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AN ULTRASONIC *C.W.* DOPPLER METHOD OF MEASUREMENT OF THE BLOOD FLOW VELOCITY

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This paper presents an analysis of factors affecting the results of measurement of the blood flow velocity by the ultrasonic continuous wave (*C.W.*) Doppler method. For the parabolic flow velocity profile the effect on the spectrum of a Doppler signal of such factors as the ratio of the width of the ultrasonic beam to the inner diameter of the blood vessel and the coincidence of the transmitted ultrasonic beam with the one received inside the blood vessel, was analyzed. To that end, two variants of the position of the transmitting transducer and the transducer receiving the ultrasonic wave, with respect to the blood vessel were considered. In the first variant the point of intersection of axes of the transmitted ultrasonic beam and of the received from the flowing blood, was outside the blood vessel. In the second variant the axes of the ultrasonic beams intersected with each other in the middle of the blood vessel, on its axis. On the basis of analysis of the spectra of a Doppler signal the value of the factor of proportionality between the frequency of zero-crossings of the amplitude of the Doppler signal, measured by a Doppler flowmeter, and the mean Doppler frequency corresponding to the mean blood flow velocity in the blood vessel, was determined. This factor is the basis for quantitative estimation of the blood flow velocity from the Doppler frequency measured by the flowmeter.

1. Introduction

Of the currently used ultrasonic Doppler methods of measurement of blood flow the *C.W.* method was historically the first to find wide application in diagnostics of diseases of the human circulation system. It permits noninvasive measurement of the mean blood flow velocity in the cross-section of the blood vessel. Information about the blood flow velocity is contained in the Doppler frequency measured by the measuring apparatus, which is the difference between the frequency of the transmitted continuous wave and its frequency received from the flowing blood.

The previous investigations of the blood flow velocity by the ultrasonic *C.W.* method were mainly qualitative and consisted in registering variations in the blood flow velocity during the cardiac cycle. This resulted from the lack of sufficient information about the quantitative relation between the measured Doppler frequency and the real mean blood flow velocity.

This paper presents a detailed analysis of factors affecting the results of measurement of the blood flow velocity by the ultrasonic *C.W.* Doppler method and attempts to determine the quantitative relation between the measured Doppler frequency and the mean blood flow velocity in the cross-section of the blood vessel.

2. The principle of measurement of the blood flow velocity

The general principle of measurement of the blood flow velocity by the ultrasonic *C.W.* Doppler method is shown in Fig. 1. A piezoelectric transmitting transducer excited to vibration by a high-frequency electric signal trans-

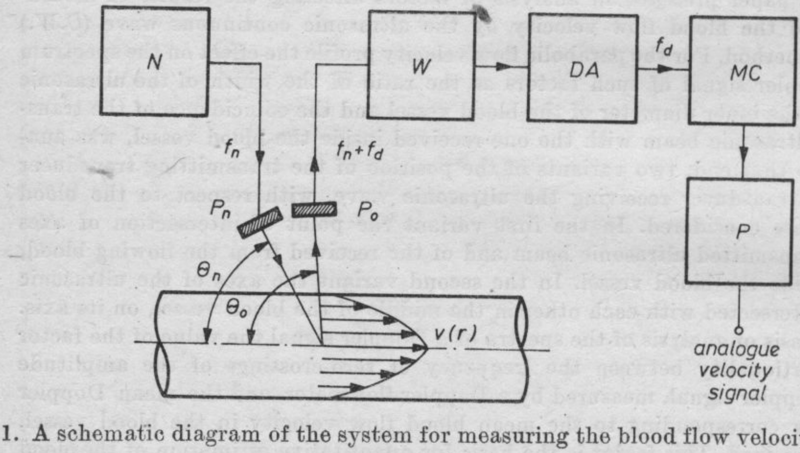

Fig. 1. A schematic diagram of the system for measuring the blood flow velocity by the *C.W.* method

N – the transmitter, P_n – the piezoelectric transmitting transducer, P – the piezoelectric receiving transducer, W – the high-frequency amplifier, DA – the amplitude detector, MC – the frequency meter, P – the frequency-to-voltage converter, f_n – the transmitted frequency, f_0 – the Doppler frequency, $v(r)$ – the velocity of blood cells in the cross-section of the vessel, θ_n, θ_o – the angles between the transmitted and received waves with respect to the axis of the vessel

mits a continuous ultrasonic wave towards the blood vessel. The ultrasonic wave scattered by flowing blood cells contains the spectrum of the Doppler frequencies f_d. The individual components of the Doppler spectrum are proportional to the velocity of blood cells flowing in the field of the transmitted ultrasonic beam, according to the relation

$$f_d = \pm f_n \frac{v(r)}{c} (\cos\theta_n + \cos\theta_0), \tag{1}$$

where c – the ultrasonic wave propagation velocity in blood, f_n – the frequency of the transmitted ultrasonic wave, $v(r)$ – the velocity of blood cells in the cross-section of the blood vessel at the distance r from its axis, θ_n – the angle between the direction of the transmitted ultrasonic beam and the blood vessel, θ_0 – the angle between the direction of the received ultrasonic beam and the blood vessel.

The Doppler frequency f_d is positive when blood cells approach the transmitting and receiving transducers in the ultrasonic head, and it is negative when they flow away from the transducers.

According to formula (1) the mean Doppler frequency f_s described by the following relation, corresponds to the mean blood flow velocity in the blood vessel of cylindrical cross-section

$$f_s = \frac{1}{\pi R^2}\left[\frac{f_n(\cos\theta_n + \cos\theta_0)}{c}\right]\int_0^R 2\pi r v(r)\,dr, \tag{2}$$

where R is the inner radius of the blood vessel.

In ultrasonic Doppler flowmeters the ultrasonic wave received from the flowing blood is transformed into an electric signal which after amplification is detected in terms of amplitude. In order to obtain information about the mean blood flow velocity the technique of measurement of the frequency of a Doppler signal by the method of zero-crossing is generally used. The mean frequency f_{zc} of the positive zero crossings of the amplitude of the Doppler signal depends on the power density spectrum $S(f)$ of the Doppler signal and is proportional to the mean velocity v_s of blood flow in the cross-section of the blood vessel, according to the relation

$$v_s = a\frac{cf_{zc}}{f_n(\cos\theta_n + \cos\theta_0)}, \tag{3a}$$

where

$$f_{zc} = \left[\frac{\int_0^\infty f^2 S(f)\,df}{\int_0^\infty S(f)\,df}\right]^{1/2}. \tag{3b}$$

The proportionality factor a in formula (3a) describes the relation between the Doppler frequency f_{zc} measured by the method of zero-crossing and the mean Doppler frequency calculated from formula (2).

