ARCHIVES OF ACOUSTICS
3, 4, 283-292 (1978)

CORRELATION FUNCTION DETERMINATION FOR INHOMOGENEITIES SCATTERING AN ACOUSTIC WAVE

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A random inhomogeneous isotropic medium filling a domain immersed in an infinitely extended homogeneous isotropic medium is considered. The formulae describing the scalar potential of the scattered field are deduced for small and large distances from the domain of the heterogeneous material. The fluctuations of density and wave propagation velocity (and also pressure in the case of a nonviscous emulsion) are treated as random variables of the space coordinates. The correlation function is calculated from the appropriate farfield solution and expressed in terms of a scalar potential for the angular distribution of the scattered wave. This general method is adapted for a non-viscous random emulsion and the correlation function is expressed in terms of the intensity angular distribution of the scattered wave.

List of symbols

V' — domain filled by the inhomogeneous medium
S' — boundary of the domain V'
$\boldsymbol{r}$ — position vector
$\psi(\boldsymbol{r}, t)$ — scalar velocity potential of an acoustic wave
ϱ_s and $\varrho_0(\boldsymbol{r})$ — density of the homogeneous and heterogeneous medium, respectively
c_s and $c_0(\boldsymbol{r})$ — wave velocity in the homogeneous and heterogeneous medium, respectively
$\langle A\rangle$ — mean value of a quantity A
$\delta A(\boldsymbol{r}) = A(\boldsymbol{r}) - \langle A(\boldsymbol{r})\rangle$ — fluctuation in a quantity A at a point $\boldsymbol{r}$
$\gamma(x)$ — autocorrelation function (called shortly correlation function)
L_c — correlation length
β — volume concentration of the grains
$\bar{F}$ — equilibrium value of a quantity F
ΔF — acoustic disturbance of a quantity F
ω — angular frequency
a — amplitude of oscillations
$\boldsymbol{n}$ — unit vector in the direction of propagation of the incident wave
θ — angle of scattering
p — pressure
η — kinematic viscosity
I_0 — intensity of the incident wave
I_{sc} — intensity of the scattered wave

1. Introduction

Several authors [1, 2, 5, 6] have considered the problem of finding the scattered intensity and the scattering cross section in a random inhomogeneous isotropic media from the incident acoustic wave and the correlation function [4]. In these previous works the scattering intensities have been calculated from the appropriate farfield solutions, and the results have been expressed in terms of correlation functions. The purpose of the present paper is to solve the inverse problem of determining the correlation function from the angular distribution of the acoustic field of a wave scattered in a random inhomogeneous isotropic medium. The inverse problem under consideration is the analogue of the light scattering problem discussed by Debye and Bueche [4].

In the present paper we start with the differential equation of motion for the scalar wave $\psi(\mathbf{r}, t)$ in a heterogeneous medium, where the wave velocity $c_0(\mathbf{r})$ is a function of the position vector $\mathbf{r}$ and is independent of the time t. As a result we obtain an integral expression for the scalar scattered wave $\psi_{sc}(\mathbf{r}, t)$. With the help of the correlation function [4] and use of the Fourier integral transformation for odd functions we obtain, in the farfield approximation, a rather simple integral formula expressing the correlation function in terms of $\langle|\psi_{sc}(\mathbf{r}, t)|^2\rangle$ or $\langle|\nabla\psi_{sc}(\mathbf{r}, t)|^2\rangle$ and of the angle of scattering θ where $\langle\cdot\rangle$ denotes an average. Next we consider the special case of acoustic wave scattering in nonviscous emulsions. For this case we obtain an integral formula expressing the correlation function in terms of the scattered intensity and the angle of scattering.

