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GENERALIZED MODEL OF AN AXIAL DYNAMIC GENERATOR*

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This paper present a model of the theoretical acoustic system of a dynamic axial generator with a horn and pressure chamber common to all channels of the stator. The model has been developed on the basis of electroacoustic analogies. This permits formulation of the wave phenomena encountered in the acoustic system of the generator.

Experimental investigations have shown that the model is correct within the range of conditions assumed during its formation.

Postulates have also been formulated which, when satisfied, will permit the optimal working of the generator over a wider frequency range.

1. Introduction

The development of experimental investigations in the non-linear acoustics of gas media still faces difficulties resulting from the lack of satisfactorily efficient and stable sound sources of high acoustic power. Such a situation also limits the use on an industrial scale of the acoustic coagulation of aerosols and of other ultrasonic technology. This particularly applies to the so-called "flow generators". The main reason for their limited use is their comparatively low acoustic efficiency and thus resulting high operational costs. Another essential reason is the lack of a complete theoretical formulation of the phenomena found within the acoustic system of the generator in the process of transforming the energy of the compressed air into acoustic energy. Thus it is impossible to design a generator of given acoustic parameters and recourse has to be made to the duplication of certainly non-optimal solutions obtained by way of experiment.

Much progress has recently been made in the investigation of static generators in terms of the explanation of their mechanism of operation [7].

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Dynamic generators may claim a more advanced theoretical description. However, they require further studies, both theoretical and experimental, the aim of which is the description of the generator's properties over the full range of its working conditions.

Jones [4], Allen and Watters [2, 3] and Wyrzykowski [13, 14] have developed a theoretical foundation for the operation of a dynamic generator. A physical model of the dynamic generator developed by them, in view of the accepted simplifying assumptions, should be in agreement with experiment over a range of comparatively low frequencies for the sound wave produced by the generator, i.e. up to several hundred of Hz. This has been confirmed by experiment both by Jones, and by Allen and Watters. Leśniak, while making measurements of the acoustic parameters of the siren KRW [14] observed a sudden drop in the sound power of this generator at higher frequencies, which until now is not explained.

2. The electrical equivalent diagram of the generator and its mathematical description

The acoustic system of a dynamic axial generator (Fig. 1) consists of a pressure chamber, *1*, of which one wall is a flat stator, *2*, with passages arranged evenly on the circumference. The stator passages enter into a chain-type horn of annular cross-section, *3*. In the chamber in front the stator there is a flat rotor, *4*, of small thickness and with holes corresponding to the passages in

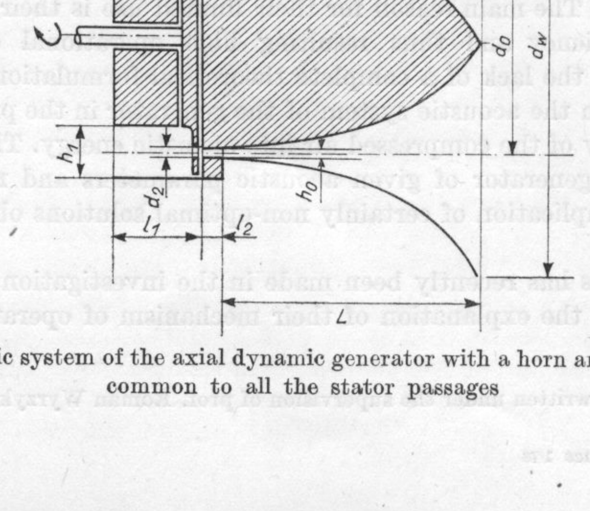

Fig. 1. The acoustic system of the axial dynamic generator with a horn and pressure chamber common to all the stator passages

the stator. The formation of a sound wave takes place in the elementary functional unit of the acoustic system of the generator, composed of the pressure chamber, the hole arrangement of the rotor and stator, the stator passage, and the horn. The number of such units in the acoustic system of the generator ranges from a few to several hundred. They are usually identical, with the pressure chamber and the horn being common to all stator passages.

Compressed air is forced into the pressure chamber in such a quantity that the thermodynamic parameters of the air in the chamber should not undergo any changes caused by the escape of air from the chamber (via the passages and the horn) into the atmosphere. The mass air flow in the acoustic system of the generator, with the rotor stopped, can be described [8] by the relationship

$$M = \alpha\varepsilon S_2 \sqrt{2\varrho_1 \Delta P}, \tag{1}$$

where

$$\alpha = \frac{1}{\sqrt{1-m^2}}$$

is the flow ratio, and

$$\varepsilon = \left[r^{2/\varkappa} \frac{\varkappa}{\varkappa - 1} \, \frac{1 - r^{(\varkappa-1)/\varkappa}}{1-r} \, \frac{1-m^2}{1-m^2 r^{2/\varkappa}} \right]^{1/2}$$

is the expansion coefficient for isentropic air flow out of the pressure chamber, with

$$\Delta P = P_1 - P_2, \quad r = P_2/P_1, \quad m = S_2/S_1.$$

For sufficiently small values (when compared with unity) of m ratio of the cross-sectional area of the stator passage S_2 to the surface of the pressure chamber S_1, and of the pressure differences ΔP of the air in the pressure chamber, P_1, and in the stator passage P_2 (i.e. smaller than $0.2 \times 10^5\ \mathrm{N/m^2}$ Fig. 2) then $\alpha = \varepsilon = 1$ [2, 4, 13] can be accepted in expression (1).

