ARCHIVES OF ACOUSTICS
5, 3, 237-250 (1980)

MUTUAL ACOUSTIC IMPEDANCE OF CIRCULAR MEMBRANES AND PLATES WITH BESSEL AXIALLY-SYMMETRIC VIBRATION VELOCITY DISTRIBUTIONS

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In this paper the mutual impedance of circular membranes and circular plates clamped at the circumference is analyzed. It was assumed that a Bessel axially-symmetric vibration velocity distribution was predetermined on the surface of the sources, and that the sources were placed in a rigid planar baffle. The impedance was calculated by a method based on a Fourier representation of the acoustic pressure. In view of the axially-symmetric vibration velocity distribution, the acoustic pressure and the subsequent formulae for the mutual impedance are given in the Hankel representation. As a result, the mutual impedance can be expressed in the form of a single integral. Practically useful formulae are derived for specific cases. The results of the calculation are also shown graphically.

1. Introduction

Most of the planar acoustic sources that have a practical usefulness, are characterized by a nonuniform vibration velocity distribution. The Bessel vibration velocity distributions have a particular significance, since they are suitable for an exact description of the problem of the vibration of a membrane or of a circular plate clamped at the circumference.

The literature on the acoustic impedance of source with nonuniform vibration velocity distributions contains only a few items.

The problem of the mutual impedance of two elastic circular pistons vibrating in the flexural mode was investigated in 1964 by PORTER [5] and CHAN [1] in 1967. They assumed that the vibration velocity distribution was axially-symmetric and could be expressed by a power series in the radial variable.

The problem of the mutual impedance of two circular sources with non-uniform axially-symmetric vibration velocity distributions, namely: gaussian, parabolic and Bessel distributions, was investigated by the author in papers [6-10] in the 1970's.

The problems presented in the present paper are connected with the mutual impedance of circular membranes and circular plates clamped at the circumference. It has been assumed in the calculation of the impedance that there is a Bessel axially-symmetric vibration velocity distribution on the surface of the sources, and that they are situated in ideal rigid planar baffles. The results of the calculations are presented graphically.

Notation

The numbers in parentheses denote the formulae giving definition or application of the symbol.

a — radius of the circle,
c — the wave velocity,
f — the vibration velocity distribution function,
$H_p^{(2)}$ — a cylindrical Hankel function of the second kind of the pth order,
$h_p^{(2)}$ — a spherical Hankel function of the second kind of the pth order (27),
i — $\sqrt{-1}$,
J_n — a cylindrical Bessel function of the nth order,
j_m — a spherical Bessel function of the mth order (24),
k — the wave number,
l_{12} — the distance between the centres of sources 1 and 2,
N_m — a cylindrical Neumann function of the mth order,
n_m — a spherical Neumann function of the mth order (26),
p_{12} — the acoustic pressure generated by source 1 acting on source 2 (10),
R — the distance between two points on the two sources,
r — the radial variable in a polar coordinate system,
t — time,
U — a characteristic function of the source (15),
v — the vibration velocity of points of the source,
v_0 — the vibration velocity of the central point on the surface of the source,
W — a characteristic function of the source for an axially-symmetric vibration velocity distribution (17),
Z_{11} — the mechanical self-impedance of one source,
Z_{12} — the mechanical mutual impedance of two sources (9),
θ_{12} — the normalised mutual resistance (11),
χ_{12} — the normalised mutual reactance (11),
ζ_{12} — the normalised mutual impedance (11),
ϱ — the density of the medium,
σ — the area of the source,
ω — the angular frequency.

2. Assumptions of the analysis

It is assumed that in an infinite ideal liquid medium of density ϱ there is an ideal rigid baffle at the plane $z = 0$, with a system N of sound sources. The sources are assumed to be harmonically vibrating circular transducers, each of thickness h and radius a (Fig. 1). The distance between the centra points of sources 1 and 2 is

$$l_{12} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\,.$$

We can consider a transducer to be a vibrating circular plate of specified thickness, rigidly clamped by its edge to the housing. For this reason we shall take as the basis for assuming a suitable vibration velocity distribution on the surface of the source, an exact mathematical solution of the problem of the free vibration of a circular plate rigidly clamped at its edge.

