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AIRCRAFT NOISE PROPAGATION THE SIMPLEST CASE

R. MAKAREWICZ and R. GOŁĘBIEWSKI

Institute of Acoustics
A. Mickiewicz University
61-614 Poznań, Umultowska 85
makaaku@amu.edu.pl

Geometrical spreading, air absorption, and refraction are the main wave phenomena that govern the propagation of aircraft noise. The results of this study make possible the calculate noise at ground level in terms of the sound exposure level, L_{AE}. The model applies to an aircraft with a nondirectional radiation pattern (e.g., helicopter, one-engine propeller) while the atmosphere is at rest. The derived equations provide some insight into the nature of noise propagation, that can be useful for teaching purposes.

1. Introduction

Several different techniques are available for the prediction of aircraft noise [7, 12, 13]. In this study we show how to calculate the noise generated by a single aircraft. To simplify calculations we start from the following conditions:

- due to the A-frequency weighting an aircraft is a nondirectional point source (e.g. helicopter, single-engine propeller),
- noise is affected by air absorption and refraction,
- Doppler and ground effects are ignored.

The above conditions restrict the range of possible applications, however, making the results useful for teaching purposes.

The noise from a single aircraft is quantified by the sound exposure level,

$$L_{AE} = 10 \log \left\{ \frac{E}{p_o^2 t_o} \right\}, \quad p_o = 20\ \mu\text{Pa}, \quad t_o = 1\ \text{s}, \tag{1}$$

and the sound exposure

$$E = \int_{-\infty}^{+\infty} p_A^2(t)\, dt, \tag{2}$$

can be calculated, when one knows the A-weighted squared sound pressure, p_A^2, at every moment of the flight. To determine $p_A^2(t)$, air absorption (Sec. 2) and refraction (Sec. 3) are taken into account. Then, equations for L_{AE} are derived for level flight (Sec. 4) and for climbing or descending (Sec. 5).

2. Air absorption

Air absorption is a function of frequency, temperature τ [°C], relative humidity h [%], and atmospheric pressure. In the free field of a point source, the squared sound pressure in the n-th frequency band can be calculated by,

$$p_n^2 = \frac{W_n \rho c}{4\pi r^2} \cdot 10^{A_n(\tau,h)\cdot r/10000}, \tag{3}$$

where W_n denotes the sound power in the n-th frequency band, $A_n(\tau, h)$ is the standardized attenuation coefficient at 1 atm. pressure [8], ρc expresses the characteristic impedance of air, and r is the source-receiver distance expressed in meters. The A-weighted squared sound pressure is

$$p_A^2 = \frac{W_A \rho c}{4\pi r^2} \cdot F_A(r, \tau, h), \tag{4}$$

where W_A expresses the A-weighted sound power and

$$F_A[r] = \sum_n 10^{\delta L_n/10} \cdot 10^{A_n \cdot r/10000}, \tag{5}$$

defines the A-weighted absorption factor. In the above equation,

$$\delta L_n = L_{Wn} - L_{WA} + \Delta L_n, \tag{6}$$

where L_{Wn} is the sound power level in the n-th frequency band, L_{WA} is the overall A-weighted power level, and ΔL_n denotes the A-weighting.

3. Refraction

For a windless atmosphere, with temperature decreasing with altitude z [m], the linear approximation of the sound speed variation can be expressed as,

$$c(z) = c(0) \cdot (1 - \zeta \cdot z), \quad \zeta > 0, \tag{7}$$

where $c(0)$ denotes the sound speed at the ground and ζ [1/m] is the sound speed gradient. The above approximation is quite useful for engineering purposes [3, 6, 9, 10, 14].

