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ON AN APPROXIMATE METHOD FOR CALCULATING FILTERS BASED ON ACOUSTIC SURFACE WAVES

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The work describes an electric filter consisting of interdigital transducers deposited on piezoelectric substrate. A method has been presented for computing the transducer parameters with an assumed amplitude-frequency characteristic.

The filters produced on the basis of calculated transducer parameters were tested and the results are given in this paper.

1. Discussion on transducers

The possibility of constructing devices based on the phenomenon of the propagation of elastic waves in a solid created considerable interest in the analysis of interdigital transducers.

There are a few methods employed for analyzing transducers, for example the method of an equivalent circuit [1, 5, 6] and that of strips.

One of the most convenient methods, because of its simplicity and reliability, is to calculate the form of an amplitude-frequency characteristic for a given shape of transducer as proposed by TANCRELL and HOLLAND [1]. This procedure is relatively simple but it requires repeated calculations by trial and error.

In this procedure it is assumed that a period of an alternating acoustical field is equal to a period of electrical field gradient. Since the electric field gradient is non-uniform an approximate assumption is that the energy sources appear only on the edge of each electrode and, therefore, they can be described by the function δ (Fig. 1).

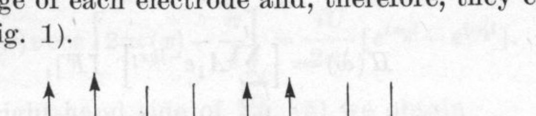

Fig. 1. The electric field gradient at the edges of electrodes

The frequency response of the transducer in this case is described by the expression

$$\sum_{l=-\infty}^{l=+\infty} \frac{r(x_l)}{4z'(x_l)} \cos\left[\frac{\pi}{2}e - \frac{\pi}{4}\right] e^{-ikx_l}$$

$$= \int_{-\infty}^{+\infty} \left\{ r(x_l)\cos\left[2\pi z(x) - \frac{\pi}{4}\right] e^{-ikx}\right\} dx + \quad \text{1st harmonic}$$

$$+ \int_{-\infty}^{+\infty} \left\{ r(x_l)\cos\left[6\pi z(x) - \frac{\pi}{4}\right] e^{-ikx}\right\} dx + \quad \text{3rd harmonic}$$

$$+ \int_{-\infty}^{+\infty} \left\{ r(x_l)\cos\left[10 z(x) - \frac{\pi}{4}\right] e^{-ikx}\right\} dx + \ldots \quad \text{5th harmonic} \tag{1}$$

where $k = \omega/v$, ω is the angular frequency, v — propagation velocity of the surface wave, l — number of successive edges on the transducer, $r(x)$ — function of electrode overlapping, the envelope of mutual penetration, $z(x)$ — function of the number of electrode pairs, $2z'(x) = 2dz(x)/dx$ the density of electrodes.

It follows from the above analysis that the higher odd harmonics will be present with the fundamental.

The frequency transmittance of a system consisting of one transmitting and one receiving transducer takes the form

$$H(\omega) = \frac{U(\omega)}{U(\omega)} = \sum_{k=1}^{K} \sum_{l=1}^{L} A_k A_l e^{i(x_l - X_k)\frac{\omega}{v}}, \tag{2}$$

where K is the total number of edges in the receiver, L — total number of edges in the transmitter, x_l — distance between the lth edge in the transmitter and the beginning of the transducer, X_k — distance between the kth edge in the receiver and the beginning of the transmitting transducer, A_k, A_l — coefficients depending on the geometry of electrodes; these are functions of the phase and the electrical field gradient.

For a system of two transducers in which one is a broadband unit with a small number of electrodes, while the other is a narrowband unit with a complex structure of electrodes, the function $H(\omega)$ can be expressed as follows

$$H(\omega) = \left[\sum_{l=1}^{l} A_l e^{-ikx_l}\right] \; [F], \tag{3}$$

where $[F]$ is a frequency response of a broadband teeth arrangement that can be treated as a constant, since it exerts no influence on the frequency characteristics of the system. The transmission band of the transmitting transducer is comprised within the transmission band of the receiving transducer. As follows from expression (3) there exists in this case a direct relation between Fourier transformation of the teeth arrangement mode and the transmittance $H(\omega)$.

2. Computation of the data for filter designing

The purpose of the present work is to propose a modification of the method [1] employed for computing filters based on acoustical surface waves.