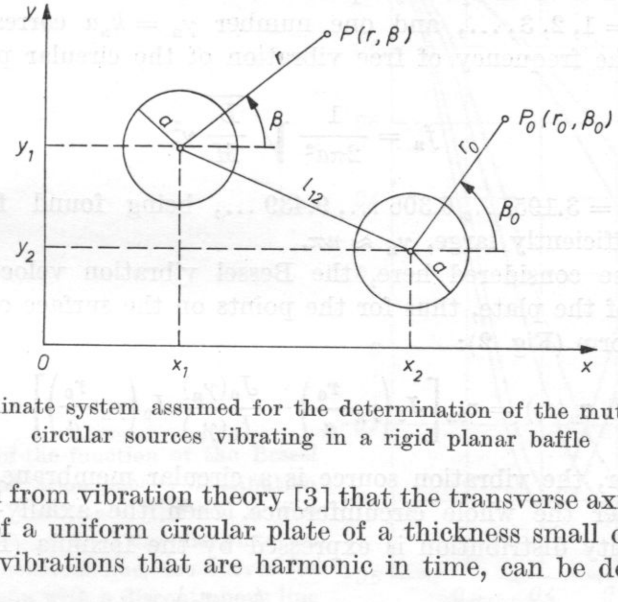

Fig. 1. The coordinate system assumed for the determination of the mutual impedance of circular sources vibrating in a rigid planar baffle

It is known from vibration theory [3] that the transverse axially-symmetric free vibration of a uniform circular plate of a thickness small compared to its radius [2], for vibrations that are harmonic in time, can be described by the equation

$$\zeta(r_0, t) = \zeta_0(r_0)e^{i\omega t} = [A_0 J_0(kr_0)+B_0 I_0(kr_0)]e^{i\omega t}, \tag{1}$$

where $\xi(r_0, t)$ is the displacement of points of the plate in the transverse direction, ω is the angular frequency, t is the time, $i = \sqrt{-1}$, $k^2 = \omega/\sqrt{M/B}$, M is the mass of the plate per unit area, B is the flexural rigidity of the plate and A_0 and B_0 are constants. The special function which occurs here, $I_0(kr_0)$ is a modified Bessel function of the first kind of the zeroth order, which can be described as a Bessel function J_0 of imaginary argument [11], i.e. $I_m(w) = i^{-m} J_m(iw)$.

The boundary conditions for $r_0 = a$ are the following:

1) the plate has zero displacements at the places of rigid clamping:

$$\zeta_0(v_0)|_{r_0=a} = 0; \tag{2}$$

2) and the plate cannot rotate:

$$\frac{d}{dr_0}\zeta_0(r_0)\bigg|_{r_0=a} = 0. \tag{3}$$

For behaviour which is harmonic in time, the vibration velocity distribution $v(r_0, t) = v(r_0)e^{i\omega t}$, with consideration of relations (2) and (3), satisfies the boundary conditions

$$v(r_0)\Big|_{r_0=a} = 0, \quad \frac{d}{dr_0} v(r_0)\Big|_{r_0=a} = 0,$$

which lead to the so-called frequency equation:

$$J_0(ka)I_1(ka) + J_1(ka)I_0(ka) = 0. \tag{4}$$

The solution of the frequency equation is an infinite series in the quantity k: $k = k_n$ for $n = 1, 2, 3, \ldots$, and one number $\gamma_n = k_n a$ corresponds to each value of k_n. The frequency of free vibration of the circular plate is

$$f_n = \frac{1}{2\pi a^2} \sqrt{\frac{B}{M}}\, \gamma_n^2 \tag{5}$$

with $\gamma_1, \gamma_2, \gamma_3 = 3.195\ldots, 6.306\ldots, 9.439\ldots$, being found for $n = 1, 2, 3$; and, if n is sufficiently large, $\gamma_n \approx n\pi$.

In the case considered here, the Bessel vibration velocity distribution for any point of the plate, thus for the points on the surface of the plate, has the following form (Fig. 2):

$$v_n(r_0) = v_{0n}\left[J_0\left(\gamma_n \frac{r_0}{a}\right) - \frac{J_0(\gamma_n)}{I_0(\gamma_n)} I_0\left(\gamma_n \frac{r_0}{a}\right)\right]. \tag{6}$$

If, however, the vibration source is a circular membrane tensioned with equal force over the whole circumference, then the axially-symmetric free vibration velocity distribution is expressed by the formula (Fig. 2)

$$v_n(r_0) = v_{0n} J_0\left(\beta_n \frac{r_0}{a}\right), \tag{7}$$

where β_n is nth root of the equation

$$J_0(\beta_n) = 0. \tag{8}$$

In this equation $\beta_1, \beta_2, \beta_3 = 2.4048\ldots, 5.5201\ldots, 8.6537\ldots$, and, generally, if n is sufficiently large [11], then $\beta_n \approx n\pi - \pi/4 \approx n\pi$.

