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APPROXIMATE METHODS FOR THE SOLUTION OF THE EQUATION OF ACOUSTIC WAVE PROPAGATION IN HORNS

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In practice, there are acoustic horns designed for which the equation of wave propagation has no exact solution of compact form. The need, therefore, arises for approximate solutions to be used. Accordingly, this investigation sought optimum methods for an approximate solution of the wave equation of a horn. It was assumed that the optimum method should combine the requirement of relatively little time-consuming calculation and the possibility of physical interpretation of the approximate formulae obtained. It was found that the WKB approximation which is recommended in the literature and has been taken directly from quantum mechanics, in general does not satisfy these requirements, and in addition it cannot be used at all in some cases. Therefore, another two approximate methods were developed and their properties analyzed.

1. Introduction

Acoustic wave propagation in horns is described by the well-known Webster equation derived under the assumption of the existence of a plane, harmonic wave that propagates without energy losess [5-9, 11, 15]. This equation, written in the so-called reduced form [1] using the dimensionless variables, is

$$\frac{d^2F}{da^2}+[\mu^2-V_{(a)}]F=0, \tag{1}$$

where F is a function defined by the sound pressure p and the cross-section area of the horn S by the formula [12];

$$F=p\sqrt{S}, \tag{2}$$

and a is the so-called dimensionless abscissa. When the axis of abscissae is the geometrical axis of the horn, a can be expressed by the formula [12]

$$a=\frac{x}{x_0}, \tag{3}$$

where x_0 is the coefficient of the divergence of the walls of the horn. The quantity μ is dimensionless frequency defined as the quotient of the absolute frequency and of a constant f_0 [4]

$$\mu = \frac{f}{f_0}, \tag{4}$$

and $f_0 = c/2\pi x_0$, where c is the adiabatic wave propagation velocity. The function $V_{(\alpha)}$ depends on the geometry of the horn and can be given by the so-called dimensionless cross-section radius of the horn ϱ by the formula

$$V_{(\alpha)} = \frac{1}{\varrho}\frac{d^2\varrho}{d\alpha^2}, \tag{5}$$

and ϱ can be defined as [12]

$$\varrho = \sqrt{\frac{S}{S_0}}, \tag{6}$$

where S_0 is the cross-section area at the in tlet of the horn.

The solution of equation (1) can be presented in the form [12]

$$F = A\exp^{\pm i\Theta}, \tag{7}$$

where i is an imaginary unit and A and Θ are functions of the variable α satisfying the equations [11, 12]

$$\mu^2 - V_{(\alpha)} + \frac{A''}{A} - \Theta'^2 = 0, \tag{8}$$

$$\frac{2A'}{A} + \frac{\Theta''}{\Theta'} = 0. \tag{9}$$

The dashes in formulae (8) and (9) denote differentiation with respect to the dimensionless abscissa α.

As a final result of considerations based on the reduced form of the Webster equation (1) a general formula for the relative unit admittance of the horn β can be derived [12]

$$\beta = -\frac{i}{\mu}\left(\frac{F'}{F} - \frac{\varrho'}{\varrho}\right). \tag{10}$$

The horns most often considered in the literature were those for which the exact solution of equations (8) and (9) could be achieved. When these equations were to be solved in an approximate manner, however, the approximation known in quantum mechanics was recommended, particulary the WKB (Wentzel, Kramers, Brillouin) method [1, 3, 9, 10]. This resulted from the formal similarity of equation (1) to the onedimensional Schrödinger equation independent of time.

However, the approximate methods used so far in the theory of horns for the solution of equation (1) most often lead to rather tedious calculations possible only when a computer was used and gave so complex approximate formulae that they were hardly useful in physical interpretation. The aim of the present investigation was to find more optimum approximate methods which combine little time-consuming calculations with the requirements of physical interpretation of expressions derived.

2. Discussion of the range of applicability of the WKB method in the theory of horns

The conditions and the range of applicability of the WKB approximation in the theory of horns have to be analysed for two reasons. Firstly, as was mentioned in section 1, the WKB method is recommended for approximate solution of the Webster equation in almost every paper on those horns for which the wave equation has no exact solution [1, 3, 9, 10]. Secondly, in the present paper this method will be a starting point for development of more optimum approximation methods.