This modification consists of computing the transducer shape corresponding to the a priori assumed amplitude-frequency characteristics.

Let us determine such a function to overlap the digits of an interdigital transducer operating in conjunction with a broadband transducer such that the frequency transmittance is of the form

$$H(\omega) = \begin{cases} C & \text{for } \omega_1 < \omega < \omega_2, \\ 0 & \text{for remaining range,} \end{cases}$$

where ω_0 is the resonance frequency, and ω_2, ω_1 — upper and lower frequency limits, respectively.

This discussion concerns only the fundamental frequency. According to (1) we have

$$H(\omega) = \sum_{l=-\infty}^{l=\infty} \frac{r(x_l)}{4z'(x_l)} \cos\left[\frac{\pi}{2} l - \frac{\pi}{4}\right] e^{-ikx_l}$$

$$= \int_{-\infty}^{+\infty} r(x) \cos\left[2\pi z(x) - \frac{\pi}{4}\right] e^{-ikx} dx$$

$$= \int_{-\infty}^{+\infty} r(x) \cos\left[2\pi z(x) - \frac{\pi}{4}\right] e^{-ikx} v dt. \qquad (4)$$

The inverse Fourier transformation of the above expression gives

$$r(x) v \cos\left[2\pi z(x) - \frac{\pi}{4}\right] = \frac{1}{2\pi} \int_{-\infty}^{+\infty} H(x) e^{i\omega t} d\omega = \frac{1}{2\pi} \int_{\omega_1}^{\omega_2} C e^{i\omega t} d\omega,$$

$$r(x) v \cos\left[2\pi z(x) - \frac{\pi}{4}\right] = \frac{iC}{2\pi t} [e^{i\omega_2 t} - e^{i\omega_1 t}]. \qquad (5)$$

By developing the right-hand side of Eq. (5) we obtain

$$r(x) v \cos\left[2\pi z(x) - \frac{\pi}{4}\right] = \frac{C}{\pi v t} [\sin \omega_2 t - \sin \omega_1 t], \qquad (6)$$

and this gives the following set of equations

$$r(x) = \frac{C}{\pi v t}\sin\frac{\omega_2-\omega_1}{2v}t, \tag{7a}$$

$$\cos\left[2\pi z(x)-\frac{\pi}{4}\right] = \cos\frac{\omega_1+\omega_2}{2v}x, \tag{7b}$$

where $t = x/v$.

It follows from (7a) and (7b) that in order tc satisfy the condition (4) the envelope of electrode overlap and the function of a number of electrode pairs should take the form

$$r(x) = \frac{C\Delta\omega}{2\pi v}\frac{\sin Bx}{Bx}, \tag{8}$$

where

$$B = \frac{\Delta\omega}{2v},\quad z(x) = \frac{\omega_0}{2\pi v}x+\frac{1}{8},\quad z'(x) = \frac{\omega_0}{2\pi v} = \frac{1}{\lambda} = \text{const}. \tag{9}$$

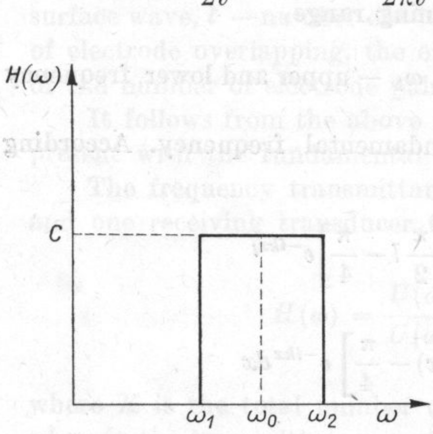

Fig. 2. The rectangular characteristics of a filter

Expression (9) shows that the transducer is non-dispersive

$$z(x) = \frac{l}{m} = \frac{l}{4} = \frac{\omega_0}{2\pi v}x_l+\frac{1}{8}, \tag{10}$$

where m is the number of edges per period, and hence

$$x_l = \frac{2\pi v}{\omega_0}\left(\frac{l}{4}+\frac{1}{8}\right) \tag{11}$$

for $l = 0, 1, 2, \ldots$

If the width of electrodes is equal to the gap between them, then

$$x_l - x_{l-1} = \frac{v}{4\omega_0}2\pi. \tag{12}$$

The electrodes are reversed at the points at which the amplitudes D_n [1] are equal to zero

$$D_n = \pm \frac{1}{\sqrt{2}} \frac{r(x_n)\,v}{4z'(x_n)_r}. \tag{13}$$

Taking into account Eqs. (8) and (9), we obtain the expression

$$D_n = \pm \frac{\pi}{2\sqrt{2}} \frac{v^2}{\omega_0} \frac{\sin\left(\frac{\Delta\omega}{2v}\,x\right)}{\frac{\Delta\omega}{2v}\,x}. \tag{14}$$

