φ_i is the acoustic potential of the incident wave, φ_M is the potential amplitude, J_M is a Bessel function of order m, m is an integer, $k_t = \omega / c_t$ is the wave number, $\omega = 2\pi f$, f is the frequency, and c_t is the wave velocity in the tissue.

The acoustic potentials of the reflected wave and of the wave penetrating into the vessel are of the forms respectively

$$\varphi_r = \sum_{0}^{\infty} A_m H_m^{(2)}(k_t r) \cos(m\theta) e^{j\omega t}, \tag{2}$$

$$\varphi_t = \sum_{0}^{\infty} B_m J_m(k_b r) \cos(m\theta) e^{j\omega t}, \tag{3}$$

where $H_m^{(2)}$ is a Hankel function of the second kind of order m, $k_b = \omega / c_b$ is the wave number, and c_b is the wave velocity in blood.

The functions $J_m(kr)$ and $H_m^{(2)}(kr)$ are solutions of the wave equation in the cylindrical coordinate system for the variable r. The function $H_m^{(2)}$ represents the wave propagating in the direction of increasing radius r. However, this function is not specified for r tending to zero. The wave penetrating into the blood vessel was therefore described by the function J_m. The constants A_m and B_m are determined from two boundary conditions, which must be satisfied on the perimeter of the vessel $r = a$. These conditions consist in the equality of the pressures p and the normal components of acoustic velocity, v_r, which are connected with the acoustic potential the by relations

$$p = \varrho \frac{\partial \varphi}{\partial t}, \tag{4}$$

$$v_r = -\mathrm{grad}_r \varphi, \tag{5}$$

where ϱ is the density of the medium.

All the tangential stresses were assumed to be equal to zero. It was thus assumed that the elastic properties of the blood vessel and the surrounding tissue are the same as those of a liquid.

This is certainly an approximation, since tangential stresses, e.g., those maintaining the cylindrical shape of the vessel, do occur in the walls of the vessel.

The acoustic pressure and velocity in the surrounding medium can be obtained from expressions (1) and (2), and for the internal medium from expres-

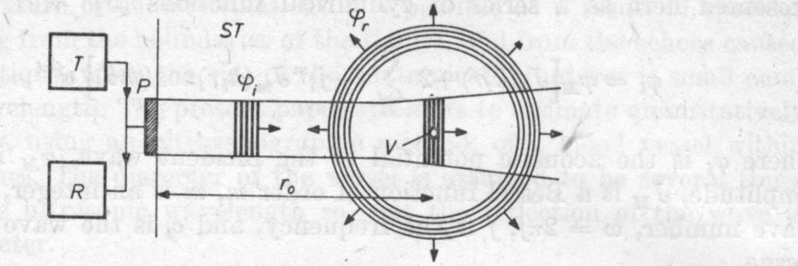

Fig. 1. Diffuse reflection of an ultrasonic pulse from a small blood vessel

T – transmitter, R – receiver, P – ultrasonic probe with a piezoelectric transducer, v – small blood vessel, φ_i – incident wave, φ_r – reflected wave (diffuse), ST – soft tissue, r_0 – distance between the piezoelectric transducer and the blood vessel

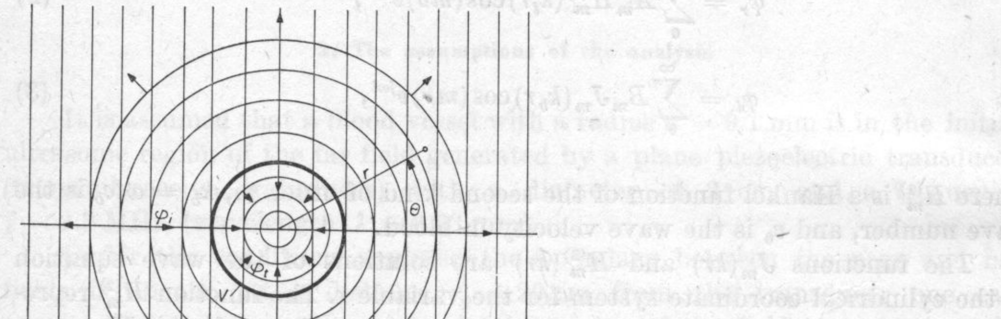

Fig. 2. A blood vessel (V) with incident (φ_i), reflected (φ_r) and penetrating (φ_t) waves

