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AUDITORY FILTERING AT LOW FREQUENCIES

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This paper is concerned with the comparison of the critical bandwidth (CB) and the equivalent rectangular bandwidth (ERB) of the auditory filters at low frequencies. The method of critical bandwidth determination based on the critical modulation frequency (CMF) has been questioned, particularly for frequencies less than 500 Hz. The CMF, which is the modulation frequency at which amplitude modulation (AM) and frequency modulation (FM) detection thresholds become identical, is confounded as a proper measure of the auditory filter's width. It refers to the modulation rate for which one of the sidebands is most detectable. For low carrier frequencies the higher sideband is most detectable whereas for higher carrier frequencies the lower sideband becomes most detectable. Thus, at least for low carrier frequencies (i.e. less than 200 Hz), the CMF does not reflect the auditory system's sensitivity for detecting the phase differences of the spectral components of the signal. These findings can account for the fact that the critical bandwidth flattens off at low carrier frequencies, whereas the equivalent rectangular bandwidth of the auditory filters continues to decrease down to very low centre frequencies. Is was also shown that, at least for very low frequencies, critical bands do not reflect directly the auditory filtering that takes place in the peripheral auditory system.

1. Concepts of peripheral filtering

Since FLETCHER [4] described his band-widening experiment, the human peripheral auditory system has often been described as an array of linear bandpass filters with continuously overlapping centre frequencies. One of the first models of the auditory system (based on the Critical Band concept) assumed that the filters were rectangular in shape with bandwidth equal to the critical bandwidth, [4, 49, 50, 52]. Using this idea researchers tried to estimate a "breakpoint" in the data relating performance to bandwidth — a point at which subjective responses change "abruptly" (see below for details). PATTERSON and MOORE [26] used a different approach to modelling the auditory system's properties. They assumed a specific shape of the auditory filter described usually by means of a rounded exponential function (roex (f)) and based on the experimental data. They attempted to estimate parameters of the filter including its bandwidth.

Critical bandwidth and the equivalent rectangular bandwidth of the auditory filters are related by a constant factor in the frequency range above 1 kHz. However, for frequencies lower than 500 Hz the critical bandwidth is constant while the equivalent rectangular bandwidth of the auditory filter decreases even for the lowest centre frequencies.

1.1. The critical band concept

FLETCHER [4] carried out an experiment, the results of which are the main basis of the Critical Band (CB) and the Auditory Filter (AF) concepts. He measured the detection thresholds of a sinusoidal signal as a function of the bandwidth of a bandpass noise masker (see Fig. 1, the left panel). The band of noise was characterised by a constant power density and its centre frequency was equal to the signal frequency. At first, the threshold of the signal increased with increasing masker width from very small values (see the right panel of Fig. 1). When the bandwidth reached a critical value (i.e. the critical bandwidth) the threshold flattened off and further increase in the noise bandwidth did not affect the signal threshold significantly.

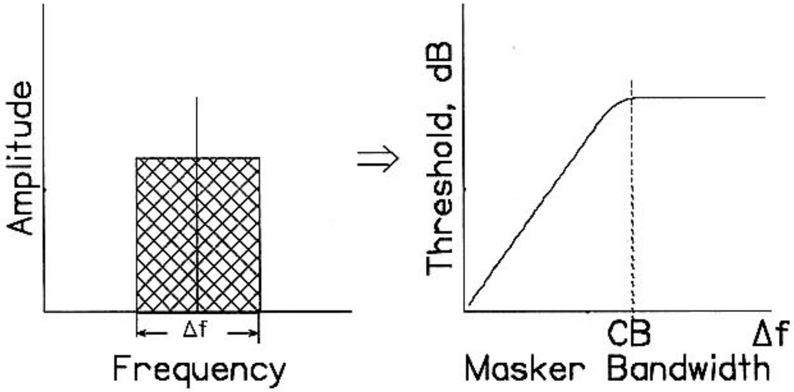

Fig. 1. A schematic illustrations of the signal spectrum and a pattern of results in FLETCHER's [4] band-widening experiment.

To account for these results Fletcher suggested that the auditory system behaved as if it contained an array of bandpass filters with overlapping passbands. The filters were assumed to be roughly rectangular in shape and their widths were equal to the CB. Based on physiological data [1] he also assumed that the basilar membrane provided the basis for these filters. Each point on the basilar membrane responds only to a very limited range of frequencies. Thus it may be considered as a filter with a specific centre frequency. Considering the detection of a signal in a background noise, it is assumed that a subject uses a filter with a centre frequency very close to the signal frequency. Only noise components passing through the filter affect the detection of the signal. At the detection threshold the power of the tone divided by the power of the noise inside the critical band (which is called the critical ratio) is constant.

Since FLETCHER [4] described the CB concept based on the band-widening experiment, results of different types of psychoacoustics experiments have confirmed this model. They gave very similar estimates of both the absolute widths of the CB and of the way that the critical bandwidth varies as a function of frequency. The model was further developed by ZWICKER [44–48, 52, 53] who showed that the CB concept could summarise a great variety of psychoacoustical data. The concept also provides a valuable tool in the planning of experiments and the analysis of the data. ZWICKER [44–46, 48, 52] presented also some types of experiments in which the CB may by be demonstrated and/or measured.

• The loudness of a complex sound was approximately independent of its bandwidth, if the bandwidth was less than the width of the one critical band. Moreover, the complex sound was judged to be about as loud as pure tone of equal intensity lying at a centre frequency of the sound. However, if the bandwidth of the complex sound was increased beyond the critical bandwidth the loudness began to increase. The results of this experiment show that the loudness of a signal depends not only on its amplitude but also depends on its spectral structure. The increase in loudness with increasing signal bandwidth became one of the crucial proofs of the CB's existence and it was also used as a critical bandwidth determination/demonstration method.

• Masking. Thresholds for detecting a narrow band of noise in the presence of two tones symmetrically situated with respect to the centre frequency of the noise (see Fig. 2, the left panel) are approximately constant for small values of the frequency separation of the tones (Fig. 2, the right panel). But, when the frequency separation reached a critical value the threshold dropped very sharply. ZWICKER [46] assumed that the threshold remained constant as long as the frequency separation between masking tones did not exceed one critical band associated with the band of noise. However, when the frequency distance between masking tones became larger than the CB, the threshold decreased. The largest value of the frequency separation, for which the threshold was still constant, was used to estimate the critical bandwidth [46]. It was also shown that combination tones, resulting from a nonlinear process in the cochlea [15, 29], might affect the detection of the noise band. It is possible that the subject may detect the combination products even though the signal itself is inaudible. When the distortion products are masked by noise the threshold does not show an abrupt decrease [7].

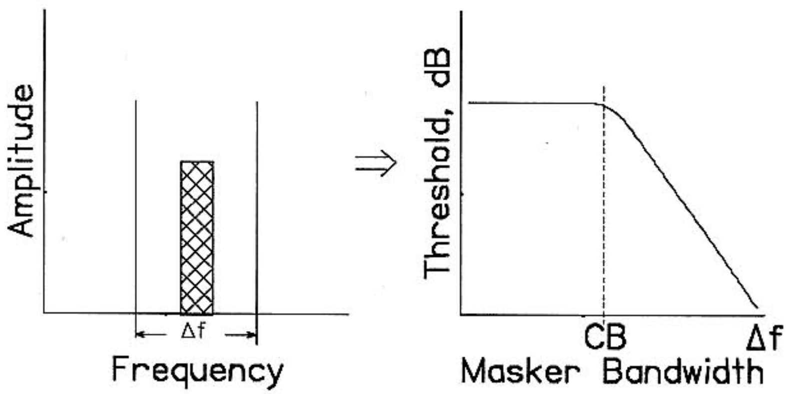

Fig. 2. A schematic illustrations of the signal spectrum and a pattern of results in ZWICKER's [51] masking experiment.

• Critical Modulation Frequency (CMF). Considering the detection of amplitude modulated (AM) and frequency modulated (FM) signals, ZWICKER [45] stated that the auditory system was capable of detecting the difference in the phase structure of the modulated signals spectra. Thresholds for detecting AM and FM, if expressed in terms of amplitude and frequency modulation indices (i.e. m and β), are markedly different for low modulation rates f_{mod}. The difference between these thresholds becomes much smaller when the modulation rate increases. When the modulation frequency reaches its critical value (called the Critical Modulation Frequency, CMF), AM and FM detection thresholds become identical and further increase in modulation rate does not affect the difference between them. ZWICKER [45] suggested, that the critical modulation frequency may be used as a CB estimator: $\text{CB} = 2 \cdot \text{CMF}$, (see Sec. 2 for further details).