The sound speed decrease with the altitude produces a shadow zone because the rays bend up (Fig. 1). If the source is at height, H, and the receiver at ground surface, $z = 0$, then noise is heard when the horizontal distance from the source does not exceed [11, 15],

$$R = \sqrt{2H/\zeta}. \tag{8}$$

In reality, noise penetrates the shadow zone due to diffraction (low frequencies) and turbulent scattering (high frequency) [1, 5]. Nevertheless, the A-weighted squared sound pressure, p_A^2, decreases rapidly within the shadow zone [2, 4]. Thus, noise might induce the feeling of annoyance, when an aircraft is inside a circle of the radius R (Fig. 2). Under the conditions of downward refraction with $\zeta < 0$, we obtain, $R = \infty$.

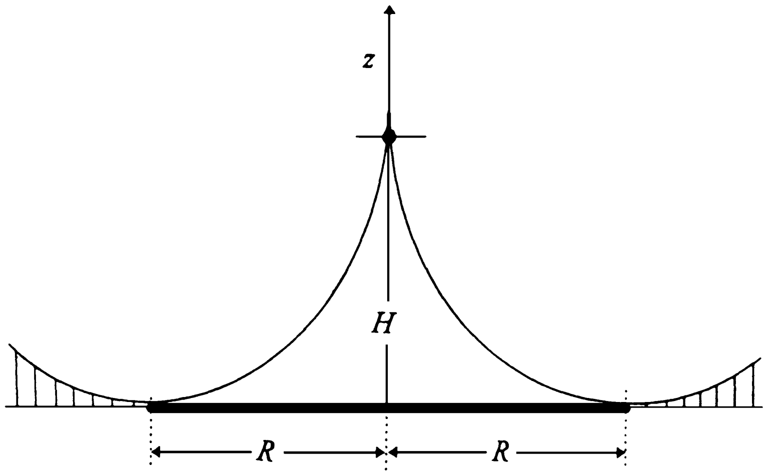

Fig. 1. Shadow zone caused by the upward refracted rays.

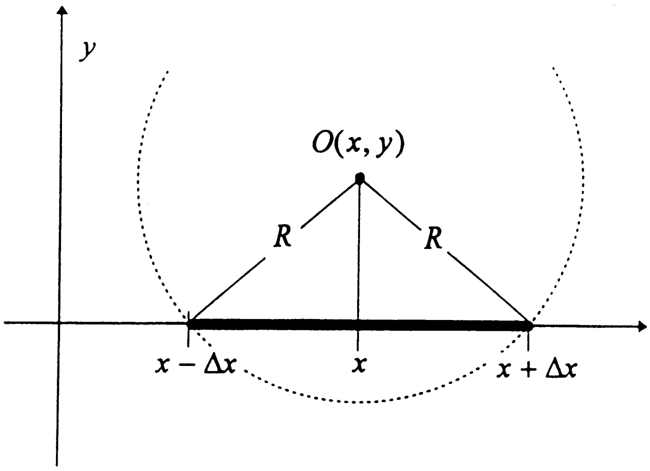

Fig. 2. Noise reaches the receiver, $O(x, y)$, when an aircraft is within a circle of radius R (Eq. (8)).

4. Level ight

Given that the receiver is on the earth's surface, reflection at the ground doubles the value of sound exposure, E. This is equivalent to the virtual doubling of the sound power, $W_A \to 2 \cdot W_A$. For a level flight along the x-axis, at altitude, H, and with constant air speed, V; the sound exposure at the receiver, $O(x, y)$, is given by the following integral (Eqs. (2), (4), (8), Fig. 2):

$$E = \frac{(2W_A)\rho c}{4\pi V} \cdot \int\limits_{x-\Delta x}^{x+\Delta x} \frac{F_A[r(x_s)]}{r^2(x_s)} dx_s, \tag{9}$$

where the instantaneous distance to the aircraft ($x = x_s$, $y = 0$, $z = H$) is

$$r = \sqrt{(x - x_s)^2 + y^2 + H^2}\,, \tag{10}$$

and the width of the noise producing path segment, $2\Delta x$, is defined by (Fig. 2, Eq. (8))

