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Research Paper

2D Modeling of Wave Propagation in Shallow Water by the Method of Characteristics

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In this paper, a 2D numerical modeling of sound wave propagation in a shallow water medium that acts as a waveguide, are presented. This modeling is based on the method of characteristic which is not constrained by the Courant–Friedrichs–Lewy (CFL) condition. Using this method, the Euler time-dependent equations have been solved under adiabatic conditions inside of a shallow water waveguide which is consists of one homogeneous environment of water over a rigid bed. In this work, the stability and precision of the method of characteristics (MOC) technique for sound wave propagation in a waveguide were illustrated when it was applied with the semi-Lagrange method. The results show a significant advantage of the method of characteristics over the finite difference time domain (FDTD) method.

Keywords: wave propagation; shallow water; MOC method; waveguide; transmission loss.

1. Introduction

The propagation of sound in the sea has been studied to an extreme degree from the beginning of the Second World War, when it was realized that insight into this matter was indispensable to the successful performance of anti-submarine warfare working. These early estimations were fastly converted into constructive, albeit primeval, forecasting tools. Naval necessities motivate progress in all features of underwater acoustic modeling, especially the modeling of sound propagation. The investigation of sound wave propagation in seawater is essential for understanding and forecasting all underwater acoustic phenomena. The essentiality of wave propagation models is intrinsic in the ranking of acoustic models is illustrated in Fig. 1.

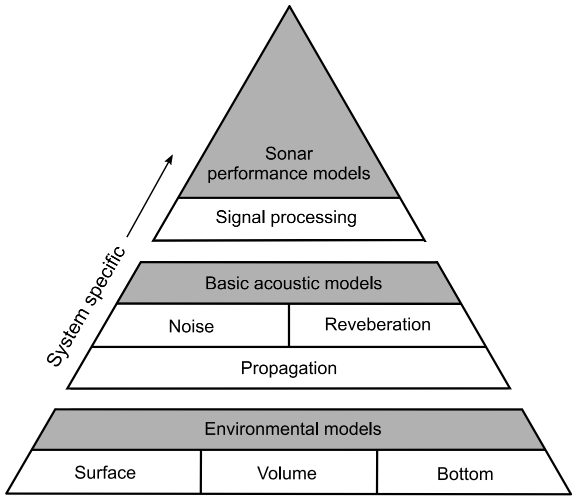

Fig. 1. Relationship between the environmental and acoustic models.

Sound propagation depends on the physical characteristics and the environment (Hosseini *et al.*, 2018). Many studies have been done on the physical characteristics of shallow waters such as the Persian Gulf. Such environments create an almost homogeneous layer of water due to the shallow depth and turbulence caused by wind and tides (Khalilabadi *et al.*, 2015; Khalilabadi 2016a; 2016b; 2016c; Mahpeykar, Khalilabadi, 2021; Mollaesmaeilpour *et al.* 2019). In this paper, a new method for simulating sound propagation within such environments is discussed. Simple intuitive developments have been given to present the physics of acoustic propagation in shallow water layer. The structure of a simple waveguide has been illustrated in Fig. 2.

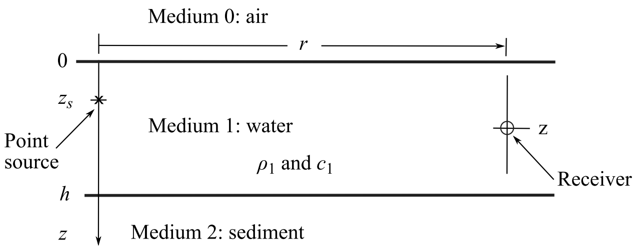

Fig. 2. Fundamental of a general waveguide.

Resulting from the computer programming evolution, numerical methods of wave propagation have been studied widely (Matsumura *et al.*, 2017). For high-performance soundwave field prediction, the progression of the precise numerical method is an essential subject (Ara *et al.*, 2011; Matsumura *et al.*, 2015; Oshima *et al.*, 2014).