2. Basic assumptions and auxiliary notions

In discussing the problem under consideration in this paper, the random heterogeneous isotropic material filling domain V' is assumed to be immersed in an infinitely extended homogeneous isotropic material of density ϱ_s, where the wave velocity c_s is known and satisfies the following inequality:

$$\left|1-\left(c_s|c_0(\mathbf{r})\right)^2\right| \equiv \left|1-\left[1+\left(c_0(\mathbf{r})-c_s\right)/c_s\right]^{-2}\right| \ll 1 \Leftrightarrow |c_0(\mathbf{r})-c_s|/c_s \ll 1. \tag{2.1}$$

Inequality (2.1) enables us to write with first order accuracy in

$$\left(c_0(\mathbf{r})-c_s\right)/c_s \equiv \left(\delta c_0(\mathbf{r})+\langle c_0(\mathbf{r})\rangle-c_s\right)/c_s,$$

the relation

$$U(\mathbf{r}) \equiv 1-\left(c_s/c_0(\mathbf{r})\right)^2 \cong \langle\tilde{U}(\mathbf{r})\rangle+\delta\tilde{U}(\mathbf{r}), \tag{2.2}$$

where

$$\tilde{U}(\mathbf{r}) = 2\left(c_0(\mathbf{r})-c_s\right)/c_s, \tag{2.3}$$

and

$$\delta\tilde{U}(\mathbf{r}) = 2\left(c_0(\mathbf{r})-\langle c_0(\mathbf{r})\rangle\right)/c_s. \tag{2.4}$$

Furthermore, it is assumed that

$$|\langle \tilde{U}(\boldsymbol{r})\rangle| \ll |\delta \tilde{U}(\boldsymbol{r})| \ll 1 \tag{2.5}$$

and

$$|\varrho_s c_s - \varrho_0(\boldsymbol{r}) c_0(\boldsymbol{r})| / \varrho_s c_s \ll 1, \tag{2.6}$$

where $\varrho_0(\boldsymbol{r})$ is the density of the medium filling domain V'.

The subsequent discussion uses a reference system with the origin at some convenient point of the domain V'. It is assumed that the volume V' of the domain filled by the inhomogeneities satisfies the inequality

$$(V')^{1/3} \gg L_c, \tag{2.7}$$

where L_c is the correlation lenght of the random inhomogeneities. The structure of the heterogeneous material filling the domain V' is described with the help of the correlation function $\gamma_{AB}(\boldsymbol{r}_1 - \boldsymbol{r}_2)$ which determines the manner in which the fluctuation δA in a quantity A at a given point $\boldsymbol{r}_1$ is correlated with that in another quantity B at a point $\boldsymbol{r}_2$. The fluctuation $\delta F(\boldsymbol{r}_0)$ in a quantity F at a point $\boldsymbol{r}_0$ is given by the formula

$$\delta F(\boldsymbol{r}_0) = F(\boldsymbol{r}_0) - \langle F(\boldsymbol{r}_0)\rangle. \tag{2.8}$$

When the material is isotropic, $\gamma_{AB}(\boldsymbol{r}_1 - \boldsymbol{r}_2)$ is a function of $|\boldsymbol{r}_1 - \boldsymbol{r}_2|$ and is independent of direction. The correlation function $\gamma_{AB}(\boldsymbol{r}_1 - \boldsymbol{r}_2)$ for an isotropic material is defined [4] by

$$\langle \delta A(\boldsymbol{r}_1)\, \delta B(\boldsymbol{r}_2)\rangle = \gamma_{AB}(x) \langle \delta A(\boldsymbol{r}_1)\, \delta B(\boldsymbol{r}_1)\rangle, \tag{2.9}$$

where

$$x = \boldsymbol{r}_1 - \boldsymbol{r}_2. \tag{2.10}$$

By comparing this equation with the condition

$$\lim_{x\to 0} \langle \delta A(\boldsymbol{r}_1)\, \delta B(\boldsymbol{r}_1 - x)\rangle = \langle \delta A(\boldsymbol{r}_1)\, \delta B(\boldsymbol{r}_1)\rangle \tag{2.11}$$

and defining the correlation length L_c by

$$\lim_{x\to L_c} \langle \delta A(\boldsymbol{r}_1)\, \delta B(\boldsymbol{r}_1 - x)\rangle = \langle \delta A(\boldsymbol{r})\, \delta B(\boldsymbol{r})\rangle / e, \tag{2.12}$$

we obtain:

$$\lim_{x\to 0} \gamma_{AB}(x) = 1, \quad \lim_{x\to L_c} \gamma_{AB}(x) = 1/e \quad (e = 2.718\ldots). \tag{2.13}$$