The calculation of the mechanical mutual impedance Z_{12} of two planar sound source is carried out on the basis of the definition [5, 9]

$$Z_{12} = \frac{1}{v_{01}v_{02}^*} \int_{\sigma_2} p_{12}(x_0, y_0) v_2^*(x_0, y_0)\, d\sigma_0, \tag{9}$$

where

$$p_{12}(x_0, y_0) = \frac{ik\varrho c}{2\pi} \int_{\sigma_1} v(x, y) \frac{e^{-ikR}}{R}\, d\sigma \tag{10}$$

is the acoustic pressure generated by source 1 and acting on source 2, v_1 is the vibration velocity distribution on the surface of source 1, v_{01} is the amplitude of the vibration velocity of the centre point of source 1, v^* is a quantity coupled in a complex manner with v, $R = \sqrt{(x-x_0)^2+(y-y_0)^2}$ is the distance from the surface element $d\sigma$ on source 1 to the point (x_0, y_0) on source 2.

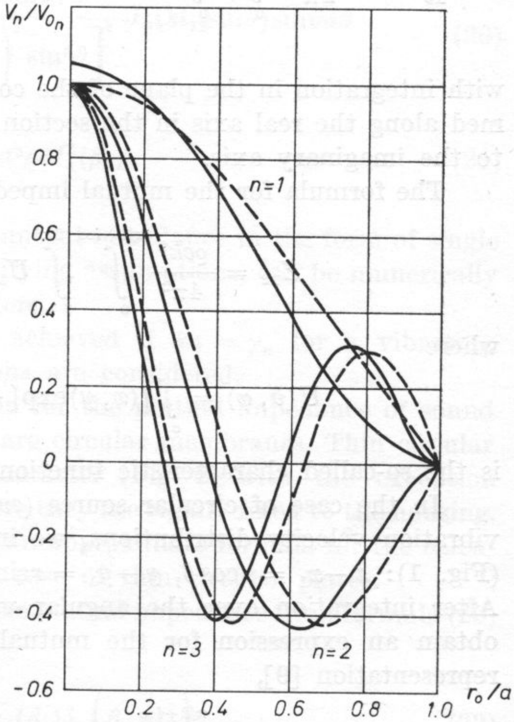

Fig. 2. The plots of the function of the Bessel axial-symmetric vibration velocity distribution for the first three modes: $n = 1$, 2 and 3. The curves of distributions for a circular plate are plotted with a continuous line, the curves for a circular membrane with a discontinuous line

In numerical calculations it is convenient to use the concept of the normalised impedance

$$\frac{Z_{12}}{\lim\limits_{k\to\infty} Z_{11}} = \zeta_{12} = \theta_{12}+i\chi_{12}, \tag{11}$$

where θ_{12}, χ_{12} are, respectively, the normalised mutual resistance and normalised mutual reactance between sources 1 and 2. For $k \to \infty$ we have $p_{11}(x, y) = \varrho c v_1(x, y)$, and then-formula (9) – we obtain

$$\lim_{k\to\infty} Z_{11} = \varrho c \int_{\sigma_1} f_1^2(x, y)\, d\sigma, \tag{12}$$

where $v_1(x, y)/v_{01} = f_1(x, y)$ is a function of the vibration velocity distribution on the surface of source 1.

3. Integral formulae for mutual impedance

In order to facilitate the calculation of the surface integrals (9) and (10), the term e^{-ikR}/R can be represented by a double integral [9],

$$\frac{e^{-ikR}}{R} = \frac{-ik}{2\pi} \int_0^{\pi/2+i\infty} \int_0^{2\pi} \exp\{-ik\sin\vartheta[(x-x_0)\cos\varphi + (y-y_0)\sin\varphi]\}\sin\vartheta d\vartheta d\varphi \tag{13}$$

with integration in the plane of the complex variable $\vartheta = \theta + i\psi$ being performed along the real axis in the section $(0, \pi/2)$ and on the ray $(0, \infty)$ parallel to the imaginary axis.