Regarding the evolutionary process of studying and modeling waves in waveguides and shallow water, especially in recent years, we can mention the latest works. Kirby and Duan (2018) used modeled the sound wave propagation in the seawater using a normal mode approach and by finite elements method. Then they used a semi-analytical method for simulating the wave propagation in a waveguide (Duan, Kirby, 2019).

Jena *et al.* (2019) proposed a new solution of wave equations arising in shallow water wave propagation. Li *et al.* (2019) presented one method based on multilayer boundary element for direct numerical modeling of acoustic wave propagation in shallow water areas.

Duan and Kirby (2020) calculated the characteristics of edge waves in 3D Plates using another numerical approach. Wang *et al.* (2020) predicted sound intensity vector field in shallow water waveguide using a prediction method. Li *et al.* (2021) determined the characteristics of sound wave propagation in shallow water waveguides for very low-frequency waves.

2. Materials and methods

The aim of this investigation is to illustrate the stability and precision of the method of characteristics (MOC) technique (Fievisohn, Yu, 2016; Liu, 2021; Mazumdar, Gupta, 2018; Song *et al.*, 2020; Subbotina, Krupennikov, 2017) with semi-Lagrange method (Jiang *et al.*, 2020; Cho *et al.*, 2021; Piao *et al.*, 2018; Saadat *et al.*, 2020) applied for sound wave propagation in a waveguide. The waveguide is a homogeneous water layer overlying a rigid sea bottom (Jihui *et al.*, 2020; Li *et al.*, 2021; Verlinden *et al.*, 2017).

In numerical method, the MOC is a method to solve the partial differential equations (Cao, Liu, 2020; Jewell, 2019; Kauffmann *et al.*, 2018; Twyman, 2018). In most cases, this technique applies to the first-order equations, albeit generally the MOC is reliable for each hyperbolic partial differential equation. The technique is reducing a partial differential equation to a group of ordinary differential equations along which the solution process can be integrated from some initial data given on an appropriate hyper-surface (Ali *et al.*, 2020; Ayas *et al.*, 2019; Gao *et al.*, 2021; Costa *et al.*, 2021).

The basic equations have been written in cylindrical coordinates. For surface and bottom boundary conditions we consider free pressure in the surface and a rigid sea bed. 1D Euler and continuity equations under the circumstances adiabatic environment can be written in these forms:

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x}, \tag{1}$$

$$\frac{\partial p}{\partial t} = -\rho c^2 \frac{\partial u}{\partial x}. \tag{2}$$

By multiplying Eq. (1) by $\pm c^2$ and collect with Eq. (2), we can obtain:

$$\frac{\partial f^+}{\partial t} + c\frac{\partial f^+}{\partial x} = 0, \tag{3}$$

$$\frac{\partial f^-}{\partial t} - c\frac{\partial f^-}{\partial x} = 0, \tag{4}$$

where

$$f^+ = \rho c u + p, \tag{5}$$

$$f^- = \rho c u - p. \tag{6}$$

Equations (1) and (2) are advection equations with soundwave speeds of $+c$ and $-c$, respectively. The parameters f^+ and f^- are advection along its characteristics. Thus new amounts at the subsequent time can be calculated by finding the up-wind amounts along with characteristics as illustrated in Fig. 3.

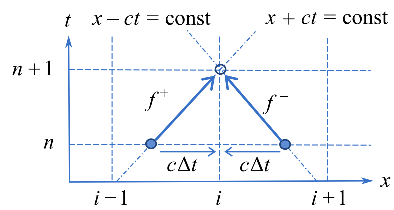

Fig. 3. Plan of advection.

