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ACOUSTIC TWO-DIMENSIONAL SCATTERING FROM CYLINDERS USING THE METHOD OF SOURCE SIMULATION TECHNIQUE

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In this work the Method of Source Simulation Technique was used to calculate the scattering of a plane wave by a circular cylinder with random distribution of the surface impedance. The scattering and radiation from vibrating bodies can be expressed by a source system that is located within the envelope of the scatterer or radiator. The Method of Comparative Sources, as is shown in this work, provides an appropriate prediction of the sources strength and consequently of the sound field. The efficiency of the method was verified through the comparison between numerical results and experimental data. The calculation of the scattering was performed for the variants of the method: the single-layer method and the one-point multipole method.

1. Introduction

The mathematical treatment of radiation and acoustic scattering represents a very old and much studied problem of mathematical physics. Both phenomena were first treated more than a century ago by Lord Rayleigh [1]. At this time, Rayleigh suggested that the sound field radiated from a transverse vibrating rigid body is built up from spherical wave functions. This is the basic idea of the source simulation technique, that is, to replace the vibrating body by a system of radiating sources, which act in an equivalent way on the surrounding medium as the original body. The sources are located inside the radiator, and the problem consists to find the sources amplitude. As long as the sources amplitudes are known, the pressure and the velocity can be mapped at each point in the field. The scattering problem can also be calculated by the method of comparative sources, since the scattering problem can be formulated as an equivalent radiation problem. Depending on the positioning of the sources inside the body and their order, different variants of the method may outcome. If the approach includes the choice of multipole expansions up to high orders at only one location, we have the one-point multipole method. On the other hand, if low order sources like monopoles are located at many points, we have the single-layer method. This paper will be dealing with these two last methods.

The field generated by the sources should approximate the prescribed field as well as possible. The error generated by this approximation should be thus minimized. In this work the least square technique was used in order to minimize the boundary error.

The method of source simulation technique has been vastly used in purely theoretical works, such as: CREMER [2], KRESS and MOHSEN [3], HECKL [4], HECKL [5] and OCHMANN [6]; and in numerical works, such as: WILLIAMS *et. al.* [7], CREMER and HECKL [8] and STEPANHISHEN [9]. However, there is no literature demonstrating the efficiency of the method through a comparison between numerical and experimental results. In this line of thought there are only one work in which the acoustical radiation of a rigid cylinder was studied [10] and only one in which the acoustical radiation of a rigid sphrerical body was studied [11] both numerically and experimentally.

This work aimed at 1) showing the formulation of the radiaton problem and the scattering problem with the method of source simulation technique and 2) presenting its variants, the one-point multipole method and the single layer method. These variants were employed in the calculation of the scattering by a rigid cylinder, an absorbent cylinder, and by a cylinder with variable surface impedance. The cylinder was always considered infinite. The numerical results thus obtained were compared to experimental data collected in an anechoic chamber.

Radiaton and scattering are present in all ondulatory phenomena (elastic waves in rigid bodies, eletromagnetic waves, surface waves on the water, etc.). The present study, however, deals only with "pure" acoustical waves, that is, acoustical waves in gases or liquids. Another important limitation is that all steps of the solution of the problem are considered linear. Consequently, the superposition principle is valid.

2. Description of the radiation and scattering problem

Consider the scatter or radiator with surface S. The interior from S is called S_i and the exterior field S_e. The normal surface $\mathbf{n}$ is directed to the exterior field S_e. In the following, only exterior problems will be treated [12].

In the exterior field S_e, the complex sound pressure p should satisfy the Helmholtz equation

$$\Delta p + k^2 p = 0, \tag{1}$$

where $k = \omega/c$ is the wave number, ω is the circular frequency, c is the speed of sound and Δ is the Laplace operator. All the variables as functions of time should obbey the function e^{+jwt}.

As long as the sound radiation in a free three-dimensional space is considered, the pressure p should also satisfy the Sommerfeld radiation condition [12, 13]

$$\mathrm{Lim}_{r\to\infty} r\left[\frac{\partial p}{\partial r} + jkp\right] = 0 \tag{2}$$

which could be considered as a boundary condition in the infinity. Here $r = |\overset{\rho}{x}|$, $\overset{\rho}{x} = (x_1, x_2, x_3)$ is a position vector and r denotes the distance from the center to each point x in the field. The solutions of Eqs. (1) and (2) are called radiating wave functions [10]. Typical functions that represent this class are called spherical wave functions [10, 12, 14], which are generated when the solution of the wave equation is obtained in spherical coordinates. For the sake of simplicity, radiating wave functions will be called *sources*.

A complete description of the problem requires a description of boundary conditions on the surface of the radiator or scatterer. The Neumann boundary conditions will be considered. In this case the normal velocity v_n and the gradient of the pressure

$$\partial p/\partial n = -j\omega\rho v_n \tag{3}$$

on S are described. In Eq. (3), ρ is the density of the medium surounding S and $\partial/\partial n$ is the derivative in the direction of normal $\mathbf{n}$ into the exterior field S_e.

The problem of acoustic radiation is obtained if the normal velocity considered on the surface of the body is different from zero -> $v_n \neq 0$. Equation (3) represents an inhomogeneous boundary condition. Equations (1) and (2) describe the radiation problem for the radiated pressure p. With respect to the scattering problem, one should consider the inciding wave p_i, which on its propagation encounters the surface S, then generating the scattered wave p_s. The scattering problem for the scattered wave p_s is described by Eqs. (1) and (2), but pressure p should be accordingly substituted by pressure p_s in both equations. Considering again the Neumann boundary value problem, the outcome is that for a totally rigid body, the surface velocity should be equal to zero, that is, $v_n = 0$. This way,

$$\partial p/\partial n = 0. \tag{4}$$

In Equation (4) the pressure p represents the total pressure $p_t = p_i + p_s$. Equation (4) thus represents an homogeneous boundary condition. The scattering problem can be formulated as a radiation problem. One should then consider velocity v_i of the inciding wave p_i on the surface S. If surface S vibrates with negative normal velocity $(-v_i)$, radiated pressure is identical to pressure p_s, originated from the incidence of p_i on S [12]. As a consequence, it is possible to write instead of Eq. (4)

$$\partial p/\partial n = -j\omega\rho(-v_i) \tag{5}$$

for the scattering problem. Equation (5), in a similar way to Eq. (3), represents an inhomogenous boundary condition. Equations (1), (2) and (5) thus describe the scattering problem in an equivalent way to a radiation problem with respect to the scattered wave p_s. The exterior problem defined by these equations is unique and complete [13].

