ARCHIVES OF ACOUSTICS
1, 2, 159-169 (1976)

DIFFRACTION OF LIGHT BY ACOUSTIC WAVES IN CRYSTALS

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Diffraction of light by acoustic waves at high frequencies, i.e. Bragg diffraction, is discussed, and the possibility of investigating the acoustic and acousto-optical properties of crystals using this method is presented. The measuring systems used, measurements of the propagation velocities and absorption coefficients of acoustic waves, and the photoelastic constants for crystals of melt quartz, TiO_2, CaF_2, Bi_{12}, GeO_{20}, and $LiNbO_3$ are presented.

1. Introduction

Investigation of the diffraction of light by acoustic waves is an important method of defining the acoustic and acousto-optical properties of solids [1, 2, 4, 7]. It is possible, however, using this method, to determine the velocity of wave propagation and thus also the corresponding elastic constants. Knowledge of these constants is necessary to evaluate the practicability of using crystals in many fields, for instance in field modulation. When investigating the diffraction of light by acoustic waves, two cases [4, 7] are usually considered:

1) Raman-Nath diffraction, observed at frequencies for which the relationship $\Lambda^2/\lambda > l$ is satisfied (where Λ and λ represent the acoustical and optical wavelengths, and l the width of the acoustic beam);

2) Bragg diffraction, observed at higher frequencies for which the relationship $\Lambda^2/\lambda < l$ is satisfied.

In this paper we shall be dealing with the latter case.

Invoking the principles of conservation of momentum and conservation of energy for a photon-phonon collision, one may write

$$\boldsymbol{k}_2 = \boldsymbol{k}_1 \pm \boldsymbol{q}, \tag{1a}$$

$$\omega_2 = \omega_1 \pm \Omega, \tag{1b}$$

where $\boldsymbol{k}_1$, $\boldsymbol{k}_2$, and $\boldsymbol{q}$ represent wave vectors of the incident light, scattered light and acoustic waves, respectively.

Since $\omega_1 \cong \omega_2 \gg \Omega$, it can be assumed that the lengths of the wave vectors $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ are changed because of the different values of optical refraction index in the directions of the incident and scattered light, k.e. $\boldsymbol{k}_1 = \boldsymbol{k}_0 n_1$ and

$\boldsymbol{k}_2 = \boldsymbol{k}_0 n_2$, where $\boldsymbol{k}_0$ is the wave vector of the light wave in a vacuum, while n_1 and n_2 are values of the optical refraction index in the directions of the incident and scattered waves.

With this assumption, and using formulae (1a), (1b) and Fig. 1, it is possible to obtain expressions for the angle of incidence θ_1 and diffraction θ_2 of light,

$$\sin\theta_1 = \frac{\lambda_0 \nu}{2n_1 v}\left[1 + \left(\frac{v}{\lambda_0 \nu}\right)^2 (n_1^2 - n_2^2)\right], \tag{2a}$$

$$\sin\theta_2 = \frac{\lambda_0 \nu}{2n_2 v}\left[1 - \left(\frac{v}{\lambda_0 \nu}\right)^2 (n_1^2 - n_2^2)\right], \tag{2b}$$

where λ_0 denotes the wavelength of light in a vacuum, and ν and v the frequency and velocity of acoustic wave propagation, respectively.

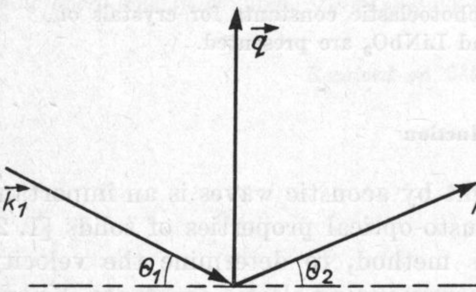


Fig. 1. Wave vector diagram for photon-phonon scattering

If $n_1 = n_2 = n$, i.e. if the medium is optically isotropic, then

$$\sin\theta_1 = \sin\theta_2 = \frac{\lambda_0 \nu}{2nv}. \tag{3}$$

In the general case it is necessary to know the coefficients of diffraction n_1 and n_2 of the angles θ_1 and θ_2, i.e. one must know the surface of wave vectors in a given crystal. This problem is very interesting and has been discussed in papers [5, 6].

