PREDICTION OF THE REVERBERATION TIME IN RECTANGULAR ROOMS WITH NON-UNIFORMLY DISTRIBUTED SOUND ABSORPTION

R. NEUBAUER

Consulting Bureau, Ingolstadt, Germany

B. KOSTEK

Technical University of Gdańsk
Faculty of Electronics, Telecommunications and Informatics
Sound & Vision Engineering Department
(80-952 Gdańsk, G. Narutowicza 11/12, Poland)

The aim of this paper is first to review the best known reverberation time formulae and then to show that the reverberation time cannot be thereby predicted accurately in cases mostly encountered in practice, where the sound field is not diffuse. Introducing a correction to Fitzroy's formula allows better prediction of the reverberation time in the case of non-uniformly distributed sound absorption. Comparison of calculation results obtained on both the basis of classical equations and the new time reverberation formula introduced is shown. In addition, the results obtained by measuring reverberation conditions *in situ* and those predicted for the same enclosure are compared and conclusions drawn.

1. Introduction

There exist many parameters that describe acoustic quality of an enclosure. The importance of some has been already established in the pioneering study of BERANEK [4] and later by other researchers [3, 32, 33, 38], but there is still no consensus on a set of parameters that should be taken into account while describing the acoustical quality of a room [22]. This is due to differences in functionality of a given room, volume of rooms, distribution of absorption, etc. [2, 27, 32, 33]. A so-called optimum reverberation time can serve as an example of such problems. Optimum values of this acoustical quantity that can be found in many literature sources differ to a large extent, which was pointed out by STRASZEWICZ in his paper [40]. In his opinion a pronounced divergence of opinions makes doubtful whether it is possible to assign the optimum reverberation time related to the given volume of a room or to the kind of sound produced only [40]. Numerous research studies show that rather than trying to achieve the optimum reverberation time for a given room, it is better to govern other acoustical parameters that influence acoustical quality. This is especially true in the case of the multifunction interiors. NIEMAS,

SADOWSKI and ENGEL formulated some requirements as to designing and estimating acoustical properties of sacral enclosures [27]. They pointed out the importance of other quantities than the reverberation time in the evaluation of the acoustical quality of such interiors [27]. Problems related to designing and estimating acoustical properties of interiors were also reviewed by other researchers [39, 42, 43], among others by RAKOWSKI [30] and SADOWSKI [32, 33]. In addition the relationship between acoustical parameters measured in a room and the acoustic quality assessed subjectively is still researched [17, 22]. However as was pointed out by VORLÄNDER, one of the most relevant sensations of the sound field in rooms is still the cognition of reverberation [42]. Reverberation appears to be responsible for the impression of being in a room as well as providing an awareness of distance from the source, whereas for example spatial impression due to lateral reflections appears to be more a source-specific effect involving the feeling to be close to the listener [3]. As BUDZYŃSKI [8] pointed out, early reflections from sidewalls are also responsible for the effect of increasing auditory distance localization.

During the past century several formulae for predicting reverberation time were developed empirically and theoretically, based on the assumption of homogeneous repartition of sound energy within the room, and consequently uniformly distributed sound absorption. The problem of the reverberation time prediction for non-uniform distribution, however, remains so far, open for discussion and for finding solutions fitted better to practical application.

The first and most remarkable approach to describe the reverberation characteristics of an enclosure was found by W.C. SABINE [31] around 1900. Sabine established his theory on the basis of practical results. To find a theoretical basis for calculating reverberation time, many researchers contributed new theories. Since Sabine published his results, several different approaches have been adopted to obtain equations that describe the reverberation characteristics. Among others, the best known researchers, who developed theories of reverberation include: FRANKLIN (1903) [14], JAEGER (1911) [15], FOKKER (1924) [13], BUCKINGHAM (1925) [7], SCHUSTER and WAETZMANN (1929) [36]. Then, EYRING (1930) [11] presented his remarkable paper, followed shortly after by MILLINGTON (1932) [24] and SETTE (1933) [37]. After a further 25 years FITZROY (1959) [12] published an empirical solution for reverberation time prediction in non-uniform rooms, his paper, however, went almost unrecognized at that time and was rather negatively perceived. In the last 30 years, SCHROEDER (1965) [34], KOSTEN (1965) [19], CREMER and MÜLLER (1978) [9], KUTTRUFF (1975) [20], NILSSON (1992) [28], TOHYAMA et al. (1995) [41] added some new issues to the theory of reverberation. In 1988 ARAU [1] presented an improved reverberation formula taking into account the non-uniform distribution of sound absorption. Lately, papers by KUTRUFF [21] and BISTAFA and BRADLEY [6], which dealt with the similar problems, appeared. Recently, within the framework of the standardization process an improvement of the estimation of the reverberation time in rooms with irregular absorption distribution based on Nilsson's model was proposed and discussed [23, 28].

A general description of the reverberation time based on Sabine's reverberation theory is still in common use. However, in the case of a room in which sound absorption

is not uniformly distributed, the reverberation time frequency characteristics cannot be predicted accurately using Sabine's or other classical reverberation theories. These theories are based on the assumption that the sound field considered is completely diffuse. This will, in general, be sufficiently diffuse if there are no large differences in the basic dimensions of the room, walls are not parallel, sound absorbing material is uniformly distributed, and most interior surfaces are divided into parts. In practice, almost none of these requirements is fulfilled. In 1959, Fitzroy introduced an empirically derived equation that considers non-uniform distribution of absorption. However, a thorough investigation of Fitzroy's equation revealed that in most cases the predicted reverberation time was generally too long. It is worth noting that acousticians are not altogether satisfied with existing formulae on reverberation time, thus the European standard concerning this issue is still an open question [10]. Furthermore, the reverberation time is an important parameter in calculating other objective descriptions of room acoustics. Since such parameters are well related to subjective assessment descriptors of room acoustics, it is most important for the acoustical design process to provide a general design tool which enables prediction of relevant sensations in the stage of planning a certain type of room [5, 17].

In the following paragraphs a review on best known formulae for predicting reverberation time is presented, along with corresponding calculations results. This is shown for the case of rectangular enclosures, because in such cases it is possible to systematically study the reverberation time related to the room volume.

2. Time reverberation formulae

Most of parameters that describe characteristics of an acoustic hall are calculated basing on the assumption that it is usually sufficient to consider the propagation of sound energy and not sound pressure or particle velocity, therefore all phase effects can be neglected. The basis of this assumption is that the dimensions of a hall should be large enough in comparison with the acoustic wavelengths. The so-called Schroeder cut-off frequency [35]:

$$f_s = 2000\sqrt{T/V} \quad [\text{Hz}] \tag{1}$$

limits the frequency under which this assumption is not justified. That is, f_s can be considered as the lower limit of frequencies at which a statistical treatment of superimposed normal modes in a room is permissible. In contrast, below the Schroeder cut-off frequency, the resonance peaks of the sound field are insufficiently dense to be analyzed statistically.

One of the most important parameters describing the quality of the room is the reverberation time. According to the classical formula reverberation time is defined as time needed to decrease energy by 60 dB from its original level after instantaneous termination of the excitation signal. This parameter, originally introduced by Sabine, is given by equation (2):

$$T_{60} = 0.16 \cdot V/A, \tag{2}$$

where: V — hall volume [m³], A — total area of absorption [m²], 0.16 — coefficient introduced first empirically, depending on propagation conditions. Therefore in the literature, values equal to 0.16, 0.161, 0.162, 0.163, 0.164 can be found. It is also worth noting that Sabine derived experimentally a value of 0.164. In this paper, contrarily to the original values that can be different, a value equal to 0.16 will be used in calculations, since such an accuracy is sufficient for comparison purpose.