3. Principle of the Method of Source Simulation Technique

The principle of the method here presented is based on a treatment of the radiation problem or the scattering problem through a system of radiating sources, which should be chosen so that they reproduce as well as possible the sound field generated by the body of Fig. 1. In the space previously occupied by body S, now the sources can be found, the source region M in Fig. 1. The sources are taken as point sources, and therefore do not represent an obstacle to the sound field. As a consequence the field generated by each one can be summed without taking into consideration interference effects. As the sources are known, i.e., their amplitudes, the sound field can then be easily calculated through the sum of the fields generated by each source individually. The true problem consists then in finding the sources that can best replace the original body. As a consequence, two important questions arise:

a) Which is the type of source to be used and how should they be placed inside the body?

b) Which optimization method should be employed for the results?

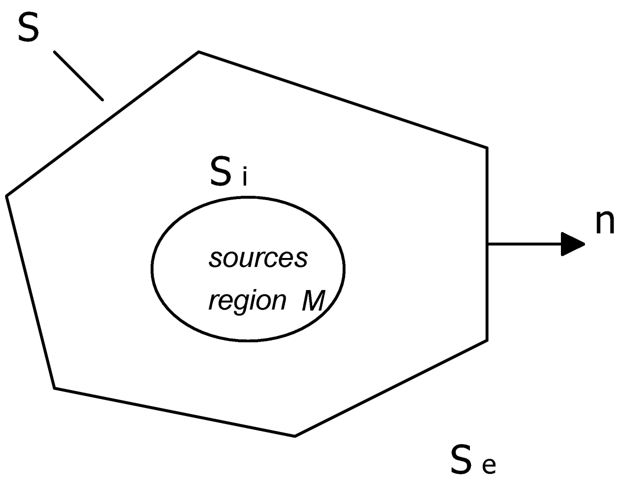

Fig. 1. Geometry of the radiation and scattering problem (OCHMANN [12]).

Mathematically the problem is based on representing the sound field by summing up the contributions of the individuals sources

$$p = \sum_{q=1}^{N_q} \sum_{m=-\infty}^{m=+\infty} A_{q,m} \Phi_{q,m}, \tag{6}$$

where p represents the scattered pressure or the radiated pressure in the field; $A_{q,m}$ is the complex source strength of the q^{th} source at a point x_q in the field; m is the order of each source and $\Phi_{q,m}$ is the sound field generated by the sources. In Eq. (6) $\Phi_{q,m}$ could also be called the source function.

Equation (6) intrinsically has the condition that each field can be represented by a sum of functions of the type $\Phi_{q,m}$. This is naturally the case, only if all functions $\Phi_{q,m}$ satisfy the wave equation and if they form a complete function system. The first condition is certainly satisfied if $\Phi_{q,m}$, represents for example the field generated by a monopole, dipole or quadrupole. The second condition, i.e., if it is possible to represent any acoustic field as a sum in the form of Eq. (6), seems to be as yet unproven with all mathematical rigor (OCHMANN, [10]). As no difficulty has been noted by other authors [2, 4, 10, and others] when miltipole sources were used for the reconstruction of the acoustic field, the same procedure will be used in the present work. In other words, the multipole sources will be used to represent the radiation or scattering problem of the original body.

We have then two distinct situations:

a) one can use a variable order multipole localized in a single point inside the body, that is, in Eq. (6) $N_q = 1$ and M is very large,

or

b) one can use only monopole sources positioned in several points inside the body, which renders N_q very large and $M = 1$ in Eq. (6).

One can also have a combination of both extreme cases presented in a) and b), that is, to use a multipole (for example, monopole + dipole) positioned in several points. Variants of the method will be discussed in the next section.

Together with the choice of the type and the positioning of the sources, the choice of the optimization criterium also imposes a fundamental question for the use of the source simulation technique. Basically the idea is to try to approximate the field generated by the sources, determining its source strength (which are ultimately the solution of the problem), to the real field generated by the original body. The error derived from those approximation should be minimized. Several methods can be used to that end, such as the null field method, the collocation method, the Cremer's method. In this work we have used the least squares minimization.

4. Variants of the Method of Source Simulation Technique

As explained in the last section, depending on the choice with respect to the number of sources and/or order of the source function, different variants of the method can be generated.

If only one point source and one multipole of larger order is used, the so called — one-point multipole method is obtained [10]

$$p = \sum_{m=-\infty}^{m=+\infty} A_m \Phi_m. \tag{7}$$

If, on the other hand, many point sources and a multipole of zero order, the so called monopole, is used, then the single-layer method is generated [6]

$$p = \sum_{q=1}^{N_q} A_{q,0} \Phi_{q,0}. \tag{8}$$

The two cases represented by Eqs. (6) and (7) can be added, resulting in the so called multi-point multipole method. In this case one has a multipole of variable order positioned on many point sources

$$p = \sum_{q=1}^{N_q} \sum_{m=-\infty}^{m=+\infty} A_{q,m} \Phi_{q,m}. \tag{9}$$

In the numerical calculation the infinite series are substituted by a finite number of functions. Then one has for Eq. (9)

$$p = \sum_{q=1}^{N_q} \sum_{m=-M}^{m=+M} A_{q,m} \Phi_{q,m}. \tag{10}$$

A double-layer method could also be constructed, if for example two layers of monopoles or a layer of monopoles plus a layer of dipoles are apllied. It should however be noted, that the one-point multipole method and the single layer method represent the extreme conditions of the Method of Source Simulation. In this work only these variants are used.

