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DIFFRACTION OF A CYLINDRICAL ACOUSTIC PULSE BY A WEDGE

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In the paper, the diffracted field of a cylindrical pulse, approximating an explosion, at a wedge with the V angle $\alpha = 3/2\pi$, was calculated. This problem was solved on the basis of the Oberhettinger theory. The drop of the acoustic pressure level at the edge of the wedge, depending on the energy of the source, and the drop of the pressure level along the wall of the wedge in the silence zone, were calculated.

1. Introduction

The protection of the environment against noise is becoming increasingly significant. Kinds of noise sources and the means of protection against noise were described synthetically in paper [2] and the bibliography given in it. Apart from the means of protection described in [1], the problem of planner-designed protection against sounds is of particular significance; this is the method of situating buildings with respect to the noise sources. The theoretical problem consists in the calculation, in the acoustical shadow zone, of the so-called diffracted field at the corner of the building. A large number of papers has so far been devoted to this problem (see [8] and the list of references given here), but they take into account the diffraction of a harmonic wave, while pulses, and among them explosion, are the most annoying forms of noise.

In the present paper, the diffracted field of a cylindrical pulse, approximating an explosion, was calculated. The diffraction of the pulse of a planar-type explosion was described in paper [9], whereas a paper on the diffraction of a spherical pulse of the same type, will be published in the near future.

The problem of pulse diffraction was considered by SOMMERFELD [6], FRIEDLANDER [2], OBERHETTINGER [4], [5] and others (see the bibliography given in [5]).

In this paper, the theoretical considerations were based on the general Oberhettinger theory [5], [7], OBERHETTINGER solved the problem of the diffraction of unitary pulses; a plane one on a wedge, and cylindrical and spherical ones on a half-plane. The problems solved are not of any greater practical significance. The subject of the present paper seems to be more practical and more general.

The theory given by OBERHETTINGER describes a linear physical phenomenon. Its application to describe the explosion-type diffraction, can raise some doubts. However, at a sufficiently long distance from the source, this assumption appears to be valid.

2. Basic assumptions

In the present paper, as shape of the pulse $f(t^*)$ (Fig. 1) was chosen, which describes the explosion with the good approximation. Physically, it describes the time distribution of the velocity potential of the acoustic field [3].

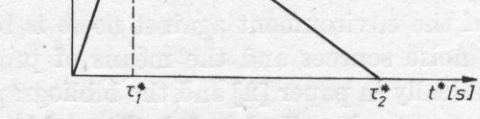

Fig. 1. The time shape of the pulse

It was assumed that the pulse changed linearly in sections, which can be written in analytical form as:

$$f(t) = \begin{cases} b_1 t & 0 < t < \tau_1 \\ -b_2(t-\tau_1)+f(\tau_1^*) & \tau_1 < t < \tau_2^* \\ 0 & \text{other } t \end{cases} \tag{1}$$

where t[m] is the reduced current time, $t = t^*c$, t^* [s]; c [m/s] is the sound velocity, $b_1 = f(\tau_1^*)/\tau_1^*$, $b_2 = -[f(\tau_1^*)/\tau_2 - \tau_1^*]$.

The geometrical aspect of the problem is shown in Fig. 2. The diffraction phenomenon occurs at a corner, i.e. on the structural element of the building.

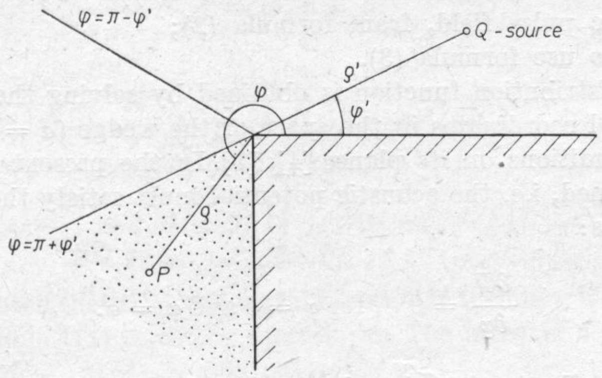

Fig. 2. The geometry of the problem: wave diffraction at a corner

The source Q was located with reference to the corner in such a way as to form a acoustical shadow zone. It was also assumed that a cylindrical wave of the zeroth order, $H_0^{(2)}(kr)$, was diffracted. This assumption seems to be valid, since the source is at some distance from the edge, and higher-order cylindrical waves are damped.