Measures of the critical bandwidth obtained from a variety of experiments generally agree reasonably well for frequencies above 1 kHz. But for lower frequencies there are considerable discrepancies between the different measurements. SCHARF [34] summarised and averaged early measurements of the CB (based mainly on ZWICKER's [48] data) and since then his presentation has been widely used. Scharf's function relating the CB to the centre frequency, which is plotted in Fig. 3, flattens off for centre frequencies below 500 Hz at a value of 100 Hz. However for these centre frequencies this function is a simplified extrapolation because experimental data for this frequency region were rather sparse. The estimates were based mainly on the measurements of the critical modulation frequency and the critical ratio [34].

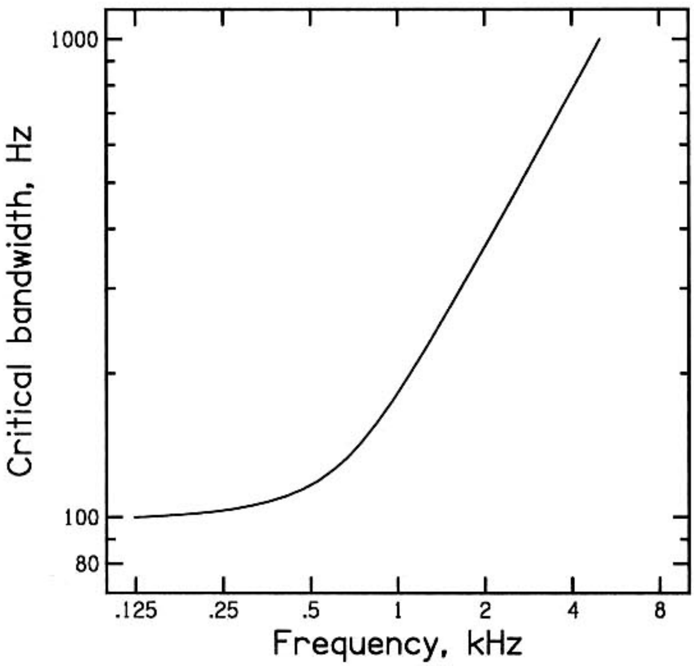

Fig. 3. Critical bandwidth as a function of centre frequency (adapted from SCHARF [34]).

There is no clear experimental evidence that the auditory filter's width is constant in the frequency range below 500 Hz. On the other hand, there is much evidence that the width of the auditory filter decreases with decreasing in frequency even at very low frequencies. The most important is the ability of the auditory system to resolve individual components in a multi-component complex sound. The frequency separation between two sinusoids at which these two sinusoids may be heard out as separate tones when presented simultaneously, depends on frequency. It gradually decreases even for the smallest values of frequency. A second piece of evidence is that frequency discrimination (i.e. the ability to detect changes in frequency over time) measured as the smallest detectable frequency difference between two successive tones, improves progressively with a decrease in frequency to very low values [42].

1.2. The auditory filter concept

An alternative approach to the modelling of the auditory filter is that proposed by Patterson and Moore [26]. They assumed that the activity of the auditory system in response to acoustic stimuli (i.e. excitation pattern) may be considered as the output signal from a bank of linear bandpass filters (called the auditory filters AF) with the overlapping passbands. The activity of the peripheral auditory system may be understood as a displacement of the basilar membrane as a function of the distance from the oval window or as the number of the neural spikes per second as a function of the neurone's characteristic frequency. The filters have the general form suggested by Patterson *et al.* [28]:

$$W(g) = (1 + pg)e^{-pg}, \qquad (1)$$

where g is the deviation from the centre frequency of the filter divided by the centre frequency, $W(g)$ is the intensity weighting function describing the shape of the filter, p is a parameter determining the sharpness of the passband of the filter, with p_l for the low- and p_u for the high-frequency skirts respectively. Assuming the power spectrum model [4, 26], Patterson and Moore suggested that the activity of the peripheral auditory system and the auditory filter shape might be derived from masking experiments, (e.g. a notched-noise masking experiment). In this case the threshold at a given signal frequency corresponds to a constant signal-to-noise ratio at the output of an auditory filter centred close to the signal frequency.

• Psychophysical tuning curves. One of the methods of determining the auditory filter involves the measurement of psychophysical tuning curves (PTCs). To determine a PTC the threshold for detecting a sinusoidal signal with a low (fixed) level is measured. The masker is usually a narrow band of noise (a tonal masker would produce low frequency beats) with a different centre frequency. The level of the masker that just masks the signal is determined. It is assumed that the PTCs express the masker level required to produce a fixed output from the auditory filter as a function of the masker frequency. Examples of the PTCs are presented in Fig. 4. Psychophysical tuning curves are very similar in a general form to neural tuning curves [29] and they were obtained based on the same method, i.e. by determining the level of the tone required to produce a fixed output from a single point of the basilar membrane or single neurone. The similarities in both the procedures and the results indicate that the shape of the PTCs reflects the activity of the auditory system and the auditory filter shape. PTCs reveal an "off frequency" listening phenomenon ([22, 27]). To detect the signal listeners use a filter whose output is characterised by the highest signal-to-noise ratio. In general case this means that subjects use a filter with a centre frequency very close to the signal frequency but not necessarily the filter with a centre frequency exactly equal to the signal frequency.

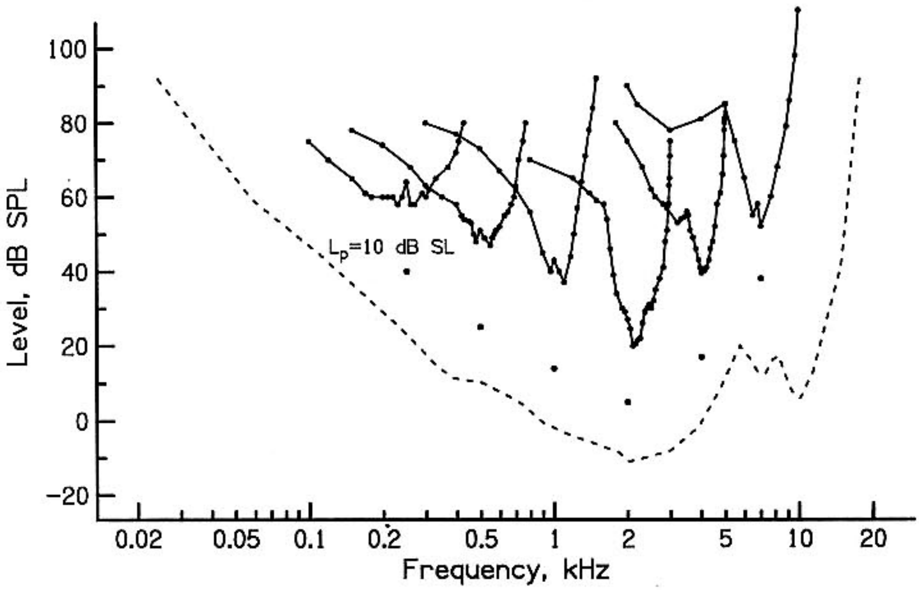

Fig. 4. Psychophysical tuning curves measured in simultaneous masking experiment (adapted from VOGTEN [41]).

• The notched noise method. PATTERSON [25] proposed a method of auditory filter shape determination based on the masking of a sinusoidal signal by means of two bands of noise symmetrically situated in the frequency domain (notched noise) with respect to the signal frequency, (see Fig. 5a). The method prevents off-frequency listening and decreases the influence of combination tones on the detection threshold. As the width of the spectral separation between the bands of noise is increased, less energy of the noise passes through the auditory filter (shaded areas in Fig. 5a) and the threshold for the signal drops. Based on the power spectrum model, the power of the signal P_s at the detection threshold may be expressed by means of the following equation:

$$P_s = K \int_0^\infty W(f) N(f)\, df, \tag{2}$$

where $W(f)$ is the intensity weighting function describing the auditory filter shape, $N(f)$ is the power spectrum of the noise and K is a constant that corresponds to the signal-to-noise ratio at the output of the auditory filter required for threshold.

Thus having the threshold values for different notch width ($2\Delta f$), and assuming the specific shape of the filter (Eq. (1)) it is possible to determine all parameters describing the shape of the auditory filter. An example of the auditory filter determined by means of this method is plotted in Fig. 5b.

