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ELECTROSTATIC SCATTERING BY STRIPS

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A system of in-plane periodic perfectly conducting strips is considered embedded in an preexisting electric field which, in applications, can result from propagation of an elastic wave in a piezoelectric material supporting strips on its surface. The spatial spectrum of charge distribution in the plane of strips is of our primary interest. In spite of the functional dependence between the spectrum and the spatial distribution of charge by means of the Fourier transform, its direct application leads to a considerable numerical error caused by poor numerical representation of the singular field at the strip edges. Carrying out the analysis in the spectral domain instead of direct evaluation of the spatial charge distribution overcomes this difficulty.

1. Introduction

A system of parallel conducting strips residing on a piezoelectric substrate surface interacts weakly with a Rayleigh wave [1]. When supplied with alternate voltages, the strips excite the wave in the media [2]. In a reciprocal phenomenon, strips detect the propagating wave by collecting electric charge. In this paper a periodic system of groups of strips is considered and evaluation of the spatial spectrum of electric charge distribution on the plane of strips is the main task of the analysis. The strips in one group can have arbitrary width and spacing, and the period of group of strips is also arbitrary. The introduced periodicity allows one to exploit the convenient fast Fourier transform in computations.

As mentioned above, there are two distinct sources of the charge distribution on strips: the first are voltages set on the strips by external time-harmonic voltage source of angular frequency ω (sometimes, the strip charge is set, like on an isolated strip of total charge zero), and the second is the incident Rayleigh wave propagating with velocity v_R accompanied by the electric field on the plane of strips. Neglecting weak mechanical interactions, the field is governed by electrostatic theory. Strips modify the field distribution that otherwise would be a spatial-harmonic function of spatial frequency ω/v_R. This is the "incident" wave of the title "electrostatic" scattering phenomenon by strips. The former problem of the supplied strips has been already discussed in literature [3]. Here, we extend the method of the analysis developed there to solve the scattering problem. For readers' convenience, the method is presented shortly in the following sections.

2. Functions of partial solution

There are two complementary identities involving Legendre polynomials P_n [4]

$$D(\vartheta) = \sum_{n=-\infty}^{\infty} P_n(\cos\Delta) e^{i(n+1/2)\vartheta} = \begin{cases} \dfrac{\sqrt{2}}{\sqrt{\cos\vartheta - \cos\Delta}}, & 0 \le \vartheta < \Delta, \\ 0, & \Delta < \vartheta < \pi, \end{cases}$$

$$iE(\vartheta) = \sum_{n=-\infty}^{\infty} S_n P_n(\cos\Delta) e^{i(n+1/2)\vartheta} = \begin{cases} 0, & 0 \le \vartheta < \Delta, \\ \dfrac{iS_\vartheta\sqrt{2}}{\sqrt{\cos\Delta - \cos\vartheta}}, & \Delta < \vartheta < \pi, \end{cases} \tag{1}$$

where $S_\nu = 1$ for $\nu \ge 0$ and -1 otherwise. Here, D is real or zero, and iE is imaginary or zero, in alternate domains. Both functions on left are well defined for either positive and negative ϑ. Replacing ϑ by Kx, $-\infty < x < \infty$, Eqs. (1) describe the periodic field distribution over x; the strip width is $2w = 2\Delta/K$ and period $\Lambda = 2\pi/K$; K is the strip wavenumber.