The frequency of the Doppler signal measured by the method of zero-crossing is transformed in the receiver of the measuring apparatus into a voltage signal whose amplitude u_0 is proportional to the measured mean blood flow velocity v_s, according to the relation

$$v_s = \frac{ab}{\eta}\frac{cu_0}{f_n(\cos\theta_n + \cos a_0)}, \tag{4}$$

where η — the constant of transformation of the measured Doppler frequency into voltage, b — a coefficient describing the relation between the measured frequency and the real frequency of zero-crossing of the amplitude of the Doppler signal.

The term $abc/\eta f_n$ in formula (4) describes the general form of the calibration coefficient of the Doppler flowmeter for a given angle at which the ultrasonic wave is transmitted and received with respect to the blood vessel. The parameters in the term, such as the ultrasonic wave velocity in blood c [10] the frequency of the transmitted ultrasonic wave f_n and the constant of transformation of the measured Doppler frequency into voltage η are known and given by the manufacturers of ultrasonic Doppler measuring apparatus. The value of the coefficient b in formula (4) is connected with the magnitude of the error occuring in measurement of the frequency of zero-crossing of the amplitude of a Doppler signal. The main causes of this error are: noise accompanying the Doppler signal and filtration of low-frequency components from the spectrum of the Doppler signal. This filtration is necessary so that components from pulsating walls of the blood vessel can be eliminated from the Doppler signal. The error in measurement of the Doppler frequency caused by the above factors is lower than 5 percent when the power ratio of the Doppler signal to noise is larger than 30 dB and when the maximum frequency of the Doppler signal is larger by the factor of ten than the lower frequency of the transmission band of the Doppler signal amplifier in the receiver of the apparatus [3,4]. A detailed analysis of errors occurring in measurement of the Doppler frequency by the method of zero-crossing was presented in many papers [3-5], therefore, it will not be considered here. Instead the object of the present analysis is the value of the proportionality factor a between the mean Doppler frequency f_s (see formula (2)) and the frequency f_{zc} of the positive zero crossing of the amplitude of the Doppler signal (see formula (3b)).

In order to determine a quantitative value of the proportionality factor a, it is necessary to analyze the relation between the power density spectrum of the Doppler signal and the blood flow velocity profile in the blood vessel, with consideration given to such factors as the ratio of the width of the ultrasonic beam to the inner diameter of the blood vessel and the geometrical orientation of piezoelectric transducers of the ultrasonic probe with respect to the blood vessel.

3. The power density spectrum of the Doppler signal

Analysis of the whole of phenomena occurring in measurement of the blood flow velocity and affecting the spectrum of the Doppler signal is very complicated. The power density spectrum of the Doppler signal depends on many factors such as the spatial distribution of the density of blood cells scattering the ultrasonic beam, the blood flow velocity profile in the blood vessel, the geometrical dimensions of the region common to the transmitted beam and the one received inside the blood vessel, and the geometrical orientation of transducers of the ultrasonic probe with respect to the blood vessel.

The spectral analysis presented in this paper concerns the measuring system with two rectangular piezoelectric transducers of dimensions $H \times B$ placed at the same side of the blood vessel. The transmitted and received ultrasonic beams pass symmetrically through the middle of the blood vessel at the angles θ_n and θ_0 with respect to its axis (Fig. 2a, b), respectively. The author will con-

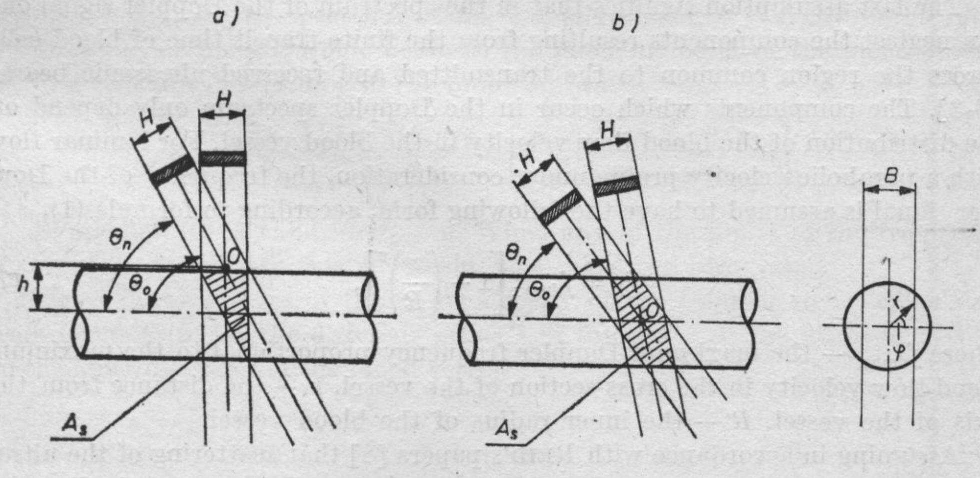

Fig. 2. The position of ultrasonic transducers with respect to the blood vessel
A_S – the region common to the transmitted ultrasonic beam and the one received inside the blood vessel, O – the point of intersection of axes of the ultrasonic beams

sider here two variants of coincidence of the transmitted ultrasonic beam with the one received inside the blood vessel. In the first variant (shown in Fig. 2a) the point of intersection of axes of the ultrasonic beams is outside the wall of the vessel at the distance h from the axis of the vessel, the distance being defined by

$$R+\frac{H\cos(\theta_n+\theta_0)/2}{2\cos(\theta_0-\theta_n)/2} \leqslant h \leqslant \frac{H\sin(\theta_0+\theta_n)/2}{2\sin(\theta_0-\theta_n)/2}-R. \tag{5}$$

In the second variant the point of intersection of axes of the ultrasonic beams is on the axis of the vessel (Fig. 2b). In this case the following assumptions were made

$$\frac{\sin(\theta_0-\theta_n)}{2\sqrt{\sin\theta_n\sin\theta_0}} \leqslant \frac{H}{B} \leqslant 1 \tag{6a}$$

and

$$0° < \theta_n < \theta_0 \leqslant 90°. \tag{6b}$$

The spectrum of the Doppler signal will be analyzed for laminar, stationary flow in a cylindrical vessel with rigid walls. The following simplifying assumptions were made:

1. the distribution of the acoustic pressure in the field of the ultrasonic beam is uniform;

2. the spatial density distribution of flowing blood cells is uniform;

3. the mean dimensions (measured in the direction of the blood flow) of the region common to the ultrasonic transmitted beam and the one received inside the blood vessel are much larger than the ultrasonic wavelength in blood.

