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HOW THE STRUCTURE OF A MEDIUM IS SEEN BY AN ACOUSTIC WAVE*

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Division of heterogeneous media into three types, depending on the dimensions of heterogeneities. Media with macroheterogeneities occurring naturally and in technological materials. Chraracterization of microheterogeneities. Frequency and energy transport ranges requiring quantum representation of waves in media with heterogeneities caused by crystal structure defects. Quantum phenomena and background noise for audible sounds. Density of phonons occurring in acoustic impulses. Quantum phenomena possible in biological substances.

1. Introduction

The acoustic wave is a unique source of information concerning the medium in which it propagates. For many centuries only waves in continuous and homogeneous media have drawn the attention of scientists. Even before the second world war investigation of the influence of the structure and heterogeneity of a medium on the propagation of acoustic waves had begun. However it is only in the last decade that considerable progress in the investigation of this problem has been made; many laboratories have concentrated their activities on the study of this subject. This situation justified the choice of the problem of the correlation between the structure of media and bodies and acoustic waves propagation as the focal point of the second FASE Congress.

The sessions of the first and second sections are dedicated to this problem, while the third section constitutes their logical complement: moving from the physical phenomena to their perception by man. I think that this is sufficient

* FASE-78, invited paper, unpublished in the Proceedings.

reason for me to dedicate the opening lecture of the Congress to certain general aspects of the influence of media structure on the acoustic field.

The acoustic wave "sees" the heterogeneity of the medium if its length is commensurable with the dimensions of the heterogeneities or with the distance between them. For a longer wave the medium "appears" to be homogeneous; however, the heterogeneities influence the value of the attenuation coefficient and, therefore, the acoustic wave gives indirect information about their nature. As far as the dimensions are concerned, we may differentiate three ranges of heterogeneities connected, respectively, with the macrostructure, microstructure and molecular-atomic structure of a medium. The rather trivial definitions of the limits between these ranges will be omitted from my considerations.

In the majority of media all the three ranges of heterogeneities exist; it is the length of the wave that determines the one that we perceive. Of course, every material medium has heterogeneities due to its molecular-atomic structure and we should therefore pay a special attention to them. The macroscopic heterogeneities may be natural or caused by technological processes. In Table 1

Table 1. The frequencies related to the macroscopic heterogeneities

Medium	Size of heterogeneities	Related frequency
Rocks	1-10 m	0.5-5 kHz
Water in ocean	0.5-1 m	1.5-3 kHz
Reinforced concerete	0.5-1 cm	0.25-1 MHz
Organic tissue	0.1-1 cm	0.2-2 MHz
Porous porcelain	0.5 mm	40 MHz

some examples are given of media of this kind and the frequencies corresponding to the wavelengths commensurable with the dimensions of the heterogeneities.

The microheterogeneities have until lately been almost exclusively of natural origin and technological processes have influenced them only indirectly. The situation has recently changed with the introduction of microcomposites, e.g. in the form of whiskers.

In Table 2 the data pertaining to the microstructure of typical media, analogous to those given in Table 1, are presented.

Table 2. The frequencies related to the microscopic heterogeneities

Nature of heterogeneities	Size of heterogeneities [μm]	Related frequency [GHz]
Magnetic domains	10-100	0.05-0.5
Dislocations in a monocrystal	10	0.5
Erythrocytes in blood	4-8	0.2-0.4
Grains in a polycrystal	1-5	1-5
Punctual defects	0.1	50

Both ranges of heterogeneities have one essential feature in common: the medium must be treated as heterogeneous but still continuous; we deal here with an acoustic wave in the traditional sense. It allows us to use common methods of description for both of these ranges.

The essential feature is the structure of the heterogeneity; the dimensions are related to the wavelength, and thus relative parameters are dependent on the frequency. We may differentiate two basic types of the structure. On the one hand — the medium with strong heterogeneities, i.e. of markedly different acoustic impedances. Usually in such a case we deal with heterogeneities in the form of inclusions in a continuous matrix. These are called *grainy media.*

On the other hand, we have granular media with weak heterogeneities, i.e. of acoustic impedances not much different from one another. Most often heterogeneities are close to one another, so that here there is no distinct matrix.

Differences in structure necessitate different mathematical methods for determining the field distribution in media of the two types. For grainy media we usually adopt the method of imaginary sources, while in granular media the method of a small parameter is used. Depending on the degree of the structural irregularity we may or may not introduce statistical methods.