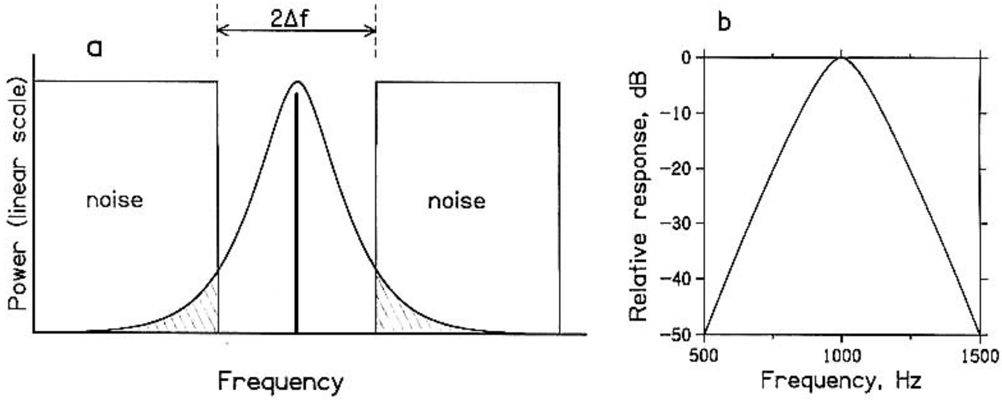

Fig. 5. PATTERSON's [25] method of auditory filter shape determination (a) and an example of the auditory filter shape at centre frequency of 1 kHz (b) determined using this method, (adapted from MOORE [15]).

Another measure of the auditory filter width is the Equivalent Rectangular Bandwidth (ERB). This expresses the bandwidth of the perfect rectangular filter whose transmission in its passband is equal to the transmission of the specified filter, and which transmits the same total power of white noise as the specified filter. The dependence of the equivalent rectangular bandwidth on its centre frequency is described by means of the equation [6, 15]:

$$\mathrm{ERB} = 24.7(4.37F + 1), \tag{3}$$

where F is a centre frequency of the filter in kHz. The dependence is well established over a wide frequency range including very low frequencies, [2, 3, 6, 14, 16, 17, 31, 32, 40, 43].

1.3. Comparison of the critical band and the ERB of the auditory filters

The comparison between the critical bandwidths CB suggested by SCHARF [34] and the equivalent rectangular bandwidth ERB [15] is presented in Fig. 6. Critical bandwidths have bigger values than the equivalent rectangular bandwidths of the auditory filters over the whole audible frequency range. In the frequency range 500 – 5000 Hz the ratio CB/ERB is approximately constant and equals about 1.22 – 1.5. The near-linear relationship between the CB and ERB in this frequency region suggests that any predictions of experimental results based on the critical band and the auditory filter concepts should be consistent. However, for frequencies less than 500 Hz, the discrepancy between the CB and the ERB gets much bigger with decreasing frequency; for a frequency of 100 Hz the ratio CB/ERB reaches a value close to 3. This discrepancy might indicate that neither the ERB nor the CB are a proper measure of the auditory filter bandwidth. However the dependency of the ERB on centre frequency is well established over whole range of audible frequencies whereas Scharf's function relating the CBs to frequency may be in error for the lowest frequencies, since the data concerning the CB for this frequency region were very sparse. However the lack of critical bandwidth data for frequencies less than 500 Hz is not a proper argument in accepting the ERB estimates as the only measure of the auditory filter width. It is necessary to show that the critical band concept does not describe the auditory filtering in a proper way or to prove that the methods of the critical bandwidth determination are in error.

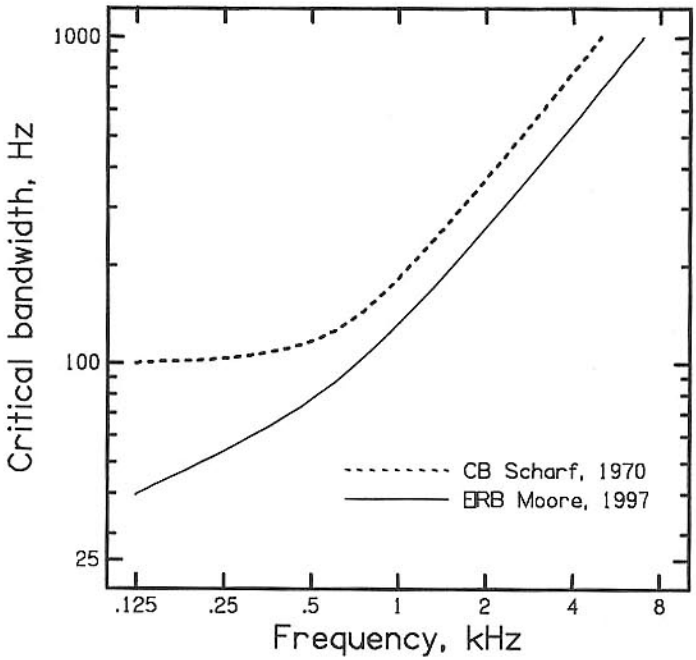

Fig. 6. Comparison between the critical bandwidth (CB) suggested by SCHARF [34] and the equivalent rectangular bandwidth (ERB) [15].

2. Critical modulation rate as a method of critical bandwidth measurement

2.1. Critical modulation rate concept

An amplitude modulated (AM) sinusoidal signal with modulation index m, and a frequency modulated (FM) sinusoidal signal with modulation index β, may each be considered as composed of three sinusoidal components, corresponding to the carrier frequency and two sidebands (the FM wave actually contains many sidebands but an approximation with two sidebands is correct for small modulation indices). A schematic structure of AM and FM signal spectra is presented over the top panel of Fig. 7. When the

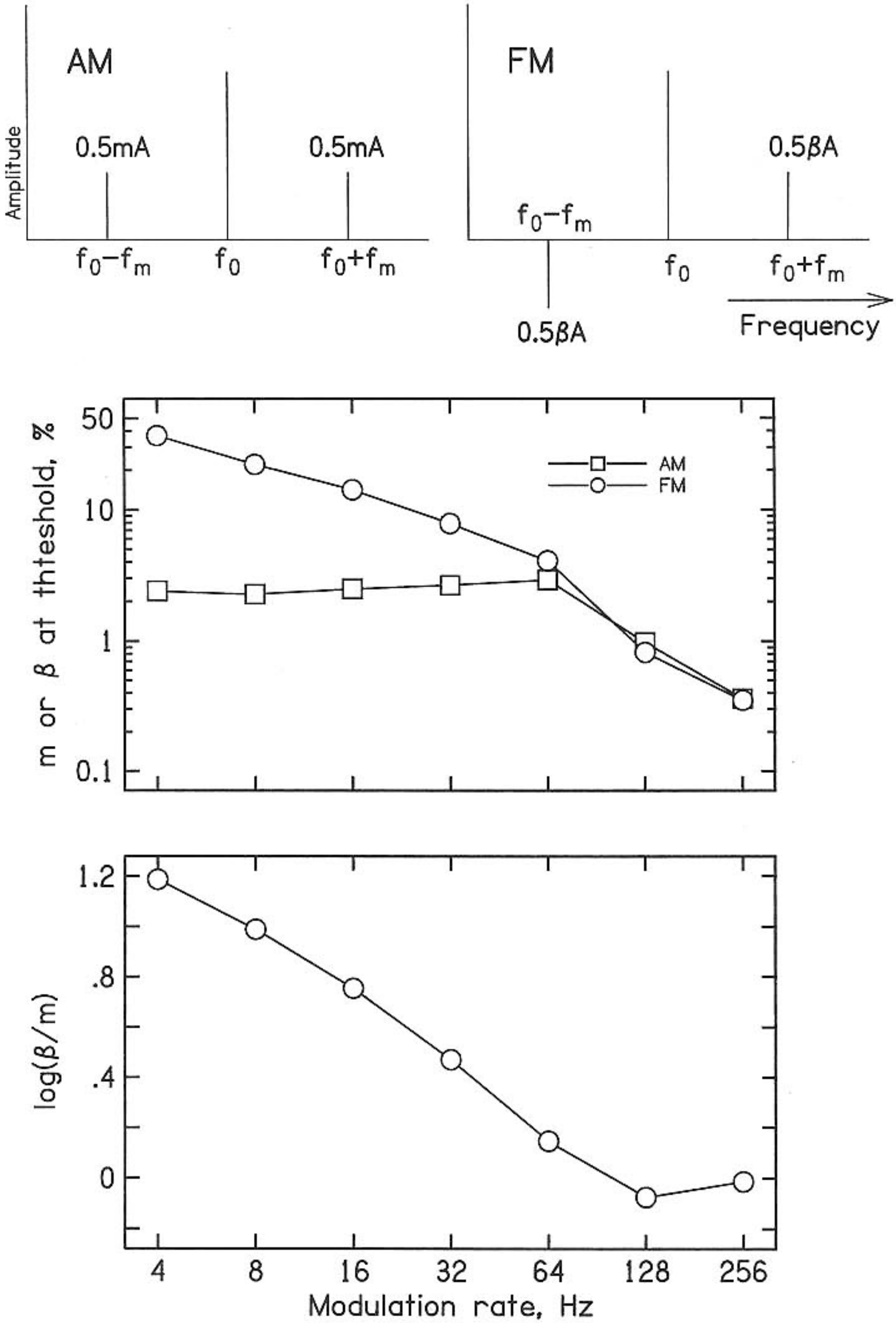

Fig. 7. The middle shows the thresholds for detecting AM or FM, expressed as the respective modulation indices, m or β, plotted as a function of modulation rate. The carrier frequency was 1000 Hz. The lower panel shows a dependence of $\log_{10}(\beta/m)$ on modulation rate which approaches zero at high modulation rates, indicating that the ratio β/m approaches unity. Data from SĘK [38].

modulation indices are equal ($m = \beta$), and when the carrier frequencies and modulation frequencies are the same, the spectral components of AM wave and FM wave are identical in frequency and amplitude. The only difference between them being in the relative phase of the components, in particular, the lower one. If, then, the two types of wave are perceived differently, the difference is likely to arise from a sensitivity to the relative phase of the components (so-called monaural phase effect [12]), which affects the temporal structure of the sound.