COROLLARY 1. *$D(x)$ and $E(x)$ are respectively the normal electric induction above the plane of strips, and electric tangential field in the plane of strips, resulting from the charge distribution on strips.*

Indeed, $D(x) = 0$ between strips, and $E(x) = 0$ on strips as required, and we only need to show that the pair (D, E) belongs to the class of field vanishing at $y \to \infty$ when extended to the whole space (x, y). Given the electric potential $\varphi(x, y) = \exp(irx - \sqrt{r^2}|y|)$, $\mathrm{Re}\{\sqrt{r^2}\} > 0$, satisfying the Laplace equation $\nabla^2\varphi = 0$, the tangential electric field on $y = 0$ plane is $E = -jr\varphi$ from the definition $E = -\nabla\varphi$. For a medium of unitary dielectric permittivity $\varepsilon = 1$ applied here, the normal induction at $y = +0$ is $D = \varepsilon E_y = \sqrt{r^2}\varphi$, yielding

$$iE = Dr/\sqrt{r^2} = S_r D. \tag{2}$$

This is exactly the relation between each harmonic component of D and iE in Eqs. (1) of the same wave number $(n + 1/2)K$; P_n is their amplitude.

Combining D and E yields the complex function $\sqrt{2}/\sqrt{\cos Kx - \cos\Delta}$

$$De(x; \Delta) = D(x) + iE(x) = \sum_{n=-\infty}^{\infty} (1 + S_n) P_n(\cos\Delta) e^{-i(n+1/2)Kx}, \tag{3}$$

the domain of which spans an entire x-axis, provided that the square-root values are chosen according to Eqs. (1). It takes real or imaginary values in alternating domains of real x. Considering one period only, $x \in (-\Lambda/2, \Lambda/2)$, De is real and different from zero if $|x| < w$, and imaginary in the domain $w < |x| < \Lambda/2$.

For convenience, the function describing the periodic system (the superscript 1 indicating one strip per period) is rewritten in the form ($\Delta_1 = Kw_1$)

$$\begin{aligned} De^{(1)}(x) &= De(x - x_1; \Delta_1) = \sum_{m=-\infty}^{\infty} (1 + S_m) F_m^{(1)} e^{i(m+1/2)Kx}, \\ D &= \sum_n F_n^{(1)} e^{i(n+1/2)Kx} \neq 0 \text{ if } \cos K(x - x_1) > \cos \Delta_1, \\ iE &= \sum_n S_n F_n^{(1)} e^{i(n+1/2)Kx} \neq 0 \text{ if } \cos \Delta_1 > \cos K(x - x_1), \\ F_m^{(1)} &= e^{-i(m+1/2)Kx_1} P_m(\cos \Delta_1), \qquad F_{-m-1}^{(1)} = F_m^{(1)*}, \end{aligned} \tag{4}$$

where asterisk means a complex conjugate value (note: $P_{-n-1} = P_n$).

For two strips per period, the function of interest is

$$\begin{aligned} De^{(2)}(x) &= De^{(1)}(x) De(x - x_2; \Delta_2) \\ &= \sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} (1+S_k)(1+S_m) P_k(\cos \Delta_2) e^{i(k+1/2)K(x-x_2)} F_m^{(1)} e^{i(m+1/2)Kx}, \end{aligned} \tag{5}$$

that takes real or imaginary values in alternating domains of x: it is real in the domains where both the multiplied functions are either real or imaginary, and it is imaginary in the domains where one of the multiplied functions is real and the other imaginary. Explicitly,

$$\begin{aligned} De^{(2)}(x) &= \sum_{n=-\infty}^{\infty} (1 + S_n) F_n^{(2)} e^{inKx}, \\ F_n^{(2)} &= 2e^{-inKx_2} \sum_{\substack{0 \le m < n \\ n \le m < 0}} P_{m-n}(\cos \Delta_2) F_m^{(1)} e^{i(m+1/2)Kx_2}, \\ \mathrm{Re}(De^{(2)}) &= \sum_n F_n^{(2)} e^{inKx}, \quad i\mathrm{Im}(De^{(2)}) = \sum_n S_n F_n^{(2)} e^{inKx}. \end{aligned} \tag{6}$$