r, θ – cylindrical coordinates

sion (3). Bearing in mind the boundary conditions we obtained from equations (1)-(3) (for $r = a$)

$$\varphi_M\Big[J_0(k_t a)+2\sum_1^{\infty}(-j)^m J_m(k_t a)\cos(m\theta)+\sum_0^{\infty}A_m H_m^{(2)}(k_t a)\cos(m\theta)$$

$$=\frac{\varrho_b}{\varrho_t}\sum_0^{\infty}B_m J_m(k_b a)\cos(m\theta); \qquad (6)$$

$$\varphi_M\Big\{-k_t J_1(k_t a)+2\sum_1^{\infty}(-j)^m\left[\frac{m}{a}J_m(k_t a)-k_t J_{m+1}(k_t a)\right]\cos(m\theta)\Big\}+$$

$$+\sum_0^{\infty}A_m\left[\frac{m}{a}H_m^{(2)}(k_t a)-k_t H_{m+1}^{(2)}(k_t a)\right]\cos(m\theta)$$

$$=\sum_0^{\infty}B_m\left[\frac{m}{a}J_m(k_b-a)k_b J_{m+1}(k_b a)\right]\cos(m\theta). \qquad (7)$$

Multiplying the above relations by $\cos(n\theta)$ $(n = 0, 1, 2, \ldots)$ and integrating with respect to θ from 0 to π we eliminate all the terms for which $n \neq m$. Only those terms remain, for which $n = m$. In addition the dependence on the angle θ disappears. Thus we can successively determine from each of relations (6) and (7), the values of the constants A_m and B_m for each m [5].

Introducing into the calculations the values corresponding to the muscle tissue of the uterus and blood [3], i.e. $\varrho_t/\varrho_b = 1$ and $c_t = 1.63$ km/s, $c_b = 1.57$ km/s we obtain the following values for the constants

$$A_0 = 0.0495\, e^{-j93^\circ}, \quad A_1 = 0.0136\, e^{j180^\circ}, \quad A_2 = 0.0003\, e^{j90^\circ}, \quad A_3 = 0\,. \tag{7a}$$

The constants A_m with higher indices quickly decrease in magnitude so they can be neglected. In order to determine the reflected wave, the expression approximating a Hankel function for large values of the parameters x [4] can be used,

$$H_m^{(2)}(x) \underset{x\to\infty}{=} \sqrt{\frac{2}{\pi x}}\, e^{-j[x-(2m+1)\pi/4]}\,. \tag{8}$$

Finally, neglecting the time factor, we obtain the acoustic potential of the reflected wave (2) in the form

$$\varphi_r = \varphi_M(A_0 e^{j\pi/4} + A_1 e^{j3\pi/4}\cos\theta + A_2 e^{j5\pi/4}\cos 2\theta + A_3 e^{j7\pi/4}\cos 3\theta)\sqrt{\frac{2}{\pi k_t r}}\, e^{-jk_t r}$$

$$= \varphi_M R D e^{-jk_t r}, \tag{9}$$

where R denotes the value of the expression in the brackets, and D is the square root expression in equation (9).

4. The ratio of the power of the signal received to the power of the signal transmitted

The power of the transmitted signal N_T can be expressed in terms of the acoustic potential in the following way

$$N_T = S\frac{|p|^2}{2\varrho_t c_t} = S\frac{\omega^2 \varrho_t}{2c_t}|\varphi_M|^2, \tag{10}$$

where S is the surface area of the piezoelectric transducer.

A similar expression can be used for the power of the signal received, N_R, by considering the blood vessel to be in the far field so that the wave falling onto the plane piezoelectric transducer is approximately a plane wave. Then the ratio of the power of the received and transmitted waves is, with consideration of (9) and (7a), equal to

$$\frac{N_R}{N_T} = \frac{|\varphi_r|^2}{|\varphi_M|^2} = R^2 D^2 = 0.051^2 \cdot 0.018^2. \tag{11}$$

Using the above formula for numerical calculations r was replaced by the distance of the vessel from the transducer $r_0 = 20$ cm, and the cosine functions were approximated-1 in formula (9), since the angle at which the piezoelectric transducer is "seen" is only 3°. R and D are the losses of the signal occurring respectively due to the reflection and the divergence of the reflected wave.