3. General theory of pulse diffraction at a wedge according to Oberhettinger

The problem was solved by applying the Laplace's transform and the diffraction problem solved for harmonic waves by OBERHETTINGER [4], [7]. According to this theory, for a harmonic source, the field diffracted at a wedge is the product of the field distribution function $U(x, y, z, \gamma) = U$ and the harmonic function $\exp(\gamma ct)$, $(\gamma = ik)$.

OBERHETTINGER proposed a similar approach to the pulse. First, the function of the pulse $f(t)$ is resolved into the sum (integral) of harmonic components (on the basis of the simple Laplace's transform), subsequently the diffracted field of each harmonic component is calculated, and these fields are summed up (integrated on the basis of the inverse Laplace's transform):

$$\Phi(x, y, z, \gamma) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \left[U \exp(\gamma ct) \int_0^{\infty} f(\tau) \exp(-\gamma c\tau)\, d\tau\right] d(\gamma c). \tag{2}$$

Formula (2) can be given in the form

$$\Phi(x, y, z, \gamma) = \int_0^{\infty} f(\tau) \Phi_D(t-\tau)\, d\tau, \tag{3}$$

where $\Phi_D(t-\tau)$ is the field of the Dirac pulse, calculated from formula (2), when $f(t)$ is taken to be the distribution of δ.

In order to find the field of any pulse in the region of the wedge, it is necessary to calculate:

a. the field distribution function $U(x, y, z, \gamma) = U$;

b. the Dirac pulse field, from formula (2);

c. finally, to use formula (3),

The field distribution function is obtained by solving the wave equation in the cylindrical coordinates in the space of the wedge ($\alpha = 3/2\pi$), with the predescribed conditions on its planes [4], [7]. In the present case, ideal rigid walls were assumed, i.e. the acoustic potential must satisfy the following boundary conditions:

$$\frac{\partial \Phi}{\partial \varphi} = 0 \qquad \text{for } \varphi = 0 \text{ and } \varphi = \alpha. \tag{4}$$

The field distribution function obtained can be given by the formula [4], [7]

$$U = \sum_{v=1}^{2}\Big[\sum_{r=r_1}^{r_2} H_0^{(2)}(kR_v) + \int_0^\infty H_0^{(2)}(kR_x)G(x, \theta_v)\,dx\Big], \tag{5}$$

where

$$R_v^2 = \varrho^2 + \varrho'^2 - 2\varrho\varrho'\cos(3\pi r + \theta_v),\; R_x^2 = \varrho^2 + \varrho'^2 + 2\varrho\varrho'\cosh x; \tag{6}$$

$$G(x, \theta_v) = -\frac{1}{3\pi}\sum_{p=1}^{2}\frac{s_{vp}}{\cosh(2/3\pi) - c_{vp}}, \qquad S_{vp} = \sin\left(\frac{3}{2}\pi \pm \theta_v\right),$$

$$C_{vp} = \cos\left(\frac{3}{2}\pi \pm \theta_v\right), \qquad \theta_{1,2} = \varphi \pm \varphi'. \tag{7}$$

The symbols r_1, r_2 are given by: $r_1 = [-2(\pi+\theta)/3\pi]$, $r_2 = [2(\pi-\theta)/3\pi]$, where the symbol $[2(\pi\pm\theta)/3\pi]$, denotes the highest integer which is less than the symbol given in the bracket. The sum $\sum_{r=r_1}^{r_2}$ is zero when $r_1 > r_2$.