In the WKB approximation the approximate solution of equation (1) has the form of (7), and A must be a slowly variable function of a. The requirement of slow variation of $A_{(a)}$ permits the assumption that $A'' \cong 0$, and accordingly equation (8) can be simplified to the form

$$\Theta'^2 = K^2, \tag{11}$$

where the quantity K^2 depends on the frequency and the geometry of the horn

$$K^2 = \mu - V_{(a)}. \tag{12}$$

It follows from (11) that

$$\Theta = \int_{a_0}^{a} K d\bar{a}, \tag{13}$$

where the variation range of the integration limits is restricted by the length of the horn.

Moreover, equation (9) can be integrated directly, thus giving

$$A^2\Theta' = C^2, \tag{14}$$

where the constant C is independent of a and can be only a function of the dimensionless frequency μ.

Consideration of relation (11) in (14) gives

$$A = \frac{C}{\sqrt{K}}. \tag{15}$$

Expression (15) shows that the requirement of slow variation of $A_{(a)}$ is closely related to the requirement of slow variation of $K_{(a)}$. Thus, according to the definition of K (cf. formula (12)) it can be stated that the WKB method can be used at those frequencies and for horns of such geometry for which $K_{(a)}$ is a slowly variable function.

In the case when $K^2 > 0$, from (7), (13) and (15) the solution of the wave equation (1) can be written for the slowly variable function $K_{(a)}$, in the form

$$F = \frac{C_1}{\sqrt{K}} \exp i \left(\int_{a_0}^{a} K d\bar{a} \right) + \frac{C_2}{\sqrt{K}} \exp \left(-i \int_{a_0}^{a} K d\bar{a} \right). \tag{16}$$

In the case, however, when $K^2 < 0$, K is imaginary and formula (16) takes the form

$$F = \frac{C_3}{\sqrt{\chi}} \exp \left(\int_{a_0}^{a} \chi d\bar{a} \right) + \frac{C_4}{\sqrt{\chi}} \exp \left(- \int_{a_0}^{a} \chi d\bar{a} \right), \tag{17}$$

where

$$K = -i\chi. \tag{18}$$

After conversion it can be stated that a differential equation of the second order satisfied exactly by the solutions of (16) and (17) has the form

$$F'' + \left[K^2 - \frac{3}{4} \left(\frac{K'}{K} \right)^2 + \frac{1}{2} \frac{K''}{K} \right] F = 0. \tag{19}$$

Comparison of (19) with (1) (with consideration of formula (12)) shows that the equation satisfied exactly by the approximate solution is different from the reduced wave equation of a horn by the term

$$\Delta = \frac{3}{4} \left(\frac{K'}{K} \right)^2 - \frac{1}{2} \frac{K''}{K}, \tag{20}$$

and this term is subtracted from K^2. This discovery suggests the subsequent approximation in which Δ should be included as a correction, and that equation (11) should have the following form

$$\Theta' = \left[K^2 + \frac{3}{4} \left(\frac{K'}{K} \right)^2 - \frac{1}{2} \frac{K''}{K} \right]^{1/2}. \tag{21}$$

This leads, however, to considerable complexity of the subsequent formulae.

Expressions (16) and (17) can permit good approximation, however, when the quantity Δ is small compared to K^2. This remark permits quantitative formulation of the application condition of the WKB approximation

$$|\gamma| = \left| \frac{\frac{3}{4} \left(\frac{K'}{K} \right)^2 - \frac{1}{2} \frac{K''}{K}}{K^2} \right| \ll 1. \tag{22}$$

On the basis of the foregoing argument in can be stated that in the case of a horn (or a family of horns) of specific shape the usefulness of the WKB method should be determined by the analysis of the function $K_{(\alpha)}$, complemented by examination of condition (22). When this analysis shows that the function $K_{(\alpha)}$ does not satisfy the requirement of slow variation, the WKB method is inefficient, even when using very time-consuming approximations (cf. formula (21)). Accordingly, the next section will give another method for approximate solution of the wave equation (1), which can be used successfully in this case, and which to the authors' knowledge has not been used to date.

3. A method of linear approximation of the function $V_{(\alpha)}$

It follows from formula (12) that the function $K_{(\alpha)}$ for a given frequency μ, is determined by the function $V_{(\alpha)}$ containing information about the geometry of the horn (cf. formulae (5) and (6)). The approach proposed in this section, consists in approximation of the function $V_{(\alpha)}$ by a broken line. In this case the horn is considered to be a multi-element one, where each element corresponds to one section of the broken line. The number of sections depends on the desired accuracy of approximation. It is interesting to note here that for horns used in practice the function $V_{(\alpha)}$ behaves so regularly that the desired accuracy of approximation can be achieved for a small number of sections of the broken line.