Henceforth we can dispense with a detailed knowledge of the Bragg diffraction geometry. It can be seen from formulae (2a) and (2b) that it is possible to determine the velocity of wave propagation in a given direction by measurement of angles θ_1 and θ_2. On the other hand, by making a measurement of the intensity of diffracted light it is possible to determine, for a given crystal, the photoelastic constants and the absorption coefficient of acoustic waves. It is known [4] that the ratio of the intensity of the light diffracted by the acoustic wave to the intensity of incident light is

$$\eta = \frac{I}{I_0} = \frac{\pi^2 n^6 p_{ij}^2 l^2 P_a}{2\lambda_0^2 \varrho v^3}, \tag{4}$$

where I_0 denotes the intensity of incident light, I — the intensity of diffracted light, P_a — the intensity of the acoustic beam, ϱ — the density of medium, and p_{ij} — the photoelastic constants.

Measurements of the photoelastic constants using formula (4) are usually made in relation to some reference substance. The method of measurements is shown in Fig. 2. The reference substance (preferably melt quartz) is glued

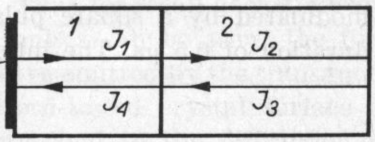


Fig. 2. Method of measurement of photoelastic constants using one transducer

1 — specimen, *2* — investigated pattern, *3* — transducer

with the substance to be tested. The acoustic wave is induced from the reference substance side. If the measurement of the intensity of diffracted light of the incident and reflected waves (I_1 and I_4) is made on the crystal-air boundary for the reference substance as well as for the tested substance (I_2 and I_3), then

$$\left(\frac{I_2 I_3}{I_1 I_4}\right)^{1/2} = \frac{n^6 p^2/\varrho v^3 \quad \text{tested substance},}{n^6 p^2/\varrho v^3 \quad \text{reference substance}.} \tag{5}$$

It is also possible to make measurements of photoelastic constants by inducing to the acoustic wave both from the side of the reference substance and from the side of the substance to be tested [3]. This method should be followed when intensive absorption of acoustic wave occurs in the medium.

As can be seen from formula (4), the intensity of diffracted light is proportional to the intensity of the acoustic beam. It is thus possible to determine the attenuation coefficient of acoustic waves. If we make measurements of the intensity of light diffracted by the acoustic wave at two distances from the transducer x_1 and x_2, the absorption coefficient can be determined from the formula

$$a\left[\frac{\text{dB}}{\text{cm}}\right] = \frac{8.686}{2(x_2 - x_1)} \ln \frac{I(x_1)}{I(x_2)}, \tag{6}$$

where $I(x_1)$ and $I(x_2)$ represent the intensities of diffracted light correspondingly to distances x_1 and x_2 from the transducer.

This measurement can be made for continuous and pulsed waves. In the former case, the crystal is moved in the direction of the acoustic wave and the intensity of the diffracted light measured for two positions of the crystal. While in the latter case, the crystal remains immobilized while the measurements are taken of the intensity of the light resulting from the acoustic wave, which has been reflected repeatedly from the faces of the crystals (see e.g. Fig. 9a). The method is particularly useful in cases where the absorption of the acoustic wave is insignificant, but it requires the use of crystals with precisely parallel faces.

2. Measuring system

The measuring system for investigating the diffraction of light by acoustic waves is shown in Fig. 3. The light source is a 10 mW He-Ne laser, and the source of acoustic waves is a transducer of lithium iodate operating at a fundamental frequency 200 MHz or a thin layer transducer of CdS, operating within the frequency range from 700 to 1300 MHz. The acoustic wave was modulated by a square pulse with a repetition rate of 10 kHz and a pulse duration of 0.5 μs. The intensity of the acoustic wave was about 0.1 W/cm².