5. The Source Function System

The basic idea of the Method of Source Simulation Technique is to replace the scatterer or the radiated body by a source function system, which acts in an acoustically equivalent way on the surrounding medium. If a source alone generated the sound pressure $A_{q,m}\Phi_{q,m}$, then the sum

$$p = \sum_{q=1}^{N_q} \sum_{m=-M}^{m=+M} A_{q,m} \Phi_{q,m} \tag{11}$$

should aproximate the original field as well as possible. Each of the individual source functions $\Phi_{q,m}$, is supposed to meet the radiation condition in Eq. (2) and the wave equation in Eq. (1) in the exterior field domain S_e. When these conditions are satisfied, one can take them from any complete function system. In practical terms, source functions which can be written in conventional coordinate systems can be used: spherical coordinates (for three-dimensional problems) and cylindrical coordinates (for two-dimensional problems). It is also possible to use spheroidal coordinates [14]. One has then for $\Phi_{q,m}$, the following representations:

• spherical coordinates — independent from ϕ

$$\Phi_m = P_m(\cos\nu) h_m^{(2)}(kr); \tag{12}$$

• spherical coordinates - generical

$$\Phi_m = h_{m_1}^{(2)}(kr) P_{m_1}^{m_2}(\cos\nu) e^{+jm_\phi}; \tag{13}$$

• cylindrical coordinates — two-dimensional

$$\Phi_m = H_m^{(2)}(kr) e^{+jm_\phi}. \tag{14}$$

In Eqs. (13) and (14) $= \omega/c$ is the wave number, ω is the circular frequency, c the sound velocity, P is the Legende polynomial, h_m is the spherical Hankel function and H_m is the cylidrical Hankel function of the second kind of order m [16].

The velocity generated by the source function system at the radiator surface is calculated by inserting Eq. (11) in Eq. (3). For reasons fo simplicity, we have taken the one-point multipole method, so that $q = 1$

$$v_n = -\frac{1}{j\omega\rho} \sum_{m=-M}^{m=+M} A_m \frac{\partial \Phi_m}{\partial n} . \tag{15}$$

We can rewrite Eq. (15), since $k = \omega/c$, as

$$v_n = -\frac{1}{\rho_0 c_0} \sum_{m=-M}^{m=+M} A_m Z_m, \tag{16}$$

where $Z_m = (1/jk)\partial\Phi_m/\partial n$ is a function defined in a similar way as in [4]. For example, the function Z_m in the commonly cylindrical coordinates for two-dimensions is given by

$$Z_m = +\frac{1}{j}\left[H_m^{\prime(2)}(kR)e^{+jm\phi}\frac{\partial r}{\partial n} + \frac{m}{k}H_m^{(2)}(kR)e^{+jm\phi}\frac{\partial \phi}{\partial n}\right], \tag{17}$$

where R is the radius from the radiated body.

For the scattering problem, the calculation go in a similar way. The total velocity generated on the scatterer surface is given by

$$v_{t(n)} = v_i + v_s, \tag{18}$$

where $v_{t(n)}$ is the total generated velocity on the scatterer surface in the direction of the normal, v_i is the velocity from a wave normaly incident at the surface of the body, and v_s is the scattered velocity when the body is present in the field

$$v_{t(n)} = -\frac{1}{j\omega\rho}\frac{\partial(p_t)}{\partial n} = -\frac{1}{j\omega\rho}\left[\frac{\partial(p_i)}{\partial n} + \sum_{q=1}^{N_q}\sum_{m=-M}^{m=+M} A_{q,m}\frac{\partial(\Phi_{q,m})}{\partial n}\right], \tag{19}$$

where p_t is the total pressure on the scatterer surface. The total pressure p_t is given by

$$p_t = p_i + p_s, \tag{20}$$

$$p_t = p_i + \sum_{q=1}^{N_q}\sum_{m=-M}^{m=+M} A_{q,m}\Phi_{q,m}, \tag{21}$$

where p_i is the pressure from the inciding wave and p_s is the scattered pressure in the field. For reasons of simplicity, we have taken again the one-point multipole method, so that $q = 1$,

$$p_t = p_i \sum_{m=-M}^{m=+M} A_m \Phi_m \tag{22}$$

and

$$v_{t(n)} = -\frac{1}{j\omega\rho}\frac{\partial(p_t)}{\partial n} = -\frac{1}{j\omega\rho}\left[\frac{\partial(p_i)}{\partial n} + \sum_{m=-M}^{m=+M} A_m Z_m\right]. \tag{23}$$

The function Z_m is the same as in Eq. (17) for the two-dimensional problem with cylindrical coordinates. If the inciding wave is a plane wave in cylindrical coordinates

$$p_i = p_0 e^{-jkR\cos(\phi)} \tag{24}$$

so, $v_{t(n)}$ is rewritten as

$$v_{t(n)} = \frac{p_0}{\rho_0 c_0}\cos(\phi)e^{-jkR\cos(\phi)} - \frac{1}{\rho_0 c_0}\sum_{m=-M}^{m=+M} A_m Z_m. \tag{25}$$

The requirement that the velocity distribution given by Eqs. (16) and (25) generated by the sources at the surface should aproximate the prescribed normal velocity as well as possible leads to a linear system of equations through which the complex source strength will be determinated.

6. Optimization criteria

Several methods can be used in order to minimize the error in the surface velocity aproximation. In [6] the mathematical description of the use of the Cremer equations can be found, as well as the null-field equations, the collocation method and the least squares minimization technique. In the literature this technique is the most commonly used and will be used also in this work. The technique consists in minimizing the surface integral error

$$\int_S |v_{t(n)} - v_b|^2 dS = \text{Min} \tag{26}$$

which sums the erros generated in the approximation of the surface velocity. In Eq. (26) S is the surface of the scatterer, dS a surface element and $v_{t(n)}$ is the velocity generated by the source simulation technique. For the special case of scattering from a rigid body, the surface velocity is zero, so that $v_b = 0$

$$\int_S |v_{t(n)}|^2 dS = \text{Min}. \tag{27}$$

The velocity $v_{t(n)}$ has the same form as in Eq. (23) for the one-point multipole mehtod. The system of equations for the determination of the sources strength A_m is obtained through the calculation of the partial derivative of the integral in Eq. (27) with respect to A_m, and making the result equal to zero

$$\frac{\partial}{\partial A_m}\left(\int_S |v_{t(n)}|^2 dS\right) = 0. \tag{28}$$

The solution of the linear system of Eq. (28) gives us the sources strength A_m, which when substituted in Eqs. (22) and (23) allow the calculation of the sound pressure and the sound velocity for each point in the acoustical field. Thus, the problem is perfectly solved.

For the somewhat general case, that the body is not rigid, but has a constant relation on the whole surface between the total sound pressure and the total sound velocity in the direction of the normal, this leads Eq. (26) to

$$\int\limits_S \left| v_{t(n)} - \frac{p_t}{Z} \right|^2 dS = \text{Min}, \tag{29}$$

where Z is the surface impedance of the scatterer.

