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ON PRACTICAL ASPECTS OF OPTIMAL MODELLING IN BOUNDARY ELEMENT METHOD

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The paper is concerned with application of the BEM in the environmental noise problems. The main step of the BEM is the discretization of the boundary into elements. Using too many elements is not efficient. To eliminate this drawback, in this paper two ideas are proposed. First of them is the optimal discretization, and the second one is the solution of the modelling problem in rotated coordinates. Since the optimal discretization theory derived for the function of one variable, then only the 2D problem is considered. The commercial code SYSNOISE is used to solve the numerical examples. The results confirm the utility of the proposed ideas.

Notations

P, Q, r	see Fig. 1,
HIE	Helmholtz Integral Equation,
BEM	Boundary Element Method,
Φ, $\Phi(Q)$	velocity potential,
$\Psi(r)$	free-space Greens function; $\Psi(r) = e^{-ikr}/r$, $r = \lvert P - Q \rvert$,
$\beta(Q)$	surface admittance; $\beta(Q) = v_n/p$,
v, $v_n(Q)$	particle velocity, vibration velocity; $v_n(Q) = -\partial\Phi(Q)/\partial n$,
p, $p(Q)$	acoustic pressure,
k	wave number; $k = \omega/c$, ω – angular frequency, c – speed of sound,
j	number of the element; $j = 1, 2, ..., n_j$,
i	number of the node on j-element; $i = 0, 1, ..., n_i$,
ν	global number of the nodes; $\nu = 1, 2, ..., n_\nu$,
(x, y)	(x, y)-coordinates,
f	frequency,
ρ_0	air density.

1. Introduction

The propagation of the waves through air is governed, in the steady state, by the Helmholtz equation (frequency domain model). Then, this equation has to be solved subject to the appropriate boundary conditions. Usually, the Helmholtz equation with boundary conditions is replaced by the Helmholtz Integral Equation (HIE). An exact solution of the HIE is possible in particular cases only. Then the HIE is solved using the numerical methods. Particularly the direct Boundary Element Method (BEM) offers an elegant and efficient solution, [5, 8, 15, 30, 36, 37]. In spite of many advantages and versatility of the BEM, there are a few drawbacks. One of them is that the standard HIE for exterior problems fails to have a unique solution at frequencies corresponding to the eigenfrequencies of the adjoint associated internal problem. But this problem was solved; see references pointed out in the Sec. 3.

Another problem is the quality of the problem solution. One way of improving this quality, i.e. the convergence of the BEM solution towards the exact one, is improving the quality of the boundary model. There are different methods to perform it:

- h-refinement; i.e. by using more elements, [22, 29, 34],
- p-refinement; i.e. by increasing the order of the polynomial, [16, 22, 29],
- r-refinement; i.e. by moving the nodes within an element, [10],
- the combination of the methods given above, particularly $h-p$, [11, 22, 23, 29, 33].

All papers have been mainly described in the mathematical sense. It seems that the advanced theory of the BEM is not widely known among acoustic engineers; more practical approach to the BEM is presented in [21].

In the papers [3, 4] the methods described above were revised, completed and presented in an engineering sense. But the acoustic test cases presented there are rather academic; the plane and fully axisymmetric source is considered. All methods improving the boundary model may be applied in engineering problems; it is the aim of this paper. To perform it, it is necessary to use the commercial software package, e.g. SYSNOISE, [35]. Unfortunately, this software code does not take into consideration many methods proposed in [3, 4]. But one of them, i.e. the optimal discretization, may be adopted. In this method, the boundary between elements is changed, without using more elements (h-refinement).

As an example, the highway noise problem is considered. Then the main aim is the estimation of the efficiency of the noise barrier, what leads to calculation of the acoustic pressure around the barrier; up to now, five different theories were applied, [13, 14, 19]. It seems that the HIE is an accurate and efficient method to do it, [8, p. 234], [20, 24]. In the following, the barrier, which was worked out in [28], is tested; see always [17]. It turned out that for this barrier, the theory of optimal discretization can not be applied directly. To cope with this difficulty, one proposes to solve the optimal modelling problem in rotated coordinates, which is a new idea. Then in the paper, to improve the quality of the boundary model (and consequently, the quality of the boundary problem solution), two methods are presented. First of them is the discretization in the rotated coordinates. The second one is the optimal discretization. Both methods improve the quality of the boundary model and consequently the proliferation of the elements and the nodes can be avoided. So they allow to reduce the amount of the computer storage and the computing time.

To demonstrate the validity, efficiency and advantages of both methods considered in the paper, the numerical examples are performed.

This paper is written for engineers, not for mathematicians. In the following sections, the authors avoid mathematical rigour in the interests of clarity.

2. Statement of the problem

Consider the boundary surfaces S of any body in an infinite acoustic medium of average density ρ_0. The body can be either a vibrating structure (in a radiation problem) or a passive obstacle (in a scattering problem). The governing differential equation in steady-state linear acoustics is the Helmholtz equation, Fig. 1,

$$\nabla^2\Phi + k^2\Phi = 0, \qquad \Phi = \Phi(P), \qquad P \in \Omega_e. \tag{1}$$

For scattering problems, the velocity potential $\Phi(P)$ can be decomposed into two parts,

$$\Phi = \Phi^{\rm inc} + \Phi^s, \tag{2}$$

where $\Phi^{\rm inc}$ – incident wave potential in the absence of the obstacle, Φ^s – scattered wave potential. Both parts have to satisfy the Helmholtz equation in Ω_e.