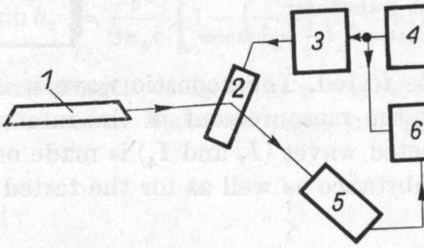
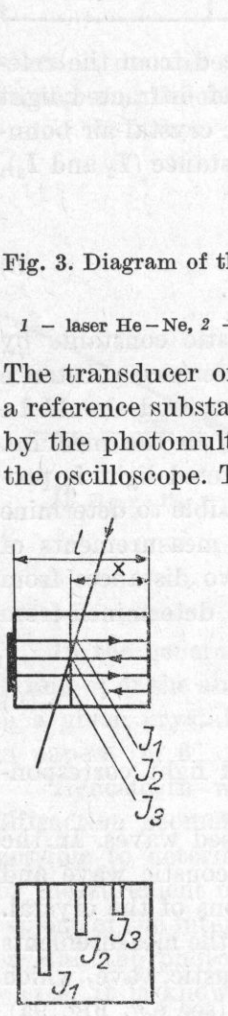


Fig. 3. Diagram of the experimental design used in the investigation of the light diffraction by acoustical waves

1 – laser He–Ne, *2* – cristall, *3* – h. f. generator, *4* – modulator, *5* – photomultipler, *6* – oscyloscope

The transducer of lithium iodate was glued to the melt quartz to be used as a reference substance. The light scattered as a result of dispersion was recorded by the photomultiplier M12FQC51 from which the signal was transmitted to the oscilloscope. The pulses obtained are shown schematically in Figs. 4 and 5.



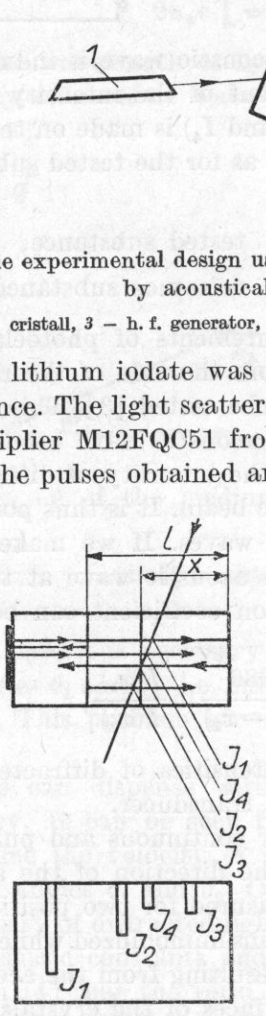


Fig. 4. Diagram of oscillogram patterns for standard sample

Fig. 5. Diagram of oscillogram for investigated substances

In case of the light diffracted in the quartz itself (Fig. 4) the first pulse (I_1) corresponds to the diffraction by the wave incident directly from the transducer, the other pulse (I_2) corresponds to the diffraction by the wave reflected from the end of the crystal, while the third pulse (I_3) — to the diffraction of the light reflected from the transducer etc. As the laser beam moves towards the end of the crystal, the distance between the pulses I_1 and I_2 decreases, but that between the pulses I_1 and I_3 remains constant at a value corresponding to the time of passing through the double crystal length. When the specimen crystal (Fig. 5) is gived to the quartz reference, the acoustic wave is reflected from the crystal—air surface. Thus for small acoustic wave absorption in a specimen crystal the following pulses can be seen: the first pulse that corresponds to the diffraction by the wave emitted by the transducer and passing directly through the reference pattern-tested crystal surface of separation. This is followed by pulses that correspond to the diffraction of light by acoustic waves reflected from following surfaces: specimen crystal — air, specimen crystal — reference substance, reference substance — transducer, etc. depending on the length of crystal and the velocity of acoustic wave propagation.