Note that the finite summation domain involved in the expression for $F^{(2)}$ is empty if $n = 0$, what means that $F_0^{(2)} = 0$. Moreover, replacing n by $-n$, one obtains that

$$F_{-n}^{(2)} = F_n^{(2)*}, \qquad F_0^{(2)} = 0. \tag{7}$$

Similarly are defined the harmonic functions for more strips, with corresponding property that $F_{-n-1}^{(N)} = 0$ if $n \in \mathcal{N} = [N_1, N_2]$; explicitly for $N = 3$

$$\begin{aligned} De^{(3)}(x) &= \sum_{n=-\infty}^{\infty} (1 + S_n) F_n^{(3)} e^{i(n+1/2)Kx}, \\ F_n^{(3)} &= 2e^{-i(n+1/2)Kx_3} \sum_{\substack{0 \le m \le n \\ n < m < 0}} P_{n-m}(\cos \Delta_3) F_m^{(2)} e^{imKx_3}, \\ F_{-n-1}^{(3)} &= F_n^{(3)*}, F_{-1,0} = 0. \end{aligned}$$

Generally, for odd ($N = 2k+1$) and even ($N = 2k$) number of strips:

$$\begin{aligned} F_{-n-1}^{(2k+1)} &= F_n^{(2k+1)*}, \quad \text{and} \quad F_n^{(2k+1)} = 0 \quad \text{if} \quad -k-1 < n < k, \\ F_{-n}^{(2k)} &= F_n^{(2k)*}, \quad \text{and} \quad F_n^{(2k)} = 0 \quad \text{if} \quad -k < n < k. \end{aligned} \tag{8}$$

It is convenient to make the expressions for D and iE similar for both cases of odd or even number of strips N; it is sufficient to replace $F_n^{(N)}$ by $\widetilde{F}_n^{(N)} = S_n F_n^{(N)}$ if N is even. Applying this and noticing that $\mathrm{mod}(N,2) = 1$ for N odd, otherwise 0, one may prove the following

Corollary 2.

$$\begin{aligned} \widetilde{F}_n^{(2k+1)} &= F_n^{(2k+1)}; \qquad \widetilde{F}_n^{(2k)} = S_n F_n^{(2k)}, \\ D &= \sum_{n=-\infty}^{\infty} \widetilde{F}_n^{(N)} e^{i(n+\nu)Kx}, \qquad \nu = \mathrm{mod}(N,2)/2, \quad \text{and} \\ iE &= \sum_{n=-\infty}^{\infty} S_n \widetilde{F}_n^{(N)} e^{i(n+\nu)Kx}, \end{aligned} \tag{9}$$

have alternate support, corresponding to electric conditions on the plane of strips: normal electric induction D vanishes between the strips and tangential field E vanishes on the strips.

Consider the periodic harmonic function

$$\begin{aligned} De^{(N)}(x)\alpha_m e^{imKx} &= \sum_{n=-\infty}^{\infty} \alpha_m (1 + S_{n-m}) \widetilde{F}_{n-m}^{(N)} e^{i(n+\nu)Kx}, \\ &\nu = \mathrm{mod}(N,2)/2, \end{aligned} \tag{10}$$

where α_m is arbitrary and m is an integer. This is a harmonic expansion of

$$\frac{\alpha_m 2^{N/2} \exp(imKx)}{\sqrt{[\cos K(x-x_1) - \cos \varDelta_1] \cdots [\cos K(x-x_N) - \cos \varDelta_N]}} . \tag{11}$$

Theorem 1.

$$\begin{aligned} D &= \alpha_m \sum_{n=-\infty}^{\infty} \widetilde{F}_{n-m}^{(N)} e^{i(n+\nu)Kx}, \\ iE &= \alpha_m \sum_{n=-\infty}^{\infty} S_{n-m} \widetilde{F}_{n-m}^{(N)} e^{i(n+\nu)Kx}, \end{aligned} \tag{12}$$

are respectively the normal electric induction above the plane of strips and the tangential electric field resulting from strips' charges, if $m \in \mathcal{M} = [M_1, M_2]$. The domains $\mathcal{M}$ for odd and even N are explicitly:

$$\begin{aligned} -(N-1)/2 \le m &\le (N-1)/2, \quad \text{for odd } N, \quad \text{or} \\ -N/2 \le m &\le N/2 - 1, \quad \text{for even } N, \end{aligned} \tag{13}$$

there are N such coefficients α_m altogether.