$$\Delta x = \sqrt{2(H/\zeta) - y^2}\,. \tag{11}$$

The largest contribution to the sound exposure, E (Eq. (9)), results from the aircraft opposite to the receiver, $x \approx x_s$, i.e., at the shortest distance to the flight path,

$$r_* = \sqrt{y^2 + H^2}\,. \tag{12}$$

With increasing distance, $r \to \infty$, the noise contribution becomes less significant, thus

$$\int\limits_{x-\Delta x}^{x+\Delta x} \frac{F_A[r(x_s)]}{r^2(x_s)}\, dx_s \approx F_A[r_*] \cdot \int\limits_{x-\Delta x}^{x+\Delta x} \frac{dx_s}{r^2(x_s)}\,, \tag{13}$$

where $F_A[r_*]$ (Eq. (5)) quantifies air absorption. The integration on the right-hand side of the above equation can be done in closed form (Eqs. (10), (11)) and the sound exposure level is (Eqs. (1), (9))

$$L_{AE} = L_{AE}^{(o)} + \Delta L_{AE}^{(a)} + \Delta L_{AE}^{(r)}, \tag{14}$$

where $L_{AE}^{(o)}$ corresponds to geometrical spreading,

$$L_{AE}^{(o)} = L_{WA} - 10 \log \left\{ 2Vt_o \sqrt{y^2 + H^2}/s_o \right\}, \quad s_o = 1 \text{ m}^2, \tag{15}$$

air absorption is described by (Eqs. (5), (12))

$$\Delta L_{AE}^{(a)} = 10 \lg\{F_A[r_*]\}, \tag{16}$$

and the negative value of

$$\Delta L_{AE}^{(r)} = 10 \lg \left\{ \frac{2}{\pi} \tan^{-1} \sqrt{\frac{2(H/\zeta) - y^2}{y^2 + H^2}} \right\}, \tag{17}$$

accounts for refraction. Note, that $\Delta L_{AE}^{(r)} \to -\infty$ for $y \to \sqrt{2H/\zeta}$, i.e., when the receiver approaches the boundary of the shadow zone (Fig. 2). On the other hand, one gets $\Delta L_{AE}^{(r)} = 0$ for $\zeta < 0$, when refraction bends the rays down.

5. Climbing and descending

It is assumed that during climbing or descending along a straight line, the aircraft speed, V, and the sound power spectrum, L_{Wn}, do not vary in time. Taking into account the aircraft height on the runway, H_o, the climbing angle, $0 \le \delta \le \pi/2$, and the ground track, $y = 0$ with $x > 0$ (Fig. 3), the instantaneous location of the aircraft is given by,

$$x = l \cos\delta, \qquad y = 0, \qquad z = H_o + l \sin\delta, \tag{18}$$

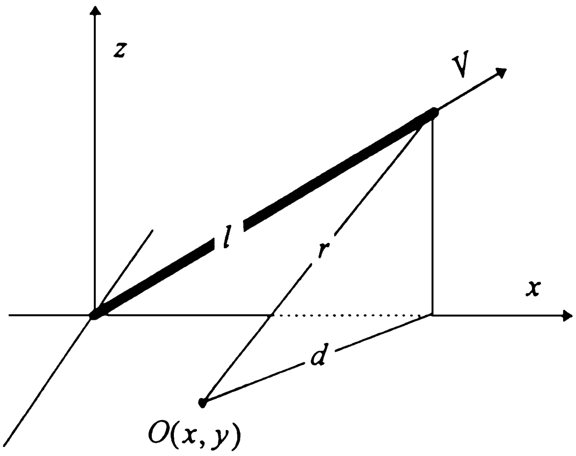

Fig. 3. The instantaneous location of an aircraft is determined by the length of climbing path, l, and the climbing angle, δ.