The formula for the mutual impedance takes the form

$$Z_{12} = \frac{\varrho ck^2}{4\pi^2} \int_0^{\pi/2+i\infty} \int_0^{2\pi} U_1(\vartheta, \varphi) U_2^*(\vartheta, \varphi) \sin\vartheta d\vartheta d\varphi, \tag{14}$$

where

$$U(\vartheta, \varphi) = \int_\sigma f(x, y) \exp\{-ik\sin\vartheta(x\cos\varphi + y\sin\varphi)\} d\sigma \tag{15}$$

is the so-called characteristic function of the sound source.

In the case of circular source (each of radius a) with axially-symmetric vibration velocity distributions, we introduce local polar coordinate systems (Fig. 1): $x - x_1 = r\cos\beta$, $y - y_1 = r\sin\beta$, $x_0 - x_2 = r_0\cos\beta_0$, $y_0 - y_2 = r_0\sin\beta_0$. After integration over the angular variables between the limits $(0, 2\pi)$, we obtain an expression for the mutual mechanical impedance in the Hankel representation [9],

$$Z_{12} = 2\pi\varrho ck^2 \int_0^{\pi/2+i\infty} W(ka\sin\vartheta)\, W^*(ka\sin\vartheta) J_0(kl_{12}\sin\vartheta)\sin\vartheta d\vartheta, \tag{16}$$

where

$$W(ka\sin\vartheta) = \int_0^a f(r_0) J_0(kr_0\sin\vartheta) r_0 dr_0. \tag{17}$$

In the sound source are circular plates, on whose surfaces Bessel type (6) vibration velocity distributions occur, then the normalized mutual impedance [9] is equal to

$$\zeta_{12}^p = \left(2\frac{ka}{\gamma_n}\right)^2 \int_0^{\pi/2+i\infty} \frac{\sin\vartheta J_0(kl_{12}\sin\vartheta)}{\left[1 - \left(\frac{ka}{\gamma_n}\right)^4 \sin^4\vartheta\right]^2} \times$$
$$\times \left[\frac{J_1(\gamma_n)}{J_0(\gamma_n)} J_0(ka\sin\vartheta) - \frac{ka}{\gamma_n}\sin\vartheta J_1(ka\sin\vartheta)\right]^2 d\vartheta \tag{18}$$

and the normalising factor has the form

$$\lim_{k\to\infty} Z_{11}^{p} = 2\pi a^2 \varrho c J_0^2(\gamma_n). \tag{19}$$

However, when the sound source are circular membranes, we obtain [9]

$$\varrho_{12}^{M} = 2\left(\frac{ka}{\beta_n}\right)^2 \int_0^{\pi/2+i\infty} \frac{J_0^2(ka\sin\vartheta)}{\left[1-\left(\frac{ka}{\beta_n}\right)^2 \sin^2\vartheta\right]^2} J_0(kl_{12}\sin\vartheta)\sin\vartheta d\vartheta \tag{20}$$

with

$$\lim_{k\to\infty} Z_{12}^{M} = \pi a^2 \varrho c J_1^2(\beta_n). \tag{21}$$

The expressions given here for the mutual impedance in the form of single integrals, without using additional simplifying assumptions, can be numerically evaluated only with the use of computers.

Considerable simplification can be achieved if $ka = \gamma_n$ for a vibrating plate, or $ka = \beta_n$ if membrane vibrations are considered.

Formula (20), which is an expression for the mutual impedance of sound sources, is exact only when the sources are circular membranes. Thin circular plates exhibit vibration of the same character (Fig. 2) with the exception of slight deviations close to the edges where they are rigidly fixed to the housing. Thus expression (20) can also be used as an approximate formula for the calculation of the mutual impedance in the case of thin circular plates.