The last assumption signifies that in the spectrum of the Doppler signal one can neglect the components resulting from the finite transit time of blood cells across the region common to the transmitted and received ultrasonic beams [2-3]. The components which occur in the Doppler spectrum only depend on the distribution of the blood flow velocity in the blood vessel. For laminar flow with a parabolic velocity profile under consideration, the frequency of the Doppler signal is assumed to have the following form, according to formula (1),

$$f_d = f_{d\max}\left[1-\left(\frac{r}{R}\right)^2\right], \tag{7}$$

where $f_{d\max}$ – the maximum Doppler frequency proportional to the maximum blood flow velocity in the cross-section of the vessel, r – the distance from the axis of the vessel, R – the inner radius of the blood vessel.

Assuming in accordance with Reid's papers [8] that scattering of the ultrasonic wave on flowing blood cells is of the first order, the power of the Doppler signal is proportional to the density of blood cells flowing across the field of the ultrasonic beam. Blood cells flowing at the distance r from the axis of the vessel at the velocity $v_1 < v < v_1 + dv$ are sources of a Doppler signal whose power dN can be expressed by the relation

$$dN = a\varrho r dr \int\limits_{\varphi_1(r)}^{\varphi_2(r)} l(r,\varphi)\, d\varphi, \tag{8}$$

where a – the proportionality factor whose value depends among other things on the intensity of the transmitted ultrasonic wave, ϱ – the density of flowing blood cells, r, φ – the cylindrical coordinates describing the position of blood cells in the cross-section of the vessel, $l(r, \varphi)$ – the length of the region common to the transmitted and received ultrasonic beams in the direction of blood flow (cf. Fig. 2a, b).

The integration limits in formula (8) are a function of the coordinate r and depend on the ratio of the width B of the ultrasonic beam to the inner diameter $2R$ of the blood vessel (cf. Fig. 2a, b).

Differentiation of expression (7) with respect to r and consideration of the results of differentiation in expression (8) give an expression of the distribution of the power density spectrum of the Doppler signal. For the measuring system shown in Fig. 2*a* where the point of intersection of the ultrasonic beams is outside the blood vessel, the power density spectrum of the Doppler signal calculated on the basis of expressions (7) nad (8) assumes the following form

$$S(f_w) = \begin{cases} \dfrac{2W}{\pi} \arcsin \dfrac{k}{\sqrt{1-f_w}} & \text{for} \quad 0 \leqslant f_w \leqslant 1-k^2, \\ W & \text{for} \quad 1-k^2 \leqslant f_w \leqslant 1, \end{cases} \tag{9}$$

where $S(f_w)$ – the power density spectrum of the Doppler signal, f_w – the relative frequency equal to the ratio of the current frequency f_d to the maximum frequency $f_{d\max}$ of the spectrum of the Doppler signal, $k = B/2R$ – the ratio of the width of the ultrasonic beam to the inner diameter of the blood vessel, W – a constant determined in the following way

$$W = \frac{\pi a \varrho H R^2}{f_{d\max}} \frac{\sin(\theta_0 - \theta_n)}{\sin\theta_n \sin\theta_0} \left[\frac{H \sin(\theta_0 + \theta_n)/2}{2\sin(\theta_0 - \theta_n)/2} - h \right]. \tag{9a}$$

Expression (9) is valid for $f_w \geqslant 0$. When any of the limits of the frequency ranges does not satisfy this condition, this takes the zero value.

Fig. 3 shows spectra of the power density of the Doppler signal calculated from formula (9) for the different ratios k of the width of the ultrasonic beam to the inner diameter of the blood vessel.

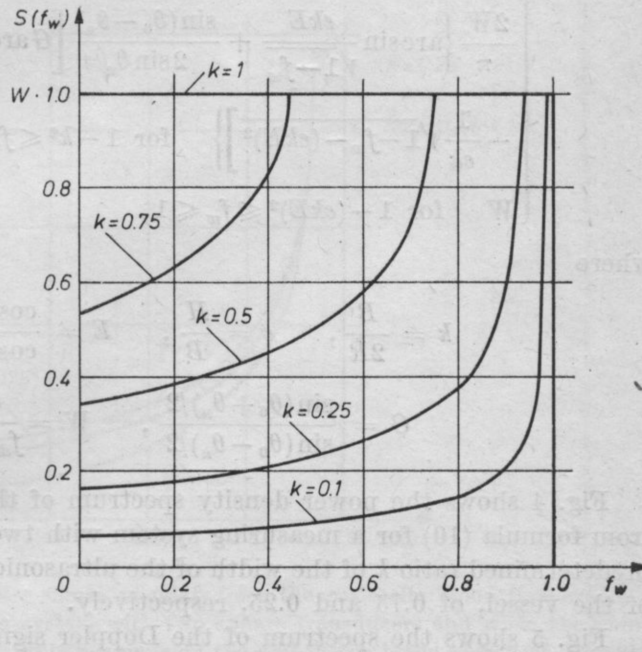

Fig. 3. The spectra of the Doppler signal for the parabolic blood flow velocity profile in the case when the point of intersection of axes of the ultrasonic beams is outside the blood vessel (cf. Fig. 2a)
k – the ratio of the width of the ultrasonic beam to the inner diameter of the blood vessel

For the measuring system shown in Fig. 2*b* where the point of intersection of axes of the ultrasonic beams is on the axis of the blood vessel, the spectrum of the power density of the Doppler signal calculated from formulae (7) and (8) takes the form described by formula (10). This expression, as also expression (9), is valid for $f_w \geqslant 0$.