ZWICKER [45, 51], SCHORER [35] and SĘK [38] measured the just-detectable amounts of amplitude or frequency modulation, for various rates of modulation. They found that for high rates of modulation, where the frequency components were widely spaced, the detectability of FM and AM was equal when the components in each type of wave were of equal amplitude ($m = \beta$). However, for low rates of modulation, when all three components fell within a narrow frequency range, AM could be detected when the relative levels of the sidebands were lower than for a wave with a just-detectable amount of FM ($m_{th} < \beta_{th}$). This is illustrated in the middle panel of Fig. 7 (based on data from [38]). Thus for small frequency separations of the components, subjects appear to be sensitive to the relative phases of the components (which is also known as a monaural phase effect [12]), while for wide frequency separations they are not.

If the threshold for detecting of modulation is expressed in terms of the modulation index, m or β, the ratio β/m decreases as the modulation frequency increases, and approaches value of unity (or zero if the value of $\log(\beta/m)$ is used). This is illustrated in the lower panel of Fig. 7. The modulation frequency at which the ratio first becomes unity is the CMF. ZWICKER [45, 51] suggested that the CMF is reached when the spectral sidebands in the stimulus first become detectable. Once the modulation is detected in this way, the relative phases of the components do not play a role. ZWICKER [45, 51] suggested further that the CMF corresponded to half the value of the CB; essentially the CMF was assumed to be reached when the overall stimulus bandwidth reached the CB. If this is correct, then the CMF may be regarded as providing an estimate of the CB at the carrier frequency.

SCHORER [35] and SĘK [38] argued that the CMF was one of the best ways of estimating the CB, especially at low centre frequencies. They gave several reasons supporting this argument:

1. The whole stimulus falls within a very restricted spectral region. At the CMF only one critical band is excited.

2. Variations in the absolute threshold with frequency have only a small influence on the results, due to small stimulus bandwidth.

3. There is no "off-frequency" listening ([22, 27]).

4. Combination tones do not appear to influence the threshold. Thus the method can be used over a wide range of sound pressure levels.

2.2. Measurements of the critical modulation frequency

Figure 8 presents data obtained by SĘK [38] and SCHORER [35] who measured the critical modulation rate for the whole frequency range and then converted it into the critical bandwidth. This figure shows also the critical bandwidth data collected by Scharf [34] and the equivalent rectangular bandwidths [15] to make a comparison between these two sets of data slightly easier. As can be seen from this picture Sęk's and Schorer's data are very similar since they used the same experimental methods. For frequencies above 1 kHz the dependence of the critical bandwidth determined based on CMF on frequency is very similar to that of Scharf. However, the values of the critical bandwidth obtained for frequencies lower than 1 kHz are slightly lower and the difference between them and those of Sharf appears to be statistically significant [36, 38]. The critical bandwidth determined by Sęk and Schorer tends to decrease gradually as frequency decreases even below 500 Hz. This is a very important result since it shows a qualitative difference between the most recent measurements and traditionally accepted data that suggest that below 500 Hz the critical bandwidth is constant and independent of frequency.

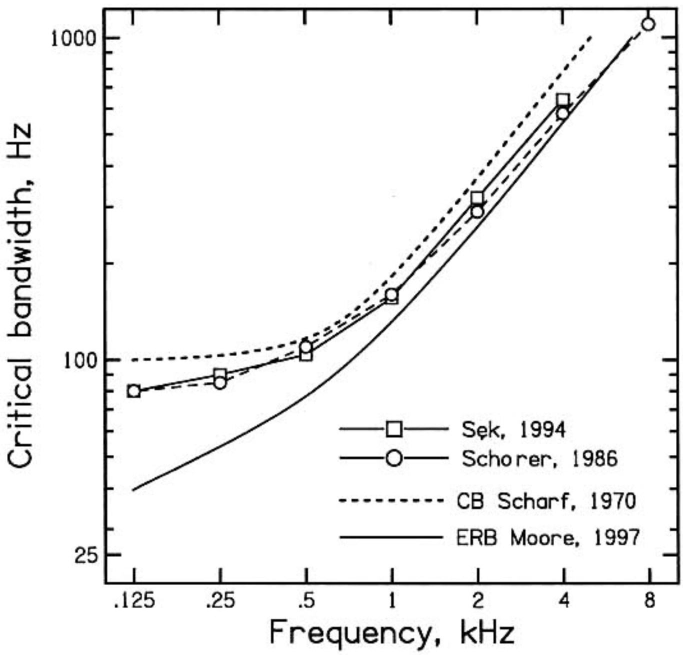

Fig. 8. Critical bandwidth as a function of frequency obtained by Schorer [35] and Sęk [36, 38] based on measurements of the critical modulation frequency. The critical bandwidth (CB) suggested by Scharf [34] and the equivalent rectangular bandwidth (ERB) [15] are also presented in the figure.

The data presented in Fig. 8 are not fully compatible with the equivalent rectangular bandwidth of the auditory filter. For frequency above 1 kHz, however, the relation between ERB and CB determined by Sęk and Schorer is approximately linear and it may be assumed that differences observed in this frequency area are not important and may be ignored. Quite a different situation is observed in the low-frequency region i.e. below 500 Hz. As frequency decreases, the difference between critical bandwidth measured be Sęk [36, 38] and Schorer [35] on one side and ERBs measured by Moore on the other becomes statistically significant and gets progressively larger; for frequency of 125 Hz the difference reaches about 25 – 30 Hz. Thus even though the estimates based on the critical modulation rate showed that the auditory filter bandwidth decreases as frequency decreases below 500 Hz there is no agreement between CB and ERB. The difference observed between them is not just quantitative but also qualitative.

The above presented comparison rises some questions. For example, is the critical band model correct or does the critical modulation frequency properly describe the auditory filtering that takes place in the peripheral auditory system? Does the critical modulation frequency properly describe a phase sensitivity of the auditory system for very low frequencies?

To check on this, some predictions based on the excitation pattern model have been made as well as results of two experiments have been compared. This is described in the following sections.

3. Further analysis of the critical modulation frequency concept

A more detailed analysis of Schorer and Sęk's arguments that the critical modulation rate is a proper estimator of the critical bandwidth was elaborated theoretically and experimentally and is presented below. It suggests that all Schorer and Sęk's arguments may not be entirely valid. Furthermore, the interpretation of the CMF proposed by ZWICKER [45] and SCHORER [35] may not be completely correct. HARTMANN and HNATH [10] suggested that the CMF corresponded to the point where the lower sideband in the spectrum first becames detectable. The threshold for detecting the lower sideband depends more on the selectivity of auditory filters centred close to the frequency of the sideband than on the selectivity of the auditory filter centred on the carrier frequency. Furthermore, the level of the sideband relative to that of the carrier may be altered by transmission through the middle ear, especially at low frequencies, where the efficiency of transmission may change markedly with frequency [33, 54].

Perhaps a more serious problem comes from the possibility that, at the CMF, detection is not always based on the lower sideband. The results of several experiments support the idea that, at medium to high centre frequencies, the lower sideband is more detectable than the upper sideband [10, 19, 24]: the lower sideband is entirely responsible for modulation detection since the upper sideband is totally masked by the carrier. However, this may not be the case at low centre frequencies i.e. below 500 Hz because the absolute threshold strongly increases as the frequency decreases. For a modulated signal with carrier frequency less than 500 Hz, the lower sideband $(f_c - f_{\mathrm{mod}})$ may by attenuated by the middle ear much more than the upper one and modulation detection may be based on the upper sideband $(f_c + f_{\mathrm{mod}})$ of the modulated signal. If the detection were based on the upper sideband at low carrier frequencies, then the monaural phase effect would not occur and the CMF would not provide a good estimate of the CB, or the ERB of the auditory filter, since the sideband on which detection is based would change as the centre frequency was changed.

The main purpose of the theoretical consideration, as well as the experimental studies reported below, was to explore the mechanism of modulation detection for low carrier frequencies and for modulation frequencies around the CMF. More specifically, the main aim was to determine whether the detection of AM and FM for modulation frequencies at and above the CMF depends on detection of the lower sideband, the upper sideband or both, and whether this changes with carrier frequency. To establish this the results of theoretical and experimental results were considered.

