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THERMAL EFFECTS IN SOFT TISSUES DEVELOPED UNDER THE ACTION OF ULTRASONIC FIELDS OF LONG DURATION *)

LESZEK FILIPCZYŃSKI

The Department of Ultrasounds of the Institute of Fundamental Technological Research
(Warszawa)

The thermal effect arising from the sonification of soft tissues by an ultrasonic beam, cylindrical in shape, with the assumption of an even distribution of heat sources in the beam, has been considered.

The analysis is based on the solution of the thermal conductivity equation, using the Laplace transformation as in the author's paper [1]. The formulae obtained permit determination of the rise in and distribution of temperature inside and outside the ultrasonic beam for sonification times longer than 20 s.

The formulae have been applied to estimate the temperature changes encountered in ultrasonic continuous wave Doppler methods used in medical diagnosis. For example, with a cylindrical ultrasonic beam of radius 2.2 mm, frequency 5 MHz, mean spatial intensity of 0.1 W/cm^2 and sonification time of 100 s, the estimated value of the temperature increase at the centre of the beam was 1.8 °C.

The values obtained are overestimated since they do not consider the transfer of heat by the circulating blood or the thermal conductivity along the ultrasonic beam, which is particularly evident for higher frequencies.

1. Introduction

In the previous work [1] the author considered thermal effects in soft tissues due to the action of focused ultrasonic fields of short duration (i.e. from microseconds to seconds). The microsecond duration ultrasonic fields occur in impulse ultrasonography used for the visualization of internal organs of the human body, e.g. in gynaecology and obstetrics.

However, in ultrasonic Doppler methods of blood flow measurements, considerably longer sonification is used, sometimes lasting up to several minutes.

*) This paper is written within the framework of the problem MR I-24.

In order to estimate the thermal effects which may occur as a result of such a long sonification, it was decided to use the results obtained in [1], extending them to the considerably longer times of sonification.

2. The temperature at the focus of the ultrasonic field

It will be assumed that the focus (of the beam) has the shape of an infinite cylinder of radius R (cm). It is further assumed that heat sources of strength $\dot{Q}_v[\mathrm{cal\ s^{-1}\ cm^{-3}}]$ [1]) are located within this volume. According to [1], the Laplace transformation for the increment T_i of temperature increase within the focus can be written in the form

$$\overline{T}_i = \frac{\dot{Q}_v}{s^2 \varrho c_w}\left[1+\frac{\pi}{2}\sqrt{\frac{s}{a}}RH_1^{(1)}\left(i\sqrt{\frac{s}{a}}R\right)J_0\left(i\sqrt{\frac{s}{a}}r\right)\right], \tag{1}$$

where s is the complex variable, ϱ – the density of medium [g cm^{-3}], c_w – the specific heat of the medium [cal g^{-1} °C^{-1}], $a = \lambda/\varrho c_w$, λ is the coefficient of thermal conductivity [cal cm^{-1} °C^{-1}], $i = \sqrt{-1}$, $H_n^{(1)}$ is a Hankel function of the first order equal to $H_n^{(1)} = J_n + i N_n$, J_n – the Bessel function, and N_n – the Neumann function. The subscripts at H, J and N denote the order of functions.

The inverse transform of expression (1) will be determined by a series expansion of the Bessel and Hankel functions. From the similarity theorem [5] between the transformed function f and its transform $\bar{f}$,

$$L[f(at)] = \frac{1}{a}\bar{f}\left(\frac{s}{a}\right), \tag{2}$$

it can be seen that large arguments of the transformed function f correspond to small arguments $i\sqrt{s/a}x$ $(x = R, r)$ of the transform $\bar{f}$. This is of interest to us due to the longer time of sonification and we shall thus use the series expansions [3, 5] (valid for all α)

$$J_0(i\alpha) = 1+\frac{(\alpha/2)^2}{1!\,1!}+\frac{(\alpha/2)^4}{2!\,2!}+\frac{(\alpha/2)^6}{3!\,3!}+\ldots, \tag{3}$$