$$S(f_w) = \begin{cases} 0 \quad \text{for } 0 \leqslant f_w \leqslant 1-k^2(1+e^2G^2); \\ \dfrac{W\sin(\theta_0-\theta_n)}{\pi\sin\theta_n}\left\{G\left[\arcsin\dfrac{ekG}{\sqrt{1-f_w}} - \arccos\dfrac{k}{\sqrt{1-f_w}}\right] + \right. \\ \left. +\dfrac{1}{ek}\sqrt{1-f_w-(ekG)^2} - \dfrac{1}{e}\right\} \quad \text{for } 1-k^2(1+e^2G^2) \leqslant f_w \leqslant 1-(ekG)^2, \\ \dfrac{W\sin(\theta_0-\theta_n)}{\pi\sin\theta_n}\left[G\arcsin\dfrac{k}{\sqrt{1-f_w}} - \dfrac{1}{e}\right] \\ \qquad \text{for } 1-(ekG)^2 \leqslant f_w \leqslant 1-k^2(1+e^2E^2), \\ \dfrac{2W}{\pi}\left\{\left[\arcsin\dfrac{ekE}{\sqrt{1-f_w}} - \arccos\dfrac{k}{\sqrt{1-f_w}}\right] + \right. \\ \left. +\dfrac{\sin(\theta_0-\theta_n)}{2\sin\theta_n}\left[G\arccos\dfrac{ekE}{\sqrt{1-f_w}} - \dfrac{1}{ek}\sqrt{1-f_w-(ekE)^2}\right]\right\} \\ \qquad \text{for } 1-k^2(1+e^2E^2) \leqslant f_w \leqslant 1-k^2, \\ \dfrac{2W}{\pi}\left\{\arcsin\dfrac{ekE}{\sqrt{1-f_w}} + \dfrac{\sin(\theta_0-\theta_n)}{2\sin\theta_n}\left[G\arccos\dfrac{ekE}{\sqrt{1-f_w}} - \right.\right. \\ \left.\left. -\dfrac{1}{ek}\sqrt{1-f_w-(ekE)^2}\right]\right\} \quad \text{for } 1-k^2 \leqslant f_w \leqslant 1-(ekE)^2, \\ W \quad \text{for } 1-(ekE)^2 \leqslant f_w \leqslant 1, \end{cases}$$

where

$$k = \frac{B}{2R}, \quad e = \frac{H}{B}, \quad E = \frac{\cos(\theta_0+\theta_n)/2}{\cos(\theta_0-\theta_n)/2},$$

$$G = \frac{\sin(\theta_0+\theta_n)/2}{\sin(\theta_0-\theta_n)/2}, \quad W = \frac{\alpha\pi\varrho HR^2}{f_{d\max}\sin\theta_0}.$$

Fig. 4 shows the power density spectrum of the Doppler signal calculated from formula (10) for a measuring system with two square transducers for the predetermined ratio k of the width of the ultrasonic beam to the inner diameter of the vessel, of 0.75 and 0.25, respectively.

Fig. 5 shows the spectrum of the Doppler signal calculated from formulae (9) and (10) and measured for the case when the point of intersection of axes of the ultrasonic beams is outside the tube (Fig. 5a, b) and when the point of in-

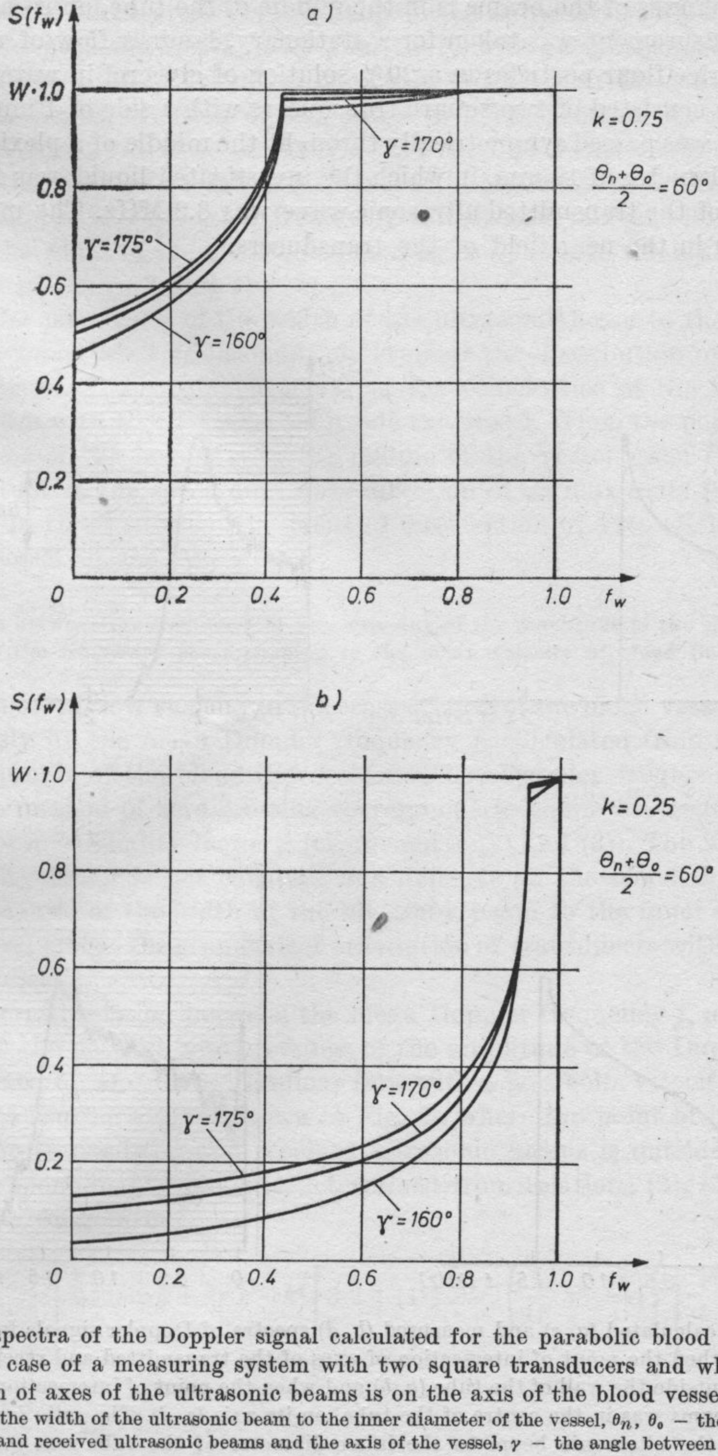

Fig. 4. The spectra of the Doppler signal calculated for the parabolic blood flow velocity profile in the case of a measuring system with two square transducers and when the point of intersection of axes of the ultrasonic beams is on the axis of the blood vessel (cf. Fig. 2b)
k – the ratio of the width of the ultrasonic beam to the inner diameter of the vessel, θ_n, θ_o – the angles between the transmitted and received ultrasonic beams and the axis of the vessel, γ – the angle between the transducers

tersection of axes of the beams is in the middle of the tube, on its axis (Fig. 5c, d). The measurement was taken for a stationary, laminar flow of a 0.01% suspension of rice flour particles in a 20% solution of glycerol in water. The ultrasonic probe consisted of two square transducers with a side of 4 mm. The ultrasonic beam was passed symmetrically throught the middle of a plexiglass tube of the inner diameter of 19 mm, in which the investigated liquid was flowing. The frequency of the transmitted ultrasonic wave was 8.2 MHz. The measurements were taken in the near field of the transducers.