When revolving the rotor periodically opens and closes the inlet hole of the stator passage, so that the active surface for air flow is a periodic function of times $S(t)$. The range of variation of this function is from zero to S_m, with S_m being the area of the wholly open stator passage. The mass air flow thus contains, in addition to the constant component M_0, a component varying with time $M(t)$:

$$M = M_0 + M(t). \tag{2}$$

The latter causes the dependence of the acoustic impedance Z, in the inlet hole of the stator passage, on the acoustic pressure $p(t)$. The pressure drop ΔP now equals

$$\Delta P' = P_1 - [P_2 + p(t)] = \Delta P - p(t), \tag{3}$$

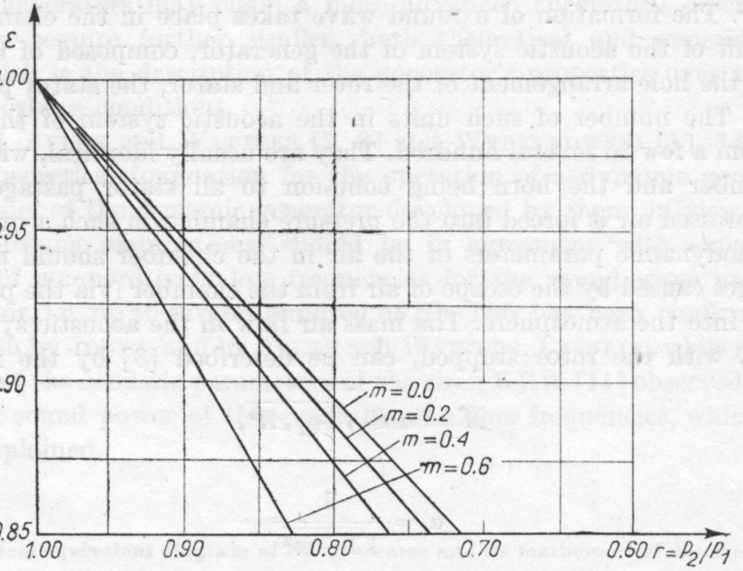

Fig. 2. The relation between the expansion coefficient and the quantities r and m

According to (1), (2) and (3) we thus have

$$S(t)=\frac{M_0+M(t)}{\sqrt{2\varrho_1[\Delta P-p(t)]}}. \tag{4}$$

In the acoustic impedance Z of the inlet hole of the stator passage we can distinguish its two components as the output impedances:

(i) of the pressure chamber

$$Z_1=\frac{p_1}{V_1}=X_1+jY_1, \tag{5}$$

(ii) of the stator passage complete with horn

$$Z_2=\frac{p_2}{V_2}=X_2+jY_2, \tag{6}$$

where p_1, p_2 and V_1, V_2 are the root mean square values of the acoustic pressure and the volume velocity of the air at the inlet, of the pressure chamber (for index 1) and of the stator passage (for index 2).

For convenience and generalization we shall in future consider the pressure chamber and the stator, including the horn, as acoustic four-terminal networks the input impedances of which are respectively Z_1 and Z_2. Let us assume that $Z=Z_1+Z_2$. This thus requires a connection in series of the inputs of these four-terminal networks. An external source of compressed air maintains the upper terminals of the four-terminal network, which represents the pressure chamber, under a constant (in time) pressure P_1. The lower terminals of this four-terminal network are under ambient pressure P_0. Assuming that the only loss of energy of the wave propagating in both four-terminal networks is the loss connected with its outward radiation, which for obvious reasons is equal to zero at zero frequency, it can be accepted that:

(i) the values of the constant component of the air pressure in the pressure chamber P_1 and in the stator passage P_2 do not depend on the position in the air flow path of air of these elements,

(ii) the constant component of the air pressure in the stator passage P_2 is equal to the ambient pressure P_0.

The inlet hole arrangement of the stator passage and of the revolving rotor can be looked upon as the periodically variable (in time) flow resistance, G, the value of which varies between the minimum value when the inlet hole of the stator passages is wholly opened, and infinit when the hole is completely closed.

Figure 3 represents the equivalent electrical circuit diagram of the acoustic system of a dynamic acoustic generator with a horn and pressure chamber common to all the passages.