If the surface area of the room is S, Sabine's average absorptivity $\overline{\alpha}$ is defined by:

$$\overline{\alpha} = A/S \tag{3}$$

and Eq. (2) assumes the form:

$$T_{60} = \frac{0.16 \cdot V}{S \cdot \overline{\alpha}}\,. \tag{4}$$

In order to complete Sabine's formula by taking into account the attenuation constant m of air, which takes into account the attenuation of sound during its free propagation, one obtains:

$$T_{60} = \frac{0.16 \cdot V}{S \cdot \overline{\alpha} + 4mV}\,. \tag{5}$$

As is seen from Eq. (4), T_{60} may be controlled by either a change in volume, or the absorption factor. Equation (4) assumes that the sound energy is equally diffused throughout the room (i.e. homogeneous and isotropic). Actually, this condition is rarely fulfilled due to the large areas existing in a hall characterized by differentiated absorption. Therefore, in practice, there are several formulae describing the reverberation time.

It was discovered by Eyring that the classical formula given by Sabine is not fulfilled when there is considerable room absorption. Eyring pointed out in his paper that Sabine's formula is essentially a "live" room formula and that the reverberation time is shape-dependent [4]. He presented the revised theory thoroughly and derived a form of the reverberation time equation, which is more general than Sabine's formula.

Eyring's formula is based on the mean free path between reflections [29, 44]. The mean free path in an enclosure characterized by a diffuse sound field is as follows [16, 18]:

$$\bar{l} = \frac{4 \cdot V}{S}\,, \tag{6}$$

where: V is the total volume of a room [m³], and S — as before.

The use of the above Eq. (6) yields:

$$T_{60} = \frac{0.16 \cdot V}{-S \cdot \ln(1 - \overline{\alpha})} \tag{7}$$

with:

$$\overline{\alpha} = \frac{1}{S} \sum_i S_i \cdot \alpha_i\,. \tag{8}$$

Eyring assumes that sound coming from a source in a room is successively reflected by boundaries having an average coefficient $\overline{\alpha}$. Each time a wave strikes one of the boundaries, a fraction $(\overline{\alpha})$ of the energy is absorbed, and a fraction $(1 - \overline{\alpha})$ is reflected. The number of reflections per second is numerically equal to the distance sound will travel

in one second divided by the average distance between reflections. As is seen from Eq. (7), Eyring assumed that energy attenuation in this case is $(1 - \overline{\alpha}_{\text{Eyring}})^n$ per second, where $\overline{\alpha}_E = A/S_{\text{bound}}$ and S_{bound} are bounding surfaces. Equation (7) is known as Eyring's reverberation formula.

Additionally, Millington [24] and Sette [37] derived an equation to predict the reverberation time that is based on similar assumptions as Eyring's formula. It differs, however, in the way in which absorption coefficients of the various portions of a wall are averaged. This leads to the Millington-Sette's formula:

$$T_{60} = \frac{0.16 \cdot V}{- \sum_i S_i \cdot \ln(1 - \alpha_i)} \,, \tag{9}$$

which, in the limit of all $\alpha_i \ll 1$, reduces to Sabine's formula with $\alpha_i = \alpha_{(\text{Eyring})i}$.

On the other hand, Fitzroy states in his paper [12] that it is possible to take into consideration not only physical, but also geometrical aspects of a sound field in an enclosure. In this way the sound field may tend to settle into a pattern of simultaneous oscillation along a rectangular room with three major axes — vertical, transverse, and longitudinal. The solution appeared to lie in some relationship within the three possible basic decay rates along these axes, each being influenced by the different average absorptivities normal to these axes, each rate being unique within its specific axis. In a rectangular room, one may consider three sets of parallel boundaries. If energy oscillates simultaneously between each pair of boundaries the average absorption in each pair would control sound waves travelling between that specific pair during the sound decay period.

It should be remembered that the Eyring's formula assumes the absorptivity to be equal in all directions and Fitzroy's equation in the case of a rectangular room, takes into account three-dimensional geometry. Thus Fitzroy derived empirically an equation in which non-uniform distribution of absorption is assumed.

$$T_{60} = 0.16 \cdot \frac{V}{S^2} \left[\frac{-x}{\ln(1 - \alpha_x)} + \frac{-y}{\ln(1 - \alpha_y)} + \frac{-z}{\ln(1 - \alpha_z)} \right], \tag{10}$$

where: x, y, z — total areas of two opposite parallel walls in m^2, α_x, α_y, α_z — average absorption coefficients of a pair of opposite walls, S — total surface area of the room in [m^2], V — total volume of the room [m^3].

Tohyama and Suzuki presented the "almost-two-dimensional" diffuse field theory in his paper [41]. The physical space inside boundaries is three-dimensional. In acoustics, there are cases, where two-dimensional acoustic field is better suited to estimate reverberation conditions. It may occur in a concert hall with hard walls perpendicular to an absorbent floor. Reverberation time in a two-dimensional diffuse field determined by "the later part of the energy decay", assuming sound velocity equals $340\,\text{m/s}$, becomes [41]:

$$T_{60} = \frac{0.128 \cdot S}{-L \cdot \ln(1 - \overline{\alpha}_{xy})} \,, \tag{11}$$

where L is the total length of the two-dimensional space and $\overline{\alpha}_{xy}$ denotes absorption coefficient in the xy-two-dimensional field.

A small regular-shaped room with a carpet-covered floor can be treated as the sound field between two- and three-dimensional [41]. In the case of a two-dimensional field reflections at the z-walls are neglected. For small-sized rooms, however, characterized by almost two-dimensional diffuse field z-walls reflections are taken into account. This field is assumed to be composed of tangential, oblique and "almost-tangential" waves. By replacing $\overline{\alpha}_{xy}$ with the averaged absorption coefficient $\overline{\alpha}_{AI-xy}$ characterizing the almost two-dimensional diffuse field, Eq. (11) becomes:

$$T_{AL-xy} = \frac{0.128 \cdot S}{-L_{xy} \cdot \ln(1 - \overline{\alpha}_{AL-xy})} \, , \tag{12}$$

where: $\alpha_{AL-xy} = \overline{\alpha}_{xy}(1-\mu) + \overline{\alpha}_z \cdot \mu$ denotes the averaged absorption coefficient, $\overline{\alpha}_z$ — averaged absorption of the z wall, $\overline{\alpha}_{xy}$ — averaged absorption in the xy-two-dimensional field, $L_{xy} = 2(L_x L_y)$ — circumference in [m], L_z – length in [m], $\mu = m \cdot c/(L_z^2 \cdot 4 \cdot f_0)$, $m = \pi \cdot S_{xy}/L_{xy}$, m — mean free path, c — speed of sound in air, f_0 — wave frequency.

In his papers, ARAU introduced a model of calculating the reverberation time for the case of asymmetrical distribution of absorption, assuming that the reverberation decay is a hyperbolic process [1]. This decay is a superposition of three contributions: initial decay, first and second linear portion of the decay, and the third linear portion. Arau considered the reverberation time of a room to be equal to the area-weighted geometrical mean of the reverberation periods in each of the rectangular directions. The absorption coefficients used in his formula are the average absorptivities of each pair of opposite walls.

$$T = \left[\frac{0.16V}{-S \cdot \ln(1 - \alpha_x) + 4mV} \right]^{S_x/S} \cdot \left[\frac{0.16V}{-S \cdot \ln(1 - \alpha_y) + 4mV} \right]^{S_y/S}$$
$$\cdot \left[\frac{0.16V}{-S \cdot \ln(1 - \alpha_z) + 4mV} \right]^{S_z/S} , \tag{13}$$

where: α_x is the area-weighted arithmetical mean of the energetic absorption coefficients of the floor S_{x1} and ceiling S_{x2} surfaces; and $S_x = S_{x1} + S_{x2}$, $\alpha_x = (\alpha_{x1}S_{x1} + \alpha_{x2}S_{x2})/S_x$, α_y and α_z are the area-weighted arithmetical mean of energetic absorption coefficients of the side-walls and front- and end-walls, respectively. $S = S_x + S_y + S_z$ is the total area of the room surfaces, m is the molecular absorption coefficient of air.