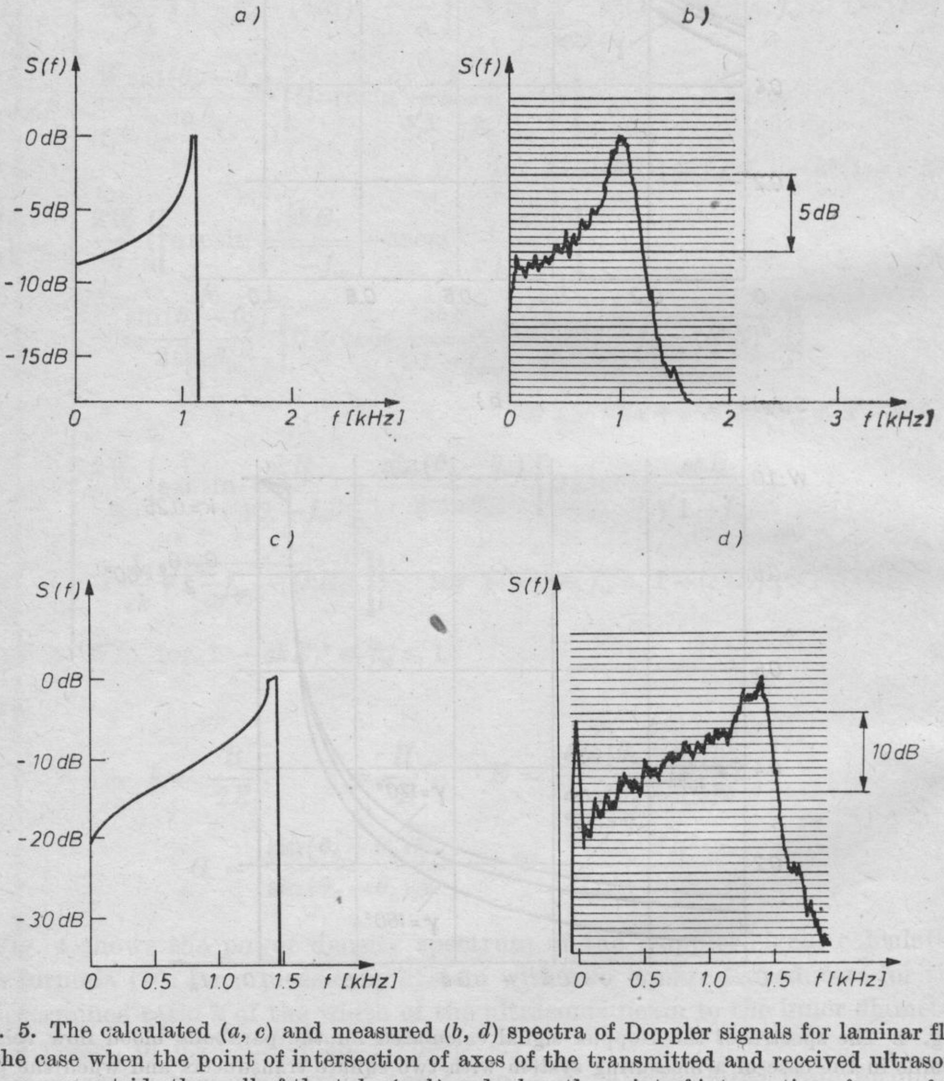

Fig. 5. The calculated (*a*, *c*) and measured (*b*, *d*) spectra of Doppler signals for laminar flow in the case when the point of intersection of axes of the transmitted and received ultrasonic beams was outside the wall of the tube (*a*, *b*) and when the point of intersection of axes of the ultrasonic beams was in the centre of the tube, on its axis (*c*, *d*). The ratio *k* of the width of the ultrasonic beam to the inner diameter of the tube was 0.21

On the basis of the theoretical analysis made and the results obtained from investigation of the spectra of the Doppler signal, the following conclusions were drawn:

(a) When the width of the ultrasonic beam is less than the inner diameter of the vessel, the distribution of the power density spectrum of the Doppler signal is affected by the ratio of the width of the ultrasonic beam to the inner diameter of the vessel. This effect can be seen in the increase in the amplitude of the spectrum over the higher frequency range. This increase is the greater the smaller the width of the ultrasonic beam is, compared to the inner diameter of the vessel (cf. Figs. 3 and 4).

(b) For the same ratio of the width of the ultrasonic beam to the inner diameter of the vessel what additionally determines the distribution of the power density spectrum of the Doppler signal is the coincidence of the transmitted ultrasonic beam with the one received inside the vessel. When the point of intersection of axes of the beams is in the middle of the vessel, then the increase in the amplitude of the spectrum in the direction of its maximum frequency is greater than in the case when the point of intersection of axes of the beams is outside the vessel (cf. Fig. 5).

4. The relation between the frequency of zero-crossing of the amplitude of the Doppler signal and the frequency corresponding to the mean velocity of blood flow

The mean blood flow velocity in the cross-section of the blood vessel is defined unambiguously by the mean Doppler frequency f_s calculated from relation (2) for the real profile of the blood flow velocity. The Doppler frequency f_{zc} measured by the method of zero crossing corresponds to the mean frequency f_s by way of the proportionality factor a (cf. formulae (2) and (3)). The value of the proportionality factor is not constant and depends on the blood flow velocity profile, on the ratio of the width of the ultrasonic beam to the inner diameter of the blood vessel and on the geometrical orientation of transducers with respect to the blood vessel.

The numerical relation between the mean Doppler frequency f_s and the frequency f_{zc} of the positive zero crossings of the amplitude of the Doppler signal will be analyzed for stationary, laminar flow with a parabolic velocity profile.

For the measuring system shown in Fig. 2a where the point of intersection of axes of the transmitted and received ultrasonic beams is outside the blood vessel the proportionality factor a determined from relations (2), (3b) and (9) takes the following form

$$a = \frac{f_s}{f_{zc}} = \begin{cases} \dfrac{\sqrt{3}}{2}\left[\dfrac{\arcsin k + k\sqrt{1-k^2}\left[3.2+(1+2k^2)\left(\dfrac{4}{15}k^2-1\right)\right]}{\arcsin k + k\sqrt{1-k^2}}\right]^{-1/2} & \text{for } k<1, \\ \dfrac{\sqrt{3}}{2} \quad \text{for } k\geqslant 1. \end{cases} \tag{11}$$

where k — the ratio of the width of the ultrasonic beam to the inner diameter of the blood vessel.