Formulae (2.9) and (2.13) are also applicable in the case of B being the same quantity as A. Then

$$\langle \delta A(\boldsymbol{r})\, \delta A(\boldsymbol{r} - x)\rangle \equiv \langle \delta A(0)\, \delta A(x)\rangle = \gamma(x) \langle (\delta A(\boldsymbol{r}))^2\rangle \tag{2.14}$$

and $\gamma(x)$ is called the *autocorrelation function*. $\gamma(x)$ measures the degree of correlation between the fluctuations at two points as a function of the distance of their separation.

The heterogeneous material considered is assumed to be a two-phase material. One of the phases consists of isolated grains randomly distributed in the matrix of the other phase in the domain V'. The fluctuations $\delta A(\boldsymbol{r})$, $\delta B(\boldsymbol{r})$, ... in quantities $A, B, \ldots$, respectively, are the result of fluctuations $\delta\beta(\boldsymbol{r})$ in the volume concentration $\beta(\boldsymbol{r})$ of the grains. For $\delta A(\boldsymbol{r})$ we have:

$$\delta A(\boldsymbol{r}) = \left(\frac{\partial A(\beta)}{\partial \beta}\right)_{\beta=0} \delta\beta \quad \text{if } \beta \ll 1. \tag{2.15}$$

Substituting equation (2.15) into (2.14) we obtain

$$\langle \delta A(0)\, \delta A(\boldsymbol{x})\rangle = \gamma(x) \left(\frac{\partial A}{\partial \beta}\right)^2_{\beta=0} \langle(\delta\beta)^2\rangle. \tag{2.16}$$

It can be verified that

$$\langle \delta A(0)\, \delta B(\boldsymbol{x})\rangle = \gamma(x) \left(\frac{\partial A}{\partial \beta}\right)_{\beta=0} \left(\frac{\partial B}{\partial \beta}\right)_{\beta=0} \langle(\delta\beta)^2\rangle. \tag{2.17}$$

Thus the autocorrelation function $\gamma(x)$ is adequate to describe all correlations if the fluctuations $\delta A(\boldsymbol{r})$, $\delta B(\boldsymbol{r})$, ... can be expressed in the form of equation (2.15). In this case an average of the type $\langle \delta A(0)\, \delta B(\boldsymbol{x})\rangle$ can be also reduced to the mean-square fluctuation $\langle(\delta\beta)^2\rangle$. As a result of the asumption (2.7) we have on the boundary S' of the domain V':

$$\gamma(x)|_{x\in S'} = 0, \qquad \left.\frac{\partial \gamma(x)}{\partial x}\right|_{x\in S'} = 0. \tag{2.18}$$

The acoustic wave under consideration in the present paper are assumed to be monochromatic, i.e. all the acoustic disturbances $\Delta F(\boldsymbol{r}, t)$ associated with the waves at a given point $\boldsymbol{r}$ are simple sinusoidal functions of time of the form

$$\Delta F(\boldsymbol{r}, t) = e^{i\omega t} \operatorname{const}(\boldsymbol{r}), \quad \Delta F = F - \bar{F}, \tag{2.19}$$

where ω is the angular frequency of the wave and $\bar{F}$ is the equilibrium value of a quantity F. It is assumed that

$$|\Delta F(\boldsymbol{r}, t)/\bar{F}(\boldsymbol{r})| \ll 1. \tag{2.20}$$

3. Angular distribution of the scalar scattered wave

The existence of a velocity scalar potential $\psi(\boldsymbol{r}, t)$ for an acoustic wave in the heterogeneous medium under consideration is postulated. The equation of motion for the scalar potential $\psi(\boldsymbol{r}, t)$ is postulated [7] to be

$$\nabla^2 \psi(\boldsymbol{r}, t) = \left(c_i(\boldsymbol{r}')\right)^{-2} \frac{\partial^2 \psi(\boldsymbol{r}, t)}{\partial t^2}, \tag{3.1}$$

where $i = 0$ if $\boldsymbol{r}$ is within the domain V', $i = s$ and $c_s(\boldsymbol{r}) = c_s = \text{const}$ it $\boldsymbol{r}$ is outside the domain V' or on the boundary S' of V'.