where $l > 0$ is the length of the climbing path. Therefore, the horizontal distance between the receiver, $O(x, y)$, and the aircraft is

$$d(l) = \sqrt{l^2 \cos^2 \delta - 2xl \cos \delta + x^2 + y^2}\,. \tag{19}$$

and the horizontal distance to the shadow zone is given by (Eq. (7))

$$R(l) = \sqrt{2(H_o + l \sin \delta)/\zeta}\,. \tag{20}$$

Noise reaches the receiver, $O(x, y)$, when it is located within the area bounded by two curves (Fig. 4),

$$y = \pm\sqrt{2x \cdot (\tan \delta/\zeta) + (\tan \delta/\zeta)^2 + 2H_o/\zeta}\,, \tag{21}$$

while the path segment producing noise (l_1, l_2) is determined by the solutions to the equation, $d(l) = R(l)$ (Fig. 5):

$$l_1 = \frac{1}{\cos \delta}\left(x + \frac{\tan \delta}{\zeta}\right) - \frac{1}{\cos \delta}\sqrt{\left(x + \frac{\tan \delta}{\zeta}\right)^2 + \left(\frac{2H_o}{\zeta} - x^2 - y^2\right)},$$

and (22)

$$l_2 = \frac{1}{\cos \delta}\left(x + \frac{\tan \delta}{\zeta}\right) + \frac{1}{\cos \delta}\sqrt{\left(x + \frac{\tan \delta}{\zeta}\right)^2 + \left(\frac{2H_o}{\zeta} - x^2 - y^2\right)}.$$

In the above equations we have (Eq. (8)),

$$2H_o/\zeta > x^2 + y^2, \tag{23}$$

because the aircraft on the runway, $z = H_o$, is not heard at the receiver, $O(x, y)$. Noise proceeds to the receiver only when the aircraft location is determined by the inequalities,

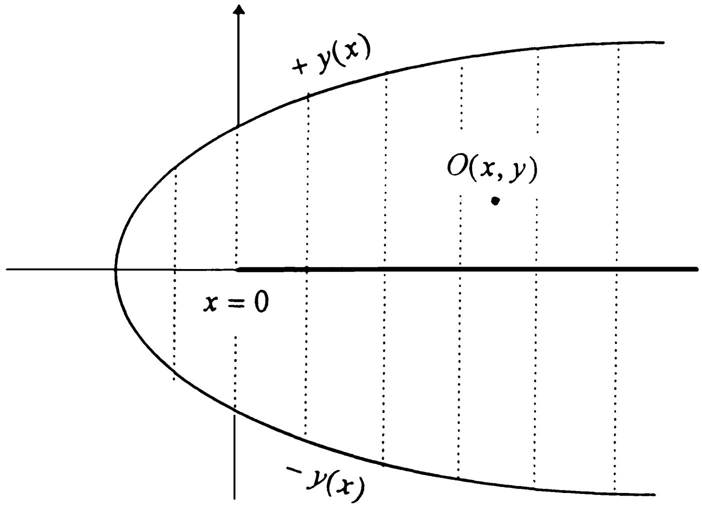

Fig. 4. The cut-off lines, $\pm y(x)$ (Eq. (21)), and the ground track, $y = 0$ with $x > 0$, for climbing or descending.

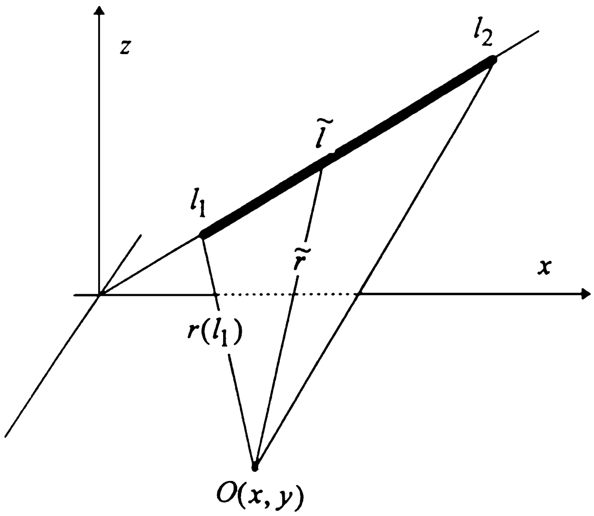

Fig. 5. Noise-producing path segment (l_1, l_2) (Eq. (22)), with the shortest distance, $r(l_1)$ (Eq. (25)).