Let us assume, as an example, that the function $V_{(\alpha)}$ has the form as in Fig. 1 and was approximated by a broken line of n sections, where $n = 1, 2, 3, \ldots, N$.

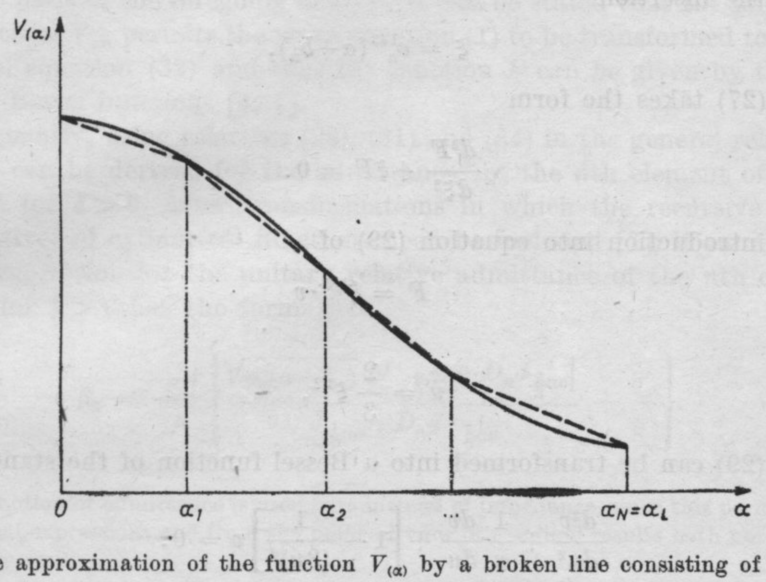

Fig. 1. The approximation of the function $V_{(\alpha)}$ by a broken line consisting of N sections

One can consider the section corresponding to an arbitrary nth section of the horn contained between the points a_{n-1} and a_n. The function $V_{(\alpha)}$ can be approximated over this section by a linear function of the form

$$V_{(\alpha)} = V_{(a_{n-1})} - c_n \alpha, \tag{23}$$

where c_n is the coefficient of directivity of the line. It can be given by the formula

$$c_n = \frac{V_{(a_{n-1})} - V_{(a_n)}}{a_n - a_{n-1}}. \tag{24}$$

After consideration of (23) the reduced wave equation (1) for $\alpha \in [a_{n-1}, a_n]$ takes the form

$$F'' + [\mu^2 - V_{(a_{n-1})} + c_n \alpha] F = 0. \tag{25}$$

Introduction of the abbreviation

$$b_n = \frac{\mu^2 - V_{(a_{n-1})}}{c_n} \tag{26}$$

permits equation (25) to be written in the following form

$$F'' + c_n(\alpha + b_n) F = 0. \tag{27}$$

One can first consider the case $\mu^2 > V_{(a_{n-1})}$, i.e. when the quantity b_n is positive. In this case equation (27) can by way of successive transformations be reduced to the form of a Bessel equation.

After the insertion

$$\xi = c_n^{1/3}(\alpha + b_n), \tag{28}$$

equation (27) takes the form

$$\frac{d^2F}{d\xi^2} + \xi F = 0. \tag{29}$$

After introduction into equation (29) of

$$F = \xi^{1/2} \cdot v \tag{30}$$

and

$$u = \frac{2}{3} \xi^{3/2}, \tag{31}$$

equation (29) can be transformed into a Bessel function of the standard form

$$\frac{d^2v}{du^2} + \frac{1}{u}\frac{dv}{du} + \left[1 - \frac{1}{(3u)^2}\right] v = 0. \tag{32}$$

The solution of this equation is Bessel functions of order $\frac{1}{3}$ [4, 7]

$$v = A_n J_{\frac{1}{3}(u)} + B_n J_{-\frac{1}{3}(u)}, \tag{33}$$

where A_n and B_n are constants.

Consideration of formulae (30), (31) and (33) gives

$$F = \xi^{1/2}\left[A_n J_{\frac{1}{3}\left(\frac{2}{3}\xi^{3/2}\right)} + B_n J_{-\frac{1}{3}\left(\frac{2}{3}\xi^{3/2}\right)}\right]. \tag{34}$$

It can now be demonstrated that the solution of (34) for $b_n > 0$ can also be obtained for $b_n < 0$.