By substituting

$$\psi(\boldsymbol{r}, t) = \varphi(\boldsymbol{r})e^{i\omega t} \equiv \varphi(\boldsymbol{r})e^{ik_s c_s t}, \quad k_s = \omega/c_s, \tag{3.2}$$

we obtain the equation

$$(\nabla^2 + k_s^2)\varphi(\boldsymbol{r}) = k_s^2 U(\boldsymbol{r})\varphi(\boldsymbol{r}), \tag{3.3}$$

which the function $\varphi(\boldsymbol{r}')$ must satisfy. $U(\boldsymbol{r})$ is given by formula (2.2); $U(\boldsymbol{r})$ being different from zero only if $\boldsymbol{r}$ is within the domain V'. Furthermore, the scalar potential $\psi(\boldsymbol{r}, t)$ is taken as the sum of a monochromatic travelling plane wave (primary or incident wave) with another one superposed (called the *scattered wave*).

The problem formulated above, of calculating $\varphi(\boldsymbol{r})$ when $\boldsymbol{k}_s$ and $U(\boldsymbol{r})$ are known, is the same, apart from the factor k_s^2 on the right-hand side of equation (3.3), as the quantum mechanical problem of finding de Broiglie waves connected with the stationary elastic scattering of spinless particles. On using a suitable method (e.g. a Green's function method) [3], the solution of equation (3.3) is found to be

$$\varphi(\boldsymbol{r}) = e^{i\boldsymbol{k}_s\cdot\boldsymbol{r}} + \varphi_{sc}(\boldsymbol{r}), \quad \boldsymbol{k}_s = k_s\boldsymbol{n}, \tag{3.4}$$

where $\boldsymbol{n}$ is the unit vector in the direction of propagation of the incident wave $e^{i\boldsymbol{k}_s\cdot\boldsymbol{r}}$. The incident wave is the solution of equation (3.3) for $U(\boldsymbol{r}) = 0$. Assuming that

$$|e^{i\boldsymbol{k}_s\cdot\boldsymbol{r}}| \gg |\varphi_{sc}(\boldsymbol{r})|, \tag{3.5}$$

we obtain for the scalar potential of the scattered wave the formula

$$\varphi_{sc}(\boldsymbol{r}) = (1/r)A(\boldsymbol{r})e^{ik_s r}, \tag{3.6}$$

where

$$A(\boldsymbol{r}) = -\frac{k_s^2 r}{4\pi}\int_{V'} \frac{[U(\boldsymbol{r}')e^{i\boldsymbol{k}_s\cdot\boldsymbol{r}'}]e^{ik_s(|\boldsymbol{r}-\boldsymbol{r}'|-r)}}{|\boldsymbol{r}-\boldsymbol{r}'|}d^3\boldsymbol{r}', \tag{3.7}$$

the integration being over the volume of element

$$dV' \equiv d^3\boldsymbol{r}' \equiv dx'\,dy'\,dz'$$

in the scattering domain V'. By $\boldsymbol{r}$ we denote the position vector of a point within the domain V', while by $\boldsymbol{r}$ – the position vector of an "observation point" ($\boldsymbol{r}$ may be within the domain V' as well as outside this domain). The additional assumption of the "farfield" approximation

$$k_s(r')^2/2r \ll 1, \quad |\boldsymbol{r}'| \ll |\boldsymbol{r}|, \tag{3.8}$$

enables us to write the expression for $A(\boldsymbol{r})$ as

$$A(\boldsymbol{r}) = -\frac{k_s^2}{4\pi}\int_{V'} U(\boldsymbol{r}')\, e^{i\boldsymbol{K}\cdot\boldsymbol{r}'}\, d^3\boldsymbol{r}', \tag{3.9}$$

where

$$\boldsymbol{K} = [\boldsymbol{n} - (\boldsymbol{r}/r)]k_s, \quad K = 2k_s \sin(\theta/2), \tag{3.10}$$

θ being the angle between the vectors $\boldsymbol{r}$ and $\boldsymbol{k}_s$ (the angle of scattering). Condition (3.8) means that we are considering only the solution valid outside the domain V' at large distances r from the inhomogeneities.