The condition imposed to impedance is that it should not have lateral couplings, that is, it should be locally reacting. This means that one part of the surface is not aware of the motion of another part, and the reaction of one part of the surface is proportional to the local pressure at that point. This condition indicates the non-inclusion of elastic surfaces (for example, surfaces where flexion waves are possible). This extremely rigid limitation should be verified in each case. It certainly is not satisfied by elastic bodies, as for example a thin-walled cylinder immersed in water. For porous materials (for example foam) one can in priciple assume that for air borne sound there is no lateral coupling, that is, the materials are locally reacting.

In the same way, we can calculate the radiation problem with the least squares technique. This leads to the surface integral

$$\int\limits_S |v_b - v_n|^2 \, dS = \text{Min} \tag{30}$$

and again the surface error should be minimized. In Eq. (30) v_b is the velocity of the vibrating body and v_n is the velocity generated from the sources. For the one-point multipole method and for the two-dimensional case in cylindrical coordinates v_n is the same as in Eqs. (16) and (17). As in the case of scattering, if the partial derivative in Eq. (30) is calculated with respect to source strength A_m and making the result equal to zero, one has a system of linear equations through which the complex sources strengths are determined. Substituting them in Eqs. (11) and (16) we have the pressure and the velocity at each point in the acoustic field. This way the acoustic radiation problem is perfectly solved.

7. Calculation of the scattering with the source simulation technique

The next issue to be addressed is the problem of calculating sound scattering for an infinite circular cylinder, in which the random distribution of the surface impedance is considered. The calculation will be performed for the one-point multipole method and for the single layer method.

7.1. One-Point Multipole Method

In this case the approach includes the choice of multipole expansions up to high orders at only one location. Using the symmetry of the circular cylinder, the location point coincides with the center of the cylinder. The condition of an infinite cylinder means that the problem is treated independently of the axial direction, that is, $\partial(...)/\partial z = 0$. Another important point is that we only consider plane harmonic waves as the inciding waves.

The total pressure p_t can be written as a sum of the inciding plane wave p_i and the scatterer wave p_s

$$p_t = p_i + s_s. \tag{31}$$

The inciding plane wave traveling in a direction perpendicular to the cylinder's axis is given by $p_i = p_0 e^{-jkx}$, where p_0 is the amplitude. The scatterer wave p_s is given by Eq. (7) and Eq. (14), so that

$$p_t = p_0 e^{-jkr\cos(\phi)} \sum_{m=-M}^{m=+M} A_m H_m^{(2)}(kr) e^{jm\phi}. \tag{32}$$

Since we used cylindrical coordinates, $x = r\cos(\phi)$, and r is the distance between the center of the cylinder and any point in the surrouding medium. In this case the medium is air.

The expression for the velocity is obtained through Eq. (3), since the normal direction coincides with the radial direction and on the surface $r = R$

$$v_{t(n)} = \frac{p_0}{Z_0}\cos(\phi)e^{-jkR\cos(\phi)} - \frac{1}{Z_0}\sum_{m=-M}^{m=+M} A_m H_m^{'(2)}(kR)e^{+jm\phi}, \tag{33}$$

where $Z_0 = \rho_0 c_0$ is the specific acoustical impedance in air and $H_m^{'(2)}(kR) = \partial\left(H_m^{(2)}(kR)\right)$ $\partial(kr)\Big|_{r=R}$. All time-varying quantities should obey the time dependence $e^{+j\omega t}$. As the exponencial factor is shared by all field quantities, it can be omitted.

As explained in the last section, it is here assumed that the surface impedance is locally reacting. Therefore, the boundary conditions for each surface element and for each angle ϕ on the surface, should obey the condition

$$v_{t(n)} = \frac{p_t}{Z}, \tag{34}$$

where Z is the surface impedance.

In this work we consider that the impedance is randomly distributed on the surface. Hence, the impedance could be infinite, that is, for a rigid surface in the interval $\phi_0 \le \phi \le \phi_1$, or could be finite, assume the value Z in the interval $\phi_1 \le \phi \le \phi_2$. The impedance Z was measured with the standing wave apparatus for a 5 cm-thick foam, and will be used in the numerical calculation as entry data in the search for the solution of the problem.

Considering the optimization criterion given by Eq. (29), we have

$$R\left[\int_{\phi_0}^{\phi_1} |v_{t(n)}|^2 d\phi + \int_{\phi_1}^{\phi_2} \left|v_{t(n)} - \frac{p_t}{Z}\right|^2 d\phi\right] = \text{Min.} \tag{35}$$

Diferentiating Eq. (35) with respect to the unknown source strength and making the result equal to zero, we obtain the following system of linear equations

$$\begin{aligned}
&+\frac{p_o R}{jZ_o^2} H_m^{(1)'}(kR) \int_{\varphi_0}^{\varphi_1} \cos(\varphi) e^{-(kR\cos(\varphi)+m\varphi)} d\varphi \\
&\quad + R\left(\frac{p_o}{jZ_o^2} H_m^{(1)'}(kR) - \frac{p_o}{Z_o Z^*} H_m^{(1)}(kR)\right) \int_{\varphi_1}^{\varphi_2} \cos(\varphi) e^{-j(kR\cos(\varphi)+m\varphi)} d\varphi \\
&\quad + R\left(\frac{p_o}{ZZ^*} H_m^{(1)}(kR) - \frac{p_o}{jZ_o Z} H_m^{(1)'}(kR)\right) \int_{\varphi_1}^{\varphi_2} e^{-j(kR\cos(\varphi)+m\varphi)} d\varphi \\
&\quad + \frac{R}{Z_o^2} H_m^{(1)'}(kR) \sum_{n=-N}^{+N} A_n H_n^{(2)'}(kR) \left(\int_{\varphi_0}^{\varphi_1} e^{-j(m-n)\varphi} d\varphi + \int_{\varphi_1}^{\varphi_2} e^{-j(m-n)\varphi} d\varphi\right) \\
&\quad + R\left(\frac{1}{ZZ^*} H_m^{(1)}(kR) - \frac{1}{jZ_o Z^*} H_m^{(1)'}(kR)\right) \sum_{n=-N}^{+N} A_n H_n^{(2)}(kR) \int_{\varphi_1}^{\varphi_2} e^{-j(m-n)\varphi} d\varphi \\
&\quad + \frac{R}{jZ_o Z^*} H_m^{(1)}(kR) \sum_{n=-N}^{+N} A_n H_n^{(2)'}(kR) \int_{\varphi_1}^{\varphi_2} e^{-j(m-n)\varphi} d\varphi = 0.
\end{aligned} \tag{36}$$

The Eqs. (36) give us the the complex sources strength, that is, the solution of the posed problem.