If the enclosed space has a non-regular distribution of absorption, irregular shapes or is filled in, to a large extent, with e.g. equipment, decorative elements, etc., the predictions of the reverberation time should be based on the NILSSON model [28]. This may improve predictions of the reverberation time for the irregular absorption distribution. An essentially rectangular space with irregular absorption distribution is quite common. In many office rooms absorption is applied only to the ceiling, all other surfaces being reflective. In such a case classical time reverberation formulae rarely solve the problem. Nilsson proposed that the sound field should be divided into the most characteristic part, i.e. tangential to the considered surface, and remaining parts of room surfaces. A practical approach based on that model, but making use of absorption data measured according to standard methods, is presented in the Annex D of the European Standard prEN 12354-6 [10].

Kuttruff considers the case of the partially diffuse field within the room and introduces the concept of the reflection coefficient $\rho = 1 - \alpha$. Basing on the assumption that absorption coefficient α and hence ρ are independent of the angles, he made use of Lambert's law of diffuse reflection. By focussing on the overall reverberation time, and neglecting details of the decay process and additionally under the assumption of an exponential law for the time dependence of the irradiation strength over the whole surface of reflecting walls, he defined an absorption exponent α^*, which is valid under the aforementioned assumption [20]. The assumption of an exponential law is reasonable since, at least in rectangular rooms, the decay process of the sound energy will decrease exponentially.

KUTTRUFF introduced a correction to Eyring's formula and showed that the absorption exponent α^* would assume its Eyring's value [20]:

$$\alpha_{\text{Eyring}} = - \ln \overline{\rho} = - \ln(1 - \overline{\alpha}), \tag{14}$$

if the irradiation strength was constant. This is true if ρ and hence the absorption coefficient α has everywhere the same value. In general, the effective absorption exponent will be smaller or larger than $(-\ln \overline{\rho})$, depending on the room shape and the distribution of the wall absorption.

$$\alpha^* = \ln \left(\frac{1}{\overline{\rho}} \right) + \ln \left(1 + \frac{\sum\limits_n \rho_n \left(\rho_n - \overline{\rho} \right) S_n^2}{\left(\overline{\rho} S \right)^2 - \sum \rho_n^2 S_n^2} \right), \tag{15}$$

where: $\overline{\rho} = 1 - \overline{\alpha}_n$ denotes the average reflection coefficient of surface area S_n, S is the total surface area of the room in [m^2].

In most cases the second term in the denominator is much smaller than the first and hence can be neglected. Expanding the second logarithm into a power series and neglecting all terms of higher than first order gives:

$$\alpha^* \approx \alpha_{\text{Eyring}} + \frac{\sum\limits_n \rho_n \left(\rho_n - \overline{\rho} \right) S_n^2}{\left(\overline{\rho} S \right)^2} . \tag{16}$$

Inserting Kuttruff's correction to Eyring's reverberation formula and completing this formula by taking into account the attenuation constant m of air, leads to:

$$T_{60} = \frac{0.16 \cdot V}{S \cdot \left[- \ln \left(1 - \overline{\alpha} \right) \right] + \Delta + 4mV} , \tag{17}$$

where Δ is:

$$\Delta = \frac{\sum\limits_i \rho_i \left(\rho_i - \overline{\rho} \right) S_i^2}{\left(\overline{\rho} S \right)^2 - \sum\limits_i \rho_i^2 \cdot S_i^2} . \tag{18}$$

Kuttruff showed that his correction to Eyring's formula could easily be applied where $n-1$ surfaces have nearly the same reflection coefficient and one surface, namely the n-th surface, e.g. the floor where the audience's seats are placed, is characterized by a different absorption coefficient. In this case very good conformity with the computer simulated results may be obtained. On the other hand, if a room has asymmetric absorption Eyring's modified formula is considerably incorrect. In this case in order to overcome this problem one can follow the reasoning presented in PIERCE's book [29].

3. Modi cation of Fitzroy s equation

In the study conducted, correction to Fitzroy's equation has been introduced in order to approximate calculated time reverberation values closer to the measured ones. A suggested modification of Fitzroy's equation was first presented in a paper by NEUBAUER [25]. Since KUTTRUFF [20] introduced a correction to Eyring's formula and FITZROY's equation [12] is based on Eyring's concept, it seems possible to introduce a similar correction to Fitzroy's equation. This can be done to achieve more accurate approximation of the reverberation time in cases where the main absorption is of the floor and the ceiling. This modification leads to Fitzroy's modified equation, called here a New Formula, which is presented and discussed in this paragraph. It has been proven that the reverberation time according to Fitzroy's modified equation is useful in cases where the sound absorption on opposite sides is substantially higher than on the remaining room surfaces. As this is the case of a large number of existing rooms, for practical use one may modify Fitzroy's equation by splitting Kuttruff's correction into two parts, namely the part of the ceiling-floor and the part of the remaining walls. This reasoning will be presented later on.

Because examples that Fitzroy presented in his paper conform with the three-term formula, it is worthwhile attempting the same manner of reasoning. Compared calculation results however, reveal no advantage over the more simple equation provided by Sabine or Eyring. In particular, simulations for a room different from those given as examples in Fitzroy's papers, generally result in higher values than these based on Sabine's or Eyring's formulae. For the most important practical case where either the ceiling and/or the floor are/is highly absorptive it results in the "almost two-dimensional" field. Generally, this occurs in a real room when the absorbing capacity of the ceiling and floor exceeds that of the remaining walls. For such a case Fitzroy's equation may be rewritten using an appropriate modification.

By dividing the room surfaces into the floor and ceiling areas, and the remaining wall areas one obtains the following expressions:

$$
\begin{aligned}
S_{cf} &= 2 \cdot l \cdot w, \\
S_{ww} &= 2 \cdot l \cdot h + 2 \cdot h \cdot w, \\
S_{\text{total}} &= 2 \cdot [h \cdot (l + w) + l \cdot w],
\end{aligned}
\tag{19}
$$

where are h, l, w — as defined before.