3.1. Theoretical consideration

In the first part of the theoretical considerations an excitation pattern model was used to evaluate the differences between the excitation patterns produced by modulated and unmodulated signals. It was done in order to show a simple subtraction between those two excitations and to check on which frequency side of the excitation pattern a bigger difference can be observed.

The excitation pattern that were used in these considerations were calculated using a program suggested by GLASBERG and MOORE [6]. The excitation is defined there as the output from each auditory filter as a function of center frequency. Each auditory filter is assumed to have the form of a rounded exponential, as in Eq. (1). This program enables to use two independent slopes for low and high frequency sides of the filter using B_u and B_l parameters [6, 13]. It also enables to control a nonlinearity on both sides of the filter independently by means of NL_l and NL_u parameters. However in this considerations the simplest case of the excitation pattern model was used. Thus the values of described above parameters were set to be equal 1, i.e. $B_u = B_l = NL_l = NL_u = 1$. The ERB spacing was assumed to be equal 0.1. Evaluations of any differences between excitation patterns produced by modulated and unmodulated signals were calculated based on those ERBs for which the excitation pattern was bigger than 10 dB, as in MOORE and SĘK [20]. They are referred to as the active channels.

GLASBERG and MOORE's program [6] also requires a correction file connected with the intensity weighing function. Thus the threshold correction file describing the ASA standard absolute threshold was used in all the theoretical considerations.

Evaluations of the differences between the excitations pattern produced by a modulated and unmodulated signal were calculated for carrier frequencies of 125 and 200 Hz. The modulating frequency was assumed to be equal to $1.1 \cdot \mathrm{CMF}$ based on SĘK's [38] results. Thus the unmodulated signal consisted of one component (f_c) whereas the modulated one consisted of three components with frequencies: $f_c - 1.1 \cdot \mathrm{CMF}$, f_c and $f_c + 1.1 \cdot \mathrm{CMF}$. The sound pressure level of the carrier was assumed to be equal to 70 dB SPL. Thresholds for detecting amplitude and frequency modulation for low carrier frequencies and for modulating frequency $1.1 \cdot \mathrm{CMF}$, expressed in appropriate modulation indices are equal to $\beta_{th} = m_{th} = 0.06$, [38, 45]. The thresholds for detecting both AM and FM do not depend significantly on carrier frequency below 1 kHz. Thus the sidebands of modulated signals were assumed to be equal to 40 dB SPL each ($m = \beta = 0.06$).

Results of these calculations were presented in Fig. 9 for carrier frequencies of 125 and 200 Hz. The upper panels present two excitation patterns. A solid line in each panel identifies the excitation pattern of unmodulated (pure sinusoid) signal whereas dotted line presents the excitation pattern of modulated signals. Those two curves are almost identical in these panels and it is difficult to see any difference between them. However, the differences of those two patterns are presented in the lower panels. The spectral structure of modulated signals was schematically presented in each panel of Fig. 9. Sidebands were marked as small arrows and the carrier was marked as a slightly bigger arrow.

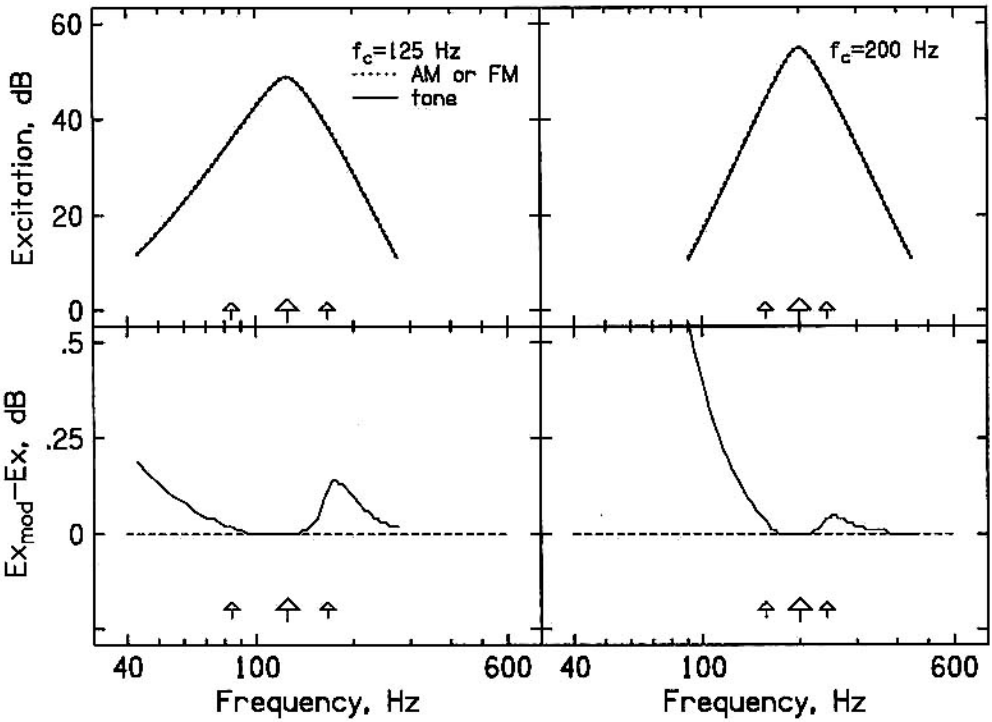

Fig. 9. Excitation patterns produced by unmodulated (solid lines) and modulated (dotted lines) carriers at frequencies of 125 Hz (left upper panel) and 200 Hz (right upper panel). Modulation rates were equal to $f_{mod} = 1.1 \cdot \text{CMF}$ and modulation index (m or β) was equal to 0.06. Lower row shows differences in the excitation patterns presented in the upper row.

The left column of Fig. 9 shows the excitation patterns (the upper panel) and their difference (the lower panel) for modulated and unmodulated signals with carrier frequency of 125 Hz. Similar differences can be observed on the low and high frequency sides of the excitation pattern. However, the difference observed on the high-frequency side can be characterized by a local maximum for frequency corresponding to the higher sideband of a modulated signal. Moreover, an exact value of this difference of excitation patterns for frequency corresponding to the higher sideband is slightly bigger than the difference corresponding to the lower sideband. Taking into account a single band model [11, 51] which assumes that detection of modulation takes place when the difference of excitation patterns of modulated and unmodulated sounds exceeds a certain amount in the single band (in this case in one active channel), it may be stated that the upper sideband of the modulated signal is decisive for modulation detection in this case.

The right column of Fig. 9 presents the results for $f_c = 200$ Hz. The difference in the excitation patterns observed on the low-frequency side is bigger than the difference on the high-frequency side. Thus, based on a single band model, it seems that the lower sideband of the modulated signal might be decisive for modulation detection in this case. Similar results were also obtained for frequencies higher than 200 Hz. A single-band model predicts that the lower component of the modulated signal is decisive for modulation detection for frequency range higher than about 200 Hz. This is consistent with experimental data obtained for carrier frequencies of 1 kHz [10, 18].

In the next step of this analysis evaluation of the differences between excitation patterns produced by modulated and unmodulated signals, based on a non-optimal multi-band excitation pattern model were carried out. Like Zwicker's model the model assumes that the detection of modulation is based in this case on a single form of information, namely changes in excitation level. However, this model assumes that information from different parts of the excitation pattern can be combined in a non-optimal way (see [20, 37] for details). The model is similar in its general form to the model proposed by FLORENTINE and BUUS [5] and is based on the integration model [8].

This model is based on the assumption that changes in amplitude or frequency of signal are detected by virtue of the changes in the excitation level that are produced in the peripheral auditory system. If the changes in excitation level in the i-th critical band or auditory filter gives rise to a value d'_i then overall value of d' is given by:

$$d' = \sqrt{\sum (d'_i)^2}. \tag{4}$$

As the first approximation it might be assumed that d'_i is proportional to the difference in the excitation level observed in this channel [37] thus the overall value of d' was assumed to be:

$$d' = K \sum_{i=1}^{n} \frac{\Delta L_i^2}{\sqrt{n}}, \tag{5}$$

where n is the number of active channels and K is a constant.

Using the above described excitation pattern program the values of $\sum \frac{\Delta L_i^2}{\sqrt{n}}$ were calculated for both the low- and the high-frequency side of the excitation pattern. Similarly to earlier considerations only those auditory filters, that gave the excitation level greater than 10 dB, were taken into account.