Fig. 6 shows the value of the proportionality factor calculated from formula (10) as a function of the ratio of the width of the ultrasonic beam to the inner diameter of the blood vessel. Variation in the value of the factor a results from variation in the distribution of the power density spectrum of the Doppler

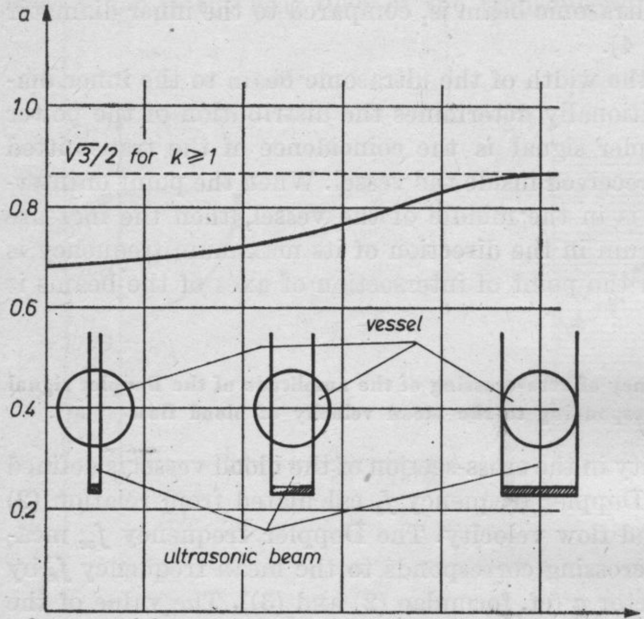

Fig. 6. The factor a of proportionality between the mean frequency and the frequency of zero crossing of the Doppler signal for the parabolic blood flow velocity profile and a measuring system in which the point of intersection of axes of the ultrasonic beams is outside the vessel (cf. Fig. 2a)
k – the ratio of the width of the ultrasonic beam to the inner diameter of the vessel

signal for the individual values of the ratio k. As was shown above (cf. Fig. 3), the spectrum of the Doppler signal does not depend on the width of the ultrasonic beam when this beam occupies the whole cross-section of the vessel. Hence the value of the proportionality factor a is constant for the ratio $k \geqslant 1$. However, when the ultrasonic beam is narrower than the diameter of the blood vessel, then the amplitude of the Doppler spectrum increases in the direction of its maximum frequency with decreasing ratio k of the width of the ultrasonic beam to the inner diameter of the vessel. This causes, in turn, an increase in the value of the Doppler frequency f_{zc} measured by the method of zero crossing for the constant mean frequency f_s. As a result the value of the factor of proportionality between the mean frequency f_s and the frequency f_{zc} measured by the method of zero crossing decreases with decreasing ratio k of the width of the ultrasonic beam to the inner diameter of the blood vessel.

The lack of the constant numerical relation between the mean frequency f_s and the measured frequency f_{zc} makes difficult interpretation of the results of measurement of blood flow in blood vessels with diameters larger than the width of the ultrasonic beam, since in this case the lack of information about the real inner diameter of the blood vessel is an essential source of error in quantitative evaluation of the mean blood flow velocity on the basis of the Doppler frequency measured by the zero crossing method.

One can now consider a case-when the point of intersection of axes of the transmitted ultrasonic beam and the one received from the flowing blood is inside the blood vessel, on its axis (Fig. 2b). The proportionality factor a calculated for this case takes, according to relations (2), (3) and (10), the following form

$$a = 0.5 \times \left[\frac{C_n + A_{kn}(1-GL) + GL(E_{kn} + A_{km} - F_{km}) + L\left(E_{km} - E_{kn} - \frac{1}{e} C_{kmn}\right) + C_{kn} + A_{kmn} + R_{kn}}{B_{kn}(1-GL) + D_{kn} + B_{kmn} - \frac{2L}{\pi e}(m^2 - n^2) + GLB_{km}}\right]^{-1/2}$$

$$\text{for } 0 \leqslant k \leqslant \frac{1}{\sqrt{1+e^2G^2}}, \tag{12a}$$

$$a = 0.5 \times \left[\frac{C_n + A_{kn}(1-GL) + GL[E_{kn} + G_{kn} + 2(A_m - B_m)]}{B_{kn}1(-GL) + GL[k^2 + H_{kn} + 2(D_m - E_m)] - \frac{4L}{3\pi}k^2 + D_{kn} - \frac{2L}{\pi e}(1-k^2-n^2) + \frac{L}{k}G_m} + \right.$$

$$\left. + \frac{-LF_{kn} + C_{kn} - \frac{2L}{3\pi e}(1-k^2-n^2)^3 + \frac{L}{k}F_m - R_{kn}}{B_{kn}(1-GL) + GL[k^2 + H_{kn} + 2(D_m - E_m)] - \frac{4L}{3\pi}k^2 + D_{kn} - \frac{2L}{\pi e}(1-k^2-n^2) + \frac{L}{k}G_m}\right]^{-1/2}$$

$$\text{for } \frac{1}{\sqrt{1+e^2G^2}} \leqslant k < \frac{1}{eG}, \tag{12b}$$

$$a = 0.5 \times \left[\frac{C_n + A_{kn}(1-GL) + GL(F_{kn} + G_{kn}) - LE_{kn} + C_{kn} - \frac{2L}{3\pi e}(1-k^2-n^2)^3 - R_{kn}}{B_{kn}(1-GL) + GL(k^2 + H_{kn}) - \frac{4}{3\pi}Lk^2 + D_{kn} - \frac{2L}{\pi e}(1-k^2-n^2)}\right]^{-1/2}$$

$$\text{for } \frac{1}{eG} \leqslant k < \frac{1}{\sqrt{1+e^2E^2}}, \tag{12c}$$

$$a = 0.5\left[\frac{C_n + 2A_n(1-GL) + 2B_nGL - \frac{L}{k}F_n + 2A_k - \frac{1}{3}(1-k^2)^3}{n + 2D_n(1-GL) + 2E_nGL - \frac{L}{k}G_n + 2D_k + k^2 - 1}\right]^{-1/2} \tag{12d}$$

$$\text{for } \frac{1}{\sqrt{1+e^2E^2}} \leqslant k < 1,$$

$$a = 0.5\left[\frac{C_n+2A_n(1-GL)+2B_nGL-F_nL\dfrac{1}{k}}{n^2+2D_n(1-GL)+2E_nGL-G_nL\dfrac{1}{k}}\right]^{-1/2} \quad \text{for } 1 \leqslant k < \frac{1}{eE}, \tag{12e}$$

$$a = \frac{\sqrt{3}}{2} \quad \text{for } k \geqslant \frac{1}{eE}, \tag{12f}$$

where $e = H/B$ — the ratio of sides of the transducer (cf. Fig. 2), k — the ratio of the width of the ultrasonic beam to the inner diameter of the blood vessel;

$$E = \frac{\cos(\theta_0+\theta_n)/2}{\cos(\theta_0-\theta_n)/2}, \quad G = \frac{\sin(\theta_0+\theta_n)/2}{\sin(\theta_0-\theta_n)/2}, \quad L = \frac{\sin(\theta_0-\theta_n)}{2\sin\theta_n},$$

$$n = ekE, \quad m = ekG, \quad A_k = \frac{1}{\pi}\left\{\frac{k}{3}\sqrt{1-k^2}\left[3.2+(1+2k^2)\left(\frac{4}{15}k^2-1\right)\right]+\right.$$
$$\left.+\frac{1}{3}\arcsin k - k^2\left(1-k^2+\frac{k^4}{3}\right)\frac{\pi}{2}\right\};$$