Introducing Eq. (19) one may rewrite Fitzroy's equation and furthermore by using Kuttruff's correction from Eq. (18) it yields a New Formula:

$$
T_{60} = \left(\frac{0.32 \cdot V}{S^2} \right) \cdot \left(\frac{h(l + w)}{\overline{\alpha}_{ww}^*} + \frac{l \cdot w}{\overline{\alpha}_{cf}^*} \right),
\tag{20}
$$

where: V, S — volume in m^3 and total surface area of the room in [m^2], h, w, l — room dimensions: height, width and length in [m], $\overline{\alpha}_{ww}^*$, $\overline{\alpha}_{cf}^*$ — average effective absorption

exponent of walls, ceiling and floor, respectively:

$$\overline{\alpha}_{ww}^* = \ln\left(\frac{1}{\overline{\rho}}\right) + \left[\frac{\rho_{w1}\left(\rho_{w1} - \overline{\rho}_{ww}\right)S_{w1}^2 + \rho_{w2}\left(\rho_{w2} - \overline{\rho}_{ww}\right)S_{w2}^2}{\left[\overline{\rho}_{ww}(S_{w1} + S_{w2} + S_{w3} + S_{w4})\right]^2}\right.$$

$$\left. + \frac{\rho_{w3}\left(\rho_{w3} - \overline{\rho}_{ww}\right)S_{w3}^2 + \ldots}{\left[\overline{\rho}_{ww}(S_{w1} + S_{w2} + S_{w3} + S_{w4})\right]^2}\right], \qquad (21)$$

$$\overline{\alpha}_{cf}^* = \ln\left(\frac{1}{\overline{\rho}}\right) + \left[\frac{\rho_c\left(\rho_c - \overline{\rho}_{cf}\right)\cdot S_c^2 + \rho_f\left(\rho_f - \overline{\rho}_{cf}\right)\cdot S_f^2}{\left[\overline{\rho}_{cf}(S_f + S_c)\right]^2}\right],$$

where: $\overline{\alpha}$ — arithmetical mean of the surface averaged absorption coefficient, $\rho = (1 - \alpha)$ — reflection coefficient.

A comparison of calculated results using the new Sabine's and Eyring's formulae is shown in Fig. 1. From Fig. 1 good conformity between all three formulae can be noted, although both Eyring reveals about 1% smaller values of the calculated reverberation time compared with the New Formula. In the consecutive figure (Fig. 2) results are presented for the case where the floor is characterized by high absorption and the remaining surfaces have low absorption. The absorbing materials have an absorption coefficient for the floor of 0.80 that for ceiling of 0.2 and other surfaces of 0.02.

Calculated Reverberation Time [s]

$\alpha_f = 0.02 \quad \alpha_c = 0.02 \quad \alpha_{ww} = 0.02$
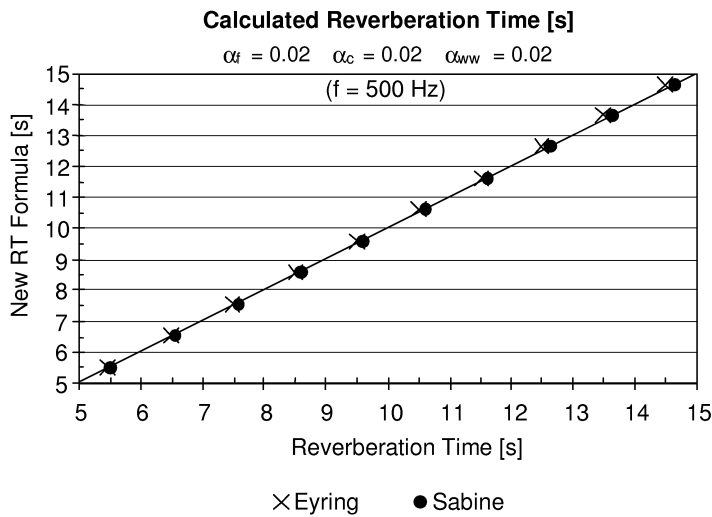
(f = 500 Hz)

Fig. 1. Calculated reverberation time for the case where all surfaces are of low absorption. The line shows values predicted using the New Formula, plotted against itself.

In this case it can be observed that the differences in the predicted reverberation time are not negligible. Both Sabine's and Eyring's formulae yield from about 25% to 35% higher values of the calculated reverberation time compared with the New Formula. This is due to unevenly distributed absorption in the room. Since both Sabine and Eyring assumed that the sound field is diffuse and the sound absorption regularly distributed

Calculated Reverberation Time [s]

$\alpha_f = 0.65$ $\alpha_c = 0.20$ $\alpha_{ww} = 0.02$
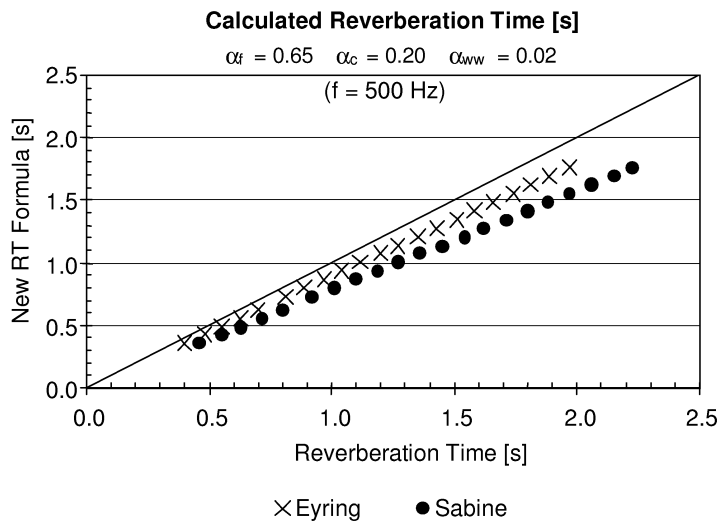
(f = 500 Hz)

× Eyring   ● Sabine

Fig. 2. Calculated reverberation time for high absorption on the floor and low absorption of remaining surfaces. The line shows values predicted using the New Formula, plotted against itself.

throughout the room, such a result is not surprising. Therefore, the advantage of using the New Formula is especially important in cases of irregularly distributed absorption. In the next figure (Fig. 3) results obtained for a room with regularly distributed absorption, but also a highly absorptive one is shown. In this case all surfaces have a sound absorption coefficient of about 0.40. From Fig. 3 it can be clearly observed that in the case of high, but evenly distributed absorption, results obtained are nearly the same by using both Eyring's and the New Formula. However, comparing the results more thoroughly reveals a deviation of about 0.16%. Values of the estimated reverberation time are smaller using

Calculated Reverberation Time [s] $\alpha_{cf} = 0.40$ $\alpha_{ww} = 0.40$
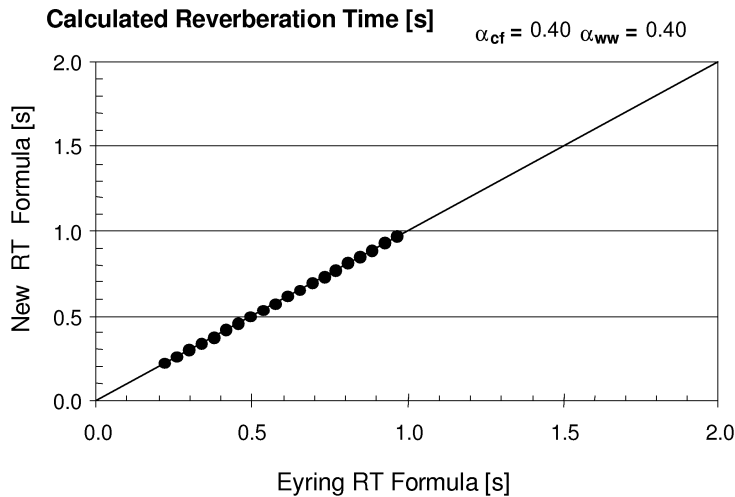

Fig. 3. Calculated reverberation time for evenly distributed sound absorption. All surfaces have an absorption coefficient of 0.4.

Eyring's formula as compared with the new one. It may therefore be assumed that the New Formula is better applicable than Eyring's formula, especially in cases of unevenly distributed absorption in the room.

4. Comparison of reverberation time predictions resulting of a new formula and other existing formulae

In this paragraph the best known formulae for predicting reverberation time are compared with the New Formula. Investigation of the prediction model for estimating the reverberation time in rectangular rooms is focused on comparison of the following formulae of: Eyring, Millington-Sette, Tohyama, Arau, model contained in the European Norm (Annex D, prEN 12354-6) and Fitzroy. Although Tohyama's formula should be properly referenced as also to SUZUKI [41], but for the purpose of result presentation this is abbreviated to Tohyama's formula only.