For $D_n = 0$ the following is valid:

$$\frac{\sin\left(\frac{\Delta\omega}{2v}\,x\right)}{\frac{\Delta\omega}{2v}\,x} = 0$$

Hence for

$$\frac{\Delta\omega}{2v}\,x \neq 0 \tag{15}$$

we have

$$\sin\left(\frac{\Delta\omega}{2v}\,x\right) = 0. \tag{16}$$

The roots of Eq. (15) are determined by

$$x_n = n\,\frac{2\pi v}{\Delta v}, \quad n = 1, 2, 3, \ldots \tag{17}$$

We see then that if we need a system with a rectangular transmittance $H(\omega)$ the method provides all data necessary for designing the appropriate transducers. This fact is important because in practice we are frequently interested in systems with such amplitude-frequency characteristics.

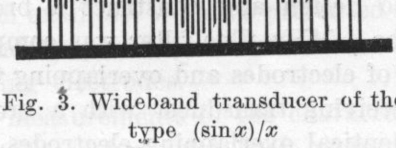

Fig. 3. Wideband transducer of the type $(\sin x)/x$

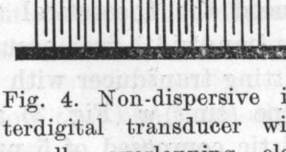

Fig. 4. Non-dispersive interdigital transducer with equally overlapping electrodes

We have also found the function for the overlapping teeth of an interdigital transducer (operating in conjunction with a broadband transducer) such that the frequency transmittance of the system is of the form

$$H(\omega) = A\frac{\sin a\omega}{a\omega}, \tag{18}$$

where A and a are constants.

From (1), taking into account only fundamental harmonics, we obtain

$$H(\omega) = \int_{-\infty}^{+\infty} r(x)\cos\left[2\pi z(x) - \frac{\pi}{4}\right] e^{-ikx}dx.$$

The inverse transformation has the form

$$r(x)v\cos\left[2\pi z(x) - \frac{\pi}{4}\right] = \frac{1}{2\pi}\int_{-\infty}^{+\infty} H(\omega)e^{i\omega t}d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} A\frac{\sin a\omega}{a\omega}e^{i\omega t}d\omega. \tag{19}$$

By integration we get

$$r(x)v\cos\left[2\pi z(x) - \frac{\pi}{4}\right] = -\frac{iA}{4\pi a}2\pi i = \frac{A}{2a}. \tag{20}$$

If an interdigital transducer is non-dispersive and its width of electrodes and the distance etween them are constant, then $\cos[2\pi z(x) - \pi/4]$ is also a constant.

In this case, the function of overlap of electrodes is also constant. We thus arrive at the conclusion that the filter consisting of non-dispersive transducers with constant overlapping digits (Fig. 4) has a frequency characteristic of the type $A(\sin a\omega/a\omega)$.

3. Construction of the filter and measurement of its characteristics

We used the theoretical studies to design and construct a broadband filter. The bandwidth was assumed to be 4 MHz. The filter was composed of a transmitting transducer with 19 pairs of electrodes and overlapping function of the type $(\sin x)/x$ (Fig. 3) and a receiving transducer with a broadband characteristic composed of 5 pairs of identical overlapping electrodes.

The maximum length of the electrodes was 6 mm, and the width of an electrode was 50 μm.

The transducers were deposited, using a photolithographic technique, onto a substrate made of piezoelectric ceramics of type PP-6. The attenuation of the input signal (in the transmission band of the system of the transducers spaced at a distance of 5 mm from each other) was 25 dB.

The results of the experiments are illustrated in Fig. 5.