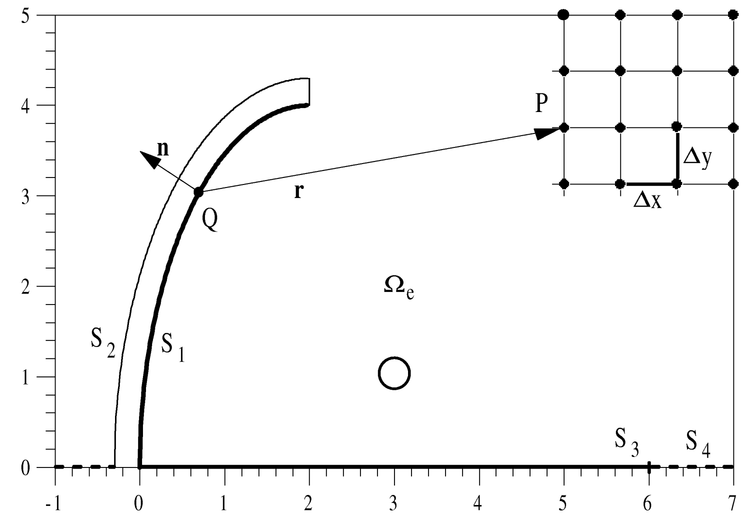


Fig. 1. Geometry of the problem: Ω_e — 2D fluid domain, $S_1 + S_2$ — barrier surface, S_3 — road surface (asphalt), S_4 — ground surface (grassland), P — observation point, ○ — source point.

Boundary conditions on the boundary surface S are of the general form

$$c_1\Phi(Q) + c_2\frac{\partial\Phi(Q)}{\partial n} = c_3, \tag{3}$$

where $n = n_Q$ – inward normal on S (directed away from the acoustic domain Ω_e), c_1, c_2, c_3 – specified constants.

The Helmholtz equation can be also written in terms of the acoustic pressure $p(P)$ and the particle velocity $v(P)$. But it is more convenient to work with the acoustic potential $\Phi(P)$, from which both $p(P)$ and $v(P)$ can be readily obtained. The acoustic particle velocity is given by $\mathbf{v}(P) = -\nabla\Phi(P)$, and the acoustic pressure can be evaluated from $p(P) = \bar{i}\rho_0\omega\Phi(P)$; $\bar{i} = \sqrt{-1}$.

In the case of acoustic radiation, the velocity potential $\Phi(P)$ must also satisfy the Sommerfeld radiation condition at infinity, but for scattering problems only the scattered potential $\Phi^s(P)$ must satisfy this condition, [31].

3. Integral formulation of the problem

By using the "direct" formulation (via either the Green second identity or the weighted residual formulation), Eq. (1) is reformulated into, [5],

$$C(P)\Phi(P) = \int\limits_S \left(\Psi(\mathbf{r}) \frac{\partial\Phi(Q)}{\partial n} - \frac{\Psi(\mathbf{r})}{\partial n}\Phi(Q) \right) dS(Q) + 4\pi\Phi^{\mathrm{inc}}(P), \tag{4}$$

where $C(P)$ – the constant that depends on the location of P,

$$C(P) = \begin{cases} 4\pi, & P \in \Omega_e, \\ \theta(P_S), & P_S = P \in S, \\ 0, & P \notin \Omega_e \cup S, \end{cases} \tag{5}$$

where $\theta(P_S)$ – exterior solid angle formed by the boundary surface.

Equation (4) with the coefficient $C(P) = 4\pi$ is the Helmholtz Huygens (HH) integral. From this integral the values of the acoustic potential $\Phi(P)$ within Ω_e can be determined, but the boundary values of the acoustic potential $\Phi(Q)$ and its derivative $\partial\Phi(Q)/\partial n$ should be known. However, for most practical acoustic problems either $\Phi(Q)$ or $\partial\Phi(Q)/\partial n$ is known. Therefore the values of $\Phi(P_S)$ at the boundary must be determined first. It may be done by solution of the HIE.

Equation (4) with the coefficient $C(P) = \theta(P_S)$ is referred to as the HIE and it is the second kind Fredholm integral equation. To derive it, first the boundary condition should be specified. Here an acoustic admittance $\beta(Q)$ is assumed on the surface S; in explicit form

$$\beta(Q) = -\frac{\partial\Phi(Q)/\partial n}{\alpha\Phi(Q)}, \qquad \alpha = \bar{i}\rho_0\omega. \tag{6}$$

The condition $\beta(Q)$ may be expressed by Eq. (3) assuming appropriate constants c_1, c_2, c_3. Substituting Eq. (6) and $C(P) = \theta(P_S)$ into (4), the HIE is obtained,

$$\theta(P_S)\Phi(P_S) + \int\limits_S \Phi(Q) \left[\alpha\beta(Q)\Psi(\mathbf{r}_S) + \frac{\partial\Psi(\mathbf{r}_S)}{\partial n} \right] dS(Q) = 4\pi\Phi^{\mathrm{inc}}(P_S). \tag{7}$$

This is the simplest HIE. However, in any acoustic problems, e.g. the CONDOR technique given by Burton and Miller, [22, p. 44], [30, p. 49], must be used in order to establish an equivalence between the HIE and the original problem given by Eqs. (1) and (3); other techniques may be found in the given references.

4. Solution technique

The numerical solution of Eq. (7) for the BEM can be divided into four steps. The first one is the division of the boundary into boundary elements (discretization). The second step is the numerical integration over the elements; these two steps lead from the HIE to the form of the algebraic system of equations. The third step is the solution of the system of equations to obtain the unknown boundary variables, here $\Phi(P_S)$. The last step is the calculation of the field values, $\Phi(P)$, from the HH integral.

At the first step, the integrals in the HIE can be replaced by the sum of integrals over surface elements S_j, $j = 1, 2, ..., n_j$. At this step no approximations are introduced. Now, it is customary to express the geometry and the distribution of the variables $(\Phi(Q), \beta(Q))$ in terms of their values at the nodes. It may be done basing on the interpolation theory.

Using piecewise polynomial interpolations, the global Cartesian coordinates $\overline{x} = (x, y, z)$ of any point on the surface elements S_j are related to the nodal coordinates $\overline{x}_i$ by

$$\overline{x}(\xi) = \sum_{i=0}^{n_i} N_i(\xi)\overline{x}_i\,, \tag{8}$$

where $N_i(\xi)$ — shape functions of the local coordinates $\xi = (\xi_1, \xi_2)$; the explicit form of $N_i(\xi)$ may be found anywhere, e.g. [5, 8, 36].