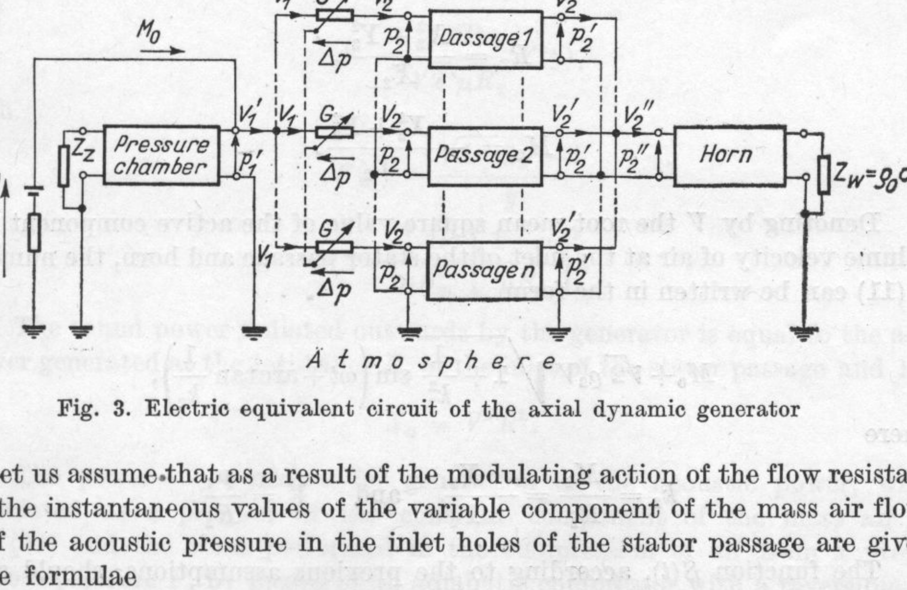

Fig. 3. Electric equivalent circuit of the axial dynamic generator

Let us assume that as a result of the modulating action of the flow resistance G, the instantaneous values of the variable component of the mass air flow, and of the acoustic pressure in the inlet holes of the stator passage are given by the formulae

$$M(t) = M_a \cos \omega t, \tag{7}$$

$$p(t) = p_a \cos \omega t, \tag{8}$$

M_a and p_a being the complex amplitudes of the above-mentioned quantities.

Taking advantage of the equation for the continuity of mass air flow ($V_1 \varrho_1 = V_2 \varrho_2$), from Fig. 3 we can write

$$M_a = \sqrt{2}\,\varrho_2 V_2 = \sqrt{2}\,\varrho_2 \frac{p_2}{Z_2}, \tag{9}$$

$$p_a = \sqrt{2}(p_1 + p_2) = \sqrt{2}\,p_2\left(1 + \frac{\varrho_2}{\varrho_1}\frac{Z_1}{Z_2}\right). \tag{10}$$

Substituting (5)-(10) into (4) we obtain

$$S(t) = \frac{M_0 + \sqrt{2}\,\varrho_2 p_2\left[\frac{X_2}{X_2^2+Y_2^2}\cos\omega t - \frac{Y_2}{X_2^2+Y_2^2}\sin\omega t\right]}{\sqrt{2\varrho_1\left\{\Delta P - \sqrt{2}\,p_2\left[\left(1+\frac{\varrho_2}{\varrho_1}\frac{X_1X_2+Y_1Y_2}{X_2^2+Y_2^2}\right)\cos\omega t + \frac{\varrho_2}{\varrho_1}\frac{X_2Y_1 - X_1Y_2}{X_2^2+Y_2^2}\sin\omega t\right]\right\}}}. \tag{11}$$

For convenience in further calculations we shall make a transformation of the components of the combined input impedance of the stator passage and horn $Z_2 = X_2 + jY_2$, connected in series, into their equivalent $Z_2 = R_2 + jM_2$, connected in parallel, according to the relations

$$R_2 = \frac{X_2^2 + Y_2^2}{X_2}, \tag{12}$$

$$M_2 = -\frac{X_2^2 + Y_2^2}{Y_2}, \tag{13}$$

Denoting by V the root mean square value of the active component of the volume velocity of air at the inlet of the stator passage and horn, the numerator of (11) can be written in the form

$$M_0 + \sqrt{2}\,\varrho_2 V\sqrt{1+\frac{1}{k_2^2}}\,\sin\left(\omega t + \operatorname{arc\,tan}\frac{1}{k_2}\right), \tag{14}$$

where

$$k_2 = \frac{M_2}{R_2} = -\frac{X_2}{Y_2} \quad \text{and} \quad V = \frac{p_2}{R_2}.$$