The field of the Dirac pulse is calculated from formula (2), if $f(t)$ is replaced by the distribution δ [4], [5], [7]:

$$\Phi_D(t-\tau) = \frac{2i}{\pi}\sum_{v=1}^{2}\Big\{\sum_{r=r_1}^{r_2}[(t-\tau)^2 - R_v^2]^{-1/2} + \int_0^\infty [(t-\tau)^2 - R_x^2]^{-1/2}G(x, \theta_v)\,dx\Big\}, \tag{8}$$

where $\Phi_D(t-\tau)$ is different from zero for $(t-\tau)^2 > R_v^2$ and $(t-\tau)^2 > R_x$. For the other values of $(t-\tau)$, $\Phi_D(t-\tau) = 0$.

4. Field of the pulse approximating the explosion

The field potential of the pulse $f(t)$ is calculated from formula (3). In view of the form of $\Phi_D(t-\tau)$, integral (3) can be given in the form of the sum

$$\Phi = \Phi_1 + \Phi_2, \tag{9}$$

where

$$\Phi_1 = \frac{2i}{\pi} \sum_{v=1}^{2} \sum_{r=r_1}^{r_2} \int_0^{\infty} f(\tau)[(t-\tau)^2 - R_v^2]^{-1/2} d\tau, \tag{10}$$

$$\Phi_2 = \frac{2i}{\pi} \sum_{v=1}^{2} \int_0^{\infty} G(x, \theta_v) \left\{ \int_0^{\infty} f(\tau)[(t-\tau)^2 - R_x^2]^{-1/2} d\tau \right\} dx. \tag{11}$$

It can be noted that the subintegral function in formula (10) and the internal integral in formula (11) have the same form. The integral Φ_1 has the solution

$$\Phi_1 = \Phi_{1a} = \frac{2i}{\pi} \sum_{v=1}^{2} \sum_{r=r_1}^{r_2} W_a, \quad 0 < t_{01} < \tau_1;$$

$$\Phi_1 = \Phi_{1b} = \frac{2i}{\pi} \sum_{v=1}^{2} \sum_{r=r_1}^{r_2} W_b, \quad \tau_1 < t_{01} < \tau_2; \tag{12}$$

$$\Phi_1 = 0 \quad \text{other } t_{01},$$

where $W_a = b_1 I_1$, $W_b = -b_2 I_1 + [b_2 \tau_2 + f(\tau_1)] I_2$, $t_{01} = t - R_v$,

$$I_1 = \int \frac{\tau}{\sqrt{(t-\tau)^2 - R_v^2}} d\tau, \quad I_2 = \int \frac{1}{\sqrt{(t-\tau)^2 - R_v^2}} d\tau.$$

The integrals I_1 and I_2 are simple inexchangable integrals. In these integrals, the integration limits are determined from the conditions $\tau < t - R_v$, $0 < \tau < \tau_1$ or $\tau < t - R_v$, $\tau_1 < \tau < \tau_2$ with R_v given by formula (6).

In order to calculate the expression Φ_2, the internal integral was first calculated in the same way as the integral Φ_1 was, whose solution is given by the symbol W_a, W_b (12), with the additional fact that the quantity t_{02} replaced t_{01}, and the quantity R_x $(t_{02} = t - R_x)$ replaced R_v.