Since, in many practical cases, the assumption of diffuse field conditions by applying Sabine's theory does not conform with the true room absorption distribution, it is of practical interest to develop a proper formula to estimate the reverberation time even for non-uniformly distributed sound absorption in the room.

In order to show the effect of computing reverberation time using various formulae, some calculation results are presented in the following figures (Figs. 4 – 6). First, two cases of the absorption coefficient value at the limits, i.e. a "live" room with little absorption
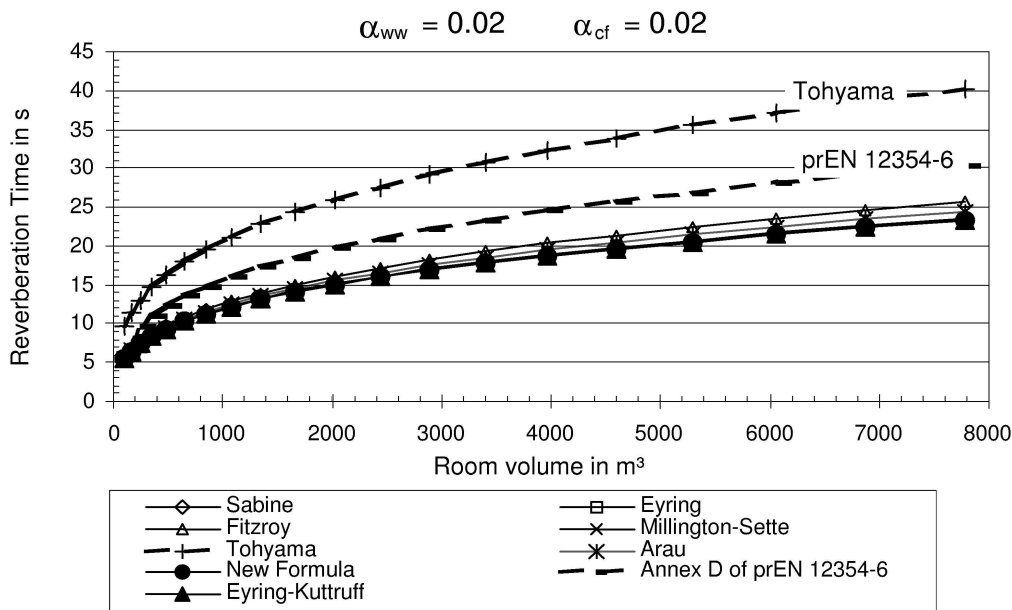

Fig. 4. Calculated reverberation times for low absorption of all room surfaces, the overall absorption is equal to 0.02 (a so-called "live" room condition).

and a "dead" room with high absorption will be discussed. In the following calculations the term $4mV$, which represents air absorption is omitted.

The simulations presented in this paragraph are performed for the case of a rectangular room of dimensions l, w, h (consecutively: length, width, and height). The rectangular shape of the enclosure room was chosen because of its systematically treatable dimensions. However, there is no need to reduce the findings of the carried out investigation to rectangular rooms only, as long no elliptical or circular rooms are considered. Moreover, the European Norm prEN 12354-6 does not distinguish between rectangular and irregular shaped rooms. The investigated room volumes were from $50\,\mathrm{m}^3$ to $8000\,\mathrm{m}^3$. The volume range used in experiments was chosen arbitrarily. But one should bear in mind that for very large rooms, especially if the ceiling is low, the RT theory is not longer suitable.
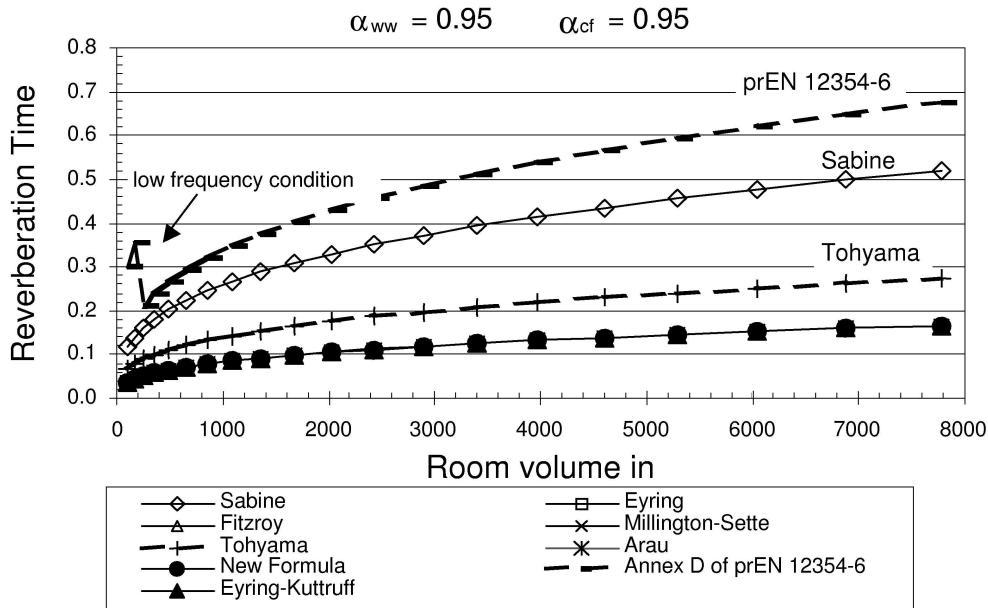

Fig. 5. Calculated reverberation times for high absorption of all room surfaces, the overall absorption is equal to 0.95 (a so-called "dead" room condition).

In the first figure (Fig. 4) the difference between various calculation formulae when all surfaces have very small absorption (live-room condition) can be seen. It is obvious because the diffusion in such a case is good. It can be observed that there is no meaningful difference in the computed reverberation time values based on various formulae except that using both Tohyama's formula and the model contained in the European Norm (Annex D, prEN 12354-6). In the next figure (Fig. 5) the difference between various formulae calculation results may easily be shown where all surfaces are of a high absorption (dead-room condition). It is also seen that the model from the European Norm, Sabine's and Tohyama's formulae are discrepant compared to other reverberation formulae.

It can be noted from Fig. 5 that all but the model from the European Norm, Sabine's and Tohyama's formulae yield more or less the same computed reverberation time values. Simulations based on the model contained in the European Norm yield the highest values followed by those calculated on the basis of Sabine's and then Tohyama's formulae.