In Table 2, examples of both types of heterogeneity are given.

2. Molecular-atomic heterogeneities

The image of the acoustical phenomena changes deeply, once we descend to the molecular-atomic range. The traditional view of the acoustic wave make sense only in connection with the movement of matter, but the space between

Table 3. Examples of two types of heterogeneities

Grainy media	Granular media
$Z_0 \lesssim Z_i$, $D \gg l$	$Z_0 \cong Z_i$, $D \ll l$
Reinforced concrete	Water in ocean
Porous porcelain	Organic tissue
Crystal lattice	Polycrystal

the molecules in fluids or the atoms in solids is not filled with a material medium. The acoustic wave is, therefore, a collection of vibrations of discretely distributed particles; this is often disregarded in technological acoustics. There is an important difference here, when compared with the electromagnetic wave, which — regardless of frequency — propagates is space in a continuous manner. The concept of the acoustic wave-length loses its physical sense in this range, becoming a formal quantity, since the same distribution of oscillations of particles may be obtained for a number of wavelengths (Fig. 1). We must, therefore, look for another solution.

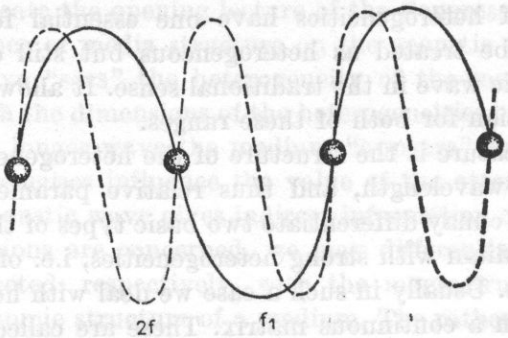

Fig. 1. Varied concept of wavelength for a chain of particles

Let us consider (Fig. 2) a simple chain of identical oscillators of masses m_{a0}, elasticities K_0 and distances a between them; the oscillator has displacement u_l and momentum P_l. Using Fourier analysis we find the displacements U_{km} and momenta P_{km} for the k_m mode of oscillation:

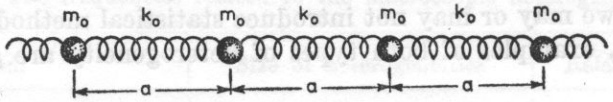

Fig. 2. A chain of oscillators of masses m_0 and elasticities K_0 between them

$$U_{km} = \frac{1}{\sqrt{Na}} \sum_{l=1}^{N} u_l e^{ik_m al},$$
$$P_{km} = \frac{1}{\sqrt{Na}} \sum_{l=1}^{N} P_l e^{-ik_m al}. \tag{1}$$

This mode of oscillation may be represented in the reciprocal space of wave vectors as a so-called *modal oscillator* moving with a velocity c_m (Fig. 3). This operation is performed on the basis of classical mechanics, but the energy transport in the chain of oscillators, representing the acoustic wave, is subject to the same universal law as the energy flux of electromagnetic wave: it does not occur in a continuous manner but in quanta of energy given by the formula

$$E = \hbar\omega\left(n + \frac{1}{2}\right), \tag{2}$$

where $\hbar$ is the universal Planck constant, and n is the number of the energy levels, i.e. the number of phonons.

The energy of the modal oscillator consists therefore of indivisible quasi-particles called, as we know, phonons; these quasi-particles are analogous to light photons, though their properties are slightly different.

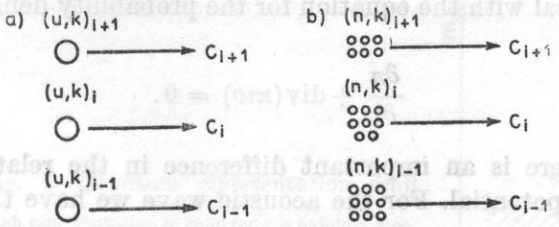

Fig. 3a. Representation of a chain of oscillators by a number of k oscillators with a velocity c_m
Fig. 3b. Representation of a chain of k oscillators as a flux of phonons nk_i

Therefore we shall present the moving modal oscillator as a flux of phonons of the same velocity c_m (Fig. 3), consequently entering the field of quantum acoustics. The probability π of the phonon of energy E, being in a given position at a given moment of time, is defined by the wave function

$$\psi = \sqrt{\pi} \cdot e^{iS}, \tag{3}$$

where $S = Et$ (energy of the system $\times$ time).