Quite similarly the geometry of the source and variables $\Phi(Q)$ and $\beta(Q)$ may be expressed; assuming the isoparametric elements

$$\left.\begin{array}{c} \widetilde{f}(\xi) \\ \widetilde{\Phi}(\xi) \\ \widetilde{\beta}(\xi) \end{array}\right\} = \sum_{i=0}^{n_i} N_i(\xi) \left\{\begin{array}{c} f_i \\ \Phi_i \\ \beta_i \end{array}\right. , \tag{9}$$

where $f_i = f(\overline{x}_i)$, $\Phi_i = \Phi(\overline{x}_i)$, $\beta_i = \beta(\overline{x}_i)$, $\overline{x}_i = Q_i \in S_j$.

After inverse transformation of $\widetilde{f}(\xi)$, $\widetilde{\Phi}(\xi)$, $\widetilde{\beta}(\xi)$, from coordinates ξ to $\overline{x}$, the approximate values $\widetilde{f}(\overline{x})$, $\widetilde{\Phi}(\overline{x})$, $\widetilde{\beta}(\overline{x})$ are obtained, respectively. These values, between the nodes, do not coincide with the original functions. Hence, an interpolation error is imposed on the approximated values. This error, for example $\widetilde{f}(\overline{x})$, is defined as follows:

$$E(\overline{x}) = f(\overline{x}) - \widetilde{f}(\overline{x}). \tag{10}$$

This error cannot be calculated exactly. Then two estimates are used. The first estimate is at the point $\overline{x}$, and the second one is in any interval. The second estimate, marked by $\|E\|_\infty$, is called as the measure of the boundary model quality, [3, 4], see always Sec. 5.

Substituting Eq. (9) into (7) one obtains

$$\theta_\nu \Phi_\nu + \sum_{j=1}^{n_j} \sum_{i=0}^{n_i} \Phi_i \left(\alpha\beta_i \int\limits_{S_j} N_i(\xi) \Psi(\mathbf{r}_S)\, dS(Q) \right.$$
$$\left. + \int\limits_{S_j} N_i(\xi) \frac{\partial \Psi(\mathbf{r}_S)}{\partial n}\, dS(Q) \right) = 4\pi \Phi_\nu^{\mathrm{inc}}, \tag{11}$$

where $\theta_\nu = \theta(P_\nu)$, $\Phi_\nu = \Phi(P_\nu)$, $\Phi_\nu^{\mathrm{inc}} = \Phi^{\mathrm{inc}}(P_\nu)$.

The evaluation of these integrals requires the use of a Jacobian $J(\xi) = \partial\overline{x}(\xi)/\partial\xi$, where $\overline{x}(\xi)$ — Eq. (8). After some calculations, the integrals in Eq. (11) are determined

$$a_{ij} = \int\limits_{S_j} N_i(\xi)\Psi(\xi) J(\xi)\, dS(\xi), \tag{12}$$

$$b_{ij} = \int\limits_{S_j} N_i(\xi) \frac{\partial \Psi(\xi)}{\partial n} J(\xi)\, dS(\xi). \tag{13}$$

The last integrals may become singular. To calculate them, the appropriate methods may be found in [18], [22, p. 93], [25], [30, p. 21], [38].

One can write Eq. (11) for every node ν defined in the discretization scheme. Hence, for the global number of the nodes n_ν, one produces n_ν simultaneous linear algebraic equations in terms of the unknowns Φ_ν, when β_ν and Φ_ν^{inc} are given at each node. Then Eq. (11) can be written in the matrix form

$$\mathbf{B\Phi} = \mathbf{\Phi}^{\mathrm{inc}}, \tag{14}$$

where the elements of the matrices $\mathbf{B}$, $\mathbf{\Phi}$, $\mathbf{\Phi}^{\mathrm{inc}}$ are the combinations of the values β_ν, Φ_ν, Φ_ν^{inc} and expressions given in Eqs. (12), (13).

The system of algebraic equations can be solved for Φ_ν using an appropriate linear equations solver routine. Once Φ_ν are known on the boundary, the acoustic field at any point $P \in \Omega_e$ can be calculated using the discrete form of the Eq. (4) with $C(P) = 4\pi$.

5. Theoretical basis of optimal elements

The optimization tools used in this paper are based upon the following mathematical properties from the theory of optimal approximation of the functions. Let $f(x)$ be a continuous function in arbitrary interval $[a, b]$, and $\widetilde{f}_n(x)$ be an approximation polynomial belonging to the class $\mathbb{P}_n$ of polynomials with degree less or equal to n. The uniform norm of the $f(x) \in C[a, b]$ is defined as $\|f(x)\|_\infty = \max\limits_{x\in[a,b]} |f(x)|$. Let $\mathbb{P}_n \subset C[a, b]$. The distance between $f(x)$ and $\mathbb{P}_n$ is defined by

$$\operatorname{dist}(f(x), \mathbb{P}_n) = \|f(x) - \mathbb{P}_n\|_\infty = \inf_{f_n \in \mathbb{P}_n} \|f(x) - \widetilde{f}_n(x)\|_\infty. \tag{15}$$

The solution of the minimization problem stated by Eq. (15) is unique and can be found by considering the following theorem, [2, p. 46] (for piecewise linear approximation) or [2, p. 189] (generally).

Theorem (revised).

Assume that the function $f(x)$ is continuous in the interval $[a, b]$, and is q times continuously differentiable at all but finitely many points in $[a, b]$, and q-th root of its q-th derivative is integrable, i.e.,

$$B = \int_a^b |f^{(q)}(x)|^{1/q} dx < \infty, \qquad f^{(q)}(x) = d^q f(x)/dx^q. \tag{16}$$

The order of approximation achievable is n_j^{-q}, for the break points $\mu_{O;j}$, which fulfil the equation

$$\int_{\mu_{O;j}}^{\mu_{O;j+1}} |f^{(q)}(x)|^{1/q} = \frac{j}{n_j} B, \qquad j = 1, 2, ..., n_j\,. \tag{17}$$

In this case, the distance between $f(x)$ and $\mathbb{P}_n$ is

$$\operatorname{dist}(f(x), \mathbb{P}_n) = \mathcal{O}(n_j^{-q}). \tag{18}$$

Note that this theorem not only claims the possibility of existence of the optimal break points, but it also indicates how they should be chosen to fulfil Eq. (18).