For $ka = \beta_n$, the real component of the mutual impedance from formula (20) takes [9] the form

$$\theta_{12}^{M} = \sum_{p=0}^{\infty} \left(\frac{a}{l_{12}}\right)^p \sigma_p(\beta_n) j_p\left(\beta_n \frac{l_{12}}{a}\right), \tag{22}$$

where

$$\sigma_0(\beta_n) = \frac{1}{2}\beta_n^2 J_1^2(\beta_n),$$

$$\sigma_1(\beta_n) = \left(\frac{\beta_n}{2}\right)^2 J_1(\beta_n) J_2(\beta_n) = \frac{1}{2}\beta_n J_1^2(\beta_n),$$

$$\sigma_p(\beta_n) = \frac{\beta_n}{2} \frac{\Gamma\left(p+\frac{1}{2}\right)}{\sqrt{\pi}} \sum_{q=0}^{p} \frac{J_{q+1}(\beta_n) J_{p-q+1}(\beta_n)}{(q+1)!(p-q+1)!}, \tag{23}$$

and [11]

$$j_m(u) = \sqrt{\frac{\pi}{2u}} J_{m+\frac{1}{2}}(u). \tag{24}$$

The imaginary component of the mutual impedance is equal [9] to

$$\chi_{12}^M = -\sum_{p=0}^{\infty}\left(\frac{a}{l_{12}}\right)^p \sigma_p(\beta_n) n_p\left(\beta_n \frac{l_{12}}{a}\right), \tag{25}$$

where [11]

$$n_m(u) = \sqrt{\frac{\pi}{2u}}\, N_{m+\frac{1}{2}}(u) \tag{26}$$

is a spherical Neumann function of the mth order. Using the definition for a spherical Hankel function [4] of the second kind

$$h_m^{(2)}(u) = j_m(u) - i n_m(u), \tag{27}$$

the normalised mutual impedance of two circular membranes, for $ka = \beta_n$, can be written in the following way:

$$\zeta_{12}^M = \sum_{p=0}^{\infty}\left(\frac{a}{l_{12}}\right)^p \sigma_p(\beta_n) h_p^{(2)}\left(\beta_n \frac{l_{12}}{a}\right). \tag{28}$$

In order to simplify numerical calculations using formulae (22), (25) or (28), the values of the coefficients $\sigma_p(\beta_n)$ are grouped in Table 1.

Table 1. The expansion coefficients $\sigma_p(\beta_n)$ calculated according to formula (23)

n	σ_0	σ_1	σ_2	σ_3	σ_4	σ_5
1	0.77932	0.32410	0.17557	0.09282	0.03327	0.02073
2	1.76398	0.31955	−0.28180	−0.46808	−0.01840	0.39125
3	2.75907	0.31883	−0.58845	−0.54459	0.34325	1.09679
4	3.75670	0.31859	−0.86487	−0.56872	0.65486	1.35270
5	4.75530	0.31849	−1.13017	−0.57930	0.93842	1.47003

5. Impedance in specific cases

In the cases where $ka \ll \beta_n$ or $ka \ll \gamma_n$, the analysis of the acoustic mutual interaction is considerably simpler since approximate formulae can then be used.

For $(ka/\gamma_n)^4 \ll 1$ we assume the following simplifications in formula (18):

$$\left[1-\left(\frac{ka}{\gamma_n}\right)^4 \sin^4\vartheta\right]^{-2} \simeq 1, \quad \left[1-\left(\frac{ka}{\gamma_n}\right)^4 \cos h^4\psi\right]^{-2} \simeq 1. \tag{29}$$

The normalised mutual impedance of two vibrating circular plates can then be reduced [9] to the form

$$\zeta_{12}^{p}=\sum_{p=0}^{\infty}\left(\frac{a}{l_{12}}\right)^{p}\left\{b_p h_p^{(2)}(kl_{12})+c_p\left[h_p^{(2)}(kl_{12})-\frac{2}{kl_{12}}\left(p+\frac{1}{2}\right)h_{p+1}^{(2)}(kl_{12})\right]+\right.$$

$$\left.+d_p\left[h_p^{(2)}(kl_{12})-\frac{4}{kl_{12}}\left(p+\frac{1}{2}\right)h_{p+1}^{(2)}(kl_{12})+\left(\frac{2}{kl_{12}}\right)^{2}\left(p+\frac{1}{2}\right)\left(p+\frac{3}{2}\right)h_{p+2}^{(2)}(kl_{12})\right]\right\}, \tag{30}$$