$$J_1(i\alpha) = i\left[\frac{\alpha/2}{0!\,1!}+\frac{(\alpha/2)^3}{1!\,2!}+\frac{(\alpha/2)^5}{2!\,3!}+\ldots\right], \tag{4}$$

$$N_0(i\alpha) = \frac{2}{\pi}\left[\left(\ln\frac{\gamma i\alpha}{2}\right)J_0(i\alpha)-\frac{(\alpha/2)^2}{1!\,1!}-\left(1+\frac{1}{2}\right)\frac{(\alpha/2)^4}{2!\,2!}-\left(1+\frac{1}{2}+\frac{1}{3}\right)\frac{(\alpha/2)^6}{3!\,3!}-\ldots\right], \tag{5}$$

[1]) Since this article is a continuation of the author's paper [1] which used the CGS-system of units, the same system will also be used here.

$$N_1(ia) = \frac{2}{\pi}\left[\left(\ln\frac{\gamma ia}{2}\right) J_1(ia) - \frac{1}{ia} - i\left(1-\frac{1}{2}\right)\frac{a/2}{0!\,1!} - \right.$$

$$\left. - i\left(1+\frac{1}{2}-\frac{1}{4}\right)\frac{(a/2)^3}{1!\,2!} - i\left(1+\frac{1}{2}+\frac{1}{3}-\frac{1}{6}\right)\frac{(a/2)^5}{2!\,3!} - \dots\right], \quad (6)$$

where $\gamma = 1.781072 \dots$ is a constant.

Substituting expansions (3)-(6) into (1), using the formula

$$\ln\frac{\gamma}{2} i \sqrt{\frac{s}{a}}\, x = \frac{\pi}{2} i + \frac{1}{2} \ln s \frac{\gamma^2 x^2}{4a} \quad (7)$$

and neglecting small order quantities, we finally obtain

$$\overline{T}_i = \frac{\dot{Q}_v}{\varrho c_w}\left[\frac{1}{4as}(R^2 - r^2) + \frac{1}{64\, a^2}(5\, R^4 + 4\, R^2 r^2 - r^4) - \right.$$

$$\left. - \frac{R^2}{4as} \ln s \frac{\gamma^2 R^2}{4a} - \frac{1}{32 a^2}(R^4 + 2R^2 r^2) \ln s \frac{\gamma^2 R^2}{4a}\right]. \quad (8)$$

Using the relations [5]

$$L^{-1}\left[\frac{1}{s}\ln ks\right] = -\ln\frac{\gamma t}{k}, \quad (9)$$

$$L^{-1}[\ln ks] = -\frac{1}{t}, \quad (10)$$

$$L^{-1}\left[\frac{1}{s}\right] = 1, \quad (11)$$

$$L^{-1}[s^n] = 0, \quad n \geqslant 0, \quad (12)$$

we calculate the inverse transform of expression (8) and obtain the final temperature within the focal volume ($r \leqslant R$) as

$$T_i = \frac{\dot{Q}_v}{\varrho c_w}\left[\frac{1}{4a}(R^2 - r^2) + \frac{R^2}{4a}\ln\frac{4a}{\gamma R^2}\, t + \frac{R^4 + 2R^2 r^2}{32 a^2}\,\frac{1}{t}\right]. \quad (13)$$

From equation (13) we can determine the maximum value T_M of the temperature, which occurs at the centre of the focus ($r = 0$) to be

$$T_M = \frac{\dot{Q}_v}{\varrho c_w}\left[\frac{R^2}{4a}\left(1 + \ln\frac{4a}{\gamma R^2}\, t\right) + \frac{R^4}{32 a^2}\,\frac{1}{t}\right]. \quad (14)$$

At the boundary of the focus ($r = R$) we have

$$T_i = \frac{\dot{Q}_v}{\varrho c_w}\left[\frac{R^2}{4a}\ln\frac{4a}{\gamma R^2}\, t + \frac{3R^4}{32 a^2}\,\frac{1}{t}\right]. \quad (15)$$

Formulae (13)-(15) fail for $a \to 0$ and for $t \to 0$ due to the higher order terms neglected in expansions (3)-(6) of the Bessel and Hankel functions.