The solution of integral (11) can be given in the form

$$\Phi_2 = \Phi_{2a} = \frac{1}{3\pi} \sum_{v=1}^{2} \sum_{p=1}^{2} s_{vp} Z_a, \quad 0 < t_{02} < \tau_1;$$

$$\Phi_2 = \Phi_{2b} = \frac{1}{3\pi} \sum_{v=1}^{2} \sum_{p=1}^{2} s_{vp} Z_b, \quad \tau_1 < t_{02} < \tau_2; \tag{13}$$

$$\Phi_2 = 0 \quad \text{other } t_{02},$$

where $Z_a = b_1(tI_3 - tI_4 - I_5)$, $Z_b = [b_2(\tau_1 - t) + f(\tau_1)](I_3^* - I_4) + b_2 I_5$; for s_{vp} see formula (7). I_3, I_4, I_5 are the symbols of definite integrals (the integration limits are determined from the conditions imposed on t_{02}) in the form

$$I_3 = \int \frac{\ln |t + \sqrt{t^2 - R_x^2}|}{M(x)} dx, \quad I_4 = \int \frac{\ln R_x}{M(x)} dx, \quad I_5 = \int \frac{\sqrt{t^2 - R_x^2}}{M(x)} dx, \tag{14}$$

where $M(x) = \cosh(2/3x) - c_{vp}$ (for c_{vp} see formula (7)). The integrals I_3 and I_5 have the respective form of I_3 and I_5, except that t should be replaced by $t - \tau_1$ in formulae (14).

After some calculations, the integrals I_3, I_4 and I_5 are reduced to hyperelliptic ones, and these, in turn, cannot be expressed by elementary functions. Therefore, numerical integration remains.

The final solution has the form:

$$\begin{aligned} \Phi &= \Phi_{1a} + \Phi_{2a}, \quad 0 < t_{01}, t_{02} < \tau_1; \\ \Phi &= \Phi_{1b} + \Phi_{2b}, \quad \tau_1 < t_{01}, t_{02} < \tau_2; \\ \Phi &= 0 \quad \text{for other } t_{01}, t_{02}. \end{aligned} \tag{15}$$

5. Numerical calculations and conclusions

It is purposeful, from the practical point of view, to calculate the drop in the pressure level at the edge of the wedge for the different maximum values of the field potential $f(\tau_1^*)$ close to the source. Since the relative value of the drop in the sound level is of interest, the point A_1 was assumed to be in the acoustical shadow zone, while the reference point B was localised in the sonicated zone (Fig. 3).

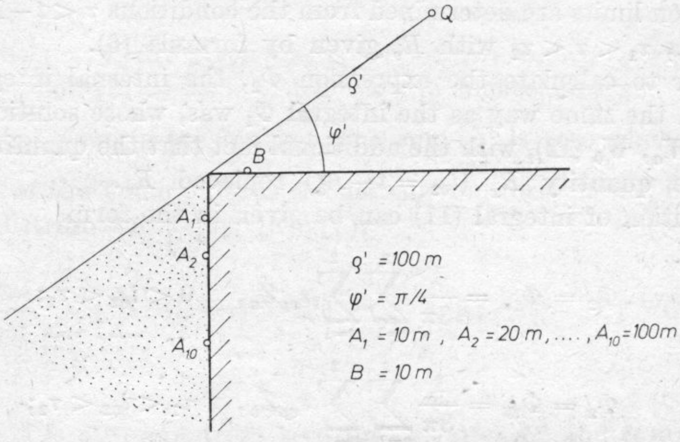

Fig. 3. The distribution of the calculation points (A_i) in the acoustical shadow zone

The pulse shape is determined by the values τ_1^* and τ_2^*; it was assumed that $\tau_1^* = 0.015$ [s] and $\tau_2^* = 0.06$ [s]. The maximum value of the field potential $f(\tau_1^*)$ at the point $Q(\varrho', \varphi')$ was assumed to vary between 1 and 10 m^2/s. The assumption of the different values of $f(\tau_1^*)$ close to the source, aims at the achievement, in the sonicated zone close to the edge, of such values of the acoustic pressure level which would correspond to a real explosion.