where

$$b_p=\left(2\frac{ka}{\gamma_n}\right)^{2}\frac{\Gamma(p+\frac{1}{2})}{\sqrt{\pi}}\frac{J_1^2(\gamma_n)}{J_0^2(\gamma_n)}\sum_{q=0}^{p}\frac{J_q(ka)J_{p-q}(ka)}{q!(p-q)!}, \tag{30a}$$

$$c_p=-\left(2\frac{ka}{\gamma_n}\right)^{3}\frac{\Gamma(p+\frac{1}{2})}{\sqrt{\pi}}\frac{J_1(\gamma_n)}{J_0(\gamma_n)}\sum_{q=0}^{p}\frac{J_{q+1}(ka)J_{p-q}(ka)}{q!(p-q)!}, \tag{30b}$$

$$d_p=4\left(\frac{ka}{\gamma_n}\right)^{4}\frac{\Gamma(p+\frac{1}{2})}{\sqrt{\pi}}\sum_{q=0}^{p}\frac{J_{q+1}(ka)J_{p-q+1}(ka)}{q!(p-q)!}. \tag{30c}$$

If $ka/\gamma_n \ll 1$, then in formula (30) we need consider only the term containing the coefficient b_p, neglecting the terms which contain the coefficients c_p and d_p. Then

$$\zeta_{12}^{p}=\sum_{p=0}^{\infty}\left(\frac{a}{l_{12}}\right)^{p}b_p h_p^{(2)}(kl_{12}). \tag{31}$$

The calculations of the mutual impedance are less complicated if the sound sources are vibrating circular membranes. In this case for $(ka/\beta_n)^2 \ll 1$ we obtain

$$\zeta_{12}^{M}=\sum_{p=0}^{\infty}\left(\frac{a}{l_{12}}\right)^{p}b_p h_p^{(2)}(kl_{12}), \tag{32}$$

where

$$b_p=a_n b'_p,\qquad a_n\equiv 2\left[\frac{\beta_n}{\gamma_n}\frac{J_1(\gamma_n)}{J_0(\gamma_n)}\right]^{2}. \tag{33}$$

For the first successive Bessel modes ($n = 1$, 2 and 3) the coefficients a_n are equal to: $a_1 = 0.7724\ldots$, $a_2 = 1.2969\ldots$, $a_3 = 1.4997\ldots$, while $\lim_{n\to\infty} a_n = 2$.

From relations (31)-(33) we obtain

$$\zeta_{12}^{p}=a_n\zeta_{12}^{M} \tag{34}$$

for $ka \ll \gamma_n$ and $(ka)^2 \ll \beta_n^2$. This means that for the first Bessel mode the mutual impedance of the plates is smaller than the mutual impedance of the membranes. However, for the second and higher modes the mutual impedance of the plates is smaller than that of the membranes.

If we assume that $ka \ll 1$, then in formulae (31) and (32) we consider only the element of the zeroth order ($p = 0$), neglecting the remaining elements as small quantities of higher orders. We then obtain

$$\zeta_{12}^{p} \simeq \theta_{11}^{p}\left[\frac{\sin(kl_{12})}{kl_{12}} + i\,\frac{\cos(kl_{12})}{kl_{12}}\right], \tag{35}$$

$$\zeta_{12}^{M} \simeq \theta_{11}^{M}\left[\frac{\sin(kl_{12})}{kl_{12}} + i\,\frac{\cos(kl_{12})}{kl_{12}}\right], \tag{36}$$

where the quantities

$$\theta_{11}^{p} \simeq \left(2\frac{ka}{\gamma_n}\right)^2 \frac{J_1^2(\gamma_n)}{J_0^2(\gamma_n)}, \qquad \theta_{11}^{M} \simeq 2\left(\frac{ka}{\beta_n}\right)^2 \tag{37}$$

are the expressions for normalised self-resistances, provided $ka \ll 1$.

6. The analysis of the results

In the numerical examples, the mutual impedance of the two kinds of source was considered:

a) circular membranes,

b) circular plates rigidly clamped at the circumference.

The impedance was calculated for Bessel axially-symmetric vibration velocity distributions shown, using formulae (7) in the case of the membranes, and formula (6) for plates.

If $ka \gg 1$, the approximate formulae (35)-(36) can be used. In practice, this approximation is valid if $ka < 0.2$.

The calculation of the mutual impedance on the basis of formula (30) or (32) involves a relative error of less than 5 % if $ka < 1/3\gamma_n,\ 1/3\beta_n$.