Fig. 6 shows the system used for measuring the absorption coefficient of the acoustic waves. The light beam reflected by the acoustic wave is divided into two parts by means of a semi-transparent mirror. One of them, after being reflected from a totally reflecting mirror, is passed through the polarizer

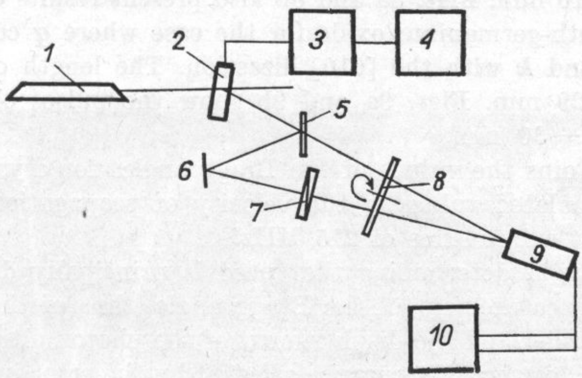


Fig. 6. Diagram of the experimental design used in the measurement of the absorption coefficient of acoustic waves

1 — laser He—Ne, *2* — tested cristall, *3* — high frequency (h. f.) generator, *4* — modulator, *5* — semi—transparent mirror, *6* — mirror, *7* — polarizer, *8* — mechanical modulator, *9* — photomultiplier, *10* — oscilloscope

and the mechanical modulator and is incident on the photomultiplier. The second part of the diffracted beam is passed through the modulator. The signal from the photomultiplier is fed to a double beam oscilloscope. The mechanical modulator, operating at a frequency of about 200 Hz, allows only one light beam to pass at a given moment onto the photomultiplier. In this manner it is possible to observe on the oscilloscope two rows of pulses on each beam. The height of pulses coming from one of the diffracted light beams can be adjusted by the polarizer. Using this facility and that of the oscilloscope for the mutual displacement of pulses, it is feasible to superimpose two sets of pulses. Knowing the path x_2-x_1 travelled by the acoustic pulse and the ratio $I(x_1)/I(x_2)$, it is possible to evaluate the absorption coefficient of acoustic wave from equation (6).

3. Discussion of experimental results

The diffraction of light by acoustic waves has been investigated in melt quartz, rutile, calcium fluoride, lithium niobate and bismuth-germanium oxide.

Experiments have been aimed at assessing the suitability of the system for measurement of the propagation velocity and the absorption coefficient of acoustic waves, and also in determining the photoelastic constants of certain crystals. The measurement accuracy of each of the above-mentioned physical quantities has also been determined.

Figs. 7-9 show some of oscillograms obtained. In these drawings the distance from the transducer to the laser beam is denoted by x. Figs. 7a and 7b are made from bismuth-germanium oxide with the acoustic wave propagation in the [100] direction and the light wave in the [010] direction, the length of crystal being 10 mm. Figs. 8a and 8b also present results of the diffraction of light in bismuth-germanium oxide for the case where $\boldsymbol{q}$ coincides with the [001] direction and $\boldsymbol{k}$ with the [010] direction. The length of crystal in this direction is $l = 29$ mm. Figs. 9a and 9b show the pulses obtained in rutile with $q \| Z$ and $l = 30$ mm.

Table 1 contains the values of the Bragg angles for crystals investigated and also the calculated values of the velocity of propagation of longitudinal acoustic waves at a frequence of 215 MHz.

The accuracy of determining the speed is principally dependent on the accuracy of the measurement of the Bragg angle, thus of the distance transducer-photomultiplier and the displacement of the photomultiplier. It amounts to about 0.2%. This accuracy may be affected by the error in determining the index of refraction. This method is very accurate and can be used for the determination of photoelastic constants for investigated crystals.

Table 2 states the values of the absorption coefficient of acoustic waves with frequencies 825 MHz measured by the above described method, Table 3 contains the values of some photoelastic constants for investigated crystals.

The accuracy of the determination of the absorption coefficient is about 15% and that of photoelastic constants — 10 to 12%. The error at the measurement of α is affected mainly by irregularities of crystal faces as well as by the divergence of an acoustic beam.

The test results indicate that the described method and used measuring systems can be successfully utilized for testing the acoustic and acoustic-and-optical properties of crystals.