Assumptions (2.1), (2.5) and formulae (2.2), (2.3), (2.4), (2.15) enable us to write, with first order accuracy,

$$\langle U(\boldsymbol{r}_1')\, U(\boldsymbol{r}_2')\rangle = \langle \delta\tilde{U}(\boldsymbol{r}_1')\, \delta\tilde{U}(\boldsymbol{r}_2')\rangle = \gamma(x)\langle(\delta\tilde{U}(\boldsymbol{r}_1'))^2\rangle, \quad \boldsymbol{x} = \boldsymbol{r}_1' - \boldsymbol{r}_2', \tag{3.11}$$

where

$$\langle(\delta\tilde{U}(\boldsymbol{r}'))^2\rangle = \left(\frac{\partial\tilde{U}}{\partial\beta}\right)^2_{\beta=0} \langle(\delta\beta)^2\rangle \quad \text{if} \quad \beta \ll 1 \tag{3.12}$$

From (3.6), (3.9), (3.10), (3.11) it follows that

$$\langle|\varphi_{sc}(\boldsymbol{r})|^2\rangle = \frac{B}{r^2}\int_{V'} \gamma(x)\, e^{i\boldsymbol{K}\boldsymbol{x}}\, d^3\boldsymbol{r}_1'\, d^3\boldsymbol{r}_2', \tag{3.13}$$

where

$$B = k_s^4 \langle(\delta\tilde{U}(\boldsymbol{r}'))^2\rangle / 16\pi^2. \tag{3.14}$$

By introducing the new variables

$$\boldsymbol{r}_0 \equiv (x_0, y_0, z_0) = \boldsymbol{r}_1' - \boldsymbol{x}/2 = \boldsymbol{r}_2' + \boldsymbol{x}/2,$$

the integrations over x_0, y_0, z_0 and over all directions can be performed. Performing these integrations ($\boldsymbol{K}$ being the polar axis), and using the Fourier integral transformation for odd functions, we obtain:

$$\gamma(x) = (r^2/2\pi^2 BV')\int_0^\infty \langle|\varphi_{sc}|^2\rangle K^2 \frac{\sin Kx}{Kx}\, dK. \tag{3.15}$$

Using (2.13) and the well-known formula

$$\lim_{x\to 0} \frac{\sin Kx}{Kx} = 1, \tag{3.16}$$

we obtain finally

$$\gamma(x) = \left[\int_0^\infty \langle|\varphi_{sc}|^2\rangle K^2 \frac{\sin Kx}{Kx}\, dK\right] \cdot \left[\int_0^\infty \langle|\varphi_{sc}|^2\rangle K^2 dK\right]^{-1}, \tag{3.17}$$

$$\omega = \text{const}, \quad r = \text{const}.$$

Using formulae (3.6), (3.9), (3.10) the expression for $\nabla\varphi_{sc}(\mathbf{r})$ can be found. Finding this expression and performing the same mathematical operations which had given equation (3.17) from equations (3.6), (3.9), (3.10), we obtain finally

$$\gamma(x) = \left[\int_0^\infty \langle|\nabla\varphi_{sc}|^2\rangle K^2 \frac{\sin Kx}{Kx} dK\right]\cdot\left[\int_0^\infty \langle|\nabla\varphi_{sc}|^2\rangle K^2 dK\right]^{-1}, \tag{3.18}$$

$$\omega = \text{const}, \ r = \text{const}.$$

Formulae (3.17) and (3.18) permit us to determine the correlation function $\gamma(x)$ from the angular distribution of the scalar potential and the gradient of the scalar potential of the wave scattered by a random inhomogeneous isotropic medium, respectively.