The function $S(t)$, according to the previous assumptions, should satisfy the following conditions:

$$S(t)_{\min} = 0, \tag{15}$$

$$S(t)_{\max} = S_m. \tag{16}$$

Thus, according to (11), (14) and (15), we can write

$$\sin\left(\omega t + \arctan\frac{1}{k_2}\right) = -1, \tag{17}$$

$$M_0 - \sqrt{2}\,\varrho_2\sqrt{1+\frac{1}{k_2^2}}\,V = 0. \tag{18}$$

Assuming that (16) occurs at $t = 0$, on the basis of (11), we have

$$S_m = \frac{M_0 + \sqrt{2}\,\varrho_2 V}{\sqrt{2\varrho_1\left[\Delta P - \sqrt{2}R_2V\left(1+\frac{\varrho_2}{\varrho_1}\,\frac{X_1X_2+Y_1Y_2}{X_2^2+Y_2^2}\right)\right]}}. \tag{19}$$

Writing

$$\gamma = \sqrt{1+\frac{1}{k_2^2}} = \sqrt{1+\left(\frac{Y_2}{X_2}\right)^2}, \tag{20}$$

$$\mu = 1 + \frac{\varrho_2}{\varrho_1}\,\frac{X_1X_2+Y_1Y_2}{X_2^2+Y_2^2}, \tag{21}$$

from (18) and (19) we obtain

$$M_0 = \sqrt{2}\,\varrho_2\gamma V \tag{22}$$

and

$$V = \frac{\Delta P}{\sqrt{2}\,\mu R_2}F(y), \tag{23}$$

with

$$F(y) = \frac{\sqrt{1+2y}-1}{y} \tag{24}$$

$$y = \frac{\varrho_2^2\,\Delta P}{\varrho_1 S_m^2}\left[\frac{1+\gamma}{\mu R_2}\right]^2. \tag{25}$$

The sound power radiated outwards by the generator is equal to the active power generated at the resistance R_2 of the inlet of the stator passage and horn:

$$N_a = V^2 R^2. \tag{26}$$

The power to be needed for generation of the acoustic power, will be calculated as a product of the constant component of the mass air flow M_0 (22) and the work performed in the compression of air from a pressure P_2 to a pressure P_1 by means of an adiabatic compressor with a reversible working cycle [8],

$$N = nM_0L_t, \tag{27}$$

where n is the number of the stator passages.

Work of compression

$$L_T = \frac{\varkappa}{\varkappa-1}\frac{P_2}{\varrho_2}\left[\left(\frac{P_1}{P_2}\right)^{\varkappa-1/\varkappa}-1\right]. \tag{28}$$

For sufficiently small pressure drops $\Delta P = P_1 - P_2$, i.e. not higher than 0.2×10^5 N/m², in the expansion

$$\left(\frac{P_1}{P_2}\right)^{\varkappa-1/\varkappa} = \left(1+\frac{\Delta P}{P_2}\right)^{\varkappa-1/\varkappa} = 1+\frac{\varkappa-1}{\varkappa}\frac{\Delta P}{P_2}+\ldots \tag{29}$$

consideration need be given only to two terms. Then

$$L_T = \frac{\Delta P}{\varrho_2} \tag{30}$$

and the supply power [4, 13]

$$N = nV_0\Delta P, \tag{31}$$

where $V_0 = M_0/\varrho_2$ is the constant component of the volume air flow in the stator passage. The acoustic efficiency of the generator

$$\eta_a = Na/N. \tag{31a}$$

3. The internal structure of the four-terminal networks

WYRZYKOWSKI [13] has proved that the optimal catenary horn to be used in conjunction with dynamic generators is one whose profile describes the equation

$$S = S_0\cosh^2\frac{z}{z_0}, \tag{32}$$

where S is the cross-sectional area of the horn at a distance z from its inlet, S_0 is the area of the horn inlet and z_0 is the opening coefficient.

The horn with a catenary profile and annular cross-section is formed of two rigid surfaces resulting from the rotation about the axis Oz of the two curves described by the equation (Fig. 1)

(i) upper

$$f_g(z) = \frac{1}{2}\left(d_0+h_0\cosh\frac{z}{z_0}\right), \tag{33}$$

(ii) lower

$$f_d(z) = \frac{1}{2}\left(d_0-h_0\cosh\frac{z}{z_0}\right). \tag{34}$$

The equation of the profile (32) of such a horn takes the form

$$h = h_0\cosh^2\frac{z}{z_0}, \tag{35}$$

where $h = f_g(z) - f_d(z)$.