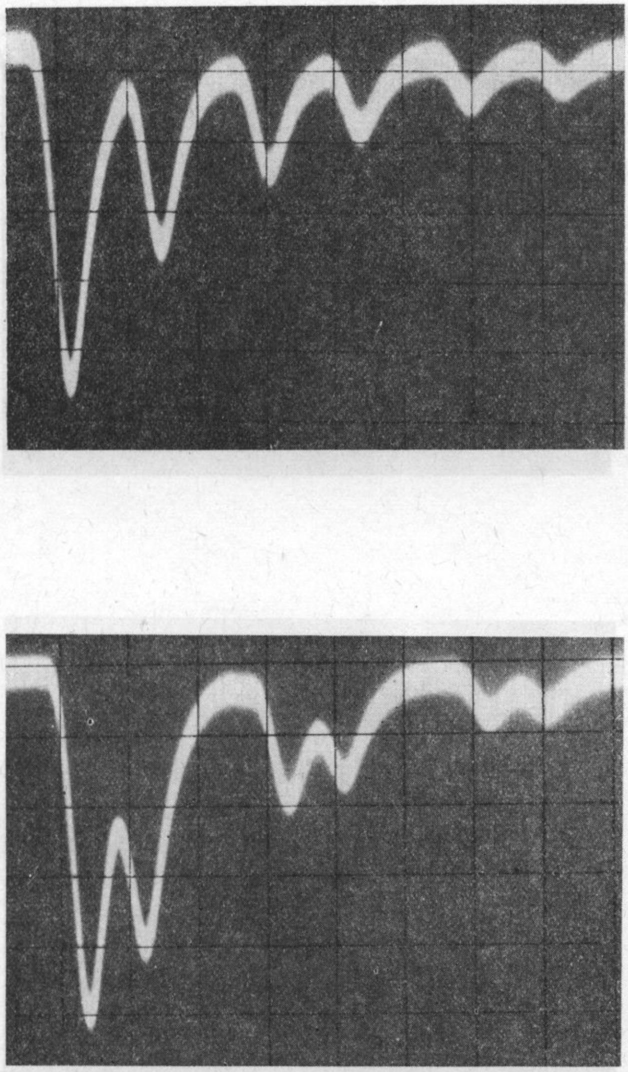
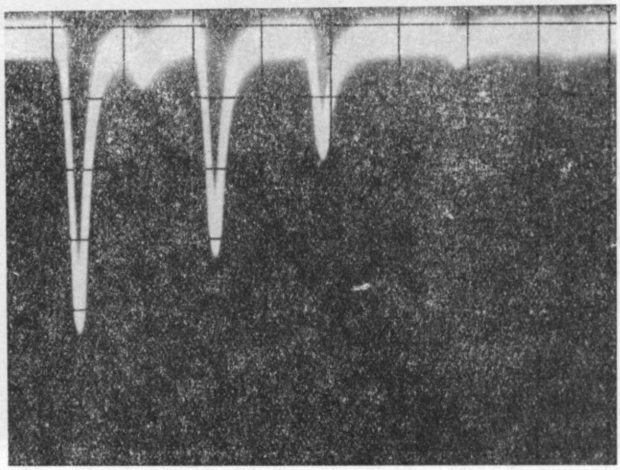


Fig. 7. Oscillogram patterns for bismuth germanium oxide; $\boldsymbol{q} \parallel X$; $\boldsymbol{k} \parallel Y$, $l = 10$ mm, time scale 2 μs/cm, a) $x = 7$ mm, b) $x = 4$ mm