P r o o f. Following Corollary 2, these functions satisfy conditions on the plane of strips. Thus we only need to prove that they are components of the field that vanish at $y \to \infty$. In the above domain of m, $S_{n-m} \neq S_n$ only when simultaneously $\alpha_m \widetilde{F}^{(N)}_{n-m} = 0$ on the strength of Corollary 2. That means that S_{n-m} can be replaced by S_n in Eqs. (12). The rest of proof exploits Eq. (2) like in Corollary 1 (here, $\widetilde{F}^{(N)}$ is the field harmonic amplitude).

REMARK. x_l and Δ_l determine the l-th strip edges and the corresponding square-root singularities of (11). It is evident that there are multiple ways of evaluation of $De^{(N)}$ by applying different x_l, Δ_l yielding singularities of (11) at the same x but not necessarily belonging to the same strip. These different choices may affect the numerical accuracy or the computation time, or both; the matter is not discussed here. It is also evident that $De^{(k)}De^{(l)} = De^{(k+l)}$.

It results from $(1 + S_n) = 0$ if $n < 0$ that the discrete spectrum of $De^{(N)}$ in Eq. (10) has a semifinite support. This can be nicely exploited in evaluation of harmonic spectra of subsequent $De^{(l)}$ (that is, evaluation of $F_n^{(l)}$) using the FFT algorithm like in a numerical code presented in the paper [5].

3. Given strip voltages or charges

Kirchhoff's laws, when applied to the system of interconnected strips, result in a number of conditions for strip voltages V_i, $i = 1, ..., N$ (N is the number of strips in one period Λ), and/or currents $I_i = i\omega 2Q_i$ flowing to strips, where $2Q$ is the total strip charge. There are sufficient number of conditions to solve the problem accounting for that the external voltage source sets the voltage differences $U_{ij} = V_i - V_j$ between strips rather than their absolute potentials V_i (both the source ports are connected to different strips in the system). An equivalent physical requirement is that the total charge on strips vanishes, meaning the system electric neutrality. Below, strips' charges and potentials are evaluated; they are necessary in formulation of $N - 1$ circuit equations resulting from Kirchhoff's laws.

An arbitrary electric field satisfying the "radiation" condition at infinity (vanishing at infinity) is constructed by superposition

$$De(x) = \sum_{m \in \mathcal{M}} \sum_{n=-\infty}^{\infty} \alpha_m (1 + S_{n-m}) \widetilde{F}^{(N)}_{n-m} e^{i(n+\nu)Kx}, \tag{14}$$

with unknown α_m which will be evaluated from the circuit equations; $m \in \mathcal{M}$ following Theorem 1. The electric charge $2Q(x)$ on the plane of strips and electric potential $V(x)$ on this plane are integrals of $2D$ and $-E$, explicitly

$$\begin{aligned} Q(x) &= -\frac{i}{K} \sum_{m \in \mathcal{M}} \sum_{-\infty}^{\infty} \frac{\alpha_m \widetilde{F}^{(N)}_{n-m}}{n + \nu} e^{i(n+\nu)Kx}, \\ V(x) &= \frac{1}{K} \sum_{m \in \mathcal{M}} \sum_{-\infty}^{\infty} S_{n-m} \frac{\alpha_m \widetilde{F}^{(N)}_{n-m}}{n + \nu} e^{i(n+\nu)Kx}, \end{aligned} \tag{15}$$

using informal notation that $\alpha_m F_{n-m}/n = 0$ at $n = 0$ provided that $\alpha_{-N/2} = 0$ is already assumed if N is even, and because of vanishing F_n according to Eq. (8). This, in fact, is the necessary and sufficient condition for the system electric neutrality. Indeed, for even number of strips, the electric field is Λ-periodic, and such is $Q(x)$. It is easily seen from Eqs. (15) that the total charge on strips over one period is $Q(\Lambda/2) - Q(-\Lambda/2) = 0$ provided that Q is finite. For odd number of strips, the field (14) satisfies the neutrality condition automatically because $Q(x + \Lambda) = -Q(x)$, thus the average charge over 2Λ period in this case vanishes.