For $z = L$ we have $f_d(z) = 0$ and usually $f_g(z) = r_w$, with r_w being the radius of the horn outlet. From (34) we have

$$z_0 = \frac{L}{\operatorname{arc\,cosh} \dfrac{d_0}{h_0}}. \tag{36}$$

The wave equation [16] of the wave propagating in a catenoidal horn of annular cross-section has so far been solved for the propagation of a tangential wave mode of zero order (i.e. for a plane wave) and this occur above the cut-off frequency [15].

$$f_0 = \frac{c}{2\pi z_0}. \tag{37}$$

The impedance of the horn inlet as regards the range of propagation of the above-mentioned mode when neglecting the wave reflection from the horn outlet, is given by the relation

$$Z_{to} = \frac{\varrho_2 c}{S_0}\left[1 - \left(\frac{f_0}{f}\right)^2\right]^{-1/2}, \tag{38}$$

whereas the transmission coefficient for acoustic pressure takes the form (Fig. 3)

$$K_{pt} = \frac{p_2'''}{p_2''} = \frac{\exp\left[-j\sqrt{\left(\dfrac{f}{f_0}\right)^2 - 1}\,\dfrac{L}{z_0}\right]}{\cosh \dfrac{L}{z_0}}, \tag{39}$$

where p_2'' and p_2''' are the acoustic pressures at the inlet and outlet of the horn, respectively.

It results from those relationships that a catenoidal horn for the propagation range of the tangential wave mode of zero order represents, for the stator passage, a mere resistance load. At the cut-off frequency the impedance of the horn inlet is infinitely great, while below this frequency it has a pure imaginary value [15]. Thus it can be concluded that below the cut-off frequency of the horn the generator should not produce a sound wave. Only an infinitely long horn can exhibit such properties. The assumption of a boundary condition at the horn outlet of finite length for describing the conditions of radiation of the horn outlet implies that for the cut-off frequency the impedance of the horn outlet must assume a finite value while below this frequency it has a real part, but with values considerably smaller than is the case above the limiting frequency [17]. This is in agreement with experiment since the generator does produce a sound wave below the cut-off frequency of the horn.

With increasing frequency the impedance of the horn inlet tends to the asymptotic value

$$Z_{to} = \frac{\varrho_2 c}{S_0}. \tag{40}$$

A drawback of this discussion is the lack of a statement at which frequency range, in a catenoidal horn, the propagation of an exclusively tangential wave mode of zero order is possible. This is tantamount to the lack of a definition of the range of application the argument presented here. The solution of this problem can only be obtained empirically.

The passage of the generator stator represents a length of acoustic wave-guide, loaded at the outlet with the impedance of the horn inlet, which is common for all stator passages. According to Fig. 3 we can write the impedance loading the outlet of each of the stator passages as

$$Z_2' = \frac{p_2'}{V_2'} = \frac{p_2''}{V_2''/n} = nZ_{to}. \tag{41}$$

In practice each stator passage satisfies the condition for plane wave propagation i.e. that the diameter $d_2 < \lambda/2$, where λ is the wavelength. Neglecting loss of the wave energy which is associated with propagation in the passage an is caused by the viscosity and thermal conductivity of the air, and assuming an ideal boundary condition on the walls of the horn, that the normal component of the vibration velocity amplitude of the medium should there be zero, the acoustic properties of the stator passage can be described by the following relations [18]:

(i) the acoustic impedance of the inlet of the stator passage is equal to

$$Z_2 = Z_{02}\frac{Z_2' + jZ_{02}\tan kl_2}{Z_{02} + jZ_2'\tan kl_2}; \tag{42}$$

(ii) the transmission coefficient for the acoustic pressure

$$K_{ps} = \frac{p_2'}{p_2} = \frac{1}{\cos kl_2 + j\dfrac{Z_{02}}{Z_2'}\sin kl_2}, \tag{43}$$

where $k = \omega/c$ is the wave number, l_2 is the length of the stator passage, and $Z_{02} = \varrho_2 c/S_2$ is the wave impedance of the passage.

Here it should be noted that above the cut-off frequency of the horn $Z_{02} > Z_2$, implying the occurence of wave reflection at the place of connection of the passage outlet and the horn inlet. For this reason the inlet impedance of the passage Z_2 possesses a non-zero imaginary part and both components are strongly frequency dependent. The maximum values of Z_2 correspond to the quarter-wave resonances of the stator passage, and the minimum values to the half-wave resonances.

The reflection of the wave at the place of the connection of the stator and the horn becomes weaker as the number of passages in the stator increases (41). However, it is not possible to completely eliminate this phenomenon without basic changes in the acoustic system of the generator in the form of individual horns for each stator passage [14]. Although the construction of such a generator is possible in practice, in view of the practical possibility of using only several horns, the attainment of the satisfactorily high sound powers with the generator at higher frequencies remains a problem.

The pressure chamber, usually annular in cross-section is, like the horn also common to all the stator passages. According to Fig. 3 the input impedance of the pressure chamber, loading each of the inlets of the stator passages is equal to

$$Z_1 = \frac{p_1}{V_1} = \frac{p_1'}{V_1'/n} = nZ_{k0}, \tag{44}$$

where Z_{ko} is the input impedance of the pressure chamber.