6. Numerical example

6.1. Barrier, source and ground data

In order to analyse the effect of optimal discretization, the 2D noise reduction problem by a long barrier is considered. The geometry of the problem is shown in Fig. 1. The cross-section of the barrier may be modelled by a part of an ellipse given by equation $(x - a)^2/a^2 + y^2/h^2 = 1$, where a, h — semi major axis of the ellipse, $a = 2$ m, $h = 4$ m, Fig. 2, [17, 28]. Hence the barrier height is 4 m. In addition, the barrier thickness is 0.3 m and its surfaces $(S_1 + S_2)$ are rigid.

Consider the sound generated by the point source, representing the vehicle noise including both the noise of the engine and of the tires. In this case the frequency $f_1 =$ 800 Hz (car) and $f_2 = 1300$ Hz (truck) may be assumed, [27, p. 163]. The source position is chosen at the elevation 1 m above the ground surface and 3 m to the right of the barrier (it is above the middle of the road). The vibration amplitude of the point source is 0.075 m.

The ground surface is divided into two parts. The first one S_3 is 6 m to the right of the barrier and is an acoustically hard surface (e.g. an asphalt); S_3 may be the model of a typical road. The latter S_4 is both on the right (more than 6 m) and on the left of the barrier and is an acoustically soft surface (e.g. a grassland); the remaining part of S_4 was

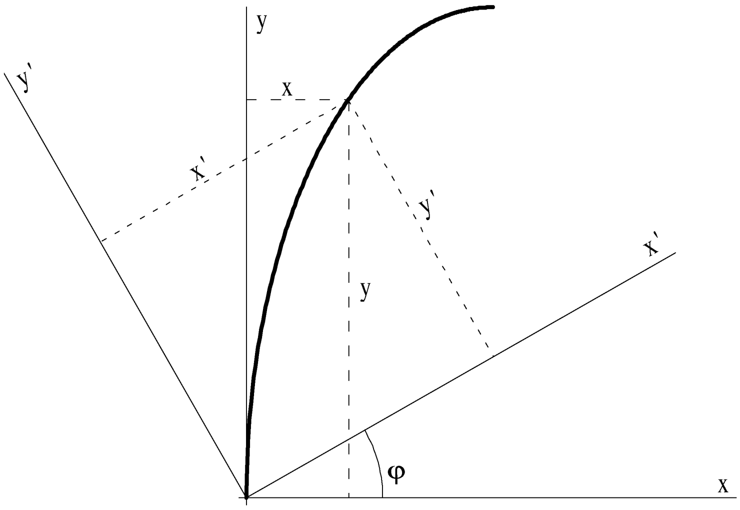


Fig. 2. Cross-section of the barrier in (x, y)- and (x', y')-coordinates.

omitted, because it had a little influence on the acoustic field. The absorptive property of the ground surface is described by surface admittance $\beta(Q)$, [6, 26], [27, p. 103]. The admittance may be expressed either as a function of the reflection factor (via the Robin condition, [8, p. 228]) or as a function of the flow resistance σ. The latter dependence is more convenient, then under the circumstances described above $\sigma_a = 1 \times 10^6\,\mathrm{Ns/m^4}$ (for asphalt) and $\sigma_g = 3 \times 10^5\,\mathrm{Ns/m^4}$ (for grassland), [12], [27, p. 119]. The normalised admittance (with $\rho_0 c$) is given by, [1, 9, 12], [27, p. 113], [32],

$$\frac{1}{\beta} = 1 + 9.08\left(\frac{1000f}{\sigma}\right)^{-0.75} + \bar{i}11.9\left(\frac{1000f}{\sigma}\right)^{-0.73}, \tag{19}$$

where $\rho_0 = 1.293\,\mathrm{kg/m^3}$, $c = 345\,\mathrm{m/s}$.

6.2. Application of the optimal discretization

An optimal discretization may not be directly applied to the chosen barrier. It is so because the function $f(x)$, which describes the barrier, is not q times continuously differentiable at $x = 0$. So the condition Eq. (16) is not fulfilled. In order to eliminate this drawback, it is proposed to solve the optimal discretization problem in rotated coordinates (x', y') and next to transform this model to (x, y), Fig. 2. It may be accomplished by applying orthogonal transformations, [7, p. 453].

One assumes two triads of mutually perpendicular unit vectors $\mathbf{e} = (e_1, e_2, e_3)$ and $\mathbf{e}' = (e'_1, e'_2, e'_3)$, i.e. $e_m e_n = 0$ and $e'_m e'_n = 0$, $m \neq n$, $m, n = 1, 2, 3$. One supposes that $\mathbf{e}$ and $\mathbf{e}'$ are the basis vectors of the rectangular coordinates $\overline{x} = (x, y, z)$ and $\overline{x}' = (x', y', z')$, respectively. The arbitrary unit vector e'_m may be expressed in $\overline{x}$-coordinates as $e'_m \sum_{n=1}^{3} \alpha_{mn} e_n$, where α_{mn} are the direction cosines of the m-th axis of $\overline{x}$-coordinates relative to $\overline{x}'$. Now, any vector $\mathbf{w}$ can be written in $\overline{x}$- and $\overline{x}'$-coordinates, i.e. $\mathbf{w} = \sum_{n=1}^{3} w_n e_n$ and $\mathbf{w} = \sum_{n=1}^{3} w'_n e'_n$, respectively. Substituting e'_m into the last expression, one finds $\mathbf{w} = \sum_{n=1}^{3} \alpha_{mn} w'_n e'_n$; hence $w_n = \alpha_{mn} w'_n$ or in a matrix form $\mathbf{W} = \mathbf{A}\,\mathbf{W}'$, where $\mathbf{W}$, $\mathbf{W}'$ are the column matrices, and $\mathbf{A}$ is a 3×3 orthogonal matrix. One can say that the matrix $\mathbf{A}$ transforms the vector from $\overline{x}'$-coordinates to $\overline{x}$. The inverse transformation is possible, i.e. $\mathbf{W}' = \mathbf{A}^{-1}\mathbf{W}$, $\mathbf{A}^{-1}$ is the inverse matrix.