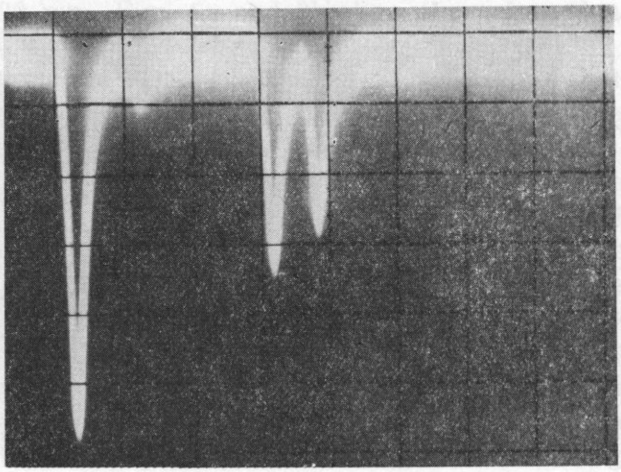
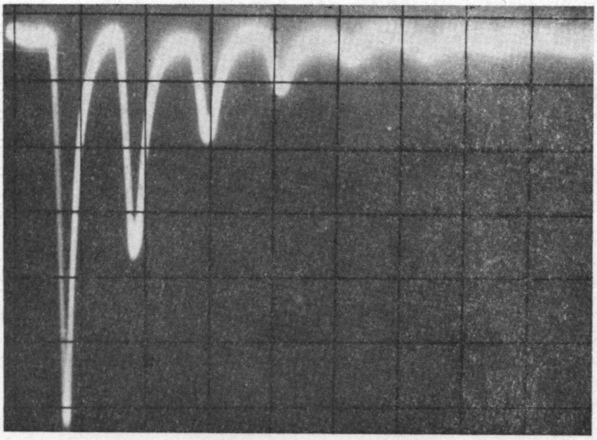


Fig. 8. Oscillogram patterns for bismuth germanium oxide; $\boldsymbol{q} \| Z$, time scale 5 μs/cm, $l = 29$ mm, a) $x = 21$ mm, b) $x = 14$ mm



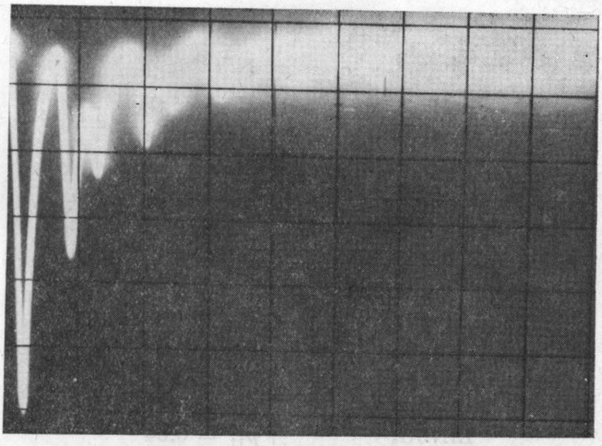


Fig. 9. Oscillograms for rutile; $\boldsymbol{q} \| Z$, time scale 5 μs/cm, $l = 30$ mm, a) $x = 19$ mm, b) $x = 11$

Table 1. Values of the Bragg angle and velocities of longitudinal acoustic waves for the crystals investigated

Substance	n	Direction	θ_B	V [m/s]
Melt quartz	1.45	—	25′50″	5890 ± 15
TiO_2	2.58	[100]	11′20″	7860 ± 20
CaF_2	1.43	[100]	22′00″	7070 ± 20
$Bi_{12}GeO_{20}$	2.55	[101]	25′50″	3300 ± 10
		[001]	26′00″	3210 ± 10
$LiNbO_3$	2.20	[100]	16′50″	6530 ± 15
		[001]	14′00″	7330 ± 12

Table 2. Absorption coefficients of longitudinal acoustic waves for the crystals investigated (f = 825 MHz, T = 293 K)

Substance	α [dB/cm]
Melt quartz	7 ± 1,5
TiO_2	0,9 ± 0,2
CaF_2	4,5 ± 0,7

Table 3. Measured values of the photoelastic constants for the crystals investigated

Substance	p_{ij}
CaF_2	$p_{11} = 0.06$
	$p_{12} = 0.23$
TiO_2	$p_{12} = 0.18$
	$p_{31} = 0.10$
	$p_{13} = 0.17$
$LiNbO_3$	$p_{11} = 0.04$
	$p_{12} = 0.08$
	$p_{31} = 0.18$

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Received on 26th May 1975