The pressure chamber of the generator (Fig. 1) can be regarded as a wave-guide with one end closed by a rigid partition. In the frequency range for which the condition for plane wave propagation is satisfied for the pressure chamber, i.e. for $h_1 < \lambda/2$, analogous assumptions have been made to those for the stator passage. Thus the input impedance of the pressure chamber is described [18] by the relation

$$Z_{k0} = -jZ_{01} \operatorname{cotan} kl_1, \tag{45}$$

where l_1 is the length of the chamber, and $Z_{01} = \varrho_1 c/S_1$ is its wave impedance.

In view of the previous assumption $S_1 \gg S_2$, and $Z_{01} \ll Z_{02}$. The resonance properties of the pressure chamber found at the higher frequencies advesely affect the sound power characteristic of the generator. This particularly applies to the anti-resonance frequency of the chamber for which $Z_{k0} = \infty$, and thus $Z_1 = \infty$. For these frequencies the generator does not radiate sound power as results from relations (21) and (23)-(25) with $\mu = \infty$.

In this situation it is necessary to attenuate the free vibrations of the pressure chamber. This is feasible through the use of an intensively absorbing end. This solution has the advantage that it does not change the conditions of flow through the chamber of the constant component of the mass air flow. The input impedance of the pressure chamber of the generator attenuated in this manner, is equal to the wave impedance Z_{01}.

In the range of higher frequencies the impedance Z_1 loading the inlet of the stator passages with the pressure chamber attenuated in this manner can assume its lowest values for various designs of the chamber [1, 2, 4] while m (1) and n (44) are still not too large.

4. Subject of investigations and its characteristics

In order to provide experimental verification of the theoretical arguments measurements have been carried out on the sound power of an axial dynamic generator detailed characteristics have been given previously [10]. Here only the basic dimensions specified in Fig. 1 are repeated: $d_0 = 100$ mm, $d_w = 200$ mm, $h_0 = d_2 = 1.5$ mm, $h_1 \cong 15$mm, $l_1 \cong 30$ mm, $l_2 = 10$ mm, $L = 150$ mm, $n = 50$. From these dimensions the following values for the characteristic frequencies of this generator may be calculated.

(i) the cut-off frequency of the horn $f_0 = 1790$ Hz

(ii) the frequency of the first resonance of the stator passage $f_{rs} = 8600$ Hz,

(iii) the frequency of the first anti-resonance of the pressure chamber $f_{ak} \cong 6000$ Hz.

In view of the rather complicated construction of the pressure chamber of the tested generator, the definition of the third of the above-mentioned frequencies could only be approximate.

In view of the fact that for the tested generator $S_1 \gg S_2$ it has been assumed that except for those frequencies near to the anti-resonance frequencies of the pressure chamber, $\mu = 1$.

5. Method of measurement of the sound power of the generator

The sound power of the generator is defined by the experimentally measured distribution of the root mean square value of the acoustic pressure at a constant distance r_0 from the horn outlet in the far field of the generator, using the relation

$$N_a = \frac{2\pi r_0^2}{\varrho_0 c} \int_0^{\pi/2} p^2(\vartheta)\sin(\vartheta)\, d\vartheta \tag{46}$$

estimated in the calculations by

$$N_a = \frac{1}{\varrho_0 c} \sum_{i=1}^{m} p^2(\vartheta_i)\, \Delta S_i, \tag{47}$$

where $S_i = 2\pi r_0^2 \sin(\vartheta_i)\,\Delta\vartheta$ is the surface area of a spherical ring of height $\Delta h_i = r_0 \sin(\vartheta_i)\,\Delta\vartheta$ (Fig. 4), and $\varrho_0\, c$ is the specific resistance of air.

If $\Delta S_i = \Delta S =$ const, and this occurs when $\Delta h_i = \Delta h =$ const, expression (47) takes a form which is convenient for calculation

$$N_a = \frac{2\pi r_0 \Delta h}{\varrho_0 c} \sum_{i=1}^{m} p^2(\vartheta_i). \tag{48}$$

tion equal to 0.01 rd. The frequency of the wave produced by the generator has been measured by means of a digital frequency meter 5, while its shape as also that of the motion produced by the generator was observed on the oscilloscope 6. The recording of the results of the measurements was done by means of the blockmeter 7 and the tape perforator 8.

The error in the determination of the value of the sound power of the generator depends mainly on the accuracy of the calibration of the apparatus used for the measurement of the root mean square value of the acoustic pressure. The second largest error occurs in the arithmetical method used for determining the value of the definite integral which occurs in the formula for the sound power of the generator (46). The magnitude of this error depends mainly on the length of the integration increment $\Delta\vartheta$. In the measurements carried out with a confidence level of 0.95, the relative error with which the value of the sound power of the generator was determined did not exceed 15 %.