The Courant–Friedrichs–Lewy (CFL) condition (Ascher, van den Doel, 2013; Domingues *et al.*, 2013; Hersh, 2013; Jeltsch, Kumar, 2013; Lax, 2013; LeFloch, 2013; Rhebergen, Cockburn, 2013; Schneider *et al.*, 2013) can be written as:

$$c_0 \Delta t \sqrt{(1/\Delta r)^2 + (1/\Delta z)^2} \le 1. \tag{7}$$

If CFL = 1, f^+ and f^- propagate the quantities from one specific cell to the next cell in time iterations.

If the CFL number is not a natural number, we can apply the constrained interpolation profile (CIP) technique (Matsumura *et al.*, 2017; Yabe *et al.*, 2001). Therefore by addition and subtracting the Eqs (5) and (6), the pressure and particle velocity can be written as:

$$p = \frac{f^+ - f^-}{2}, \tag{8}$$

$$u = \frac{f^+ - f^-}{2\rho c}. \tag{9}$$

In 2D cases, we can solve these equations by a directional splitting technique (Gendre *et al.*, 2017; Nakamura *et al.*, 2001). At the first, the equation of advection can been solved in the range direction, then this equation can be solved in depth direction.

The numerical model designed in this study, uses square grids. In this model, all of the physical quantities (the particle velocity and pressure) are collocated.

3. Results and discussion

The numerical model prepared in this research was implemented in a homogeneous seawater layer overlying a rigid seabed with a depth of 100 m. The projector and the receiver established at a same depth (50 m).

Figure 4 illustrates the sound wave propagation in this waveguide during running the model program. The model also calculate the transmission loss [dB] versus range [km].

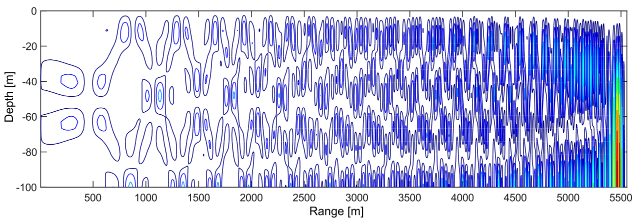

Fig. 4. Sound propagation in the waveguide during running the program.

The comparison of transmission loss between the model and theory is shown in Fig. 5. As seen in this figure, the range lowers than about 1 km where all modes have propagated, the model is well matches the theory. As the range increases the difference between theory and model increases.

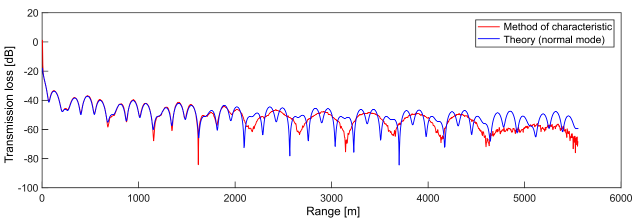

Fig. 5. Comparison between theory and model for transmission loss.

Then we changed the setup and put the source at the bottom. Figure 6 shows the sound propagation in the waveguide during running the program in this condition, and Fig. 7 shows the comparison of transmission loss between model and theory for this status. As discussed above, in the lower ranges where all modes have propagated, the model well matches the theory, and as the range increases the difference between theory and model increases.

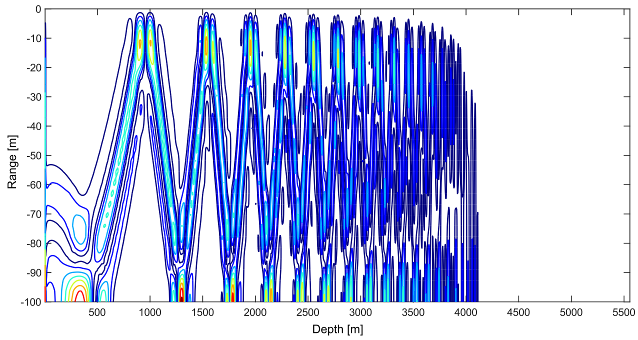

Fig. 6. Sound propagation in the waveguide during running the program.