Figure 2 shows the problem described above:

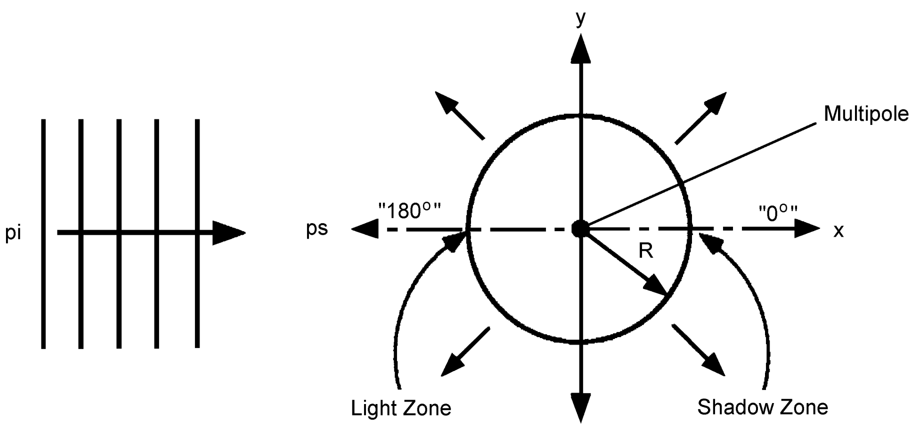

Fig. 2. One-point multipole method for the rigid and absorbing circular cylinder.

7.2. Single Layer Method

In this case the approach is to take several monopole sources and to position them on an auxiliary surface. The auxiliary surface is positioned inside the body. Note that the auxiliary surface should not coincide with the surface of the original body. If the auxiliary surface coincides with the surface of the body, we have another problem that cannot be solved by the source simulation technique, but by the boundary element method (BEM). The auxiliary surface has the same form as the surface of the body being studied, that is, the circular cylinder.

For the total pressure we have

$$p_t = p_0 e^{-jkR\cos(\phi)} + \sum_{q=1}^{N_q} A_q H_{0,q}^{(2)}(kr), \tag{37}$$

where r is the distance between a point with polar coordinates (R, ϕ) on the cylinder surface and a source point q with the polar coordinates $(r_{(q)}, \phi_{(q)})$. R is the radius from the circular cylinder.

The cossinos law (see Fig. 3) gives us $r = \sqrt{R^2 + r_{(q)}^2 - 2Rr_{(q)}\cos(\phi_{(q)} - \phi)}$ and the normal component of the velocity on the surface at a point (R, ϕ) is

$$v_{t(n)} = \frac{p_0}{Z_0}\cos(\phi)e^{-jkR\cos(\phi)} - \frac{1}{Z_0}\sum_{q=1}^{N_q} A_q H_{0,q}^{'(2)}(kr)r'(R) \tag{38}$$

where $r'(R) = \partial(r)/\partial R$ and $H_0^{'(2)}(kr)$ is the derivative of the Hankel function of the second kind of zero order. Inserting Eqs. (37) and (38) into Eq. (35), the partial derivatives with respect to the unknown source strength are equated to zero, and then we obtain a system of linear equations symilar as Eqs. (36). This system of equations give us the the complex sources strength, that is, the solution of the problem.

Figure 3 shows us schematically the posed problem.

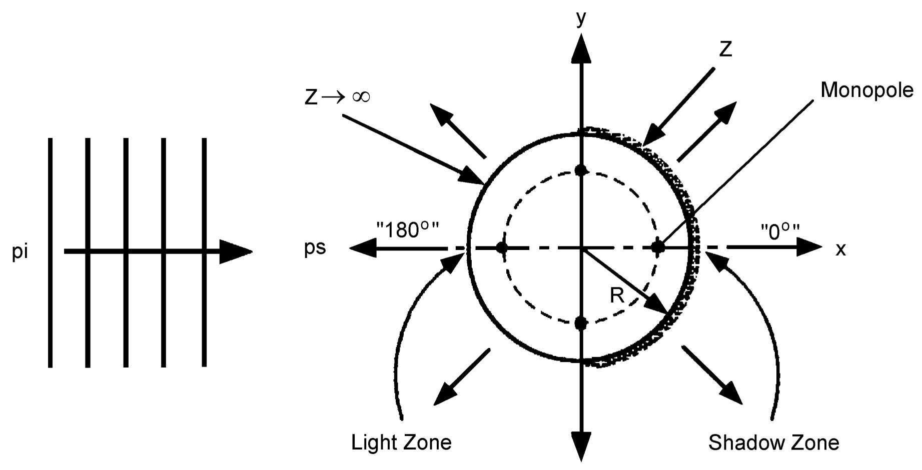

Fig. 3. Single Layer Method for the rigid and absorbing circular cylinder.

8. Experimental methodology

The assays were performed in an anechoic chamber with a rigid cylinder 3 m long and with a radius of 15 cm (see Fig. 4). Posteriorly the cylinder surface was covered with a porous absorbing material 5 cm thick. The impedances of the aborbing material were measured for different frequencies (200 – 8000 Hz) by a standing wave apparatus. The absorbing cylinder was covered in a half of its perimeter with a metal plate, thus resulting in a half-rigid/half-absorbing cylinder. This characterization depends on which face of the cylinder is primarily in contact with the incident wave. The sound was generated by a noise generator in 1/3 Octave and after being amplified it was irradiated through