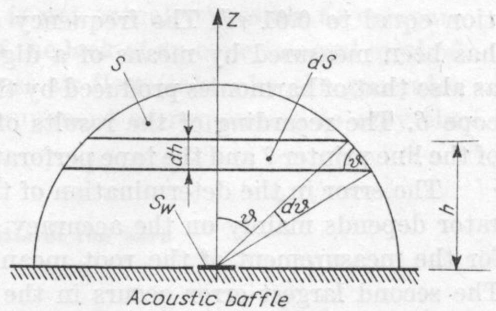

Fig. 4. Geometrical illustration of the determination method of the generator sound power

The angles ϑ_i at which the measurements of the acoustic pressure should be made are defined by the relation

$$\vartheta_i = \arccos(1 - i/m), \tag{49}$$

where $i = 0, 1, \ldots, m$.

Measurements of the sound power of the generator have been carried out in an anechoic chamber *10* (Fig. 5) using an apparatus composed of a 1/4″ — condenser microphone *1*, an analyser *2*, and a digital voltmeter *3*. The microphone was placed a distance of 0.9 m from the horn outlet of the generator and fixed to the arm. The latter permits rotation of the microphone, from the outside of the chamber, in a plane passing through the main axis of the generator through an angle of $\pi/2$ *rd*. The analog to digital convertor being coupled to the axis of rotation of the arm while cooperating with the reversion counter *4*, enables digital measurement of the angle of the microphone with a resolu-

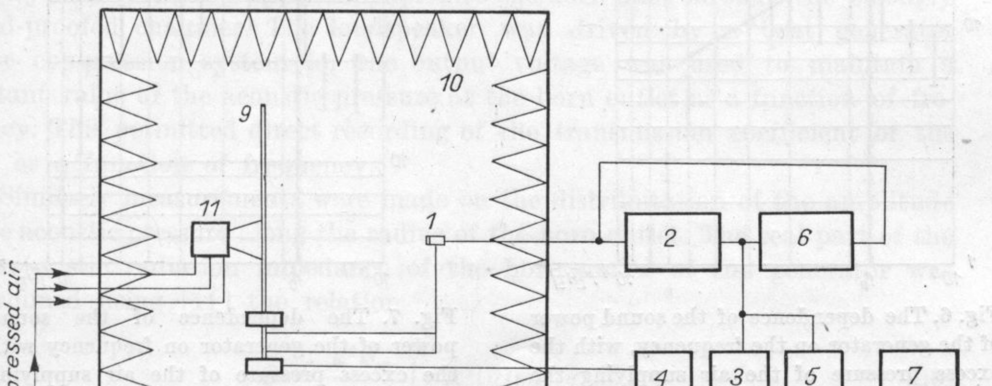

Fig. 5. The diagram of the measuring system for the determination of the sound power of the generator

tion equal to 0.01 *rd*. The frequency of the wave produced by the generator has been measured by means of a digital frequency meter *5*, while its shape as also that of harmonics produced by the generator was observed on the oscilloscope *6*. The recording of the results of the measurements was done by means of the line printer *7* and the tape perforator *8*.

The error in the determination of the value of the sound power of the generator depends mainly on the accuracy of the calibration of the apparatus used for the measurement of the root mean square value of the acoustic pressure. The second largest error occurs in the approximate numerical method for determining the value of the definite integral which occurs in the formula for the sound power of the generator (46). The magnitude of this error depends mainly on the length of the integration increment Δh. In the measurements carried out with a confidence level of 0.95, the relative error with which the value of the sound power of the generator could be determined did not exceed 19 %.

The results of the measurements of the sound power of the generator as a function of frequency for the excess pressures of the generator air supply of 0.2 and 0.5×10^5 N/m are shown in Figs. 6 and 7. In these figures a continuous

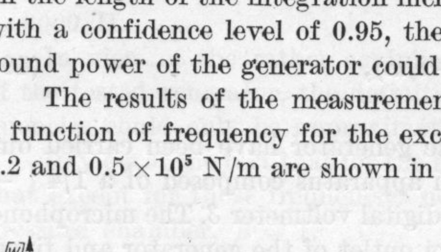

Fig. 6. The dependence of the sound power of the generator on the frequency, with the excess pressure of the air supplying the generator equal to 0.2×10^5 N/m²

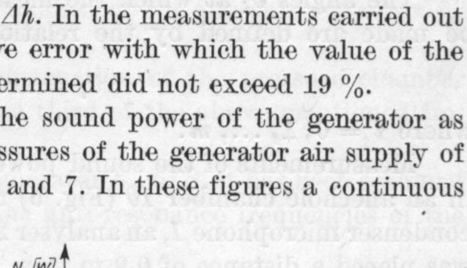

Fig. 7. The dependence of the sound power of the generator on frequency with the excess pressure of the air supplying the generator equal to 0.5×10^5 N/m²

line marks the results of the calculations obtained on the basis of the equivalent electric diagram of the generator used for the experimental investigations, with the assumption that $\mu = 1$. The broken line is used where the properties of the horn and the pressure chamber deviate from the previously accepted assumptions.