4. Angular distribution of the intensity of the acoustic wave scattered by a nonviscous emulsion

The case of a nonviscous emulsion will be considered as an example of a two-phase random heterogeneous material filling the domain V'. In the present model an emulsion is considered as a mixture of two chemically non reacting and nonviscous fluids, one of which is not soluble in the another. One fluid is coherent and volumetrically dominant and the other is dispersed in the forms of grains randomly distributed in the matrix fluid. The fluctuations $\delta c_0(\mathbf{r}')$ and

$$\delta\bar{\varrho}_0(\mathbf{r}') = \bar{\varrho}_0(\mathbf{r}') - \langle\bar{\varrho}_0(\mathbf{r}')\rangle \tag{4.1}$$

(where $\bar{\varrho}_0(\mathbf{r}')$ is the equilibrium value of the density $\varrho_0(\mathbf{r}')$ within the domain V') are the results of fluctuations $\delta\beta$ in the volume concentration β of the grains. In accordance with the basic assumptions of the present paper, the emulsion filling the domain V' is assumed to be immersed in an infinitely extended fluid of density ϱ_s, where the wave velocity $c_s(\mathbf{r}) = c_s = \text{const}$ is known. It is thus also assumed that inequalities (2.1), (2.5) and (2.6) are valid. Furthermore, it is assumed that

$$|\delta\varrho_0(\mathbf{r}')|/\langle\bar{\varrho}_0(\mathbf{r}')\rangle \ll 1. \tag{4.2}$$

The linearized acoustic equations of the system under consideration may be obtained from the general equations of flow, by omitting all the higher order terms in small acoustic disturbances. The acoustic disturbances under consideration in the present paper are assumed to be the periodic fluctuations, of the form given by equations (2.19)–(2.20), in the density $\Delta\varrho(\mathbf{r}, t)$, pressure $\Delta p(\mathbf{r}, t)$ and the velocity $\mathbf{v}(\mathbf{r}, t)$ of the liquid about the equilibrium values

$$\bar{\varrho}(\mathbf{r}, t) = \bar{\varrho}(\mathbf{r}), \quad \bar{p}(\mathbf{r}, t) = p_0 = \text{const}, \quad \bar{\mathbf{v}}(\mathbf{r}, t) = 0, \tag{4.3}$$

respectively. The fluctuations are assumed to be adiabatic, i.e.

$$\frac{dp(\boldsymbol{r},t)}{dt} = c_i^2(\boldsymbol{r})\frac{d\varrho(\boldsymbol{r},t)}{dt}, \tag{4.4}$$

subject to the rules given under equation (3.1) for $i = 0$ or $i = s$.

It is assumed [1] that the equations of flow in the case under consideration have the following form:

$$\frac{d\varrho(\boldsymbol{r},t)}{dt} + \varrho(\boldsymbol{r},t)\operatorname{div}\boldsymbol{v}(\boldsymbol{r},t) = 0, \tag{4.5}$$

$$\varrho(\boldsymbol{r},t)\frac{d\boldsymbol{v}(\boldsymbol{r},t)}{dt} + \nabla p(\boldsymbol{r},t) = 0. \tag{4.6}$$

In order to determine the range of applicability of the nonviscous emulsion approximation given by equations (4.5) and (4.6), we have to introduce appropriate dimensionless variables into Navier-Stokes equation. In this way it can be verified that the nonviscous emulsion approximation is justified if

$$L_c^2\omega/\eta \gg 1, \quad L_c\langle c_0(\boldsymbol{r}')\rangle/\eta \gg 1, \quad 2\pi\langle c_0(\boldsymbol{r}')\rangle/\omega \gg a, \tag{4.7}$$

where η is the kinematic viscosity and a is the amplitude of the oscillations. Combining equations (4.4), (4.5) and (4.6) we obtain [1] the first order acoustic equation

$$\nabla^2\big(\Delta p(\boldsymbol{r},t)\big) = \big(c_i(\boldsymbol{r})\big)^{-2}\frac{\partial^2\Delta p(\boldsymbol{r},t)}{\partial t^2} + \nabla\ln\bar{\varrho}(\boldsymbol{r})\cdot\nabla\big(\Delta p(\boldsymbol{r},t)\big), \tag{4.8}$$

where $c_i(\boldsymbol{r}) = c_0(\boldsymbol{r}')$ and $\nabla\ln\bar{\varrho}(\boldsymbol{r}) \neq 0$ if $\boldsymbol{r}$ is within the domain V', and $c_i(\boldsymbol{r}) = c_s = \text{const}$ and $\nabla\ln\bar{\varrho}(\boldsymbol{r}) = 0$ if $\boldsymbol{r}$ is outside the domain V' or on the boundary of V'.