A_n and Q_m are described by expression A_k, when the parameter k is replaced with the parameters n or m, respectively;

$$D_k = \frac{1}{\pi}\left[k\sqrt{1-k^2}+\arcsin k - k^2\frac{\pi}{2}\right];$$

D_n and D_m are described by expression D_k when the parameter k is replaced with the parameters n or m, respectively;

$$C_k = \frac{1}{3}[1-(1-k^2)^3], \quad C_n = \frac{1}{3}[1-(1-n^2)^3], \quad B_n = \frac{1}{6}(1-n^2)^3,$$

$$B_m = \frac{1}{6}(1-m^2)^3, \quad E_n = \frac{1}{2}(1-n^2), \quad E_m = \frac{1}{2}(1-m^2),$$

$$G_n = \frac{4}{3\pi}\sqrt{(1-n^2)^3}, \quad G_m = \frac{4}{3\pi}\sqrt{(1-m^2)^3}, \quad F_m = \frac{32}{105\pi}\sqrt{(1-m^2)^5},$$

$$F_n = \frac{32}{105\pi}\sqrt{(1-n^2)^5},$$

$$A_{kn} = \frac{2}{\pi}\left\{\frac{nk}{3}\left[3+(k^2+3n^2)\left(\frac{4}{15}n^2-1\right)+\frac{1}{5}(k^2+n^2)^2\right]+\right.$$
$$\left.+(k^2+n^2)\left[1-(k^2+n^2)+\frac{1}{3}(k^2+n^2)^2\right]\arcsin\frac{n}{\sqrt{k^2+n^2}}-\frac{\pi}{2}n^2\left(1-n^2+\frac{n^4}{3}\right)\right\};$$

$$B_{kn} = \frac{2}{\pi}\left[(k^2+n^2)\arcsin\frac{n}{\sqrt{k^2+n^2}} + nk - \frac{n^2}{2}\pi\right],$$

$$C_{kn} = \frac{2}{\pi}\left\{\frac{nk}{3}\left[3+(n^2+3k^2)\left(\frac{4}{15}k^2-1\right)+\frac{1}{5}(k^2+n^2)\right]+\right.$$

$$\left.+(k^2+n^2)\left[1-(k^2+n^2)+\frac{1}{3}(k^2+n^2)^2\right]\arcsin\frac{k}{\sqrt{k^2+n^2}} - \frac{\pi}{2}k^2\left(1-k^2+\frac{k^4}{3}\right)\right\},$$

$$D_{kn} = \frac{2}{\pi}\left[(k^2+n^2)\arcsin\frac{k}{\sqrt{k^2+n^2}} + nk - \frac{k^2}{2}\pi\right],$$

$$E_{kn} = \frac{2}{\pi}k^2\left[\frac{2}{15}(5-10n^2-6k^2)+\frac{2}{7}(k^2+n^2)^2+\frac{8}{105}n^2(5n^2+3k^2)\right],$$

$$F_{kn} = \frac{1}{3}\left[(1-n^2)^3-(1-n^2-k^2)^3\right],$$

$$G_{kn} = \frac{2}{\pi}\left\{\frac{k}{3}\sqrt{1-k^2}\left[3.2+(1+2k^2)\left(\frac{4}{15}k^2-1\right)\right]+\frac{1}{3}\arcsin k-(k^2+n^2)\times\right.$$

$$\times\left[1-(k^2+n^2)+\frac{(k^2+n^2)^2}{3}\right]\arcsin\frac{k}{\sqrt{k^2+n^2}} - \frac{kn}{3}\left[3+(n^2+3k^2)\left(\frac{4}{15}k^2-1\right)+\right.$$

$$\left.\left.+\frac{(k^2+n^2)^2}{5}\right]\right\},$$

$$H_{kn} = \frac{2}{\pi}\left[\arcsin k + k\sqrt{1-k^2} - nk - (k^2+n^2)\arcsin\frac{k}{\sqrt{k^2+n^2}}\right],$$

$$R_{kn} = \frac{1}{3}\left[(1-k^2)^3-(1-k^2-n^2)^3\right],$$

$$A_{kmn} = \frac{2}{\pi}\left\{(k^2+m^2)\left[1-(k^2+m^2)+\frac{1}{3}(k^2+m^2)^2\right]\arcsin\frac{k}{\sqrt{k^2+m^2}} -\right.$$

$$-\frac{km}{3}\left[3+(m^2+3k^2)\left(\frac{4}{15}k^2-1\right)+\frac{1}{5}(k^2+m^2)^2\right]-(k^2+n^2)\left[1-(k^2+n^2)+\right.$$

$$\left.\left.+\frac{(k^2+n^2)^2}{3}\right]\arcsin\frac{k}{\sqrt{k^2+n^2}} - \frac{kn}{3}\left[3+(n^2+3k^2)\left(\frac{4}{15}k^2-1\right)+\frac{1}{5}(k^2+n^2)^2\right]\right\},$$

$$B_{kmn} = \frac{2}{\pi}\left[(k^2+m^2)\arcsin\frac{k}{\sqrt{k^2+m^2}} + k(m-n) - (k^2+n^2)\arcsin\frac{k}{\sqrt{k^2+n^2}}\right];$$

$$C_{kmn} = \frac{2}{3\pi}\left[(1-k^2-n^2)^3-(1-k^2-m^2)^3\right],$$

A_{km}, B_{km}, E_{km}, F_{km} are described by expressions A_{kn}, B_{kn}, E_{kn}, F_{kn}, respectively when the parameter n is replaced with the parameter m. The expressions given above (12a-f) take a very complicated form which results from successive transformations of relations (2), (3) and (10). The present investigation is the first attempt in the world literature to solve this problem analytically and, therefore, it seems purposeful to present the whole form of the solution.