It follows from formula (28) that when $|b_n| < a$ we have $\xi > 0$ despite a negative b_n and the foregoing argument (formulae (28)-(34) are still valid).

When, however, $|b_n| > a$, ξ is negative and equation (29) takes the form

$$\frac{d^2F}{d\xi^2} - \xi F = 0. \tag{35}$$

In this case, without changing (30) a substitution different from that in (30) must be used

$$u = -i\frac{2}{3}|\xi|^{3/2}. \tag{36}$$

This substitution permits (35) to be rewritten in the form of (32), and the expression of F takes a form analogous to formula (34)

$$F = \xi^{1/2}\left[A_n J_{\frac{1}{3}\left(-i\frac{2}{3}|\xi|^{3/2}\right)} + B_n J_{-\frac{1}{3}\left(-i\frac{2}{3}|\xi|^{3/2}\right)}\right]. \tag{37}$$

On the basis of the foregoing analysis it can be stated that the linearization of the function $V_{(a)}$ permits the wave equation (1) to be transformed to the form of a Bessel equation (32) and thus the function F can be given by the known tabulated Bessel functions [4, 7].

Subsequently, using relations (28), (31) and (34) in the general relation (10) a formula can be derived for the admittance* of the nth element of the horn considered for $\xi > 0$. After transformations in which the recursive formulae for derivatives of cylindrical functions need be included [7], it can be stated that the expression for the unitary relative admittance of the nth element of the horn for $\xi > 0$ has the form

$$\beta_n = \frac{-i}{\mu}\left\{\frac{\sqrt{c_n(a+b_n)}\,\left[J_{-\frac{2}{3}(u)} - D_n J_{\frac{2}{3}(u)}\right]}{J_{\frac{1}{3}(u)} + D_n J_{-\frac{1}{3}(u)}} - \frac{\varrho'}{\varrho}\right\}. \tag{38}$$

* The notion of admittance is used here instead of impedance, since this permits simpler mathematical expressions and from the point of view of the final results both notions can be used equally well.

The constant $D_n = B_n/A_n$ which occurs in formula (38) can be determined from the boundary condition at the outlet of the nth element

$$\frac{-i}{\mu}\left\{\frac{\sqrt{c_n(a_n+b_n)}\,[J_{-\frac{2}{3}(u_n)}-D_nJ_{\frac{2}{3}(u_n)}]}{J_{\frac{1}{3}(u_n)}+D_nJ_{-\frac{1}{3}(u_n)}}-\frac{\varrho'_{(a_n)}}{\varrho_{(a_n)}}\right\} = Re(\beta_{0n+1})+Jm(\beta_{0n+1}), \quad (39)$$

where, from (28) and (31),

$$u_n = \frac{2}{3}\sqrt{c_n(a_n+b_n)^3}, \quad (40)$$

and $\beta_{0_{n+1}}$ is the inlet admittance of the $(n+1)$th element of the horn.

The admittance of the other elements of the horn can be represented similarly as for the nth elements, e.g. the inlet acoustic admittance of the whole horn can be given by the formula for the inlet acoustic admittance of the first element

$$\beta_{0_1} = \frac{-i}{\mu}\left\{\frac{\sqrt{c_1b_1}\,[J_{-\frac{2}{3}(u_0)}-D_1J_{\frac{2}{3}(u_0)}]}{J_{\frac{1}{3}(u_0)}+D_1J_{-\frac{1}{3}(u_0)}}-\left(\frac{\varrho'}{\varrho}\right)_{(a=0)}\right\}, \quad (41)$$

where, from (28) and (31)

$$u_0 = \frac{2}{3}\sqrt{c_1b_1^3}. \quad (42)$$

In practice β_{0_1} is calculated in several stages, from the outlet to the inlet. First the inlet admittance of the end element must be determined, considering it as the load of the outlet of the previous element. Subsequently the inlet admittance of this element etc. must be calculated.

Now the case when $\xi = 0$ $(u = 0)$ will be considered. It follows from formula (28) that this case occurs when b_n is negative and satisfies the equation

$$a = |b_n|. \quad (43)$$

After expansion of the functions $J_{-\frac{1}{3}(u)}$ and $J_{-\frac{2}{3}(u)}$ into series [7] and transformations, formula (38) within the limits for $u = 0$ takes the form

$$\beta_n = \frac{-i}{\mu}\left[\frac{(3c_n)^{1/3}\,\Gamma\left(\frac{2}{3}\right)}{D_n\Gamma\left(\frac{1}{3}\right)}-\frac{\varrho'}{\varrho}\right], \quad (44)$$

where Γ is an Euler function [2, 4].