By substituting

$$\Delta p(\boldsymbol{r},t) = \Delta p(\boldsymbol{r})e^{i\omega t} = \Delta p(\boldsymbol{r})e^{ik_sc_st}, \tag{4.9}$$

we obtain

$$(\nabla^2 + k_s^2)\Delta p(\boldsymbol{r}) = k_s^2 U_p(\boldsymbol{r})\Delta p(\boldsymbol{r}), \tag{4.10}$$

where $U_p(\boldsymbol{r})$ is given, to the first order, by the expression

$$U_p(\boldsymbol{r}) = U(\boldsymbol{r}) + \big(1/k_s^2\langle\bar{\varrho}(\boldsymbol{r})\rangle\big)\,\nabla\big(\delta\bar{\varrho}(\boldsymbol{r})\big)\cdot\nabla, \tag{4.11}$$

k_s and $U(\boldsymbol{r})$ are given by formulae (3.2) and (2.2), respectively.

The pressure disturbance $\Delta p(\boldsymbol{r},t)$ is taken as a sum of a monochromatic travelling plane wave (the incident wave) $P_0e^{i(\omega t + \boldsymbol{k}_s\boldsymbol{r})}$ superposed on another, called the *scattered wave*, $\Delta p_{sc}(\boldsymbol{r})e^{i\omega t}$. Thus the solution of equation (4.10) is taken as

$$\Delta p(\boldsymbol{r}) = P_0e^{i\boldsymbol{k}_s\cdot\boldsymbol{r}} + \Delta p_{sc}(\boldsymbol{r}). \tag{4.12}$$

The incident wave $P_0 e^{i\boldsymbol{k}_s\cdot\boldsymbol{r}}$ is the solution of equation (4.10) for the case of $U_p(\boldsymbol{r}) = 0$. The assumption

$$|\Delta p_{\mathrm{sc}}(\boldsymbol{r})| \ll |P_0 e^{i\boldsymbol{k}_s\cdot\boldsymbol{r}}| \tag{4.13}$$

leads to

$$\Delta p_{\mathrm{sc}}(\boldsymbol{r}) = (1/r)\, A_p(\boldsymbol{r})\, e^{ik_s r}, \tag{4.14}$$

where $A_p(\boldsymbol{r})$ is given by formula (3.7) if $U(\boldsymbol{r})$ is replaced by $U_p(\boldsymbol{r})$, and $e^{i\boldsymbol{k}_s\cdot\boldsymbol{r}'}$ is replaced by $P_0 e^{ik_s r'}$. Using the assumptions (2.1), (2.5), (2.6), and (3.8), we arrive at the approximate (to first order) integral expression

$$A_p(\boldsymbol{r}) = \frac{-k_s^2 P_0}{4\pi}\int\limits_{V'}\left[2\,\frac{\delta c_0(\boldsymbol{r}')}{c_s} + \frac{i\boldsymbol{k}_s}{\langle\bar{\varrho}(\boldsymbol{r}')\rangle k_s}\cdot\nabla\big(\delta\bar{\varrho}(\boldsymbol{r}')\big)\right] e^{i\boldsymbol{K}\cdot\boldsymbol{r}'}\, d^3 r, \tag{4.15}$$

where $\boldsymbol{K}$ is given by formula (3.10). These same assumptions together with the assumptions of (2.18) and formulae (2.16), (3.11) and (3.12) enable us to write, with first order accuracy,

$$\langle|\Delta p_{\mathrm{sc}}(\boldsymbol{r})|^2\rangle = \frac{B_p}{r^2}\int\limits_{V'}\gamma(x)\, e^{i\boldsymbol{K}\cdot\boldsymbol{x}}\, d^3\boldsymbol{x}, \tag{4.16}$$