Formula (11) is only valid for a plane wave. When the vessel v is small (Fig. 3) the reflected wave is a cylindrical wave according to expression (9).

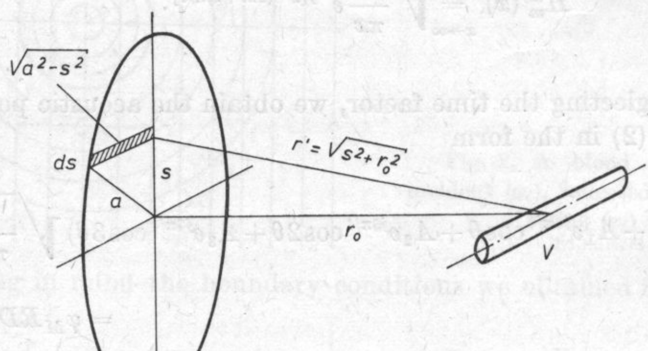

Fig. 3. Geometrical relationships in a cylindrical wave reflected from a vessel
P – a plane piezoelectric transducer, v – a blood vessel, r_0 – the distance from the vessel to the transducer, a – the radius of the transducer, s – the current coordinate

This wave propagating along the path r' falls onto a planar transducer T with a different phase than the wave propagating on the path r_0. The phase difference between the waves is

$$\frac{2\pi}{\lambda}(r' - r_0) = \frac{2\pi}{\lambda}(\sqrt{s^2 - r_0^2} - r_0).$$

As a result of this phenomenon a decrease of the elementary cylindrical waves reflected from the vessel v and falling onto the piezoelectric transducer T, occurs. Thus an electric signal, which is proportional to the value of the acoustic pressure averaged over the whole surface area of the transducer, occurs on the electrical terminals. From such averaging we obtain a coefficient of proportionality

$$N = \sqrt{\frac{4}{\pi a^2}\int\limits_0^a e^{-j\frac{2\pi}{\lambda}\sqrt{s^2+r_0}-r_0}\sqrt{a^2-s^2}\,ds}. \qquad (11a)$$

It is responsible for the lack of parallelism between the equiphasal surface of the reflected wave and the surface area of the transducer. For the conditions assumed here with $r_0 = 20$ cm it takes the value $N = 0.92$.

Thus finally formula (11) can be expanded by the introduction of the factor N and represented on a logarithmic scale (with the numerical data, respectively),

$$\frac{N_R}{N_T} \doteqdot R\,[\mathrm{dB}]+D\,[\mathrm{dB}]+N\,[\mathrm{dB}] = -29\mathrm{dB}-35\mathrm{dB}-1\mathrm{dB} = 65\mathrm{dB}. \qquad (12)$$

5. The electrical parameters of the apparatus

When one assumes $U_o = 10\ \mu$V as a typical voltage sensitivity of the ultrasonograph receiver, and the output voltage of the transmitter as $U_n = 250$ V, then the ratio of these two voltages is equal to $W = U_n/U_o = 2.5\cdot 10^7 \doteqdot 148$ dB. Considering the losses due to the transducing in the transmission and reception of the electric and ultrasonic pulses (including the diffraction losses) to be equal to $T = -15$ dB, then the ratio of the amplitude of the minimum detectable echo to the amplitude of the pulse radiated corresponds to $W-T = 133$ dB.

Since in the present case this value is higher them the value of (12) expressed in dB, the blood vessel would be detectable. However, it is necessary to account for the attenuation of the tissue.

6. Detectability of the vessel

A detailed survey of the literature on the ultrasonic properties of tissues and organs shows over 140 papers containing data on attenuation [3]. One should, however, state a general lack of data obtained from measurements "in vivo". The present author therefore decided to use the "in vivo" measurements obtained in a typical situation for ultrasonic visualization in obstetrics [1]. Attenuation in the penetrated tissues of the abdomen was found on average to be 1.8 dB/cm at a frequency of 2.5 MHz. In this case the signal loss on the path of 40 cm (from the transducer to the blood vessel and back) would be $A = -72$ dB.