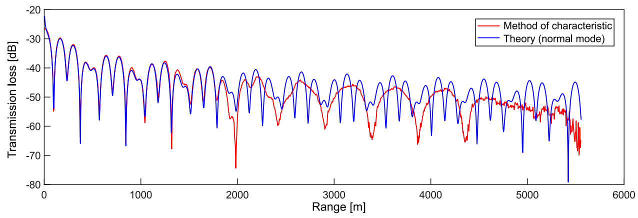

Fig. 7. Comparison of transmission loss between model and theory.

4. Conclusions

In this research, the propagation of acoustic waves within a waveguide is modeled using the MOC. The results compared with the finite difference time domain (FDTD) method. By examining the findings and modeling results, the following facts can be drawn:

- There is no difference in computational time between the MOC method and the FDTD per iteration, however, the FDTD is bound by more severe CFL conditions.
- When a rectangular mesh is used, the maximum amounts are so large that FDTD will take at least 1.4 times the calculation time required.
- In the method of characteristics, the phase properties of are more precise than the FDTD method. At low frequencies, this difference is not significant, but with increasing frequency, this difference becomes significant.

- It is determined by comparison of transmission losses that in the lower ranges where all modes have propagated, the model and the theory will be compatible. But as the range increases, the difference between model and theory will increase.

References

1. Ali K.K., Yilmazer R., Baskonus H.M., Bulut H. (2020), Modulation instability analysis and analytical solutions to the system of equations for the ion sound and Langmuir waves, *Physica Scripta*, **95**(6): 065602, doi: 10.1088/1402-4896/ab81bf.

2. Ara Y., Okubo K., Tagawa N., Tsuchiya T., Ishizuka T. (2011), A novel numerical simulation of sound wave propagation using sub-grid CIP-MOC method, *2011 IEEE International Ultrasonics Symposium*, pp. 760–763, doi: 10.1109/ULTSYM.2011.6293349.

3. Ascher U., van den Doel K. (2013), Fast Chaotic Artificial Time Integration, [in:] *The Courant–Friedrichs–Lewy (CFL) Condition: 80 Years After Its Discovery*, C.A. de Moura, C.S. Kubrusly [Eds], pp. 147–155, Birkhäuser, Boston, doi: 10.1007/978-0-8176-8394-8_10.

4. Ayas H., Chabaat M., Amara L. (2019), Dynamic analysis of a cracked bar by the method of characteristics, *International Journal of Structural Integrity*, **10**(4): 438–453, doi: 10.1108/IJSI-01-2018-0001.

5. Cao F., Liu J. (2020), Nonlinear partial differential equation model-based coordination control for a master-slave two-link rigid-flexible manipulator with vibration repression, *Journal of Computational and Nonlinear Dynamics*, **16**(2): 021007, doi: 10.1115/1.4049219.

6. Cho S.Y., Boscarino S., Russo G., Yun S.-B. (2021), Conservative semi-Lagrangian schemes for kinetic equations. Part I: Reconstruction, *Journal of Computational Physics*, **432**: 110159, doi: 10.1016/j.jcp.2021.110159.

7. Costa G., Montemurro M., Pailhčs J. (2021), NURBS hyper-surfaces for 3D topology optimization problems, *Mechanics of Advanced Materials and Structures*, **28**(7): 665–684, doi: 10.1080/15376494.2019.1582826.

8. Domingues M.O., Gomes S.M., Roussel O., Schneider K. (2013), Space-time adaptive multiresolution techniques for compressible Euler equations, [in:] *The Courant–Friedrichs–Lewy (CFL) Condition: 80 Years After Its Discovery*, C.A. de Moura, C.S. Kubrusly [Eds], Birkhäuser, Boston, pp. 101–117, doi: 10.1007/978-0-8176-8394-8_7.

9. Duan W., Kirby R. (2019), Guided wave propagation in buried and immersed fluid-filled pipes: Application of the semi analytic finite element method, *Computers Structures*, **212**: 236–247, doi: 10.1016/j.compstruc.2018.10.020.