For phonons not dispersed by external factors this function fulfils the Schroedinger equation in the form

$$i\hbar \frac{\partial \psi}{\partial t} + \frac{\hbar^2}{2m_f} \nabla^2 \psi = 0, \tag{4}$$

where m is the effective mass of the phonon.

This is compatible with the equation for the potential of an acoustic wave in a non-dissipative medium:

$$\frac{\partial^2 \Phi}{\partial t^2} - c^2 \nabla^2 \Phi = 0. \tag{4a}$$

It is evident that the quantum equation is of the first order, while the acoustic equation is of the second order; the quantum equation describes the flux of phonons moving with velocity w,

$$\boldsymbol{w} = \frac{1}{m_f} \operatorname{grad} S, \quad m_f = \frac{\hbar k}{w}, \tag{5}$$

while the classical equation concerns the local movement of real particles of instantaneous velocity:

$$\boldsymbol{v} = -\operatorname{grad} \Phi. \tag{5a}$$

The continuity of flux equation in a continuous medium, describing the instantaneous density ϱ of the medium,

$$\frac{\partial \varrho}{\partial t} + \operatorname{div}(\varrho \boldsymbol{v}) = 0, \tag{6}$$

is formally identical with the equation for the probability density of the phonon distribution:

$$\frac{\partial \pi}{\partial t} + \operatorname{div}(\pi v) = 0. \tag{6a}$$

However, there is an important difference in the relations between the velocity and the potential. For the acoustic wave we have the Euler equation

$$\varrho \frac{dv}{dt} + \operatorname{grad} p = 0, \tag{7}$$

and for the phonon flux the quantum acoustical equation

$$m_f \frac{dv}{dt} + \operatorname{grad} Q = 0, \tag{7a}$$

where Q is the quantum potential:

$$Q = -\frac{\hbar^2}{2m_f} \frac{\nabla^2 \sqrt{\pi}}{\sqrt{\pi}}. \tag{8}$$

The gradient of the quantum potential defines the uncertainty of the position of the phonons, that is the dispersion of their flux. In the case of the flux of classical particles, having precisely determined positions, the quantum potential is obviously equal to zero.

The relations presented are an illustration of the degree of similarity and at the same time of difference of the phenomena occurring in the micro- and macroworlds. The acoustician crossing the boundary between these two worlds must be fully aware of the complexity of the problems which he tackles.

The energy of a single phonon is given by formula

$$E_f = \hbar \omega_f. \tag{9}$$

In light flux the scintillations corresponding to the individual photons had been observed by the beginning of our century. It must be expected that in view of the development of the technology of hypersound, for which a single phonon has a relatively high energy (for instance, for a frequency of $f = 10^{12}$ Hz, the energy $E = 10^{-15}$ erg), a similar observation of individual phonons will be possible. However, this does not seem to be possible at lower frequencies, since we deal there with enormous number of phonons. For instance, the weakest audible sound impulse at a frequency of 4000 Hz and with a duration of 1 msec carriers 10^{11} phonons. We have, therefore, a double quantum limitation on the range of applicability of traditional acoustics. The quantization in space occurs when the wavelength is commensurable with the distance between the oscillators, and in the case of a spatial system — commensurable with the mean free path of the phonon. It is independent of the wave energy and thus on the (ω, E)-diagram the limit is a vertical line (Fig. 4).

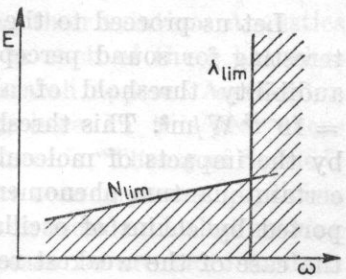

Fig. 4. Quantum representation limit

Such representation is valid for the hatched area; coordinates: ω – frequency, E – system transported energy

As for the quantization of energy, it may be sensed only at low intensities, when the number n is small, e.g. equal to 10. Then the limit on the (ω, E)-diagram is an oblique straight line.

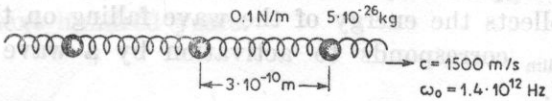

Fig. 5. Vibration transport in the internal ear represented by a chain of oscillators

For the chain of oscillators previously presented (Fig. 5) the limit of quantization thus defined corresponds to a transported power of

$$N_{\text{lim}} \cong [10^{-26} \omega_0^2 \sin k_m^a] \quad \text{W}, \tag{10}$$

where $\omega_0 = \sqrt{k_0/m_0}$ is the frequency of natural oscillation of the resonator, and $k_m = \omega m/c$.