In the case of the frequencies of resonance, i.e. for $ka = \beta_n$, formula (28) is convenient for the calculation of the mutual impedance of the membranes.

If $ka \gg \gamma_n, \beta_n$, the real component of the normalized mutual impedance is close to unity; the imaginary component, however, is close to zero.

In the other cases, when the approximations cannot be used, or when great exactness of results is required, the calculation can be performed using computers on the basis of the integral formulae (18) and (20).

The practical formulae for numerical calculations of the self-impedance of one membrane or one plate, can be obtained from the integral formulae (18) and (20). This problem was analyzed in reference [9].

On the basis of the results obtained from the investigation of the problem of the mutual impedance of the Bessel axially-symmetric modes of vibrating circular plates, clamped at the circumference, or vibrating circular membranes, the following conclusions can be formulated:

1. The mutual impedance of two sound sources depends predominantly on the distance l_{12} between the central points of the sources.

The real and imaginary components of the mutual impedance oscillate around a value of zero, i.e. take extreme values that are alternately positive and negative, where the oscillations decay as the distance between the sources increases (Fig. 3). The curves of the mutual resistance are shifted relative to the reactance curves (Figs. 3-5). When the linear dimensions of the sources are smaller than the acoustic wavelength, then the mutual impedance depends on the distance l_{12} through the function $e^{-ikl_{12}}/kl_{12}$, where $k = 2\pi/\lambda$. This regularity is valid in the case of the mutual reactance with an additional assumption that $l_{12} > 0$.

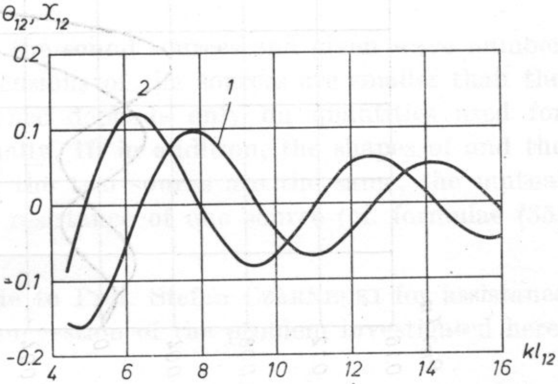

Fig. 3. The normalised mutual impedance of two vibrating circular membranes for the first axial-symmetric Bessel mode ($m = 0$, $n = 1$), depending on the parameter kl_{12} for $ka = 2.2$

Curve 1 – resistance, curve 2 – reactance

2. The mutual impedance for a given distance between sources and given sizes of sources, is an oscillating function of frequency, oscillating about a value of zero.

The extreme values of the mutual impedance of vibrating circular plates or vibrating circular membranes, occur for values of ka close, respectively, to γ_n or β_n (Figs. 4 and 5).

In the case of higher Bessel functions, i.e. when n is sufficiently large, the approximation $\beta_n \simeq \gamma_n \simeq n\pi$ gives us the simple relation $n\lambda \simeq 2a$ in place of $(2\pi/\lambda) \simeq a\beta_n$ and $(2\pi/\lambda)a \simeq \gamma_n$. It also seems characteristic that the acoustic interaction decays rapidly for wavelengths only slightly different from the value $2a/n$. For example, for the fifth axially-symmetric Bessel mode ($n = 5$, $\lambda \simeq 2a/5$), the largest acoustic interaction occurs for a narrow range of wavelengths which are only slightly different from the value $\lambda = 2a/5$. Outside this narrow radiation band, the acoustic interaction decays violently.

3. The form of the vibration velocity distribution on the surface of the sources affects the size and behaviour of the variation in the mutual impedance.

For given source size and separation the acoustic interaction becomes greater, as the active surfaces of the sources become greater.