10. Duan W., Kirby R. (2020), A numerical approach for calculation of characteristics of edge waves in three-dimensional plates, *Journal of Theoretical and Computational Acoustics*, **29**(02): 2050014, doi: 10.1142/S2591728520500140.

11. Fievisohn R.T., Yu K.H. (2016), Steady-state analysis of rotating detonation engine flowfields with the method of characteristics, *Journal of Propulsion and Power*, **33**(1): 89–99, doi: 10.2514/1.B3610.

12. Fukuda A., Okubo K., Oshima T., Tsuchiya T., Kanamori M. (2018), Numerical analysis of three-dimensional acoustic field with background flow using constrained interpolation profile method, *Japanese Journal of Applied Physics*, **57**(7S1): 07LC09, doi: 10.7567/jjap.57.07lc09.

13. Gao W., Veeresha P., Prakasha D.G., Baskonus H.M. (2021), New numerical simulation for fractional Benney–Lin equation arising in falling film problems using two novel techniques, *Numerical Methods for Partial Differential Equations*, **37**(1): 210–243, doi: 10.1002/num.22526.

14. Gendre F., Ricot D., Fritz, G., Sagaut P. (2017), Grid refinement for aeroacoustics in the lattice Boltzmann method: A directional splitting approach, *Physical Review E*, **96**(2): 023311, doi: 10.1103/PhysRevE.96.023311.

15. Hersh R. (2013), Mathematical intuition: Poincaré, Pólya, Dewey, [in:] *The Courant–Friedrichs–Lewy (CFL) Condition: 80 Years After Its Discovery*, C.A. de Moura, C.S. Kubrusly [Eds], pp. 9–30, Birkhäuser, Boston, doi: 10.1007/978-0-8176-8394-8_2.

16. Hosseini S.H., Akbarinasab M., Khalilabadi M.R. (2018), Numerical simulation of the effect internal tide on the propagation sound in the Oman Sea, *Journal of the Earth and Space Physics*, **44**(1): 215–225, doi: 10.22059/jesphys.2018.221834.1006867.

17. Jeltsch R., Kumar H. (2013), Three-dimensional plasma arc simulation using resistive MHD, [in:] *The Courant–Friedrichs–Lewy (CFL) Condition: 80 Years After Its Discovery*, C.A. de Moura, C.S. Kubrusly [Eds], pp. 31–43, Birkhäuser, Boston, doi: 10.1007/978-0-8176-8394-8_3.

18. Jena R.M., Chakraverty S., Baleanu D. (2019), On new solutions of time-fractional wave equations arising in shallow water wave propagation, *Mathematics*, **7**(8): 722, doi: 10.3390/math7080722.

19. Jewell J. (2019), Higher-order Runge-Kutta type schemes for the method of characteristics, *UVM Student Research Conference*, https://scholarworks.uvm.edu/src/2019/program/355.

20. Jiang T., Guo P., Wu J. (2020), One-sided on-demand communication technology for the semi-Lagrange scheme in the YHGSM, *Concurrency and Computation: Practice and Experience*, **32**(7): e5586, doi: 10.1002/cpe.5586.

21. Jihui W., Guijuan L., Bing J., Zhenshan W., Rui W. (2020), Numerical computation on the scattering sound field distribution of rigid sphere in shallow water waveguide, *IOP Conference Series: Materials Science and Engineering*, **780**: 032058, doi: 10.1088/1757-899X/780/3/032058.

22. Kauffmann T., Kocar I., Mahseredjian J. (2018), New investigations on the method of characteristics for the evaluation of line transients, *Electric Power Systems Research*, **160**: 243–250, doi: 10.1016/j.epsr.2018.03.004.