Let the strips' centers be placed at $\dot{x}_i$, $i = 1, ..., k$, and the spacings' centers between strips be at $\overline{x}_i$, assuming that $\overline{x}_0 = -\Lambda/2$ and $\overline{x}_{k+1} = \Lambda/2$ are points on both sides of the discussed group of strips, outside the strips. Thus

$$U_{ij} = V(\dot{x}_i) - V(\dot{x}_j), \qquad Q_i = Q(\overline{x}_i) - Q(\overline{x}_{i-1}), \quad i = 1, ..., k, \tag{16}$$

depend on unknown α_m. There are $N - 1$ unknowns if N is even ($\alpha_{-N/2} = 0$ has been already set), or N unknowns if N is odd, what follows from (13). Taking into account that there are $N-1$ circuit equations resulting from the circuit theory, the system of equations for even number of strips is complete and can be solved, while the system of odd number of strips requires one additional equation. This can be the condition that the group of strips is electrically neutral over the period Λ: $Q(\Lambda/2) - Q(-\Lambda/2) = 0$ (they are neutral automatically over 2Λ so that this is a new and independent condition; however, one can apply any other independent condition, by setting strip voltages directly in this case instead of voltage differences between strips, for instance). This completes the system of equations in this case, too.

Summarizing, the system of equations resulting from the circuit theory

$$\sum_m A_{lm}\alpha_m = B_l, \tag{17}$$

where B_l are known U_{ij} or Q_i, and the matrix $[A_{lm}]$ describes the strip interconnections, can be solved for α_m, which substituted into Eqs. (15), allows us to evaluate the other circuit quantities (strip currents and voltages), and the main objective of this analysis that is the Fourier coefficients D_n

$$D(x) = \sum_{-\infty}^{\infty} D_n e^{i(n+\nu)Kx}, \qquad D_n = \alpha_m \widetilde{F}_{n-m}^{(N)}, \tag{18}$$

of the discrete spatial spectrum of charge distribution. The computationally most difficult task is met in evaluation of the sums (15) which are not fast convergent (numerical hint: evaluation of $\sum_1^{\infty} F_{n-m} \exp i(n + \nu)Kx_i$ suffices).

4. The "scattering" problem

It is convenient to consider the preexisting field $(D^I, E^I) \exp ipx$, $p \neq 0$, on the plane of strips like an incident wave field in the theory of scattering of waves: while the "scattered" field resulting from the induced strip charges obeying the "radiation condition" vanish at infinity, the preexisting or "incident" field does the opposite, grows at infinity. This results in the different relations between the electric field harmonic components at the plane of strips (D at $y = +0$, above the plane) in both cases: Eq. (2) for the "scattered" field resulting from the strips' charges, and

$$iE^I = -S_p D^I, \tag{19}$$

for the "incident" field, that is for the preexisting field.

In the "scattering" problem considered here, both D^I and E^I are known, and the full field being the sum of incident and scattered fields

$$\{D, E\} = \{D^I, E^I\} + \{D^s, E^s\} \tag{20}$$

must obey the conditions on strips: not only the normal induction $D(x)$ must vanish between the strips and tangential field E must vanish on the strips, but also the resulting strips' voltages and charges must obey Eq. (17) resulting from the circuit theory.