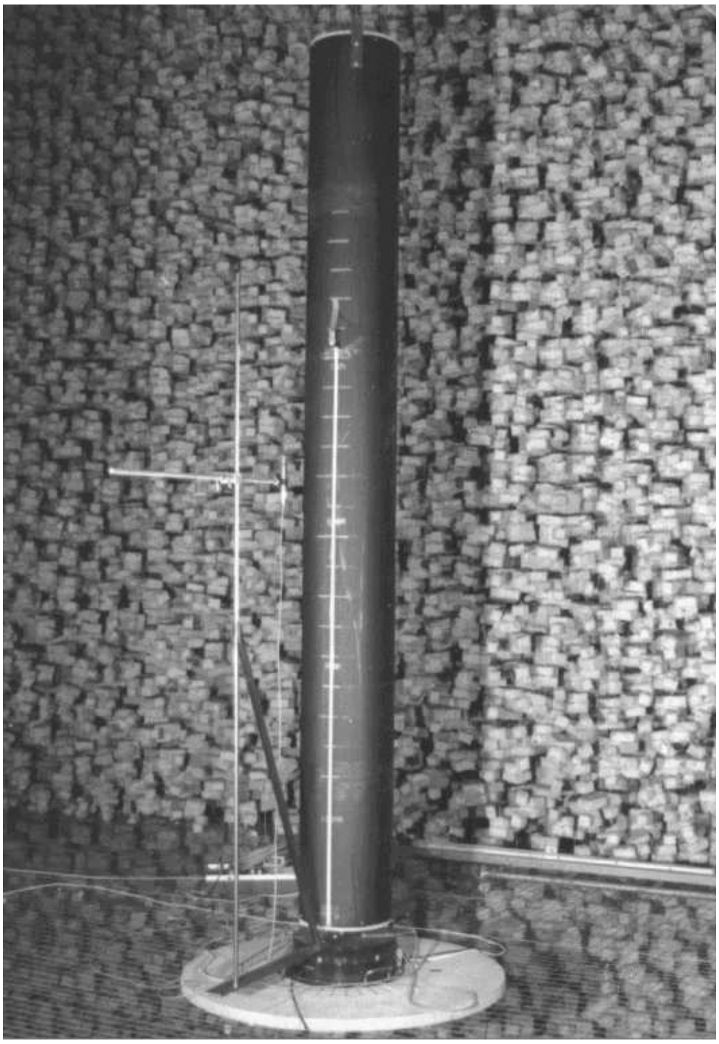

Fig. 4. The rigid cylinder and the turning table.

a loudspeaker. Sound was measured by a microphone mounted on a turning table which can face either the shadow zone or the light zone. The sound pressure levels were measured at each 10° of approach of the turning table, first without the cylinder in the field and next with the cylinder in the field. The difference between these measurements gives us sound attenuation due to the presence of the cylinder, which is dependent on the frequency, the surface impedance, and the distance from the microphone (measuring point) to the center of the cylinder.

9. Results and discussion

In order to verify the efficiency of the Method of Source Simulation Technique, the results it had generated for the classical case of a rigid infinite circular cylinder are compared to the analytical solution. Figures 5 and 6 show the directivity pattern of the scattered wave from a rigid circular cylinder of radius R calculated with the source simulation technique (left side) and the analytical solution (right side) obtained by MORSE [17]

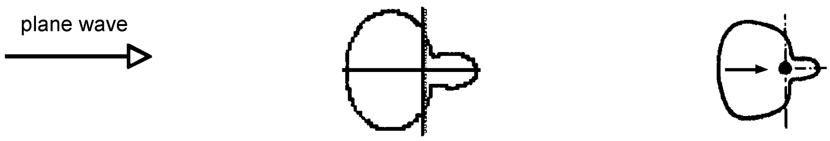

Fig. 5. Comparison between the calculation of sound scattering with the source simulation technique and the analytical solution for $\lambda = 2\pi R$.

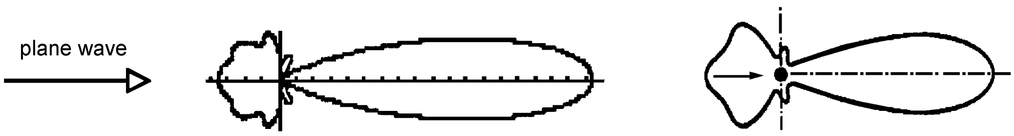

Fig. 6. Comparison between the calculation of sound scattering with the source simulation technique and the analytical solution for $\lambda = (2/5)\pi R$.

As observed, the results obtained with the source simulation technique are in very good agreement with the analytical solution. Another possibility for the verification of the efficacy of the Method of Source Simulation Technique for the calculation of acoustical fields is to compare the numerical results with the experimental data. In this sense, sound attenuation (shadow zone) produced by the presence of the cylinder in the field was both measured and calculated. It is impossible to present all the results obtained due to the great number of factors involved, such as frequency, distance from the point of measurement to the center of the cylinder, position angle, and surface impedance. Only some results obtained for the 1) rigid cylinder, 2) absorbing cylinder, 3) absorbing and rigid, and 4) mixed case will be presented here. The coming results were calculated using the single-layer method, which were not essentially different from the ones obtained using the one-point multipole method.

Figures 7 to 11 show that although there is a very good agreement between the numerical results obtained with the Method of Comparative Sources and the experimental data, some discrepancies can be noted. Possible causes are discussed in the following text. One possible reason for the differences in the values obtained for the sound attenuation in Figs. 7 to 11 is that the calculation was undertaken for a bi-dimensional problem, while the measurements were performed in a three-dimensional model.

The numerical calculation is always valid for a single frequency. However, in the measurements the sound generated by a band of 1/3 octave was used. This fact can lead to the appearance of problems in the numerical calculation, as the acoustical field generated by the scatterer changes too rapidly with the frequency and the angle of the measuring point. This happens especially in high frequencies, as can be observed in Fig. 12.

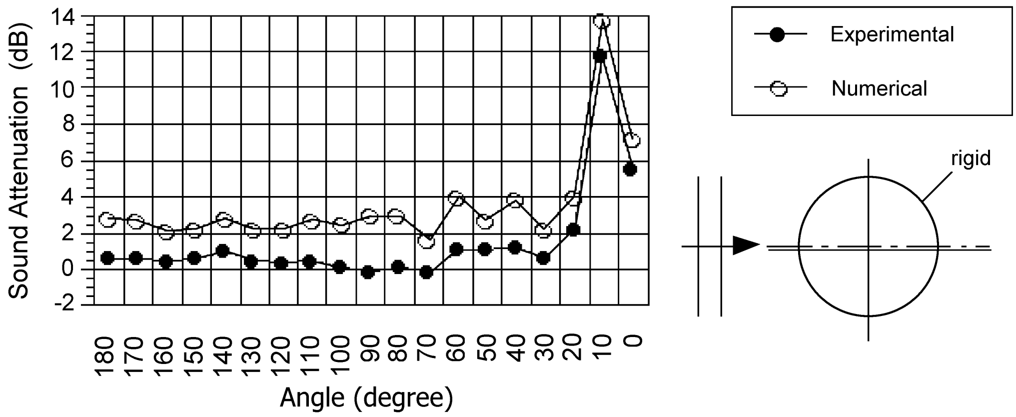

Fig. 7. Sound attenuation and scattering from a rigid cylinder: $kR = 13.7$; distance from a point in the field to the center of the cylinder: 47 cm, number of monopoles: $N = 55$.