The highest values of the mutual impedance occur for the first mode (Fig. 4), for successively higher Bessel modes (cf. Figs. 4 and 5) the extreme values become lower and are shifted towards shorter wavelength. This property

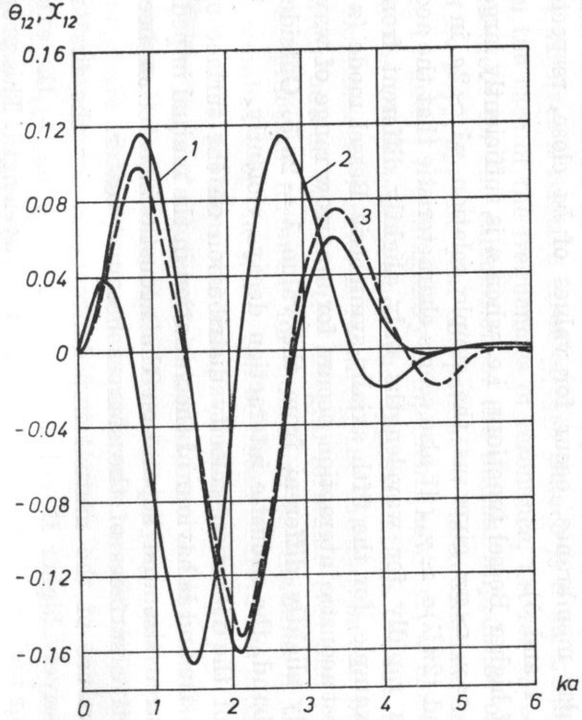

Fig. 4. The normalised mutual impedance of two circular sound sources for the first axial-symmetric Bessel mode ($m = 0$, $n = 1$)

Curve *1* – the resistance of the membrane, curve *2* – the reactance of the membrane, curve *3* – the resistance of the plate clamped at the circumference. It was assumed that $a/l_{12} = 0.44$

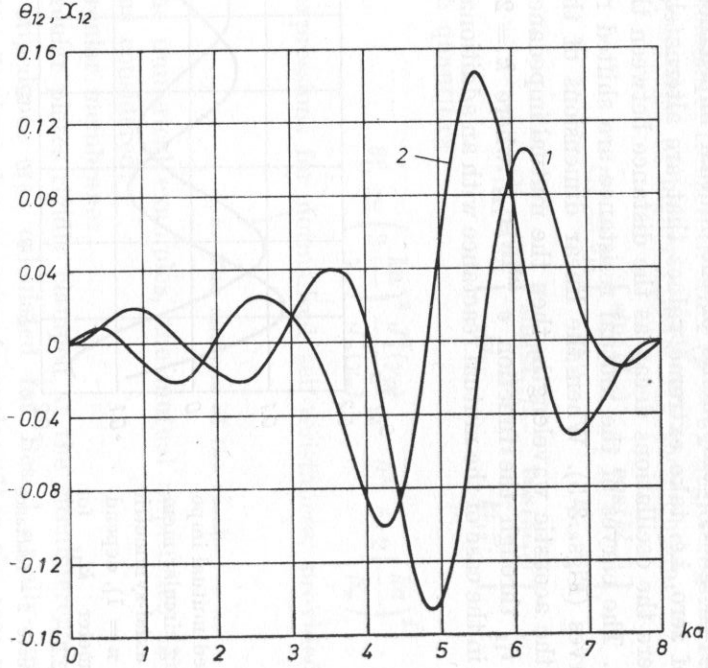

Fig. 5. The normalised mutual impedance of circular membranes for the second axial-symmetric Bessel mode ($m = 0$, $n = 2$)

Curve *1* – resistance, curve *2* – reactance. It was assumed that $a/l_{12} = 0.44$

can be clearly seen for the second and higher Bessel modes, for which the elements of the surface of the source (circular rings) vibrate with opposite phases. This is conditioned by the size of the active area of the sources, which becomes smaller with higher order Bessel modes.

4. Increasing the sizes of sources can be used to obtain an increase in the extreme values of the mutual resistance and reactance.

The limited range of variation in this quantity should be considered in the investigation of the dependence of the mutual impedance on the linear dimensions of the sources. For example, for circular sources (each of the radius a), with the centre points of the sources separated by l_{12}, the condition $0 \leqslant 2a \leqslant l_{12}$ occurs.

For a given distance between the sound sources and given wave number $k = 2\pi/\lambda$, provided the linear dimensions of the sources are smaller than the wavelength λ, the mutual impedance depends only on quantities used for characterizing each source individually. If, in addition, the shapes of and the vibration velocity distributions on the two source are the same, the mutual impedance depends on the self – resistance of one source (cf. formulae (35) and (36)).

I wish to express my gratitude to Prof. Stefan CZARNECKI for assistance in the scientific research and the suggestion of the problem investigated here.

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Received on March 27, 1979; revised version on November 22, 1980.