In some respect it may be interesting to compare results when only one surface is highly absorptive and all others are characterized by low absorption. This may happen if the floor is covered with seats, but walls and ceiling are bare. In this case the average absorption coefficient resulting from audience seats is over 0.8 and the other surfaces of about 0.02 yield results which are depicted in the consecutive plot (Fig. 6). It can be seen from Fig. 6, that Fitzroy's formula followed by Arau's equation produces the highest values of the reverberation time. Based on Tohyama's and Sabine's equations one may obtain close results followed by Eyring's equation based values. Millington-Sette's formula provides the lowest values.
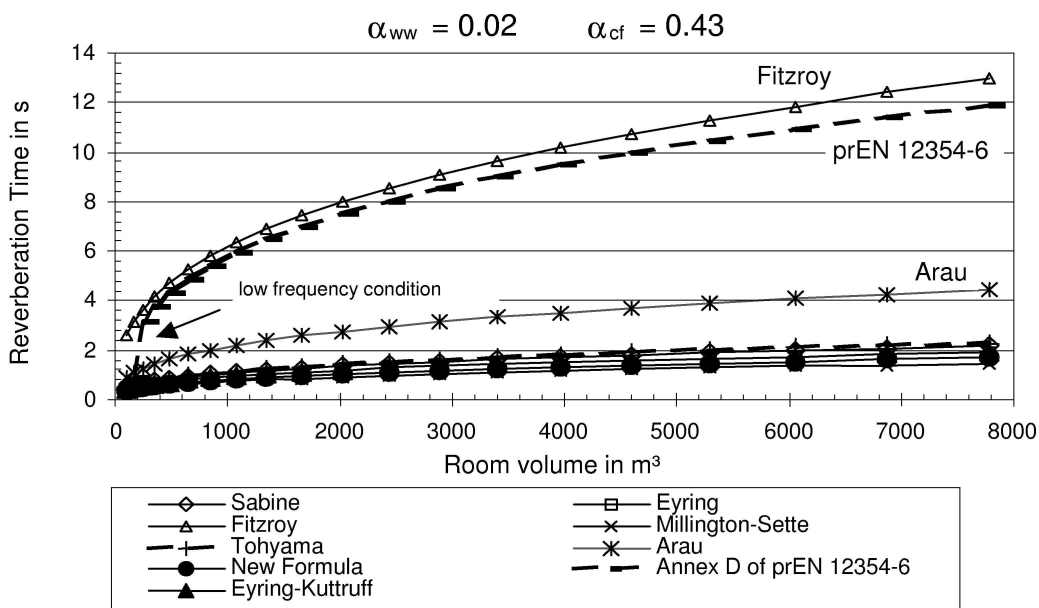

Fig. 6. Calculated reverberation time using various calculation formulae for high absorption of the floor and low absorption of other surfaces (there is an average absorption applied to the ceiling).

As already mentioned, results shown in Fig. 6 are derived by using an absorption coefficient of the floor of 0.8 ($\alpha_{cf} = 0.43$) and all other surfaces 0.02. This yields an overall absorption coefficient dependent on the method of averaging, whether by means of arithmetic or geometric averaging. Using the arithmetical mean yields an absorption coefficient of $0.24 - 0.22$ depending on the room volume and on the other hand using the geometrical mean (for Millington-Sette) this operation results in an absorption coefficient of $0.46 - 0.43$ also depending on the room volume.

5. Di erences between measured results and predicted values

In order to observe the quality of the New Formula in the case of non-uniformly distributed absorption, one should compare results of calculations based on various time reverberation formulae with values of the reverberation time values measured *in situ*.

For practical reasons, reverberation time measurement was performed in unoccupied rooms (no audience present). Since sound absorption coefficients of the surfaces are rarely known in places already built, thus in order to compare measured with calculated values it is necessary to estimate the individual sound absorption coefficients of respective surfaces of the room. This can be done basing on two assumptions. First, 10% accuracy is often adopted in engineering applications aimed at reverberation time predictions. Secondly, as shown by BISTAFA and BRADLEY [6] and other researchers any absorption coefficients enable prediction of reverberation time of a bare room within this 10% range accuracy. Thus it may be assumed that the adequate values of the unknown sound absorption of the room surfaces can be obtained from measurements using either Sabine or Eyring classical formula. Such an assumption can be also found in norms (e.g. DIN 52212, DIN EN 20354, ISO 354). Moreover, NEUBAUER estimated the error that is caused by such an approximation procedure and showed that it can be neglected [26]. In the experimental procedure Sabine formula was adopted. This was done by "calibrating" the calculated reverberation time using Sabine's formula and comparing the obtained results with the measured reverberation time yielding the respective sound absorption coefficients. The individual sound absorption coefficients were then used to calculate the respective reverberation time. Such methodology is often adopted in engineering applications and has proved satisfactory [6].

5.1. Measurement procedure

In order to measure the reverberation time *in situ*, measurement procedure was performed according to the requirements of the German Standard DIN 52216 (ISO 344) or German Standard DIN EN 20354 (ISO 354). Rooms in which the measurements took place were exposed to the "pink noise" signal filtered in the 1/3 octave band center frequency range between 50 Hz and 5 kHz. Measurements were taken at several, at least 3 locations with at least two microphone positions. The experimental configuration consisted of a 1/2″ microphone and noise source mounted inside the room. The subsequent analysis was achieved conveniently on a dual channel Real-Time Analyzer: Norsonic RTA Type 840-2. The reverberation time was calculated after the standard ISO/DIS 3382 at a rate of decay given by the linear least-squares regression of the measured decay curve from a level 5 dB below the initial level to 35 dB below. The reverberation time was calculated automatically by evaluating ensemble-averaged decay curves using linear regression.

It should be remembered that the reverberation time is defined as time needed to decrease energy by 60 dB from its original level after instantaneous termination of the excitation signal, but since this dynamic range is often not obtainable in measurements, in practice, reverberation time measurements are based on the upper 20 or 30 dB of the dynamic range.

5.2. Prediction of the sound absorption coefficient values

The measured reverberation time (T_{60}) at 500 Hz was introduced to Sabine's formula in order to find the absorption coefficient. The accuracy obtained is shown in the graph in Fig. 7.
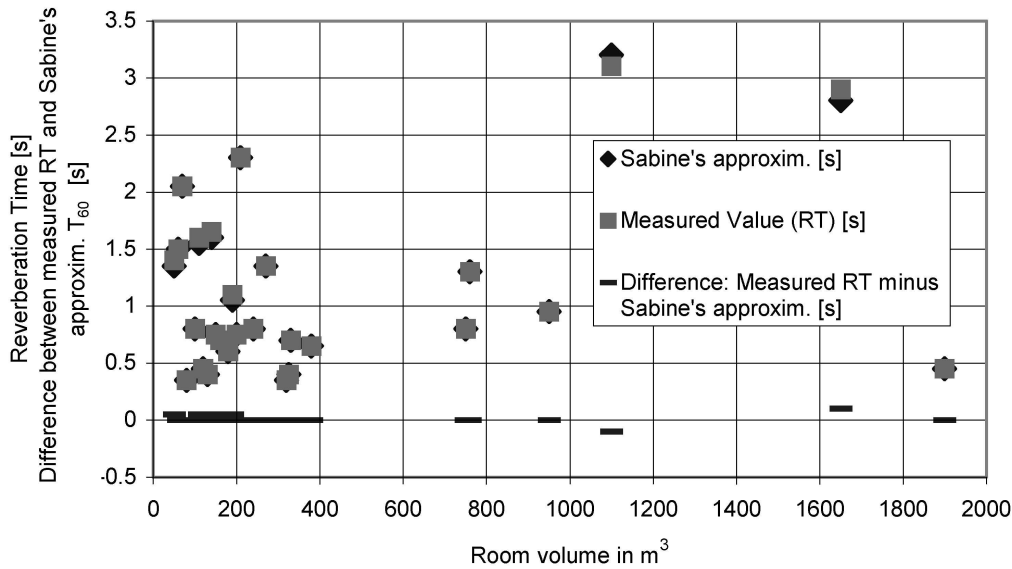

Fig. 7. Differences between "approximated" and measured reverberation time.

Figure 7 shows good conformity between "approximated" and measured reverberation time values. One of the causes of the observed difference is due to the fact that the absorption coefficients were approximated using second decimal order. Such a way of reasoning was also used by BISTAFA and BRADLEY [6] and proved to be effective in cases of unknown absorption coefficients of a room in which the reverberation time was measured. Based on such an approximation procedure the room-related individual sound absorption coefficients were taken then in order to compare calculated reverberation time values using various formulae. The obtained sound absorption coefficient was used for all formulae, i.e. all surfaces have the same respective "approximated" sound absorption coefficients.