where

$$B_p = \frac{k_s^4 P_0^2 V'}{16\pi^2}\langle(\delta\beta)^2\rangle\left[\frac{2}{c_s}\left(\frac{\delta c_0(\boldsymbol{r})}{\partial\beta}\right)_{\beta=0} + \frac{\boldsymbol{k}_s\cdot\boldsymbol{K}}{k_s^2\langle\bar{\varrho}(\boldsymbol{r}')\rangle}\left(\frac{\partial\bar{\varrho}(\boldsymbol{r}')}{\partial\beta}\right)_{\beta=0}\right]^2, \tag{4.17}$$

and $\boldsymbol{x}$ is given by formula (3.11). In the case of liquids (emulsion) the following inequality [3] is valid:

$$\left|\frac{\boldsymbol{k}_s\cdot\boldsymbol{K}}{k_s\langle\bar{\varrho}(\boldsymbol{r})\rangle}\left(\frac{\partial\bar{\varrho}(\boldsymbol{r}')}{\partial\beta}\right)_{\beta=0}\right| \ll \left|\frac{2}{c_s}\left(\frac{\partial c_0(\boldsymbol{r}')}{\partial\beta}\right)_{\beta=0}\right|. \tag{4.18}$$

Thus B_p may be calculated, with the desired degree of accuracy, from the following formula:

$$B_p = \frac{k_s^4 P_0^2 V'}{16\pi^2}\langle(\delta\beta)^2\rangle\left[\frac{2}{c_s}\left(\frac{\partial c_0(\boldsymbol{r}')}{\partial\beta}\right)_{\beta=0}\right]^2. \tag{4.19}$$

Formula (4.16) then agrees with the relevant formula given in [5].

Substituting into (4.16), (4.19) the relations

$$I_0 = AP_0^2, \qquad I_{\mathrm{sc}} = A\langle|\Delta p_{\mathrm{sc}}(\boldsymbol{r})|^2\rangle, \qquad A = \mathrm{const}, \tag{4.20}$$

we obtain, after integrating in all directions ($\boldsymbol{K}$ being the polar axis):

$$I_{\mathrm{sc}} = I_0\frac{k_s^4 V'}{4\pi r^2 K}\langle(\delta\beta)^2\rangle\left[\frac{2}{c_s}\left(\frac{\partial c_0(\boldsymbol{r}')}{\partial\beta}\right)_{\beta=0}\right]^2\int\limits_0^\infty x\gamma(x)\sin Kx\, dx, \tag{4.21}$$

where I_0 and I_{sc} denote the intensity of the incident and scattered waves, respectively. This relation can be regarded as an integral equation for the correlation function $\gamma(x)$. Using the Fourier integral transformation for odd function and formulae (2.13), (3.16) we finally obtain

$$\gamma(x) = \left[\int_0^\infty I_{sc} K^2 \frac{\sin Kx}{Kx}\, dK\right] \cdot \left[\int_0^\infty I_{sc} K^2\, dK\right]^{-1}, \qquad (4.22)$$

$$\omega = \text{const}, \quad r = \text{const}.$$

Formula (4.22) enables us to determine the correlation function $\gamma(x)$ from the angular distribution of the intensity of the wave scattered by a random isotropic nonviscous emulsion.

5. Final remarks

It has been shown that it is possible to determine the correlation function $\gamma(x)$ from the angular distribution of the scattered scalar potential. This may be done using formula (3.17). However, formula (3.17) has rather theoretical value. In contrast to formula (3.17) the basic results of sections 3 and 4 have a practical value. With the help of formulae (3.18) and (4.22), their value can be seen in the fact that those enable us to determine the correlation function $\gamma(x)$ from the appropriate measurements of the angular distribution of the intensity of the wave scattered by a random isotropic granular medium. The function $\gamma(x)$ which drops from 1 to 0 indicates the average extension of inhomogeneities. As a measure for their size we could adopt the value L_c of x for which $\gamma(x)$ becomes equal to $1/e$.

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Received on 10th September 1977