The first condition is satisfied by expanding D and iE into the series like (12), accounting for the Theorem 1, that is replacing $(n+\nu)K$ by $nK + r$, where $r = \nu K$ is the reduced wave number of the incident field

$$p = IK + r, \qquad 0 \le r < K, \tag{21}$$

and I is integer. (Within the presented theory, the wavenumber of the incident wave can take only the values allowed by the spectrum of general solution (14). Thus r must take value either 0 or $K/2$, that is $r = \nu K$.) Explicitly,

$$\begin{aligned} D &= \sum_{m,n} \alpha_m \widetilde{F}_{n-m} e^{i(nK+r)x}, \\ iE &= \sum_{m,n} \alpha_m S_{n-m} \widetilde{F}_{n-m} e^{i(nK+r)x}, \end{aligned} \tag{22}$$

with summation over $n \in (-\infty, \infty)$ and over m as discussed below.

Applying Eq. (2) to harmonic components of the scattered field evaluated from Eq. (20) and using Eq. (19), results in

$$\sum_m (1 - S_{n-m}) \widetilde{F}_{n-m} \alpha_m = 2D^I \delta_{nI}, \tag{23}$$

for each $n \in -\infty, \infty$ separately, where δ is Kronecker delta. The matrix of this system of equation has a very specific triangular form presented below for $n \in [n^- < 0, n^+ > 0]$

and $m \in [m_1^- < 0, m_2^+ > 0]$

$$[(1-S_{n-m})\widetilde{F}_{n-m}] = \begin{bmatrix} \bullet_{n^-,m_1^-} & & & & 0\cdots 0 & & & & 0 \\ \cdots & & & & & & & & \\ \cdots & \bullet & & & & & & & \\ \bullet & \cdots & \bullet & & 0\cdots 0 & & & & \\ \bullet_{-1,m_1^-} & \cdots & \bullet & \bullet_{-1,m_2^-} & 0\cdots 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0\cdots 0 & {}_{0,m_1^+}\bullet & \cdots & \bullet & {}_{0,m_2^+}\bullet \\ & & & & & & \bullet & \cdots & \cdots \\ 0 & & & & 0\cdots 0 & & & \bullet & \bullet \\ 0 & & & & 0\cdots 0 & & & & {}_{n^+,m_2^+}\bullet \end{bmatrix} \tag{24}$$

as can be proved by inspection, where the pairs of indices (n, m) are presented for significant elements discussed below. Only the components marked by bullets are different from zero.

Indeed, triangular form results from $1 - S_{n-m} = 0$ if $n \geq m$, that is if either $m > n < 0$ or $m \leq n \geq 0$. The vertical strip of zeros results from $\widetilde{F}_n = 0$ if $n \in \mathcal{N}$, see Corollary 2. Detailed analysis shows that

$$\begin{aligned} m_1^- &= -1 - M_2 - n^-, & m_2^- &= -M_2 - 2, \\ m_1^+ &= 1 - M_1, & m_2^+ &= 1 - M_1 + n^+, \end{aligned} \tag{25}$$

with $M_{1,2}$ defined in Theorem 1, Eq. (13). For $n = -1$, all elements with $m > m_2^-$ vanish; similarly for $n \leq -1$, all elements with $m > -1 - n + m_2^-$ vanish. Analogously, all elements vanish if $n = 0$ and $m < m_1^+$, as well as if $n \geq 0$ and $m < n + m_1^+$, because either $1 - S_{n-m} = 0$, or $\widetilde{F}_{n-m} = 0$. It is clearly seen from Eqs. (24), (25) that none of α_m, $m \in \mathcal{M}$, are involved in Eqs. (23) governing the scattered field.