Results of these calculations are presented in Fig. 10 and 11. Figure 10 presents differences (expressed as $\sum \frac{\Delta L_i^2}{\sqrt{n}}$) of the excitation patterns of modulated and unmodulated signals as a function of a carrier frequency. In this case modulation frequency was equal to $f_{\mathrm{mod}} = 1.1 \cdot \mathrm{CMF}$, sound pressure level of the carrier signal 70 dB SPL and sound pressure level of the sidebands was equal to 40 dB SPL each. A continuous line in this figure presents a difference between excitation levels of modulated and unmodulated signals on the high-frequency side whereas the dotted line presents this difference observed on the low-frequency side of the excitation pattern.

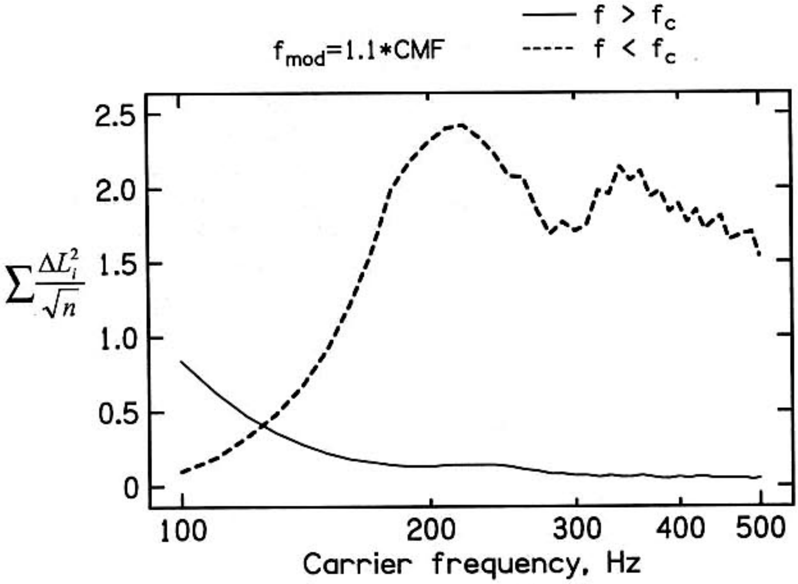

Fig. 10. Differences in the excitation pattern (expressed as $\sum \frac{\Delta L_i^2}{\sqrt{n}}$) produced by modulated and unmodulated signals as a function of carrier frequency. Modulation frequency was equal to $f_{\text{mod}} = 1.1 \cdot \text{CMF}$, sound pressure level of the carrier signal 70 dB SPL and sound pressure level of the sidebands 40 dB SPL each. Continuous line in this figure presents difference between excitation levels of modulated and unmodulated signals on the high-frequency side whereas the dashed line presents this difference on the low-frequency side of the excitation pattern.

The difference obtained for the high-frequency side of the excitation pattern is bigger than the difference observed on the low-frequency side only in a very restricted region of carrier frequencies i.e. up to 140 Hz only. Therefore it may be assumed that for this carrier frequency range the upper sideband is decisive for modulation detection which is not consistent with the HARTMANN and HNATH [10] model. However, for carrier frequencies greater than 140 Hz the difference obtained for the low-frequency side of the excitation pattern is much bigger than the difference on the high-frequency side. It means that above 140 Hz and for modulation rates above the critical modulation frequency, the lower sideband of the modulated signal might be responsible for modulation detection. So the predictions of the single-band and multi-band models are similar.

Figure 11 presents differences in the excitation patterns (expressed as $\sum \frac{\Delta L_i^2}{\sqrt{n}}$) observed on the low- and high-frequency sides of the excitation pattern obtained for modulated and unmodulated signals as a function of modulation rate for carrier frequencies of 125 Hz, and 250 Hz. Dashed lines in this figures denote the differences observed on the low-frequency side and continuous lines denote differences observed on the high-frequency side of the excitation pattern.

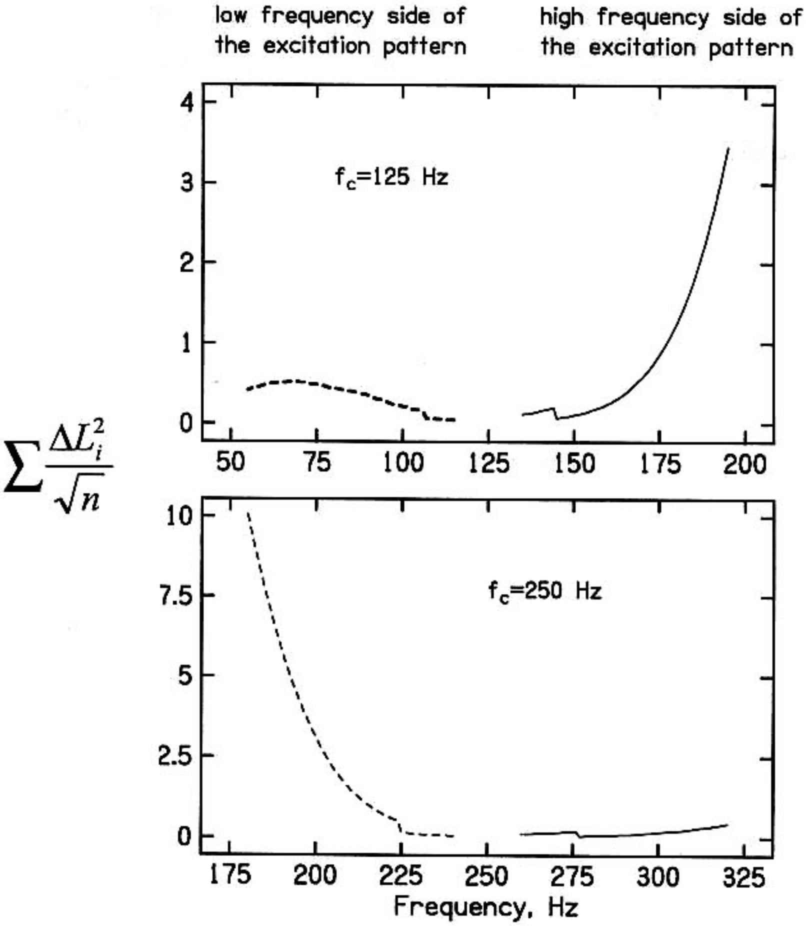

Fig. 11. Differences in the excitation patterns (expressed as $\sum \frac{\Delta L_i^2}{\sqrt{n}}$) observed on the low- and high-frequency side of the excitation pattern (for modulated and unmodulated signals) as a function of modulation rate, for carrier frequencies equal to 125 Hz and 250 Hz. Dashed lines denote the differences observed on the low-frequency side and continuous lines — on high-frequency side of the excitation pattern respectively.

As can be seen from the top panel of Fig. 11 the difference observed on the high-frequency side of the excitation pattern is much bigger than the difference on the low-frequency side when a carrier frequency was equal to 125 Hz. It can be stated that for modulation frequencies greater than the CMF the detection of modulation for this particular carrier frequency might be based on changes in the high-frequency side of the excitation pattern. These changes were evoked by the higher sideband of the modulated signal. So this sideband might be decisive for modulation detection in this case.

A quite oppsite situation can be observed for carrier frequencz 250 Hz (see Fig. 11 the bottom panel). The difference of excitation patterns evoked bz modulated and unmodulated signals is bigger on the low-frequencz side than on the high-freqencz side of the excitation pattern. The lower sideband of the modulated signal might be responsible for modulation detection in this case.

Based on the above considerations it can be stated that modulation, either AM or FM, might be detected not only by means of the lower frequency sideband of modulated signal's spectrum, as HARTMANN and HNATH's [10] model predicts. In the case when carrier frequency is equal to 125 Hz and modulation frequency is slightly bigger than the CMF modulation detection might be based on the higher sideband. It might be also expected that for carrier frequencies from the range of 125 – 160 and for modulation frequencies bigger than the CMF, a contribution of the upper sidebands of the modulated signal's spectrum in the detection of modulation might be bigger than a contribution of the lower one. In this frequency range, modulation might be detected based on the upper sideband.

3.2. Results of some experimental studies

SĘK [36] has presented evidence that the interpretation of the CMF proposed by ZWICKER [45], and generally accepted in the literature, is not quite correct. For medium to high frequencies the critical modulation frequency appears to correspond to the point where the lower sideband in the spectrum of the modulated signal first becomes detectable. This is often referred to as the HARTMANN and HNATH [10] model. However, SĘK and MOORE [39] showed, that for very low carrier frequencies the upper sideband becomes first detectable before the lower sidebands. So it seems that the lower sideband is not always decisive for modulation detection.