The orthogonal transformation from (x, y) to (x', y') in a plane, and *vice versa*, is simply a rotation of the axes through some angle φ, Fig. 2, then the transformation matrix is

$$\mathbf{A} = \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix}, \qquad \varphi = \mathrm{tg}^{-1}(b/a). \tag{20}$$

In order words, the barrier can be transformed from (x, y) to (x', y'). There, the optimal model is constructed. Next, this model is transformed from (x', y') to (x, y).

7. Calculations, results

In the paper the sound in a noise region is more interesting than in the silence one. From a practical point of view it means that the reflection of sound towards the road is considered. In such a case, the main influence on the noise in this region has the inside part of the barrier S_1 and the part of the ground on the right of the barrier, $S_3 + S_4$. In the paper only S_1 is optimally discretized; other surfaces $(S_2 + S_3 + S_4)$ are refined discretized.

The surface S_1 is discretized into two elements. Hence, the special case of the theorem about optimal discretization is applied. Here $n_j = 2$ and the parabolic interpolation is chosen, $q = 3$, then

$$\int_a^{\mu_{O;1}} |f^{(3)}(x)|^{1/3} dx = \frac{1}{2} B, \qquad B = \int_a^b |f^{(3)}(x)|^{1/3} < \infty. \tag{21}$$

In the paper three models are considered. The models are built by the discretization either x-axis or x' one, Fig. 2. The first model M_1 is built in (x, y); $x \in [0, a]$ is divided into two equal elements. Then the regular model is obtained. The second model M_2 is built in (x', y') by the discretization $x' \in [0, b/\sin\varphi]$ always into two equal elements; it leads to the regular model too. The last model M_3 is built in (x', y') applying the optimal discretization; it leads to the optimal model. The models M_2 and M_3 in (x', y') are presented in Fig. 3. After transformation of M_2 and M_3 from (x', y') to (x, y), all models are depicted in Fig. 4.

Examining the Figs. 3 and 4, it may be noted that the optimal model M_3 is the most convergent to the barrier. The remaining models are less convergent; model M_1 is less

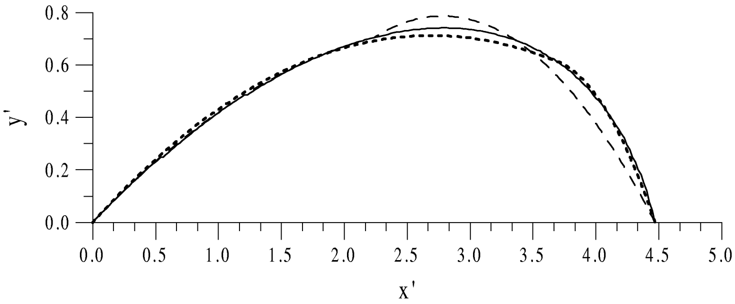


Fig. 3. Barrier and it models in (x', y')-coordinates: solid line — barrier, long dashed line — M_2, short dashed line — M_3.

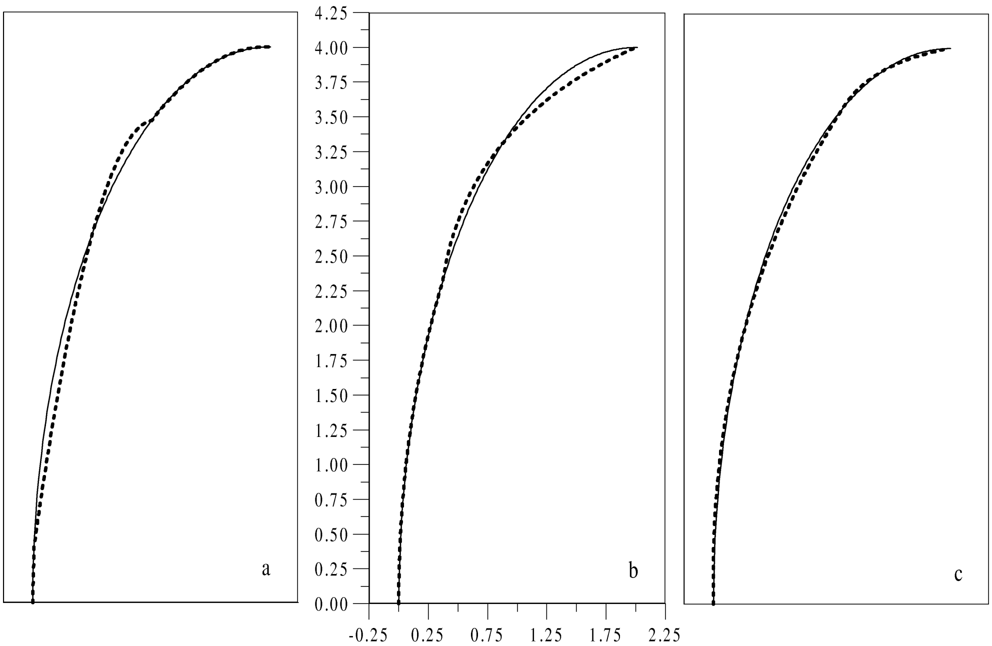


Fig. 4. Barrier and it models in (x, y)-coordinates: solid line — barrier, dashed line — model; a) M_1, b) M_2, c) M_3; M_2 and M_3 were transformed from (x', y')-coordinates.

convergent in the lower part of the barrier but M_2 in the upper part. This conclusion confirms the boundary model errors, Eq. (10), and their estimations, Fig. 5.

It is evident that the boundary model error has the influence on the efficiency of the barrier model. To check it, the acoustic field (in the sense of insertion loss in dB), for three described models, is calculated using the software package SYSNOISE. The results are compared with the nearly exact acoustic field, which is obtained for the model consisting of very many elements. Consequently, the acoustic fields of the models are compared one another. Then, the solution of the noise problem for three models are compared one another too.