23. Khalilabadi M.R. (2016a), A numerical study of internal tide generation due to interaction of barotropic tide with bottom topography in the Oman Gulf, *Journal of the Earth and Space Physics*, **42**(1): 645–656, doi: 10.22059/jesphys.2016.57903.

24. Khalilabadi M.R. (2016b), The effect of meteorological events on sea surface height variations along the northwestern Persian Gulf, *Current Science (00113891)*, **110**(11): 2138–2141, doi: 10.18520/cs/v110/i11/2138-2141.

25. Khalilabadi M.R. (2016c), Tide–surge interaction in the Persian Gulf, Strait of Hormuz and the Gulf of Oman, *Weather*, **71**(10): 256–261, doi: 10.1002/wea.2773.

26. Khalilabadi M.R., Sadrinassab M., Chegini V., Akbarinassab M. (2015), Internal wave generation in the Gulf of Oman (outflow of Persian Gulf), *Indian Journal of Geo-Marine Sciences*, **44**(3): 519–527.

27. Kirby R., Duan W. (2018), Modelling sound propagation in the ocean: A normal mode approach using finite elements, [in:] *Australian Acoustical Society Annual Conference, AAS 2018*, pp. 530–539, Australian Acoustical Society, http://hdl.handle.net/10453/139710.

28. Lax P.D. (2013), Stability of difference schemes, [in:] *The Courant–Friedrichs–Lewy (CFL) Condition: 80 Years After Its Discovery*, C.A. de Moura, C.S. Kubrusly [Eds], pp. 1–7, Birkhäuser, Boston, doi: 10.1007/978-0- 8176-8394-8_1.

29. LeFloch P.G. (2013), A framework for late-time/stiff relaxation asymptotics, [in:] *The Courant–Friedrichs–Lewy (CFL) Condition: 80 Years After Its Discovery*, C.A. de Moura, C.S. Kubrusly [Eds], pp. 119–137, Birkhäuser, doi: 10.1007/978-0-8176-8394-8_8.

30. Li C., Campbell B.K., Liu Y., Yue D.K. (2019), A fast multi-layer boundary element method for direct numerical simulation of sound propagation in shallow water environments, *Journal of Computational Physics*, **392**, 694–712, doi: 10.1016/j.jcp.2019.04.068.

31. Li N., Zhu H., Wang X., Xiao R., Xue Y., Zheng G. (2021), Characteristics of very low frequency sound propagation in full waveguides of shallow water, *Sensors*, **21**(1), 192, doi: 10.3390/s21010192.

32. Liu Z. (2021), 5 – The method of characteristics, [in:] *Deterministic Numerical Methods for Unstructured-Mesh Neutron Transport Calculation*, L. Cao, H. Wu [Eds], pp. 73–108, Woodhead Publishing, doi: 10.1016/B978-0-12-818221-5.00010-6.

33. Mahpeykar O., Khalilabadi M.R. (2021), Numerical modelling the effect of wind on water level and evaporation rate in the Persian Gulf, *International Journal of Coastal and Offshore Engineering*, **6**(1): 47–53.

34. Matsumura Y., Okubo K., Tagawa N., Tsuchiya T., Ishizuka T. (2015), Hybrid MM-MOC-based numerical simulation of acoustic wave propagation with non-uniform grid and perfectly matched layer absorbing boundaries, *2015 IEEE International Ultrasonics Symposium (IUS)*, pp. 1–4, doi: 10.1109/ULTSYM.2015.0443.

35. Matsumura Y., Okubo K., Tagawa N., Tsuchiya T., Ishizuka T. (2017), Evaluation of numerical simulation of acoustic wave propagation using method of characteristics-based constrained interpolation profile (CIP-MOC) method with non-uniform grids, *Acoustical Science and Technology*, **38**(1): 31–34, doi: 10.1250/ast.38.31.

36. Mazumdar T., Gupta A. (2018), Application of Krylov acceleration technique in method of characteristics – based neutron transport code, *Nuclear Science and Engineering*, **192**(2): 153–188, doi: 10.1080/00295639.2018.1499340.