The proportionality factor a described by expressions (12a-f) depends on the ratio k of the width of the ultrasonic beam to the inner diameter of the blood vessel, and in addition on the ratio H/B of the width of the ultrasonic beam along the blood vessel to its width across the vessel and on the angles θ_n and θ_0 at which the ultrasonic wave is transmitted and received with respect to the blood vessel. Fig. 7 shows values of the factor a calculated from relations (12a-f) for a measuring system with two square transducers placed at the angle $\gamma = 170°$ with respect to each other.

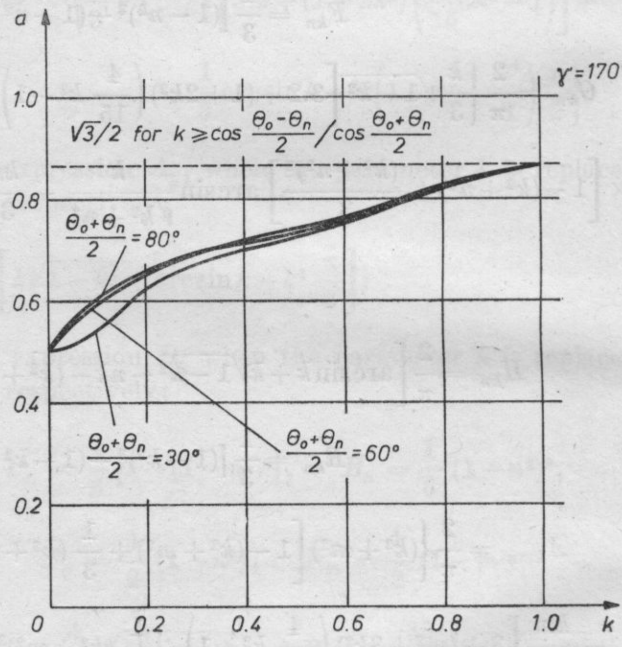

Fig. 7. The factor a of proportionality between the mean frequency and the frequency of zero crossing of the Doppler signal for the parabolic blood flow velocity profile and a measuring system with two square transducers, in which the point of intersection of axes of the ultrasonic beams is on the axis of the vessel (cf. Fig. 2b)

k – the ratio of the width of the ultrasonic beam to the inner diameter of the vessel, θ_n, θ_0 – the angles between the transmitted and received ultrasonic beams and the axis of the vessel, γ – the angle between the transducers

5. Limitations of the ultrasonic *C.W.* Doppler method in measurement of the absolute blood flow velocity

The ultrasonic *C.W.* Doppler method does not permit quantitative information about the blood flow velocity profile to be obtained. In the case when it is impossible to determine the profile analytically, then error can be committed in calculation of the mean blood velocity v_s on the basis of the Doppler frequency measured by the method of zero-crossing (formula (3a)). This results from the fact that without quantitative information about the blood flow velocity profile it is impossible to determine the exact value of the factor of proportionality a between the measured Doppler frequency and the mean frequency f_s corresponding to the mean blood flow in the cross-section of the blood vessel (cf. formula (2)). Under the assumption that the mean blood flow velocity profile during the cardiac cycle is contained between the parabolic and flat profiles, the mean a_s of the factor a can be calculated from the relation [6, 7]

$$a_s = \frac{2a}{1+a}, \tag{13}$$

where a is a factor for a parabolic profile.

Assuming the factor a_s as the basis for calculating the mean blood flow velocity, the error which can result from the lack of information about the blood flow profile is contained in the following interval

$$0 \leqslant \left| \frac{v_0 - v_s}{v_s} \right| < (1-a_s)\cdot 100\,\%, \tag{14}$$

where v_0 — the mean blood flow velocity calculated from the measured Doppler frequency, v_s — the real mean blood flow velocity. The numerical value of this error can be determined by calculation of the value of the factor a_s from relations (11) — (13). For the measuring system where the point of intersection of axes of the ultrasonic beams is outside the blood vessel (Fig. 2a), the error described by expression (14) is less than 7.2 % when the ultrasonic beam occupies the whole cross-section of the vessel.

An additional source of error in transcutaneous measurement of blood flow is the lack of information about the inner diameter of the blood vessel. This is mainly the case when the diameter of the blood vessel is greater than the width of the ultrasonic beam, since then the calibration coefficient of a Doppler flowmeter considerably changes its value, depending on the ratio k of the width of the ultrasonic beam to the inner diameter of the blood vessel (cf. Figs. 6 and 7).

The error in measurement of the mean blood flow velocity, determined from relations (11), (13) and (14) and resulting from the lack of information about the blood flow velocity profile and the diameter of the blood vessel, is less than 15 % for $k > 0.5$. From the point of view of diagnostics of the human circulation system this error is negligible, however, since blood flow velocities in normal and pathological cases are different by several times. The sources of error in measurement of the blood flow velocity also include the factors affecting the accuracy of measurement of the Doppler frequency by the method of zero-crossing (cf. section 2) and in addition the lack of information about the angle between the ultrasonic beam and the blood vessel. The problem of measurement of this angle was solved for peripheral vessels by application in the measurement of the blood velocity of an ultrasonic probe consisting of two independent pairs of transmitting and receiving transducers placed at a known constant angle to each other [1, 6, 7].

6. Conclusions

In summary of the foregoing considerations of the ultrasonic *C.W.* Doppler method it should be stated that quantitative evaluation of the blood flow velocity by this method is very difficult and requires specification of conditions of the measurement, which is not always possible. It follows therefore that the technique of measurement of the Doppler frequency by the method of zero crossing, used in ultrasonic flowmeters, does not give direct information about the mean Doppler frequency proportional to the mean blood flow velocity in the cross-section of the vessel.

The sought information can be obtained from determination of the factor a of proportionality between the frequencies mentioned above. The value of the factor is not constant, however, and depends on such factors as the blood flow velocity in the blood velocity, the ratio of the width of the ultrasonic beam to the inner diameter of the vessel and the possition of the transducers transmitting and receiving the ultrasonic wave with respect to the blood vessel. The lack of information about the above parameters is a source of error in quantitative evaluation of the blood flow velocity on the basis of the Doppler frequency measured by the method of zero crossings. This error is smallest when the mean proportionality factor a_s described in this paper is taken as the basis of calculation of the mean blood flow velocity and when the ultrasonic beam occupies the whole cross-section of the blood vessel. In such a case for a measuring system in which the point of intersection of axes of the transmitted ultrasonic beam and the one received from flowing blood is outside the blood vessel, the error under consideration is less than 7.2% when the blood flow velocity varies and takes shapes contained between parabolic and flat profiles.

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