When $(a+b_n)$ in formula (28) is negative the case $\xi < 0$ occurs. In this case expression (36) must be inserted into formula (38) instead of u and it must be considered that $\sqrt{c_n(a+b_n)}$ is an imaginary number. This gives

$$\beta_n = \frac{-i}{u}\left\{\frac{-i\sqrt{c_n|a+b_n|}\,\left[J_{-\frac{2}{3}}\left(-i\frac{2}{3}|\xi|^{3/2}\right) - D_n J_{\frac{2}{3}}\left(-i\frac{2}{3}|\xi|^{3/2}\right)\right]}{J_{\frac{1}{3}}\left(-i\frac{2}{3}|\xi|^{3/2}\right) + D_n J_{-\frac{1}{3}}\left(-i\frac{2}{3}|\xi|^{3/2}\right)} - \frac{\varrho'}{\varrho}\right\}. \tag{45}$$

The approximation in the present section leads as a rule to considerable simplification of calculations, compared to the WKB approximation, since it permits the admittance of a horn (i.e. also its impedance) to be determined from the known tabulated Bessel functions [4]. In addition, when compared with the WKB method it has the essential advantage that it can be used in the case when $K_{(a)}$ is not slowly variable.

4. Approximation of the zeroth order

Approximation of the zeroth order can be used practically in estimation of the properties of horns when high accuracy is not necessary. It is assumed in this approximation that $A = \text{const}$. It follows then from equation (9) that Θ' must be constant, and thus Θ is a linear function of a.

At the same time, in view of that from equation (8) A is constant, the application of definition (12) gives

$$\Theta'^2 = K^2 = \text{const}. \tag{46}$$

Thus K must take a constant value independent of a. This value will be given below as $\bar{K}$.

It can be suggested that $\bar{K}$ should be defined as the square root of the mean value of the function $K^2_{(a)}$ in the interval $[0, a_l]$ corresponding to the length of the horn. This gives

$$\bar{K} = \left[\frac{1}{a_l}\int_0^{a_l} K^2 da\right]^{1/2}. \tag{47}$$

It follows from (46) that knowing $\bar{K}$ the phase Θ can be calculated

$$\Theta = \bar{K}a. \tag{48}$$

Thus, solution (7) of the reduced wave equation (1) will in this approximation have the form

$$F = A_1\exp(i\bar{K}a) + A_2\exp(-i\bar{K}a), \tag{49}$$

where the first term of the sum in formula (49) corresponds to the wave travelling from the inlet to the outlet of the horn, while the second term corresponds to the reflected wave.

It can be noted that when $\bar{K}^2 < 0$ formula (49) takes the form

$$F = A_3 \exp(\bar{\chi}a) + A_4 \exp(-\bar{\chi}a), \tag{50}$$

where

$$\bar{K} = -i\bar{\chi}. \tag{51}$$

The present section can be concluded with formulae for the admittance of the horn in the zeroth approximation. Formulae (49) and (50) in the general form of (10) can be used for this purpose. The present consideration is limited to the most frequent case when the wave reflected from the outlet of the horn is neglected. This is the case of the so-called horn of infinite length [5, 8, 12, 15]. Accordingly the second term of the sum can be neglected in formulae (49) and (50) and the formula for admittance, (10), takes for $K^2 > 0$ the form

$$\beta = \frac{\bar{K}}{\mu} + \frac{i}{\mu}\frac{\varrho'}{\varrho}. \tag{52}$$

In turn, for $\bar{K}^2 < 0$

$$\beta = \frac{-i\bar{\chi}}{\mu} + \frac{i}{\mu}\frac{\varrho'}{\varrho}, \tag{53}$$

and for the boundary case $\bar{K}^2 = 0$

$$\beta = \frac{i}{\mu}\frac{\varrho'}{\varrho}. \tag{54}$$

The formulae obtained for the admittance have a similar form to the relations used generally in the literature for waveguides of a constant value of K [8, 9, 12]. It follows therefore that the approximation given in this section lies essentially in the substitution for a horn for which K is a function of position, by a hypothetical horn of a constant, i.e. averaged, value of K. This procedure can give satisfactory results only when the properties of the horn as a whole are of interest. This is the most frequent case in practice where as a rule only the frequency response of the inlet impedance of a horn is analysed. However, the application of the approximation of the zeroth order for study of phenomena occurring inside a waveguide is in the authors' opinion a useless attempt.