Let $I < 0$, thus we apply $n^- = I$ and account only for equations (23) with $I \leq n < 0$, allowing $m \in [m_1^-, m_2^-]$. The resulting system can be solved sequentially starting from $\alpha_{m_1^-}$ up to the last $\alpha_{m_2^-}$. Similarly, if $I = n^+ \geq 0$: accounted for are equations with $0 \leq n < I$ and $m_1^+ \leq m \leq m_2^+$, and the system is solved starting from $\alpha_{m_2^+}$. Accounting for more equations (23), with $n^- < I < 0$ or $n^+ > I \geq 0$, with α_m from a wider domain of m, is superfluous and results only in $\alpha_m = 0$ for all m outside the domain specified above. Summarizing the solution to Eq. (23), the scattered field at $y = 0$ is

$$\begin{aligned} D^s(x) &= \sum_m \alpha_m \sum_{n=-\infty}^{\infty} \widetilde{F}_{n-m} e^{i(nK+r)x}, \\ iE^s(x) &= \sum_m \alpha_m \sum_{n=-\infty}^{\infty} S_{n-m} \widetilde{F}_{n-m} e^{i(nK+r)x}, \end{aligned} \tag{26}$$

with summation extended over all α_m evaluated above, which after integration, yields $Q^s(x)$ and $V^s(x)$ like in Eqs. (15) but with $(n+\nu)K$ replaced by $nK+r$

$$\begin{aligned} Q^s(x) &= -i\sum_m\sum_{-\infty}^{\infty}\frac{\alpha_m\widetilde{F}^{(k)}}{nK+r}e^{i(nK+r)x}, \\ V^s(x) &= \sum_m\sum_{-\infty}^{\infty}S_{n-m}\frac{\alpha_m\widetilde{F}^{(k)}}{nK+r}e^{i(nK+r)x}. \end{aligned} \tag{27}$$

Now, the strip charges and potentials can be evaluated like in Eq. (16).

The solution (27) has exactly the form of (14), and superposition of properly chosen solutions from Sec. 3 with the above scattered field can satisfy Eq. (17) resulting from circuit theory. This superposed solution includes α_m, $m\in\mathcal{M}$, and these are evaluated from Eq. (23). The superposed domain of m is then $[-1-N_2-I, M_2]$ for $I<0$, or $[M_1, 1-N_1+I]$, for $I\geq 0$.

The above mentioned "properly chosen" α_m, $m\in\mathcal{M}$ are the solution to Eq. (17) with its right-hand side modified by the earlier evaluated V^s_{ij} and Q^s_i resulting from Eqs. (27) (with specific r as assumed above). The case $r=0$ needs further discussion, however, concerning integration of the superposed fields, (27) and (15). Naturally, to make the integration possible, the superposed fields resulting in $n=0$-harmonic component must be set to zero, yielding the condition of $\sum\limits_m \alpha_m\widetilde{F}_{n-m}=0$ if $n=0$. It can be satisfied by proper choice of $\alpha_{-N/2}$ (the same $\alpha_{-N/2}$ that was set to zero in Sec. 3). This is the condition of the strip electric neutrality, modified here by the charge generated by the incident field, or rather by the charge resulting from normal induction of preexisting field. Like in Sec. 3, this additional condition (17) makes the system complete, and all α_m can be evaluated. Substitution into Eqs. (22) (with all the α_m accounted for) yield the spatial spectrum of electric charge at the plane of strips.

5. Conclusions

The applied systems of strips, the interdigital transducers of Rayleigh waves, may have tens or even hundreds of strips, and evaluation of Fourier integrals must, practically, be based on the fast Fourier transform (FFT). This requires discretization of $V(r)$, $Q(r)$ in a grid of equally spaced $r_i=i\Delta r$ over the domain $(0,K)$. This corresponds to the analysis of spatial periodic system of groups of strips, with period $\Lambda=1/\Delta r$. This practical necessity is respected in the above analysis from the beginning. Moreover, while for finite (aperiodic) system of strips the Bessel functions (which are the Fourier transforms of $(w^2-x^2)^{-1/2}$ [5]) would be involved in the analysis and subsequent computations, in the present theory the Legendre polynomials P_n take their place, which are easier for computation.

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