In order to simplify calculations, it was assumed that the averaged sound absorption coefficient of the walls was equal to 0.05. Since all room dimensions are measured, one can use Sabine's equation to evaluate the measured reverberation time to obtain the averaged sound absorption coefficient of the ceiling and floor. Such calculations for a frequency band of 500 Hz are given in Table 1. All notations in Table 1 are as previously defined. It can be observed that averaged absorption coefficient values of the floor and ceiling are in most cases at least one order higher than those of the room walls.

Table 1.   Sound absorption coefficient values resulted from the approximation procedure (500 Hz frequency band).

Length [m]	Width [m]	Height [m]	Volume [m³]	S_{ww} [m²]	S_{cf} [m²]	S [m²]	$\overline{\alpha}_{ww}$	$\overline{\alpha}_{cf}$	$\overline{\alpha}$	α_{AL-xy}	RT [s] (Measured)
4.45	3.30	3.55	52.13	55.03	29.4	84.40	0.07	0.10	0.08	0.08	1.25
5.19	3.72	3.215	62.07	57.29	38.6	95.90	0.04	0.11	0.07	0.07	1.44
5.80	4.60	2.55	68.03	53.04	53.0	106.40	0.04	0.06	0.05	0.05	1.91
6.30	5.40	2.77	94.24	64.82	68.0	132.86	0.20	0.09	0.14	0.12	0.80
7.12	6.89	3.04	149.13	85.18	98.1	183.29	0.06	0.10	0.08	0.09	1.28
9.24	7.00	3.00	194.04	97.44	130.5	227.80	0.10	0.32	0.23	0.30	0.62
16.00	7.52	3.15	379.01	148.18	236.9	388.82	0.09	0.35	0.25	0.39	0.67
15.00	10.00	5.00	750.00	250.00	300.0	550.00	0.05	0.28	0.17	0.23	1.19
18.65	17.00	3.00	951.15	213.90	632.4	848.00	0.07	0.24	0.19	0.45	1.30
17.20	16.00	6.00	1651.20	398.40	550.4	948.80	0.02	0.15	0.10	0.15	2.25

5.3. Comparison of measured and predicted results

Using the obtained sound absorption coefficient one can compare the results from measurements with those of calculations (predicted results). Such calculations were performed for the whole frequency band characteristics, however for comparison purpose all values presented in Figs. 8 and 9 are shown only for a mid-frequency range of 500 Hz.
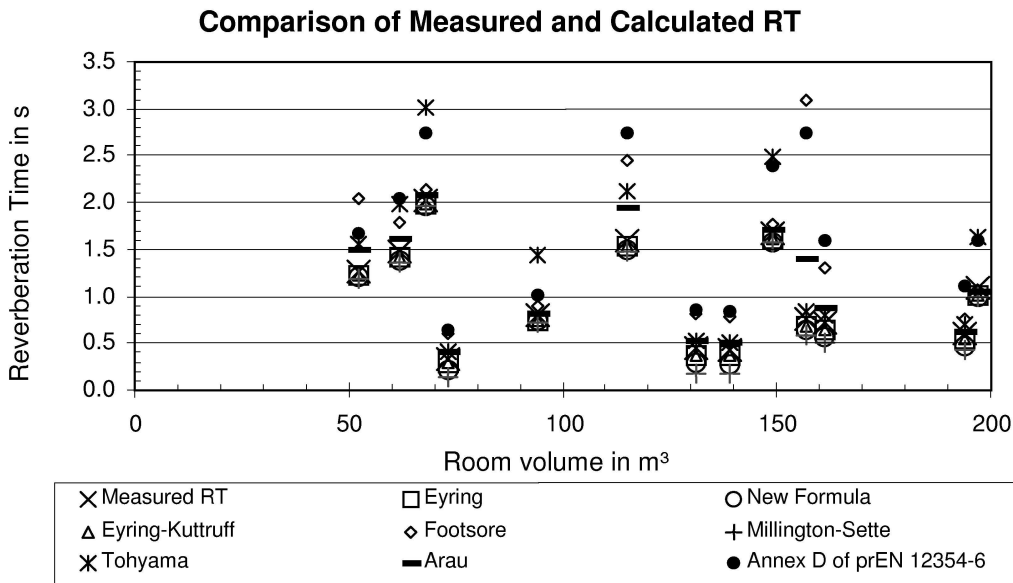

Fig. 8. Comparison of measured and predicted reverberation time values for a room with a volume range from 50 to 200 m³.

The obtained results are given for two ranges of room volumes. Although values given by Eyring's formula are close to the measured ones, those obtained on the basis of the New Formula conformed better with the "real values" of the reverberation time. In addition, it may be observed from Figs. 8 and 9 that by using Fitzroy's, the model contained in the European Norm (Annex D, prEN 12354-6), Arau's and Tohyama's equations no proper approximation to the measured reverberation time values is obtained.
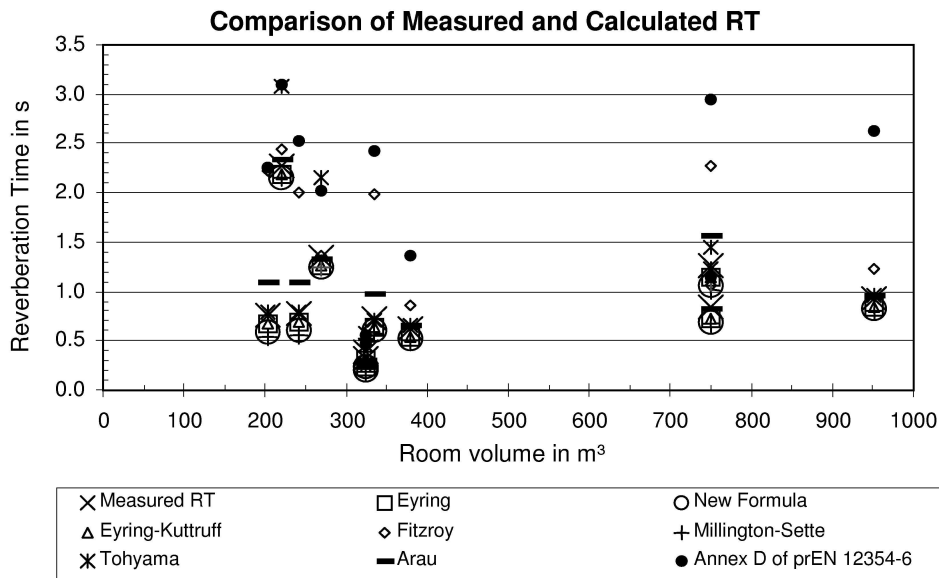

Fig. 9. Comparison of measured and predicted reverberation time values for a room with a volume range from 200 to 1000 m³.

To summarize the obtained results it can be observed that the reverberation time values calculated using the model from the European Norm (Annex D, prEN 12354-6), Tohyama or Fitzroy formulae significantly deviate from the measured values, whereas the New Formula provides values within a range of approximately ±28% and is always within the same range as the measured reverberation time.

6. Conclusions

Based on the study conducted, the following conclusions may be derived:

— In the case of low absorption there is no meaningful difference in the computed reverberation times except for those calculated using Tohyama's formula and the model contained in the European Norm (Annex D, prEN 12354-6). There exists a difference between various calculation formulae results in the case of high absorption. It was also observed that values calculated on the basis of the model from the European Norm, Sabine's and Tohyama's formulae differ considerably as compared with the other reverberation formulae.

— Prediction based on the proposed New Formula generally conforms much better than classical formulae with the reverberation time values obtained from *in situ* measurements. This is also true for various room volume values.