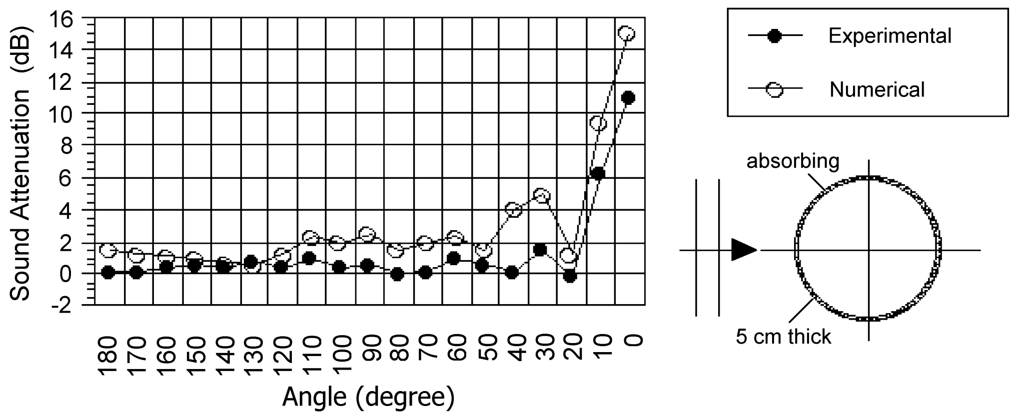

Fig. 8. Sound attenuation and scattering from an absorbing cylinder: $kR = 14.65$; distance from a point in the field to the center of the cylinder: 130 cm, number of monopoles: $N = 59$, surface impedance: $Z = 250.3 - \mathrm{j}\ 200$.

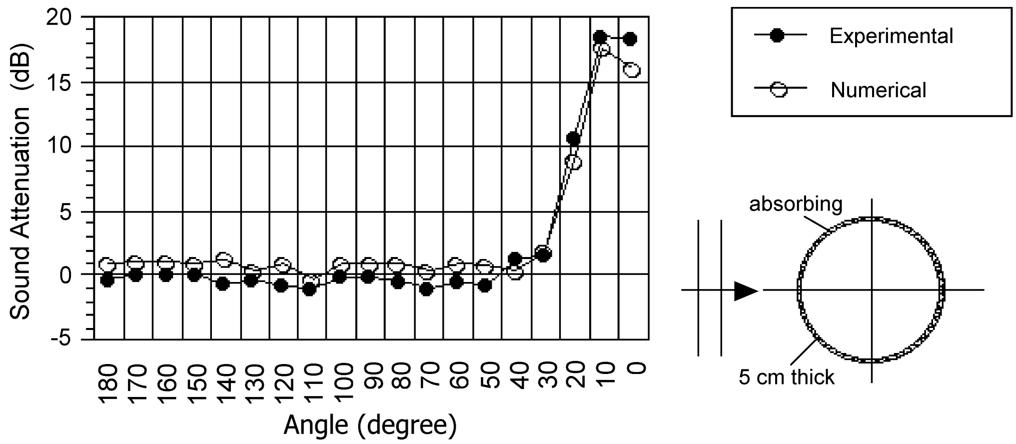

Fig. 9. Sound attenuation and scattering from an absorbing cylinder: $kR = 18.3$; distance from a point in the field to the center of the cylinder: 132 cm, number of monopoles: $N = 73$, surface impedance: $Z = 267.3 + \mathrm{j}\ 92.8$.

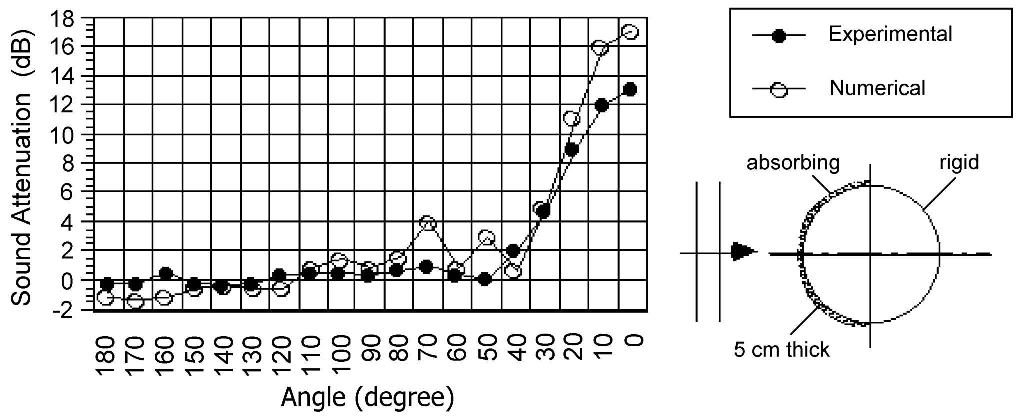

Fig. 10. Sound attenuation and scattering from a half-absorbing and half-rigid cylinder: $kR = 14.65$; distance from a point in the field to the center of the cylinder: 47 cm, number of monopoles: $N = 59$, surface impedance: $Z = 250.3 - \mathrm{j}\ 200$.