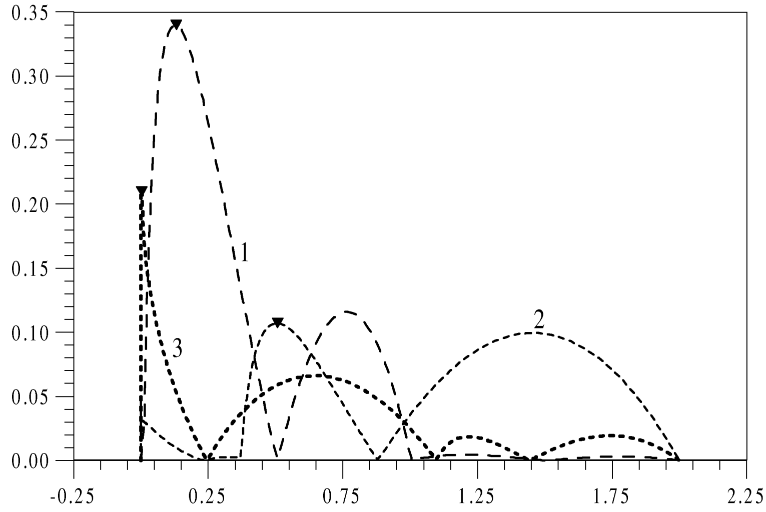


Fig. 5. Boundary errors and their estimations: 1 — $|E_1(x)|$, ▼ = $\|E_1\|_\infty = (0.131, 0.340)$; 2 — $|E_2(x)|$, ▼ = $\|E_2\|_\infty = (0.043, 0.284)$; 3 — $|E_3(x)|$, ▼ = $\|E_3\|_\infty = (0.035, 0.210)$.

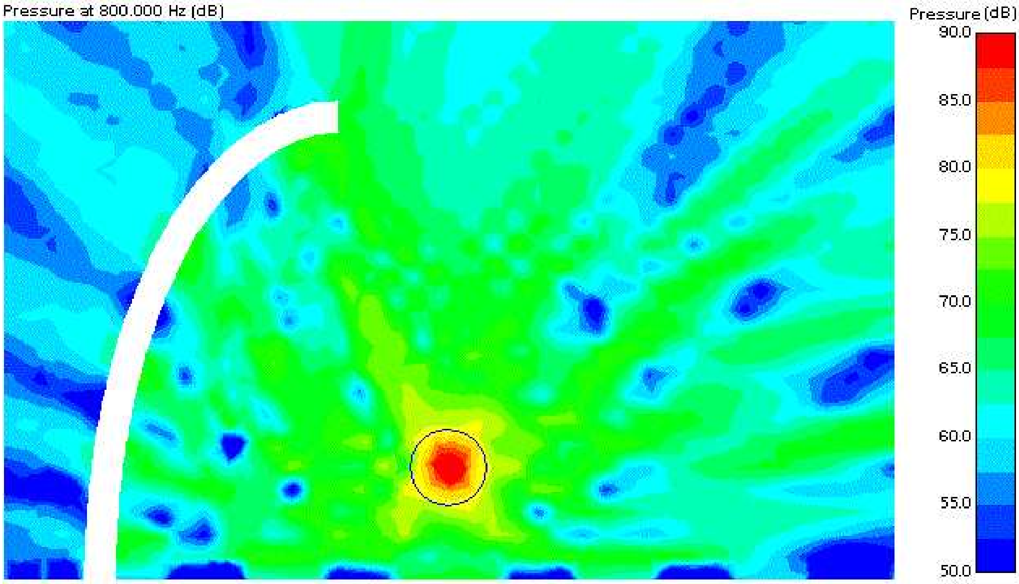
Two numerical studies are carried out; the former is for $f_1 = 800\,\mathrm{Hz}$ and the latter is for $f_2 = 1300\,\mathrm{Hz}$. The results are presented in the Figs. 6 and 7, respectively. As it can be seen from both figures, the acoustical fields of the models M_2 and M_3 coincide better with the exact one than that of the model M_1. It confirms the idea of modelling in rotated coordinates. Besides, in details, the acoustic field of M_3 is more convergent to the exact one than that of M_2. Then, the optimal discretization of only S_1 allows to build a better model. Furthermore, comparing the results presented in Figs. 6 and 7, one can note that a better effect is achieved for higher frequency.

8. Discussion, conclusions

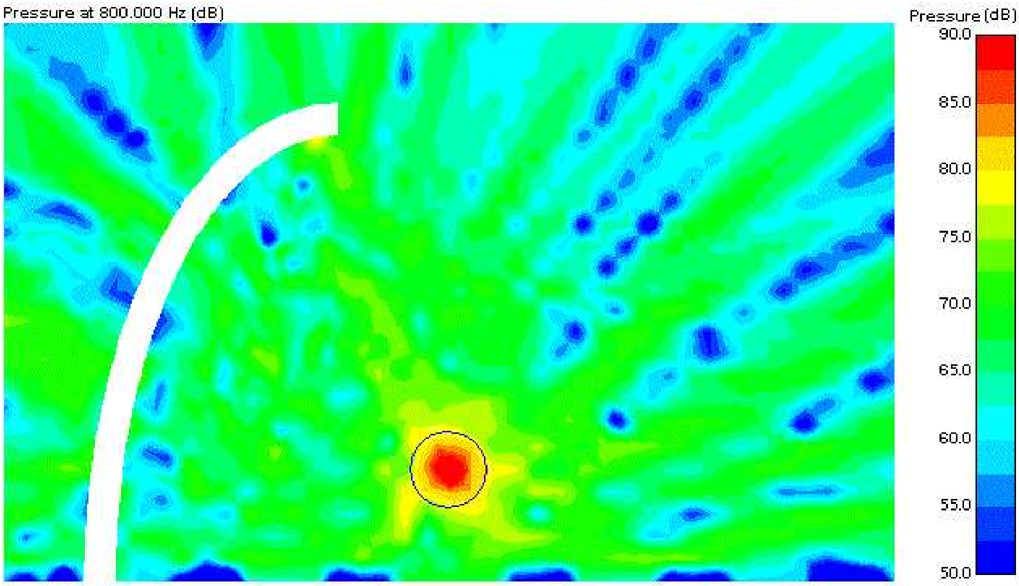
The BEM is an effective numerical technique for solving acoustic radiation/diffraction problems. But many practical acoustic problems described by good quality models consist even of thousands of elements and nodes. It is a burden to the user and is also not computationally efficient. Then one undertakes the efforts avoid this difficulty. The major aim of this paper is to search an economic and an effective discretization of the boundary. There are many methods, which reduce the number of elements keeping the quality of the boundary model. Two of them are presented in this paper.