To determine which sideband is decisive for modulation detection SĘK and MOORE [39] compared the results of two separate experiments. In the first one thresholds were determined for the detection of amplitude modulation and frequency modulation using several low carrier frequencies, namely f_c = 125, 160, 200 and 250 Hz. The following modulation rates were used: f_{mod} = 20, 30, 40, 50, 60 and 70 Hz. They expressed measured thresholds for detection either AM or FM in terms of the sound pressure levels of the sidebands i.e. L_{AM} and L_{FM} for amplitude and frequency modulation. In the second experiment thresholds were measured for detecting a single sinusoid, corresponding to either the lower (L_l) or the upper (L_u) sidebands of the modulated signal used in the first experiment, in the presence of a sinusoidal masker corresponding to the carrier frequency of the modulated signal. A schematic structure of the signal spectra they used is presented in Fig. 12. If a component corresponding to a given sideband, say the lower one, has a lower threshold than a component corresponding to the other sideband, i.e. $L_l < L_u$ at the threshold, then the given sideband should be more detectable in a situation where both sidebands are present, as for example, in the modulation detection task. Moreover, if modulation detection thresholds are determined by the threshold for the most detectable sideband, then the thresholds measured for that sideband in the second experiment should correspond with those determined in the first experiment.

The results of these experiments are presented in Fig. 13. Each panel shows the data averaged across three subjects for one carrier frequency. The value of the carrier fre-

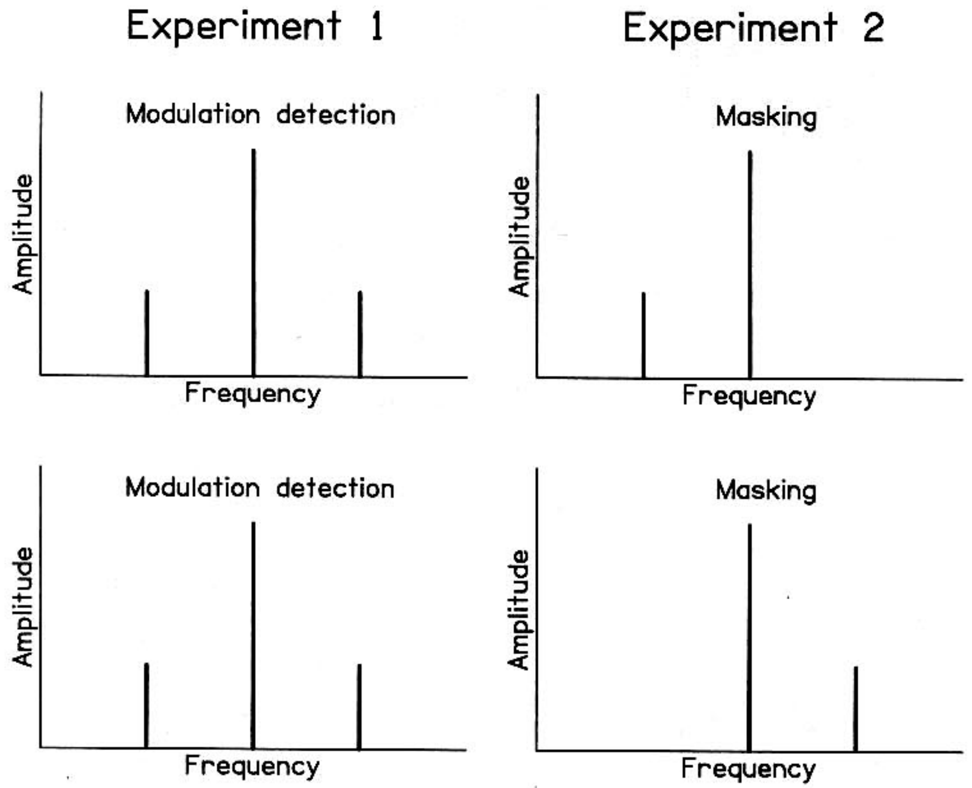

Fig. 12. A schematic illustration of a spectral structure of stimuli used by SĘK and MOORE [39] in two separate experiments.

quency is marked in each panel by an arrow. Dashed lines show absolute thresholds (standard threshold and the subject's threshold measured directly in the experiment). Squares and circles in each panel show the results of the first experiment 1. The level of each sideband at the modulation detection threshold, i.e. the values of L_{AM} and L_{FM}, is plotted as a function of its frequency. Since the lower and upper sidebands for both type of modulation were always equal in level, each measured threshold is presented twice giving a pattern symmetrical about carrier frequency. For the higher modulation frequencies used, the AM and FM thresholds coincide ($L_{\mathrm{AM}} = L_{\mathrm{FM}}$), whereas for the lower values used, i.e. when sideband were closer to the carrier, the thresholds for detecting FM are higher than the thresholds for detecting AM, ($L_{\mathrm{FM}} > L_{\mathrm{AM}}$). This is consistent with previous findings, see for example ZWICKER [45], SCHORER [35] or SĘK [36] and Fig. 8, showing that the CMF for low carriers is typically is about 40 Hz. This also reflects the fact that the modulation frequencies were chosen properly to span the critical modulation frequency.

The stars and diamonds in Fig. 13 show the results of the second experiment (i.e. the values of L_u and L_l, for the lower and the upper sideband respectively) where the thresholds were measures for a single sinusoid corresponding to the lower or the upper

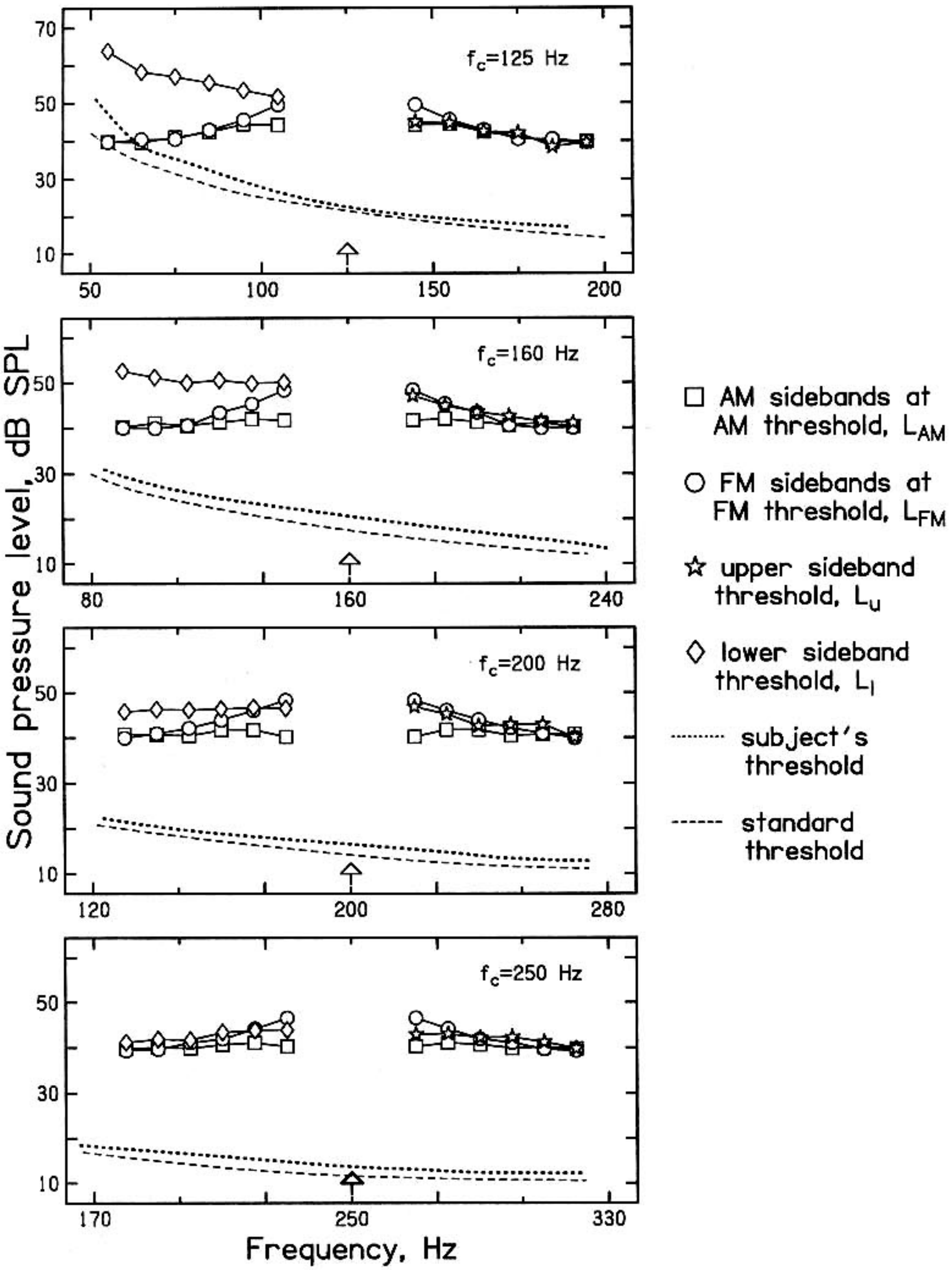

Fig. 13. Each panel shows data averaged across three normal hearing subjects for one carrier frequency. Modulation detection thresholds are expressed in terms of the levels of the sidebands.

sidebands in the spectrum of the modulated signal. The thresholds obtained for the signals with frequencies below the frequency of the carrier (diamonds) were generally higher than those for signals with frequencies above the carrier ($L_u > L_l$). This suggests that when both sidebands were present simultaneously, as in the experiment concerned with the detection of modulation, detection was based on the upper sideband rather than on the lower sideband. This is consistent with the above presented consideration based on the excitation pattern model.