The distance between the molecules or atoms and, therefore, the limiting frequency depends on the temperature and pressure. In Table 4 some characteristic examples are given.

In gases the molecules are moving at random, there is no order and the mean free path of the molecule is the determining factor. In liquids there is a close order, while in solids – a distant order; the mean free path of the phonon is determined in this case.

Table 4. Limit frequency as function of the molecular structure

Medium	Temperature T [K]	Pressure P [atm]	Mean free path l [m·10^{-7}]	Limit frequency f [GHz]
Ideal gas	300	1	2	2
	100	1	0.7	6
	20	1	0.1	40
Liquids				
CO_2	273	70	480	0.05
CS_2	298	1	7.4	3.25
SO_2	273	10	34	0.7
C_6H_6	273	1	6.5	3.7
CH_3Cl	273	1	17	1.43

Let us proceed to the quantum limitation of energy — it is especially interesting for sound perception. A man of very keen hearing has a minimum audibility threshold of about $p_{tr} = 2 \cdot 10^{-6}$ N/m² at 4000 Hz; hence $i_{tr} = 10^{-14}$ W/m². This threshold coincides with the level of thermal noise caused by the impacts of molecules of air against the ear-drum. On the other hand, certain quantum phenomena also occur in the internal ear, the energy is transported by chains of oscillators composed of molecules of lymphatic fluid. In the case of the weakest received signal one such chain is activated. The data of such a chain are the following: $m_0 = 5 \cdot 10^{-26}$ kg, $K_0 = 0.1$ N/m, $\omega_0 = 1.4 \cdot 10^{12}$ Hz, $a = 3 \cdot 10^{-10}$ m, $c = 15000$ m/s. Introducing these parameters into formula (10) for a signal of frequency $\omega/2\pi = 4000$ Hz, we obtain for the limiting power N_{lim}, transported by the chain of molecules, a value of $3.3 \cdot 10^{-18}$ W.

This chain collects the energy of the wave falling on the ear-drum. The limiting power N_{lim} corresponds to activation by a wave of intensity $i_{\text{lim}} = 1.7 \cdot 10^{-14}$ W/m².

Table 5. Limit frequency as function of the atomic structure

Medium	Temperature T [K]	Mean free path l [m]	Limit frequency f [GHz]
Si	273	$6.8 \cdot 10^{-9}$	214
	77	$4.3 \cdot 10^{-7}$	3.4
	20	$6.5 \cdot 10^{-7}$	0.023
SiO_2	273	$1.8 \cdot 10^{-9}$	590
	77	$2.4 \cdot 10^{-8}$	38
	20	$1.3 \cdot 10^{-5}$	0.076
Ge	273	$5.2 \cdot 10^{-9}$	164
	77	$5.2 \cdot 10^{-8}$	16.4
	20	$7.1 \cdot 10^{-6}$	0.12
CaF_2	273	$1.4 \cdot 10^{-9}$	818
	77	$1.6 \cdot 10^{-8}$	58.7
	20	$1.6 \cdot 10^{-6}$	0.587
NaCl	273	$1.07 \cdot 10^{-9}$	710
	77	$0.8 \cdot 10^{-8}$	95
	20	$3.7 \cdot 10^{-}$	2.05

The convergence of that quantity with the audibility threshold i_{tr} confirms the universal character of the quantum limitation.

The phenomena of oscillation transport by the biomolecules of the nervous system is as yet unexplored to the full.

The natural frequencies of the biomolecules are considerably lower than those of the lymphatic liquid and are of the order of 10^8-10^9 Hz, and this fact affects the oscillatory processes in the neurons.

I have mentioned several problems still awaiting their solution. Acoustics is still being developed, it developes branches and in each of these branches new problems worthy of scientific and technical research appear. A reflexion taken from the past seems to be particularly fitting here. In this year there occurs the centenary of the publication of Lord RAYLEIGH's "Theory of Sound". Shortly after this book had been published one of the contemporary physicists said that it contained everything that could be written on sound and that nothing was left to be done in acoustics. After a hundred years we are more modest. In Europe alone there are several journals dedicated solely to acoustics and the papers on our speciality are dispersed in journals of many disciplines, beginning with linguistics and musicology and ending with architecture and metallurgy. I think that both we and our successors will have enough subjects to pursue for at least the next hundred years.

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