The results of the foregoing test cases show the capability of the optimal discretization of the scatterer. In particular shape of the scatterer, the optimal discretization may be performed in rotated coordinates. To obtain the best model, the coordinates should be so rotated that one axis will be located in immediate vicinity of the scatterer. The regular discretization in suitably rotated coordinates leads always to better results. Despite very smooth scatterer was considered, both methods gave better results.

a)

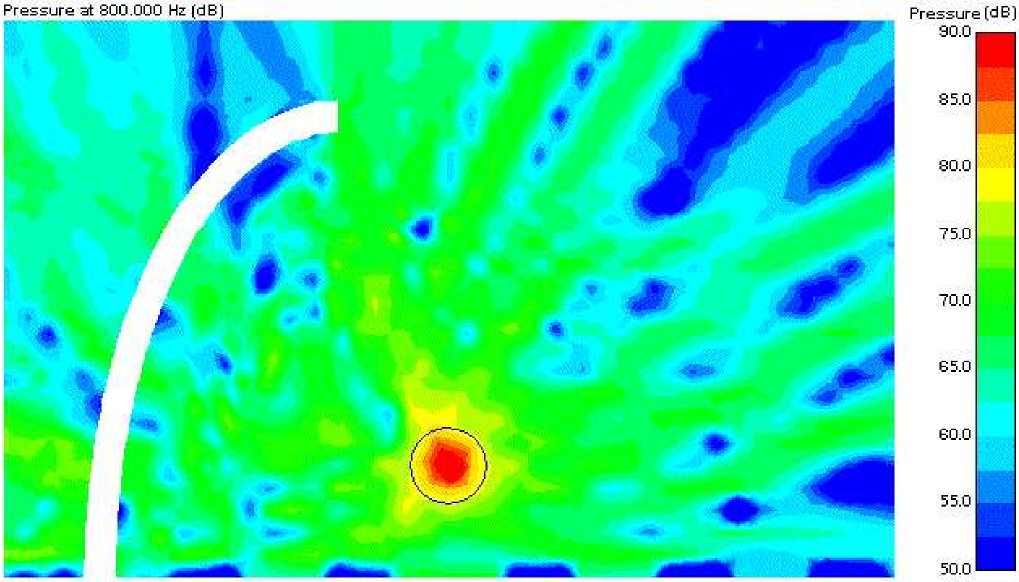

Pressure at 800.000 Hz (dB)
Pressure (dB)
90.0
85.0
80.0
75.0
70.0
65.0
60.0
55.0
50.0

b)


Pressure at 800.000 Hz (dB)
Pressure (dB)
90.0
85.0
80.0
75.0
70.0
65.0
60.0
55.0
50.0

[Fig. 6 a, b]

c)



d)

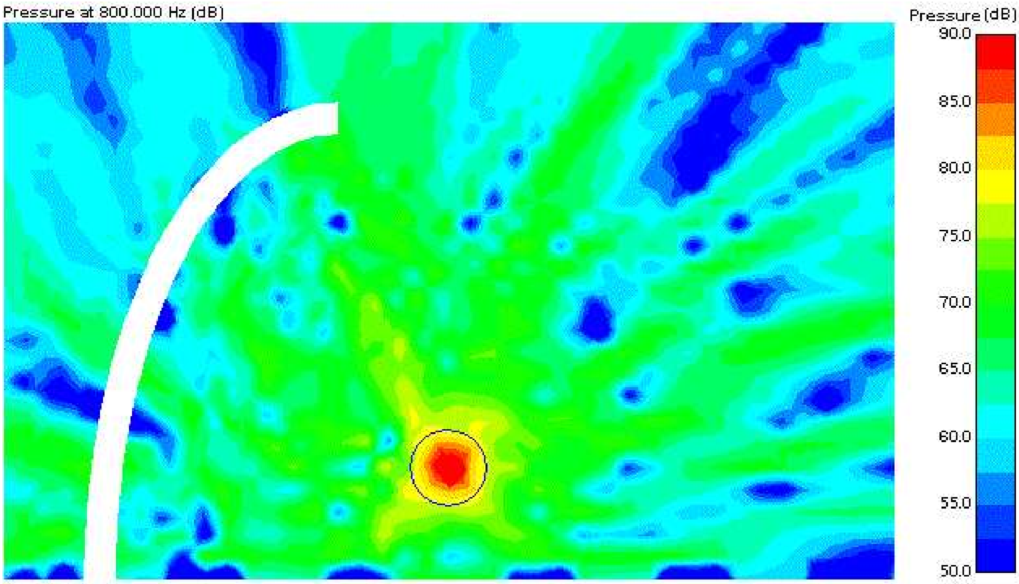
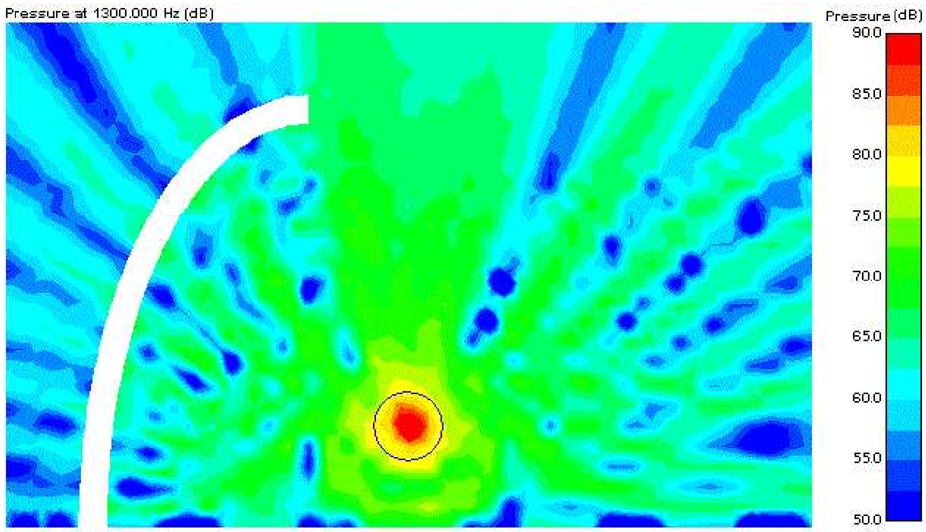