$l_1 \le l \le l_2$. Therefore, the sound exposure can be calculated from (Eq. (9), Fig. 5)

$$E = \frac{W_A \rho c}{2\pi V} \int_{l_1}^{l_2} \frac{F_A[r(l)]}{r^2(l)}\, dl, \tag{24}$$

where the instantaneous distance between the aircraft (Eq. (18)) and receiver, $O(x, y)$, is

$$r^2(l) = l^2 + 2l \cdot (H_o \sin\delta - x\cos\delta) + x^2 + y^2 + H_o^2. \tag{25}$$

Using reasoning analogous to that described in the derivation of Eq. (13), we write

$$\int_{l_1}^{l_2} \frac{F_A[r(l)]}{r^2(l)}\, dl \approx F_A[r(l_1)] \cdot \int_{l_1}^{l_2} \frac{dl}{r^2(l)}\,, \tag{26}$$

where $r(l_1)$ is the shortest distance to the noise producing path segment (l_1, l_2). Finally, the sound exposure for climbing or descending is (Eqs. (1), (24), (26))

$$L_{AE} = L_{AE}^{(o)} + \Delta L_{AE}^{(r)} + \Delta L_{AE}^{(a)}, \tag{27}$$

where geometrical spreading and refraction are quantified by

$$L_{AE}^{(o)} = L_{WA} + 10 \lg \left\{ \frac{s_o}{2\pi V t_o \tilde{r}} \left[\frac{\pi}{2} + \tan^{-1} \frac{\tilde{l}}{\tilde{r}} \right] \right\}, \tag{28}$$

and

$$\Delta L_{AE}^{(r)} = 10 \lg \left\{ \frac{\tan^{-1}\left[(l_2 - \tilde{l})/\tilde{r}\right] + \tan^{-1}\left[(\tilde{l} - l_1)/\tilde{r}\right]}{\pi/2 + \tan^{-1}\left[\tilde{l}/\tilde{r}\right]} \right\}, \tag{29}$$

respectively. In the above equations, the geometrical distance to the flight path $(0 < l < \infty)$ is

$$\tilde{r} = \sqrt{y^2 + (x \sin\delta + H_o \cos\delta)^2}\,, \tag{30}$$

the corresponding path distance equals (Fig. 5),

$$\tilde{l} = x \cos\delta - H_o \sin\delta, \tag{31}$$

and the quantities l_1 and l_2 can be calculated from Eq. (22). The effect of air absorption is described by (Eqs. (5), (25))

$$\Delta L_{AE}^{(a)} = 10 \lg\{F_A[r(l_1)]\}. \tag{32}$$

6. Conclusions

If we know the sound power spectrum, L_{Wn}, air temperature τ [°C], relative humidity h [%], and the sound speed gradient ζ [1/m], then we can predict the sound exposure level, L_{AE}, for level flight (Eq. (14)) and for climbing or descending (Eq. (27)). The former equation holds for both upward- and downward refraction. The latter holds only when the rays are bending up, $\zeta > 0$. Both equations make no provision for the ground effect, with exception of the virtual doubling of sound power, $W_A \rightarrow 2W_A$, due to the reflection from the ground close to the receiver.

The L_{AE} calculation is based on the approximate counting of air absorption for the shortest distance to the noise-producing path segment and on the linear sound speed profile. These equations provide some insight into the nature of noise propagation, that can be useful for teaching purposes.

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