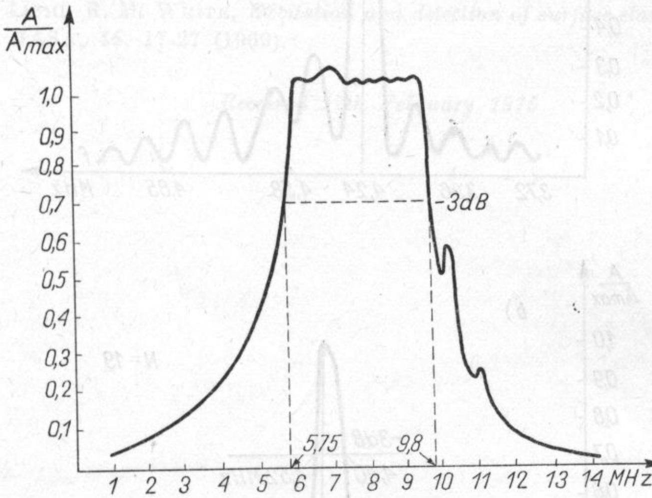

Fig. 5. The filter frequency characteristics

The transmitting transducers of the type $(\sin x)/x$ – the number of electrode pair $N = 19$. The simple receiving transducer – the number of electrode pair $N = 5$

The experimental investigations of the filters consisting of non-dispersive transducers with identically overlapping electrodes confirm the theoretical discussion with respect to their shape and frequency transmittance.

The characteristics were measured for transducers with the numbers of pair of electrodes $N = 11$, 19 and 23.

In spite of the fact that for 11 and 19 pairs of electrodes there is a discrepancy in certain frequency ranges between the characteristic measured and that assumed to be $A = (\sin a\omega)/(a\omega)$, the agreement between the theory and experiment for $N = 23$ is quite satisfactory.

As an example, we have presented the amplitude-frequency characteristics for 23 and 19 pairs of electrodes in non-dispersive transducers with constant overlapping electrodes.

The measurements of the characteristics show that the three-dimensional waves produced have not been attenuated.

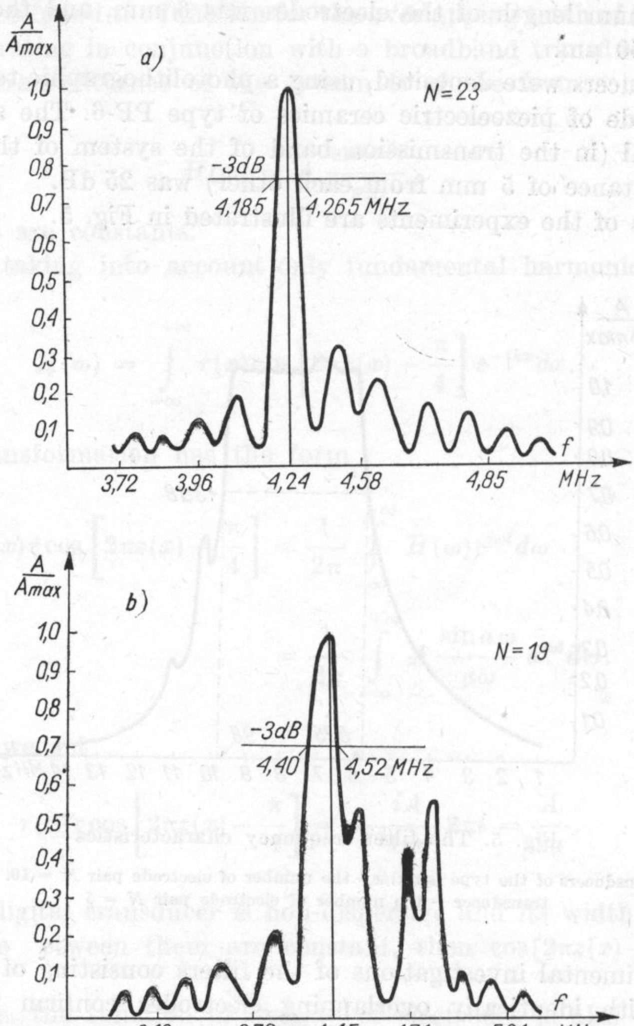

Fig. 6. The characteristics of a transducer system on a piezoelectric ceramic base

4. Conclusions

One of the factors responsible for the discrepancies between theory and experiment is the fact that only the fundamental harmonic has been considered. However, for a transducer with a greater number of electrodes, the contribution of higher odd harmonics in the frequency transmittance is low.

Thus it may be concluded that the method of determining the shape of electrode arrangement for an amplitude-frequency characteristic assumed a priori is convenient because of its simplicity, and the computing accuracy is satisfactory for numerous practical applications.

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