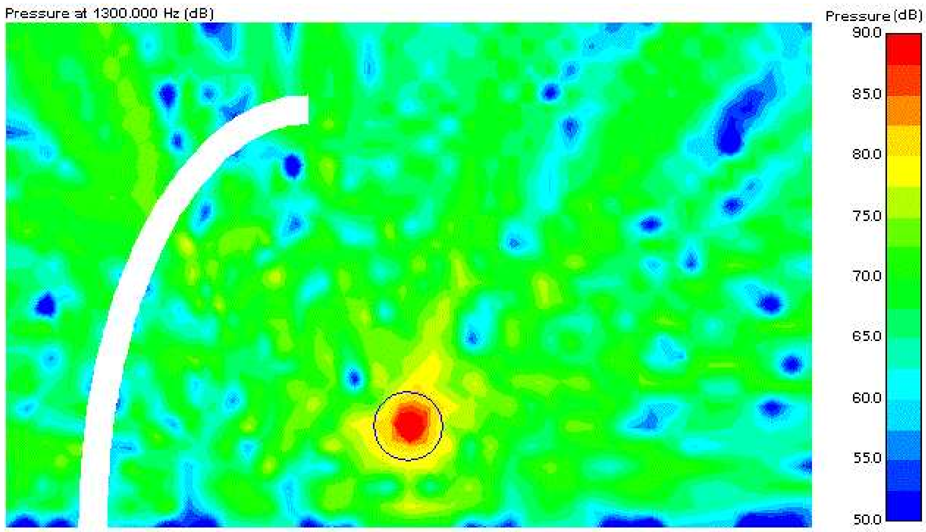
Fig. 6. Distribution of the acoustic fields, $f_1 = 800\,\mathrm{Hz}$: a) exact field, b) M_1, c) M_2, d) M_3.

a)



Pressure at 1300.000 Hz (dB)
Pressure (dB)
90.0
85.0
80.0
75.0
70.0
65.0
60.0
55.0
50.0

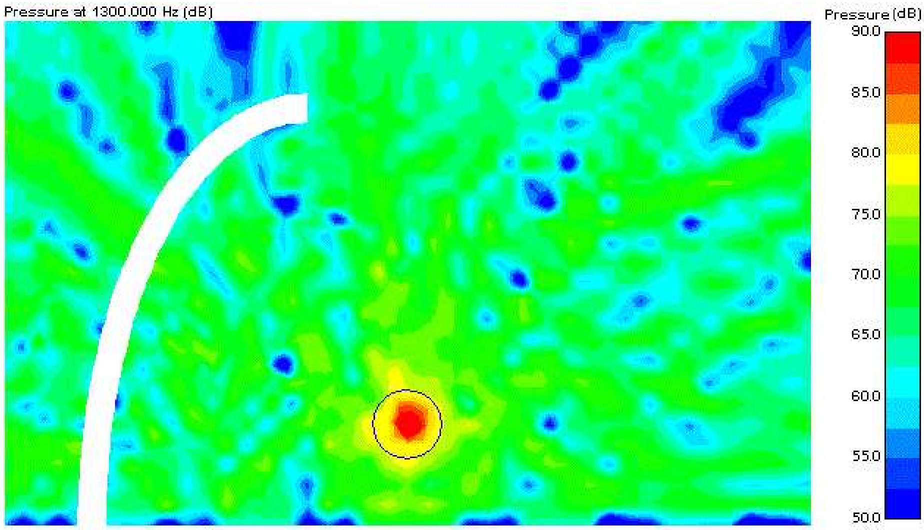
b)



Pressure at 1300.000 Hz (dB)
Pressure (dB)
90.0
85.0
80.0
75.0
70.0
65.0
60.0
55.0
50.0

[Fig. 7 a, b]

c)



d)

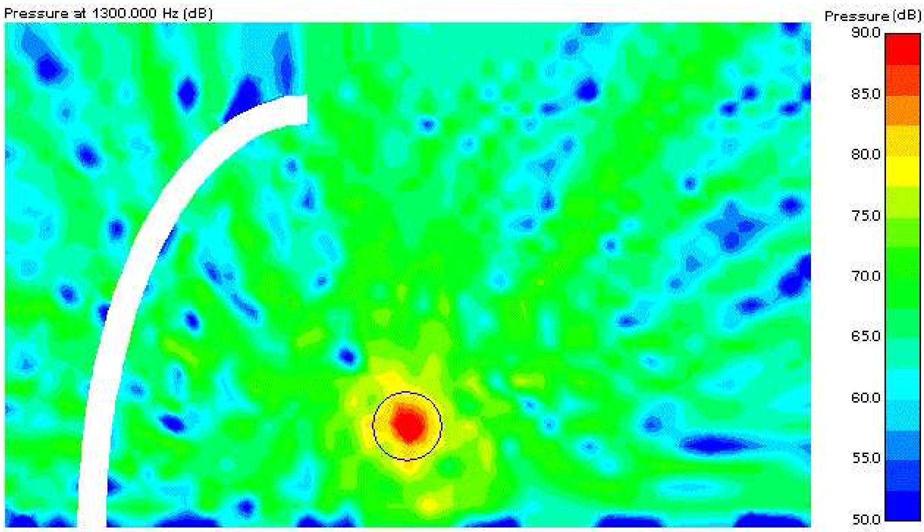


Fig. 7. Distribution of the acoustic fields, $f_1 = 1300\,\mathrm{Hz}$: a) exact field, b) M_1, c) M_2, d) M_3.

Other methods improving the solution quality of the practical acoustic problems may be examined after completing and revising the commercial software. Although 2D problem has been studied it seems that the proposed strategy can be extended to 3D problems. However, it needs much effort in the engineering formulation of the optimal discretization.

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