3. The temperature distribution outside the focal region

The temperature outside the focal region will be determined on the basis of formula (3) of paper [1]. Thus we have.

$$\overline{T}_o = \frac{\dot{Q}_v}{s^2 \varrho c_w} \frac{\pi}{2} \sqrt{\frac{s}{a}} R J_1\left(i\sqrt{\frac{s}{a}}R\right) H_0^{(1)}\left(i\sqrt{\frac{s}{a}}r\right). \tag{16}$$

The inverse transform of this expression will be determined as before using expansions (3)-(6). Substituting these into expression (16) and neglecting the higher order terms, we obtain the transform of the temperature outside the focal region as

$$\overline{T}_o = \frac{\dot{Q}_v}{\varrho c_w}\left[\frac{R^2 r^2}{8a^2} - \frac{R^2}{4sa}\ln s\frac{\gamma^2 r^2}{4a} - \frac{R^2(R^2+2r^2)}{32a^2}\ln s\frac{\gamma^2 r^2}{4a}\right]. \tag{17}$$

Using relations (9)-(12) we can calculate the inverse transform in the form

$$T_o = \frac{\dot{Q}_v}{\varrho c_w}\left[\frac{R^2}{4a}\ln\frac{4a}{\gamma r^2}t + \frac{R^2(R^2+2r^2)}{32a^2}\frac{1}{t}\right]. \tag{18}$$

For $a \to 0$, $t \to 0$ and $r \to \infty$ formula (18) fails for the same reason as before.

It can easily be seen that on the boundary of the focal region ($r = R$) we obtain from formula (18), as expected, the expression already given by formula (15).

The temperature distributions, calculated from formulae (13) and (18) inside and outside the focal region, are shown in Fig. 1. Fig. 2 shows the temperature at the centre of the focus ($r = 0$) as a function of the time of sonification, calculated on the basis of formula (14) for times $t > 20$ s and $t < 10$ s. The latter were taken from [1]. These latter distributions were calculated for the conditions encountered in pulse-echo ultrasonic obstetric and gynaecological diagnostic investigations [2]. The focus of the ultrasonic beam was in this case approximated by a cylinder of diameter of 0.125 cm and length 6.2 cm.

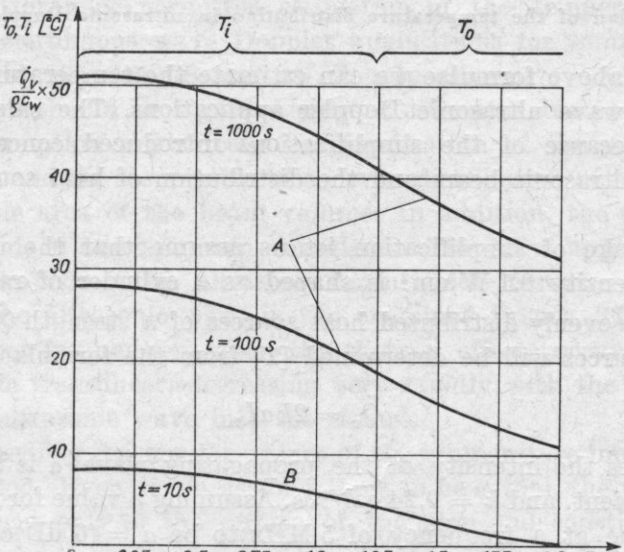

Fig. 1. The distribution of the temperature inside ($r \leqslant 1.25$ mm) and outside ($r \geqslant 1.25$ mm) the focal region at different times of sonification t as a function of r, calculated from formulae (13) and (18) (curve A), and taken from [1] (curve B)

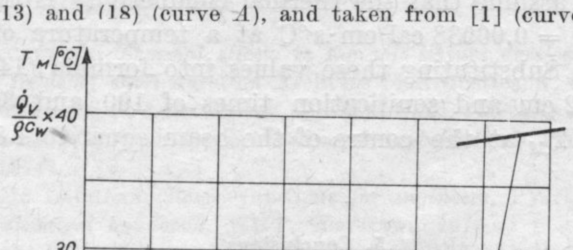

Fig. 2. The temperature at the centre of the focus of the ultrasonic beam, for sonification times $t \geqslant 20$ s, calculated on the basis of formula (14) (curve A) and taken from [1] for sonification times $t \leqslant 10$ s (curve B)