Graph 4 (left) shows the levels of the signals, starting from the signal transmitted, to the signal received by the ultrasonic pulse device (echoscope) with consideration of all potential sources of signal loss. It can be seen that it is not possible to detect a blood vessel with a radius of 0.1 mm when it is at a distance $r_0 = 20$ cm. The signal received has then a level 4 dB lower than the sensitivity level of the apparatus. However, when one decrease the distance of the vessel to the value $r_0 = 10$ cm, the attenuation losses in tissues are only $A = -36$ dB, and the losses due to the divergence of the reflected beam would decrease to a value of $D = -32$ dB. The losses due to the lack of parallelism between the surface of the equiphasal reflected wave with respect to the surface area of the transducer would, however, increase to the value of $N = -6$ dB. It follows from graph 4 (right) that the blood vessel would be detected, and the signal would be 30 dB above the noise level.

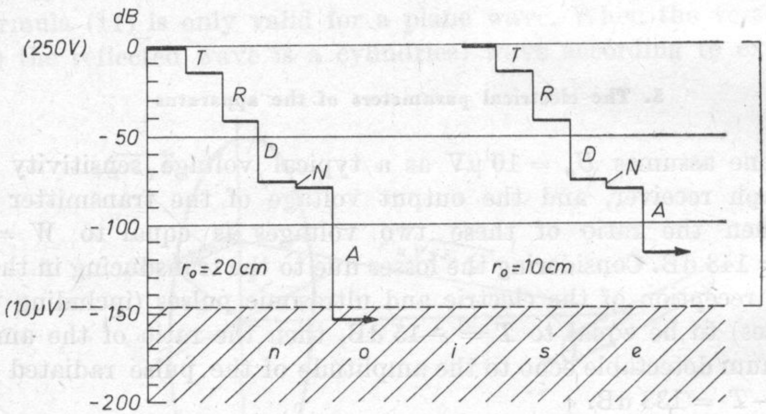

Fig. 4. The signal losses in the detection of a blood vessel with a radius $a = 0.1$ mm at different distances r_0 from the transducer

T — the electroacoustic transducing losses (of transmission and reception together), R — the losses in the reflection from the blood vessel, D — the losses in the divergence of the reflected wave (diffuse wave), A — the losses in the attenuation of the wave in tissue, N — the losses due to the lack of parallelism between the equiphasal wave reflected and the surface of the transducer, W — the ratio of the output voltage of the transmitter to the voltage sensitivity of the receiver

The fact that in reality the tissues penetrated are heterogeneous should, however, be considered. A number of echos occur due to the scattering of the ultrasonic wave from small structures, such as muscle fibres (diameter 10-150 μm, length 1-20 mm), or other small blood vessels (arterioles and capillaries of a diameter of 8-200 μm). The signal reflected from the single blood vessel considered may then be undetectable amongst the other signals with a random spatial distribution and similar amplitude.

7. Detectability of a plane boundary in soft tissues

Assuming normal incidence for an ultrasonic wave impinging on the plane boundary between two soft tissues of densities (ϱ) and wave velocities (c) that are only slightly different from each other so that their characteristic acoustic impedances are equal to ϱc and $\varrho c' = \varrho c + \Delta \varrho c$, one can determine the ratio of the potential of the reflected wave to the potential of the incident wave from the formula

$$\frac{\varphi_r}{\varphi_M} = \frac{\varrho c' - \varrho c}{\varrho c' + \varrho c} \cong \frac{\Delta \varrho c}{2 \varrho c}. \tag{13}$$

Assuming accordingly the distance of the boundary of the tissues from the transducer to be $r_0 = 20$ cm, and thus the attenuation losses in the tissue to be $A = -72$ dB, one can determine the minimum detectable value of the ratio of the potentials

$$\frac{\varphi_r}{\varphi_M} \doteqdot (W - T - A) \quad [\mathrm{dB}] = -61\ \mathrm{dB}. \tag{14}$$

From a comparison of (13) and (14) we finally obtain a minimum detectable change in the acoustic impedance of the tissue, i.e. $\Delta \varrho c / \varrho c = 1.8 \cdot 10^{-3}$. At a distance $r_0 = 10$ cm, however, when the attenuation losses in the tissue are only $A = -36$ dB, this quantity is $\Delta \varrho c / \varrho c = 2.8 \cdot 10^{-5}$.