— In addition, predicted reverberation time using the New Formula reveals a shorter reverberation time than measured. This is especially true in the case of higher octave band frequencies. However, in the case of investigated data best results were observed below or equal to the octave center frequency of 1 kHz, which is very important for obtaining adequate reverberation characteristics.

References

[1] H. ARAU-PUCHADES, *An improved reverberation formula*, Acoustica, **65**, 163–180 (1988).

[2] *Proc. of the Conference on Measurements of Reverberation Phenomena in Halls*, (Warsaw, April 1967), Archives of Acoustics, **3**, 2, 99–245 (1968).

[3] M. BARRON and A.H. MARSHALL, *Spatial impression due to early lateral reflections in concert halls: The derivation of a physical measure*, J. Sound and Vibration, **77**, 2, 211–232 (1981).

[4] L.L. BERANEK, *Concert and opera halls. How they sound*, Acoust. Soc. Amer. (1996).

[5] L.L. BERANEK, *Concert hall acoustics – 1992*, J. Acoust. Soc. Amer., **92**, 1–39 (1992).

[6] S.R. BISTAFA and J.S. BRADLEY, *Predicting reverberation times in a simulated classroom*, J. Acoust. Soc. Am., **108**, 4, 1721–1731 (2000).

[7] E. BUCKINGHAM, *Theory and interpretation of experiments on the transmission of sound through portion walls*, Bur. Standards, Sci. Paper No. 506 (1925).

[8] G. BUDZYŃSKI, *Theory of the reflective localization of sound sources*, Archives of Acoustics, **11**, 1, 13–24 (1986).

[9] L. CREMER and H.A. MÜLLER, *Die wissenschaftlichen Grundlagen der Raumakustik*, Band 1 S. Hirzel Verlag, Stuttgart 1978.

[10] European Standard prEN 12354-6 (CEN/TC126/WG2 N241), final draft, May 2001.

[11] C.F. EYRING, *Reverberation time in "dead" rooms*, J. Acoust. Soc. Amer., **1**, 217–241 (1930).

[12] D. FITZROY, *Reverberation formulae which seems to be more accurate with non-uniform distribution of absorption*, J. Acoust. Soc. Amer., **31**, 893–897 (1959).

[13] A.D. FOKKER, *Over Den Nagalm*, Physica, **4**, 262–273 (1924).

[14] W.S. FRANKLIN, *Derivation of equation of decaying sound in a room*, Phys. Rev., **16**, 372–374 (1903).

[15] A. JAEGER, *Zur Theorie des Nachhalls*, Wiener Akad, Ber., Math.-Naturwiss, Klasse, Bd. **120**, Abt. IIa, 613–634 (1911).

[16] W.B. JOYCE, *Sabine's reverberation time and ergodic auditoriums*, J. Acoust. Soc. Amer., **58**, 3, 643–655 (1975).

[17] B. KOSTEK, *Soft computing in acoustics, applications of neural networks, fuzzy logic and rough sets to musical acoustics*, Studies in Fuzziness and Soft Computing, Physica Verlag, Heidelberg, New York 1999.

[18] C.W. KOSTEN, *The mean free path in room acoustics*, Acustica, **10**, 245–250 (1960).

[19] C.W. KOSTEN, *New method for the calculation of the reverberation time of halls for public assembly*, Acustica, **16**, 6, 325–330 (1965/66).

[20] H. KUTTRUFF, *Nachhall und effektive Absorption in Räumen mit diffuser Wandreflexion*, Acustica, **35**, 3, 141–153 (1976).

[21] H. KUTTRUFF, *Energetic sound propagation in rooms*, Acustica – Acta Acustica, **83**, 622–628 (1997).

[22] H. KUTTRUFF, *On acoustics of auditoria*, Institute of Technical Acoustics, Technische Hochschule, 27-48, Aachen.

[23] Memorandum to CEN/TC 126/WG2, *Possibilities to improve the estimation of reverberation time in rooms*, TNO Institute of Applied Physics, Netherlands, HAG-MEMO-990076, 1–8 (1999).

[24] G. MILLINGTON, *A modified formula for reverberation*, J. Acoust. Soc. Amer., **4**, 69–82 (1932).

[25] R.O. NEUBAUER, *Prediction of reverberation time in rectangular rooms with a modified Fitzroy equation*, ISSEM'99, Proc. 8th International Symposium on Sound Engineering and Mastering, A. CZY§EWSKI, B. KOSTEK [Eds.], 115–122, Gdańsk 1999.

[26] R.O. NEUBAUER, *Classroom acoustics — Do existing reverberation time formulae provide reliable values?*, 17th International. Congress Acoust., Italy, Rome 2001.

[27] M. NIEMAS, Z. ENGEL and J. SADOWSKI, *Acoustic issues of sacral structures*, Archives of Acoustics, **23**, 1, 87–104 (1998).

[28] E. NILSSON, *Decay process in rooms with non-diffuse sound fields*, Report TVBA-1004, Lund Institute of Technology, Depart. of Eng. Acoustics (1992).

[29] A.D. PIERCE, *Acoustics*, Acoust. Soc. of Amer., second printing, Chap. 6, 262 (1991).

[30] A. RAKOWSKI, *Factors governing the acoustical quality of concert halls* [in Polish], Warsaw Academy of Music, 63–92.

[31] W.C. SABINE, *Collected papers on acoustics*, University Press Harvard, (1922); Reprinted by Dover, New York 1964.

[32] J. SADOWSKI, *Akustyka architektoniczna* [in Polish], Arkady, Warszawa 1971.

[33] J. SADOWSKI, *Akustyka architektoniczna* [in Polish], PWN, Warszawa 1976.

[34] M.R. SCHROEDER, *Some new results in reverberation theory and measurement methods*, 5e Congres International D'Acoustique LIÉGE 7-14 Sept., G31 (1965).

[35] M.R. SCHROEDER, *The statistical parameter of frequency curves of large rooms*, Acustica, **4**, 594–600 (1954).

[36] K. SCHUSTER and E. WAETZMANN, *Über den Nachhall in geschlossenen Räumen*, Ann. d. Physik, **1** (1929), 671; also Textbook by Muller-Pouillet, on Acoustics Chapter VII, 456–460.

[37] W.H. SETTE, *A new reverberation time formula*, J. Acoust. Soc. Amer., **4**, 193–210 (1933).

[38] G.A. SOULOUDRE and J.S. BRADLEY, *Subjective evaluation of new room acoustic measures*, J. Acoust. Soc. Amer., **98**, 1 (1995).

[39] K. SRODECKI, *Evaluation of the reverberation decay quality in rooms using the autocorrelation function and the cepstrum analysis*, Acustica, **80**, 3, 216–225 (1994).

[40] W. STRASZEWICZ, *Some remarks on the reverberation time criterion and its connection with acoustical properties of room*, Archives of Acoustics, **2**, 3, 99–107 (1968).

[41] M. TOHYAMA and A. SUZUKI, *Reverberation time in an almost-two-dimensional diffuse field*, J. Sound Vib., **111**, 3, 391–398 (1986).

[42] M. VORLÄNDER, *Objective characterization of sound fields in small rooms*, 15th Audio Eng. Soc. Int. Conf., Copenhagen, Denmark 1998.

[43] S. WEYNA, *An image of the energetic acoustic field in reduced parallelepiped room models*, Acustica – Acta Acustica, **82**, 1, 72–81 (1996).

[44] R.W. YOUNG, *Sabine reverberation equation and sound power calculations*, J. Acoust. Soc. Amer., **31**, 7, 912–921 (1959).