4. The estimation of the temperature distribution in ultrasonic Doppler applifications

From the above formulae we can estimate the temperatures encountered in continuous wave ultrasonic Doppler applications. The calculation will be an estimate because of the simplifications introduced concerning the geometry of the ultrasonic beam and the distribution of heat sources within the beam.

For the sake of simplification let us assume that the ultrasonic beam of uniform intensity 0.1 W/cm² is shaped as a cylinder of radius 2.2 mm in which there are evenly distributed heat sources of a strength $\dot{Q}_v$. The strength of the heat sources will be determined [1] from the formula

$$\dot{Q}_v = 2k\alpha I, \tag{19}$$

where I denotes the intensity of the propagating wave, α is the pressure absorption coefficient, and $k = 0.24$ cal/Ws. Assuming a value for the coefficient α of muscle tissue at a frequency of 5 MHz to be $\alpha = (6\ \mathrm{dB/cm}) \times (8.67\ \mathrm{dB})^{-1} = 0.69\ \mathrm{cm}^{-1}$, we obtain the values $\dot{Q}_v/k = 0.14\ \mathrm{W/cm^3}$ and $\dot{Q}_v/\varrho c_w = 0.033\ ^\circ\mathrm{C/s}$.

In view of the fact that water constitutes 75% of the content of the cells of soft tissues, we assume that the thermal conductivity is the same as that for water. Thus $\lambda = 0.00038\ \mathrm{cal/cm \cdot s\ ^\circ C}$ at a temperature of 30°C [4] and $a = 0.00038\ \mathrm{cm^2/s}$. Substituting these values into formula (14), with a beam radius of $R = 0.22$ cm and sonification times of 100 and 300 s, we obtain temperature rises T_M at the centre of the beam equal to 1.8 °C and 2.8 °C, respectively.

5. Conclusions

The formulae given in this paper permit an estimation of the temperature increases caused by cylindrical ultrasonic fields. In practice, this may be a focused ultrasonic beam with a focus in the form of an elongated cylinder or a cylindrical ultrasonic beam. The formulae are derived for long sonification times compared to those for the formulae in [1]. Figure 2 shows a comparison of these with the calculated temperature increases T_M on the focal axis (obtained for the conditions encountered in ultrasonic gynaecological and obstetric diagnostic investigations) for sonification times shorter than 10 s and longer than 20 s. The extrapolations of these curves tend to coincidence. However, the comparatively narrow time range $10\ \mathrm{s} < t < 20\ \mathrm{s}$ is not covered by the calculations, which yield different values due to the different approximations used for the Hankel and Bessel functions in the case of short and long sonification times.

The character of the temperature distribution inside and outside the focal region (Fig. 1) for long times of sonification (curves A) corresponds to the character of the curves calculated for short times of sonification (curves B).

The above formulae permitted estimation of the temperature rises T_M in the case of continuous wave Doppler applications for sonification times t equal to 100 and 300 s as 1.8 and 2.8 °C, respectively.

These values should be regarded only as estimates due to the number of simplifying assumption introduced: the idealization of the geometry of the ultrasonic beam and the assumption of an even distribution of the heat sources within the whole area of the beam volume. In addition, the transfer of heat by circulating blood has not been considered. The same applies to the flow of heat along the axis of the ultrasonic beam which increases with increasing absorption of the ultrasonic wave in the examined tissues. This is especially evident at higher frequencies, when the thermal effects occur at the surface of the ultrasonic transducer, decreasing very rapidly with the depth of penetration of the ultrasonic wave into the tissues.

For the foregoing reasons the values of the temperature increases obtained are overestimated. Even if we assume them to be actual, the values of a few degrees may occur only at the surface of the skin and constitute no danger to the patient.

References

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