5. A numerical example

The object of calculations illustrating as an example the results of the foregoing considerations was a horn of catenoidal profile and annular cross-section whose area is defined by the formule

$$S_0 = S_0 \cosh \mathrm{h}\alpha. \tag{55}$$

Considerations in [13, 14] showed that the wave equation for a horn of this geometry has no exact solution in a compact form. An analysis of the usefulness of the WKB approximation, made according to section 2 shows that the function $K_{(a)}$ cannot be considered as slowly variable, while the coefficient γ (cf. formula (22)) reaches the value of several score percent. In this case the approximate methods proposed in the present paper were used. Fig. 2 shows the results calculated for the real part of the inlet admittance of the horn type under consideration, with the dimensions: the width of the inlet ring $-\ 1.5\cdot10^{-3}$ m, the length $-\ 15\cdot10^{-2}$ m, the diameter of the outlet $-\ 2\cdot10^{-1}$ m.

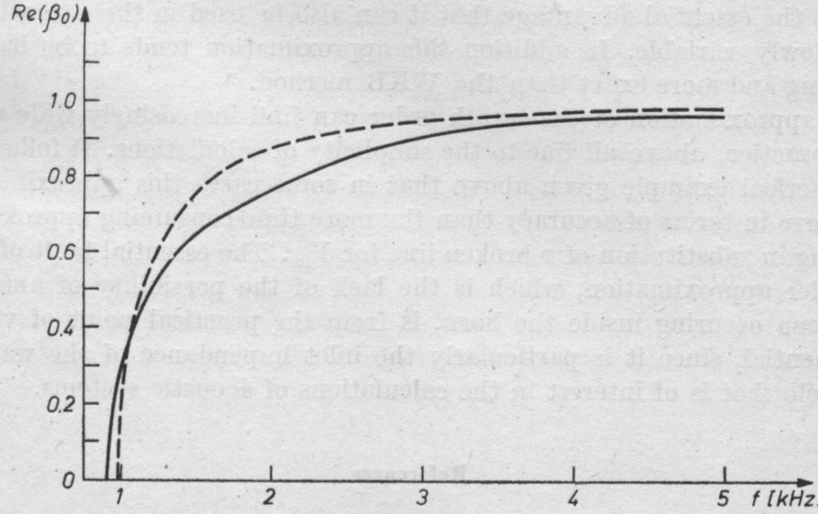

Fig. 2. The real part of the relative unit inlet admittance of a catenoidal horn of annular cross-section

– – – – the approximation of the zeroth order, —— the approximation of the function V_a by a broken line

The calculations were made with neglecting the effect of the wave reflected from the outlet. The continuous line in Fig. 2 shows the results obtained when the method of linear approximation of the function $V_{(a)}$ was used, while the dashed line represents the calculations in the approximation of the zeroth order. It can be seen that in the case of the horn under consideration the approximation of the zeroth order gives satisfactory results, compared with the much more exact approximation by the linearization of the function $V_{(a)}$ since the deviation does not exceed 15 percent.

6. Conclusions

The WKB approximation recommended in the acoustical literature for approximate solution of the equation of wave propagation in horns [1, 3, 9, 10], can be used above all for qualitative analysis of the transmission properties of horns in the cut-off frequency region. Its use in numerical calculations is, however, limited, since it cannot be applied in the case of horns for which $K_{(a)}$ does not satisfy the requirement of slow variation. In addition, even in its applicability range, the WKB method may give very complicated formulae preventing their physical interpretation and good for computer calculations only.

When compared with the WKB method, the approximation proposed in the present paper, which consists in approximation of the function $V_{(a)}$ by a broken line, has the essential advantage that it can also be used in the case when $K_{(a)}$ is not slowly variable. In addition this approximation tends to be less time-consuming and more exact than the WKB method.

The approximation of the zeroth order can find increasingly wide application in practice, above all due to the simplicity of calculations. It follows from the numerical example given above that in some cases this approximation is little worse in terms of accuracy than the more time-consuming approximation consisting in substitution of a broken line for $V_{(a)}$. The essential fault of the zeroth order approximation, which is the lack of the possibility of analysis of phenomena occuring inside the horn, is from the practical point of view not very essential, since it is particularly the inlet impendance of the waveguide as a whole that is of interest in the calculations of acoustic systems.

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