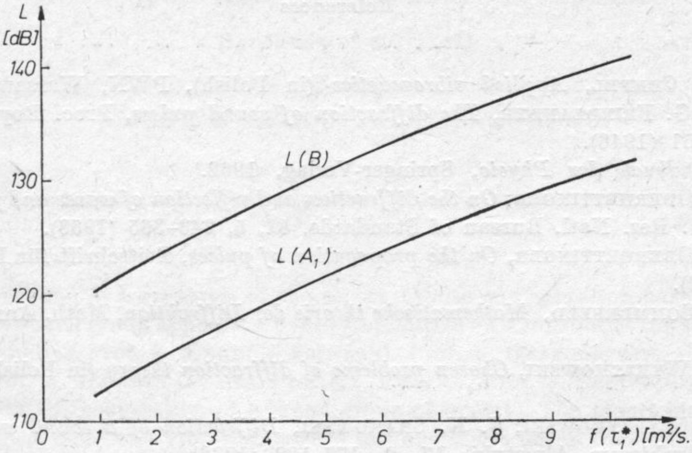

Fig. 4. The distribution of the acoustic pressure level before the edge of the corner in the sonicated zone (B) and behind it, in the silence zone (A_1), as a function of the peak value of the pulse

Fig. 4. shows the value of the acoustic pressure level $L(A_1)$ at the point A_1, and $L(B)$ at the point $B(L(A_1))$ is calculated from the formula $L(A_i) = 20 \log(p_i/p_0)$ [dB], where $p_0 = 2 \times 10^{-5}$ Pa. It follows from Fig. 4 that this drop is about 9 dB and is independent of the maximum value of the velocity potential $f(\tau_1^*)$ close to the source.

The drop in the pressure level along the wall of the wedge in the silence zone, was also calculated. With the view to the application of the results, the following calculation points were selected: $A_1 = 10$ m, $A_2 = 20$ m ... $A_{10} = 100$ m. The position of the point A_i less than 10 m from the edge of the corner has no practical justification, while at a distance of more than 100 m from the edge, it is tantamount to the design of a building that long. Buildings longer than 100 m are not often met and the assumption $A_{10} = 100$ m seems sufficient. In the calculations, the value of $f(\tau_1^*)$, varying between 1 to 10 m^2/s, was assumed.

It follows from the calculations that the drop in the pressure level $L(A_{10}) - L(A_1)$ along the wall of the wedge in the silence zone is independent, up to significant places, on $f(\tau_1^*)$ (over the investigated range of $f(\tau_1^*)$); it is about 10 dB and is approximately linear.

It follows from the whole of the calculations that the corner can be considered as an element protecting against noise. When the drop in the pressure level at the edge of the corner and that along its wall in the shadow zone are considered, its total is dozen-odd [dB], and this value is already of practical significance.

References

[1] Cz. CEMPEL, *Applied vibroacustics* (in Polish), PWN, Warsaw-Poznań, 1978.

[2] F. G. FRIEDLANDER, *The diffraction of sound pulses*, Proc. Roy. Soc. London, **A 186**, 322–351 (1946).

[3] *Handbuch der Physic*, Springer-Verlag, 1962.

[4] F. OBERHETTINGER, *On the diffraction and reflection of waves and pulses by wedges and corners*, J. Res. Natl. Bureau of Standards, **61**, 5, 343–365 (1958).

[5] F. OBERHETTINGER, *On the propagation of pulses*, Zeitschrift für Physik, **Bd 146**, 423–435 (1956).

[6] A. SOMMERFELD, *Mathematische theorie der Diffraktion*, Math. Ann., **47**, 317–341 (1896).

[7] R. WYRZYKOWSKI, *Chosen problems of diffraction theory* (in Polish), unpublished ms.

[8] R. WYRZYKOWSKI, J. K. SNAKOWSKI, *Diffraction of a sound wave at a corner* (in Polish), Archiwum Akustyki, **17**, 2, 177–188 (1982).

[9] R. WYRZYKOWSKI, A. BRAŃSKI, *Diffraction of a planar acoustic pulse at a corner* (in Polish), Proc. VIth Symp. Vibration Technique and Vibroacoustics, Cracow, 1982.

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