In conclusion, it can be stated that it is not actually possible to compare the supply power of the generator because of the lack of an experimental method permitting definition of the share in the mass air flow (taken by the generator from the compressor supplying it) of the component corresponding to only the first harmonic of the acoustic pressure [8].

6. Transmission properties of the horn

In view of the lack of definition of the range of application of relevant theoretical considerations, as has already been indicated in the theoretical part, experimental investigations have been carried out regarding the transmission properties of the horn of the generator. Their main aim was the determination of the range in which it is possible to propagate exclusively a tangential wave mode of the zero order in the horn of the generator. It has been assumed that the plane wave propagates in the horn over the frequency range in which the following criteria are simultaneously satisfied:

(i) the impedance of the radiation of the horn outlet as a function of frequency is that described by RAYLEIGH [15],

(ii) the distribution of the amplitude of the acoustic pressure on the surface of the horn outlet does not depend on position,

(iii) the modulus of the transmission coefficient of the horn (39) does not depend on the frequency

The investigations were carried out in an anechoic chamber, with the horn outlet placed in a plane acoustic baffle. A sound wave was radiated into the horn by means of a loudspeaker coupled to the horn inlet through the strongly sound-proofed chamber. The loudspeaker was driven by a beat generator whose compression system for the output voltage was used to maintain a constant value of the acoustic pressure at the horn outlet as a function of frequency. This permitted direct recording of the transmission coefficient of the horn as a function of frequency.

Similarly measurements were made on the distributation of the amplitude of the acoustic pressure along the radius of the horn outlet. The real part of the characteristic radiation impedance of the horn outlet of the generator was determined using [11] the relation

$$\Theta = \frac{k^2 S_w}{4\pi} \int_0^{\pi/2} R^2(\vartheta)\sin(\vartheta)\,d\vartheta, \tag{50}$$

where k is the wave number, S_w is the surface area of the horn outlet, and $R(\vartheta)$ is the radiation directivity coefficient of the horn outlet [15].

The distribution of the amplitude of the acoustic pressure in the far field, at a constant distance from the horn outlet, was determined by means of the apparatus used previously for the determination of the sound power of the generator. The value of the definite integral in expression (50) was calculated by SIMPSON's method on a digital computer. Each of the results of the calculations was based on 76 measurements of the acoustic pressure.

Figs. 8, 9 and 10 show in succession the results of the measurements of the real part of the characteristic radiation impedance of the horn outlet as a function of frequency, the amplitude distribution of the acoustic pressure along the

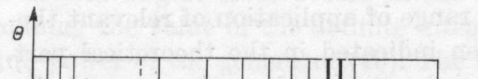

Fig. 8. The dependence of the real part of unitary relative impedance of the radiation of the horn outlet on the frequency

Fig. 9. The distribution of the amplitude of the acoustic pressure along the radius of the horn outlet

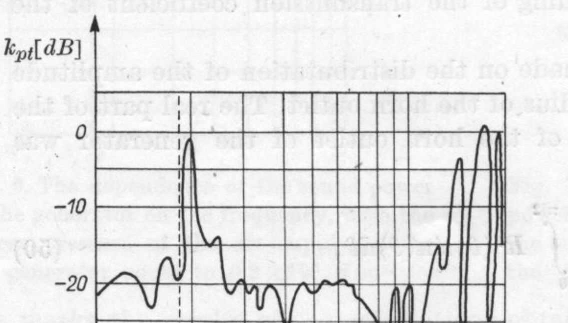

Fig. 10. The dependence of the transmission coefficient of a catenoidal horn with an annular cross-section on the frequency

radius of the horn outlet, and the the transmission coefficient of the horn as a function of frequency.

On the basis of the criteria accepted previously, and the results of the measurements it can be said that the propagation of a plane wave at frequencies up to 8 kHz is possible in the horn of the dynamic generator. Above this frequency wave modes of higher orders are produced in the horn.

7. Conclusions

On the basis of the measurement results it can be said that the model of the dynamic generator used for the theoretical considerations is correct for the limited range of conditions accepted at its formation, i.e. for small values of the air pressure in the pressure chamber (smaller than 0.2×10^5 N/m^2), and for the frequency range in which only plane wave propagation is possible in the acoustic system of the generator. The reason for the observed drop in sound power should be looked for in the resonance properties of the pressure chamber, and the stator passage, as well as in the formation of higher wave modes in the horn and pressure chamber of the generator.

The influence of the above-mentioned factors on the acoustic parameters can be reduced by the use of

(i) the largest number of passages in the stator,
(ii) stator passages as short as possible,
(iii) a strongly absorbing end to the pressure chamber,
(iv) slightly opened horns of.

The satisfaction of the above-mentioned criteria in the construction of a dynamic generator permits the optimal operation of the generator over a wide frequency range.

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