Comparing the results of these two experiments it is clear that, at least for the two lowest carrier frequencies used (125 and 160 Hz), the thresholds for the lower sideband alone (diamonds) are markedly higher than the levels of the lower sideband at the modulation detection threshold (circles and squares). This indicates that, at the modulation detection threshold, the lower sideband would have been undetectable. Indeed, for the highest modulation frequency used the lower sideband would have been below absolute threshold. In contrast, for the modulation frequencies above the critical modulation frequency the levels of the upper sideband at the modulation detection threshold (circles and squares) are almost identical with the detection thresholds for the upper sideband when presented alone i.e. $L_{\mathrm{AM/FM}} = L_u$ (stars). This is true for all carrier frequencies used. Taken together, these results indicate, that modulation detection thresholds for modulation frequencies above the critical modulation frequency and carrier frequencies of 125 and 160 Hz were determined by the threshold for detecting the upper sideband. This is also consistent with the above presented theoretical predictions based on the excitation pattern model.

The results of SĘK [36] suggest that for carrier frequencies from 200 to 250 Hz there is a certain transition region. For modulation rates above the critical modulation frequency and for carrier below that region modulation detection threshold were determined by the threshold for detecting the upper sideband. This is clearly seen in the two upper panels of Fig. 13 that present the results for carrier frequencies of 125 and 160 Hz. However, for carriers above that region detection is probably based mainly on the threshold for detecting the lower sideband. This is consistent with the above presented theoretical consideration (see Sec. 3.1 and Figs. 9 – 11) and with previous data of HARTMANN and HNATH [10], OZIMEK and SĘK [23] and MOORE and SĘK [18]. However, in that region both sidebands may contribute to detection of the modulation. It may be assumed that modulation is detected by means of the auditory filter at a centre frequency close to the frequency of the decisive (the most detectable) sideband.

The fact that the most detectable sideband switches from the lower one to the upper one as the carrier frequency decreases means clearly that the critical modulation frequency is not a satisfactory measure of the auditory filter bandwidth or the frequency selectivity of the auditory system, since the auditory filter centred on the upper sideband has a broader bandwidth than the auditory filter centred on the lower sideband.

4. Conclusions

The auditory system is usually considered an array of bandpass overlapping linear filters, called auditory filters. This basic point of view has been confirmed in a variety of experiments and at present is well established. One of the most important factors describing the properties of the auditory filter is its bandwidth. So far two measures have been proposed: the Critical Bandwidth CB and the Equivalent Rectangular Bandwidth ERB. These two measures are consistent in a high-frequency region, i.e. for frequencies higher than 1 kHz where the ratio of CB to ERB is approximately constant. However these two measures are considerably different in a low-frequency region, i.e. for frequencies lower than 1 kHz.

Estimates of the critical bandwidth, as collected by SCHARF [34], were based on the results of band-widening and two-tone masking experiments while the equivalent rectangular bandwidth were based on the data collected in notched noise masking or psychophysical tuning curves experiments. ZWICKER [45] and SCHORER [35] have suggested that the critical band reflects also a sensitivity of the auditory system to phase differences of spectral components of stimulus, as observed in so-called monaural phase effect [12]. If a bandwidth of a given stimulus with a centre frequency f is less than the critical bandwidth for this frequency, then its phase structure is very important and influences a sensation evoked by the stimulus. However, when a bandwidth of a stimulus is greater than one critical band then its phase structure is less important: a change in the relative phase of components in the stimulus may have not influenced the sensation evoked by the stimuli. Such a situation has been observed in a case of modulation detection. The phase of the lower component of amplitude and frequency modulated signal is different. Due to this difference a marked discrepancy in AM and FM thresholds are observed, particularly for low modulation frequencies. However, when modulation rate reaches so-called critical modulation frequency (CMF) the threshold for detecting AM and FM becomes identical. It has been suggested that the CMF determines that point at which one of the sidebands of the modulated signal's spectrum first becomes detectable. As long as the lower sideband (i.e. that one whose phase is different in AM and FM case) first becomes detectable and responsible for modulation detection the critical modulation frequency may be considered as a measure of the phase sensitivity of the auditory system or a proper measure of the auditory filter bandwidth.

However, as shown in this paper, for very low carrier frequencies the thresholds for detecting both AM or FM were not determined by the detection of the lower sideband in the modulated signal's spectrum.

The CMF is defined as the modulation frequency at which AM and FM thresholds first become identical. However, the results considered in this paper show that for very low values of the carrier frequency the threshold for detection of either AM or FM was not determined by the detection of the lower sideband in the modulated signal's spectrum. Thus the CMF does not reflect the ability of the auditory system to detect the phase difference between the spectral component of the complex sound, since upper components have the same phases. The CMF, then, is confounded as a proper estimate of the critical bandwidth since, at least for low frequencies, it does not say anything about phase sensitivity of the auditory system.

The above considerations also give an explanation of the discrepancy observed between the critical bandwidth and the equivalent rectangular bandwidth for very low carrier frequencies. For carrier frequencies less than 200 Hz modulation detection was entirely based on detection of the upper sideband in the modulated signal's spectrum, (i.e. at frequency $f_c + f_{\text{mod}}$). Excitation evoked by this frequency component was associated with the activity of an auditory filter with a centre frequency higher than the frequency of the carrier signal i.e. with a frequency close to this component. Thus modulation detection in this carrier frequency region was based exclusively on the auditory filter with centre frequency higher than the carrier frequency.

For frequencies 200 – 250 Hz both sidebands, the upper one and the lower one, contribute to the detection of modulation. Excitation changes associated with these components have approximately the same value. It is difficult to state which auditory filter (i.e. either with lower or with higher centre frequency than the carrier) is responsible for modulation detection in this case. This carrier frequency area is probably a transition area in which the activity of the auditory filters with centre frequencies lower and higher than the carrier are nearly the same. These filters are equally responsible for modulation detection.

For the frequency range above 250 Hz modulation detection was entirely based on the detection of the lower spectral component ($f_c - f_{\text{mod}}$) of the modulated signal. Excitation evoked by this sideband was associated with the activity of the auditory filter with a centre frequency close to the frequency of this component. Thus modulation detection in this case was exclusively based on an auditory filter with a centre frequency lower than the carrier frequency.

If the modulation detection had always been based on the auditory filter with centre frequency lower than the carrier, then the critical bandwidth would have decreased even for very low carrier frequencies. However, when the carrier frequency decreases, in the transition area (i.e. 200 – 250 Hz), the auditory filter that is responsible for modulation detection switches from a filter at a centre frequency lower than the carrier to a filter at a centre frequency higher than the carrier. The equivalent rectangular bandwidth of the filter at a centre frequency higher than the carrier is bigger than the ERB of the filter at a centre frequency lower than the carrier. Thus the change of the filter is the reason that dependence of the CBs on the frequency flattens off while ERBs decrease even for very low carrier frequencies.

There are several factors that might cause the lower sideband to be less detectable for low carrier frequencies. These include the effect of the transfer function of the middle ear [54], the possibility that the cochlea is characterised by a relatively high level of internal noise, particularly at low frequencies [21] and the possibility, that the signal-to-masker ratio required for the threshold increases at low frequencies. Is seems likely that the main detection cue used for signal detection in the case when a single sideband was presented were beats produced by the interaction of a signal (imitating one of the sidebands) and masker. Harris [9] showed that a "depth of modulation" produced by beats required for threshold increased mainly at low frequencies. In addition, the detectability of beats decreases with an increasing beats rate above 3 – 4 Hz and with decreasing in level, [30].

These factors provide further justification for a claim that the critical modulation frequency is confounded as a measure of the auditory filter bandwidth or frequency selectivity of the auditory system. None of these factors is directly connected with frequency selectivity, yet may influence the value of the critical modulation frequency for low carrier frequencies. The above presented results show that, taking these factors into account, the values of the critical modulation frequencies for low carrier frequencies can be reconciled, at least to a first approximation, with the estimates of auditory filter bandwidth and shape summarised by MOORE [15].

The critical bands determined based on the critical modulation rate are confounded as a measure of auditory filter's bandwidth for very low frequencies. They do not reflect directly the frequency selectivity of the auditory system. The assumption that the auditory filter has a constant width, as used in the critical band concept presented by SCHARF [34] and used up to the present for this frequency region, is in error.

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