37. Mollaesmaeilpour S., Mohammad Mahdizadeh M., Hasanzade S., Khalilabadi M.R. (2019), The study of hydrophysical properties of the northern Arabian Sea during monsoon: A numerical study, *Hydrophysics*, **5**(1): 47–59.

38. Nakamura T., Tanaka R., Yabe T., Takizawa K. (2001), Exactly conservative semi-Lagrangian scheme for multi-dimensional hyperbolic equations with directional splitting technique, *Journal of Computational Physics*, **174**(1): 171–207, doi: 10.1006/jcph.2001.6888.

39. Oshima T., Hiraguri Y., Imano M. (2014), Geometry reconstruction and mesh generation techniques for acoustic simulations over real-life urban areas using digital geographic information, *Acoustical Science and Technology*, **35**(2): 108–118, doi: 10.1250/ast.35.108.

40. Piao X., Kim P., Kim D. (2018), One-step $L(\alpha)$-stable temporal integration for the backward semi-Lagrangian scheme and its application in guiding center problems, *Journal of Computational Physics*, **366**: 327–340, doi: 10.1016/j.jcp.2018.04.019.

41. Rhebergen S., Cockburn B. (2013), Space-time hybridizable discontinuous Galerkin method for the advection–diffusion equation on moving and deforming meshes, [in:] *The Courant–Friedrichs–Lewy (CFL) Condition: 80 Years After Its Discovery*, C.A. de Moura, C.S. Kubrusly [Eds], pp. 45–63, Birkhäuser, Boston, doi: 10.1007/978-0-8176-8394-8_4.

42. Saadat M.H., Bösch F., Karlin I.V. (2020), Semi-Lagrangian lattice Boltzmann model for compressible

flows on unstructured meshes, *Physical Review E*, **101**(2): 023311, doi: 10.1103/PhysRevE.101.023311.

43. Schneider K., Kolomenskiy D., Deriaz E. (2013), Is the CFL condition sufficient? Some remarks, [in:] *The Courant–Friedrichs–Lewy (CFL) Condition: 80 Years After Its Discovery*, C.A. de Moura, C.S. Kubrusly [Eds], pp. 139–146, Birkhäuser, Boston, doi: 10.1007/978-0-8176-8394-8_9.

44. Song P., Zhang Z., Zhang Q., Liang L., Zhao Q. (2020), Implementation of the CPU/GPU hybrid parallel method of characteristics neutron transport calculation using the heterogeneous cluster with dynamic workload assignment, *Annals of Nuclear Energy*, **135**: 106957, doi: 10.1016/j.anucene.2019.106957.

45. Subbotina N.N., Krupennikov E.A. (2017), The method of characteristics in an identification problem, *Proceedings of the Steklov Institute of Mathematics*, **299**(1): 205–216, doi: 10.1134/S008154381709022X.

46. Twyman J. (2018), Transient flow analysis using the method of characteristics MOC with five-point interpolation scheme, *Obras y Proyectos*, **24**, 62–70, doi: 10.4067/s0718-28132018000200062.

47. Verlinden C.M.A., Sarkar J., Cornuelle B.D., Kuperman W.A. (2017), Determination of acoustic waveguide invariant using ships as sources of opportunity in a shallow water marine environment, *The Journal of the Acoustical Society of America*, **141**(2): EL102–EL107, doi: 10.1121/1.4976112.

48. Wang W., Yang D., Shi J. (2020), A prediction method for acoustic intensity vector field of elastic structure in shallow water waveguide, *Shock and Vibration*, **2020**: article ID 5389719, doi: 10.1155/2020/5389719.

49. Yabe T., Xiao F., Utsumi T. (2001), The constrained interpolation profile method for multiphase analysis, *Journal of Computational Physics*, **169**(2): 556–593, doi: 10.1006/jcph.2000.6625.