8. Conclusions

This analysis shows that even very small blood vessels of a radius 0.1 mm give potentially detectable signals at a frequency of 2.5 MHz. The level of these signals is close to the noise level of the apparatus and depends critically on the distance of the blood vessel from the transducer because of attenuation in the tissues penetrated.

The present calculations have an approximate character in view of a number of simplifying assumptions, which were necessary for the analysis to be performed.

Detection of blood vessels of larger radii is not difficult. Fig. 5 shows a longitudinal ultrasonogram of a pregnant woman with a visible placenta including a number of blood vessels of very small diameters [2]. Fig. 6 shows an ultrasonogram of the abdominal aorta with a large aneurism. Both ultrasonograms were obtained at a frequency of 2.5 MHz.

The detectability of plane boundaries of soft tissues situated perpendicular to the direction of the falling waves is very large. At a distance 20 cm from the transducer, differences in the acoustic impedances of even 0.2 % are potentially detectable. At shorter distances this detectability rapidly increases due to the smaller attenuation losses of the wave in the tissues penetrated.

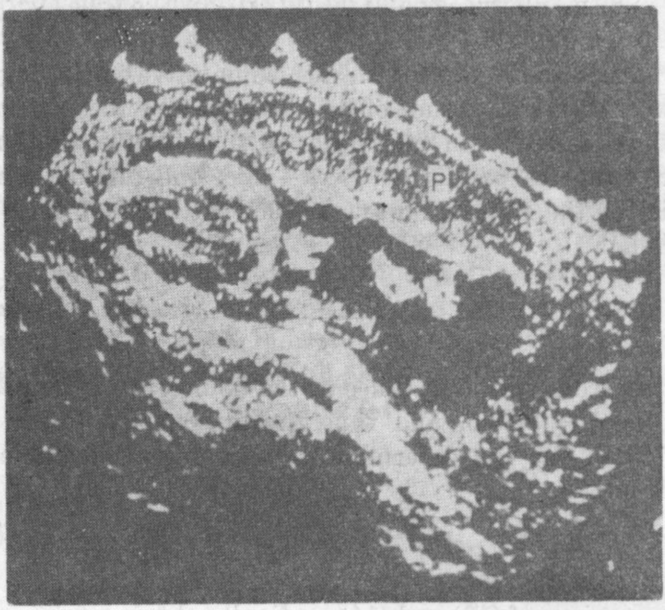

Fig. 5. A longitudinal ultrasonogram of a pregnant woman with a visible placenta P, obtained with a Polish USG – 10 apparatus [2]

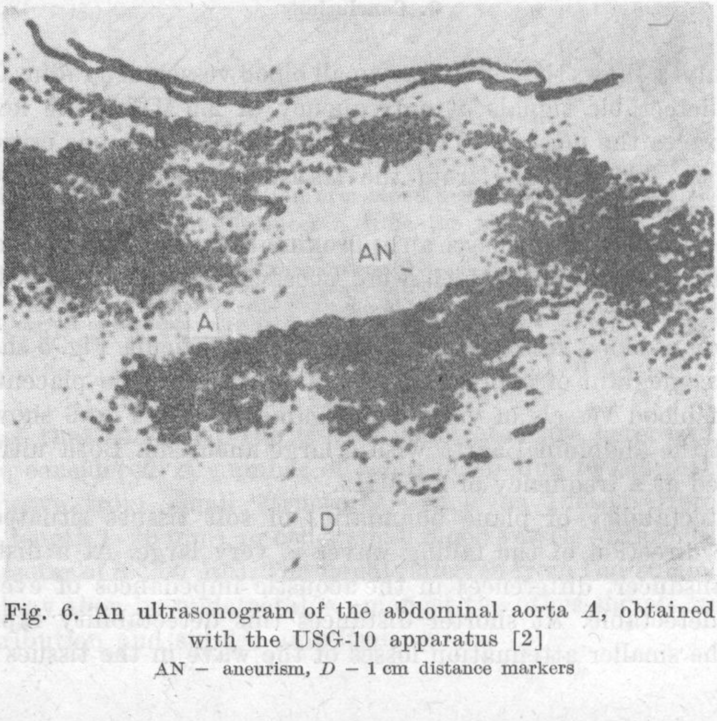

Fig. 6. An ultrasonogram of the abdominal aorta A, obtained with the USG-10 apparatus [2]

AN – aneurism, D – 1 cm distance markers

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