This numerical difficulty can be avoided if we take the mean of the results for several frequencies inside each band of frequencies. In other words, the central frequency of the band of interest is considered and the frequencies below and above the central frequency are harmonically calculated, as is shown next:

$$\ldots\ldots \frac{f_c}{2^{1/4}},\ \frac{f_c}{2^{1/8}},\ \frac{f_c}{2^{1/16}},\ f_c,\ f_c 2^{1/16},\ f_c 2^{1/8},\ f_c 2^{1/4} \ldots\ldots,$$

where f_c is the central frequency of the band. Several numerical tests were performed and we could reach the conclusion that a good approximation of the theoretical and experimental results was obtained when the mean was calculated out of 5 frequencies

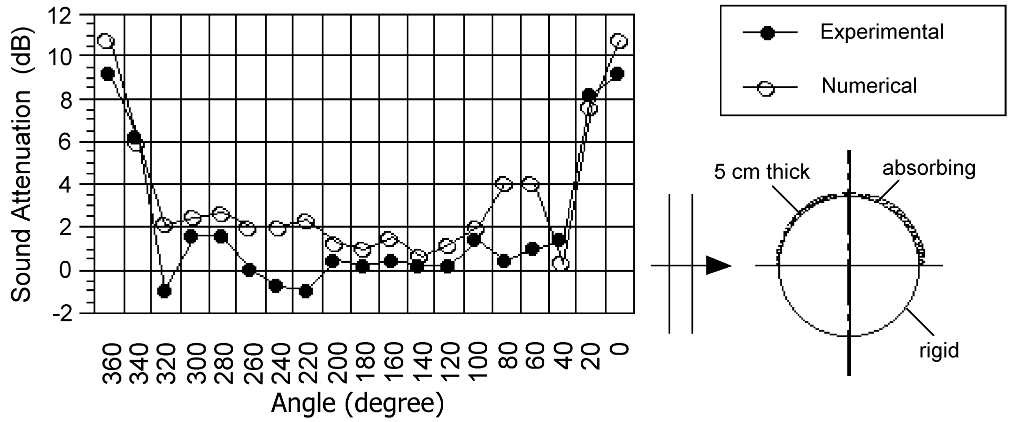

Fig. 11. Sound attenuation and scattering for a mixed case: $kR = 9.16$; distance from a point in the field to the center of the cylinder: 47 cm, number of monopoles: $N = 37$, surface impedance: $Z = 564 + \text{j}\ 542.3$.

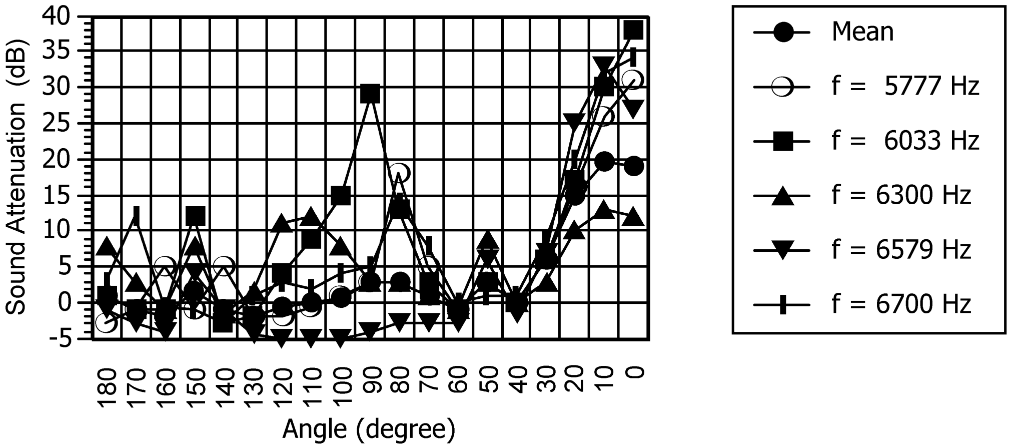

Fig. 12. The mean of five frequencies.

(see Fig. 12). However, one should not discard the possibility that, in some cases, the mean should be calculated from a larger number of frequencies, especially in high frequencies.

As in the shadow zone ($\phi = 0$) sound attenuation reached values above 20 dB, it was questioned whether possible reflections from the walls of the chamber could be interfering with the experimental data. The cylinder was placed in other positions, at longer distances from the walls, but the new measurements have not shown any significant difference from the results obtained previously.

Equation (35) has its fundament in the hypothesis that the surface impedance is locally reacting. This hypothesis was also considered for the 5 cm thick foam with which the cylinder was covered, thus generating an absorbing cylinder. In order to confirm that hypothesis, fissures were made in the surface of the foam, but the sound pressure levels measured afterwards showed no significant modification when compared to the values obtained previously. One may thus conclude that the material used behaves as locally reacting.

An ideal agreement between calculation and measurement is obtained if the indexes M in Eq. (32) or the index N_q in Eq. (37) grow infinitely. That is, however, in practical terms impossible because of the immense computer time it would consume. Numerical simulations have shown that a good agreement between calculation and measurement is found for $M_{\max} = 4\pi E/\lambda$, and for $N_{q(\max)} = 6\pi R/\lambda$, where R is the cylinder's radius and λ is the wavelength. Exceptions to this rule are some regions in the shadow zone between $-10° \leq \phi \leq +10°$.

Another important reason for the diferences between the numerical results and the experimental data resides in the fundamental principle of the method, that is, of not reconstructing exactly at each surface element the given boundary conditions, but to minimize the error through an integration, like that in Eq. (26), over the whole perimeter of the body. Equation (26) corresponds to the optimization of the error in the surface velocity approximation in the least mean square procedure. Equation (26), and thus the Method of Comparative Sources, allow the control of the error as they satisfy the boundary conditions for every computation. This is a very important characteristic of this method, mainly when an analytical solution to the problem is not available. HECKL [5, 18] and OCHMANN [6] indicate this property of the Method of Source Simulation as the great advantage of this method with respect to the other method often employed in the calculation of the radiation and scattering problems, the Boundary Element Method - BEM. The BEM doesn't display this property. For practical cases, however, it would be important to assure a controlled accuracy not only of the surface velocity as in Eq. (26), but also in the determination of the sound power. The use of an infinite number of sources would certainly allow the precise reconstruction not only of the surface velocity, but also of the sound power. Nevertheless, in the use of a finite number of sources (a maximum value for M and N_q), it is possible that this number would be sufficient for the reconstruction of the surface velocity, but not for the determination of the sound power. This should be the case, especially, if there is a too rapid variation with respect to the position and the distribution of the surface velocity. Keeping in mind this limitation, one has in hand a very efficient method for the reconstruction of the acoustic field, as has been shown in the works of CREMER [11] and OCHMANN [10] for the radiation problem, and here for the scattering problem.

10. Conclusions

This work has presented the study of the scattering in a rigid and infinite cylinder with variable surface impedance, both numerically and experimentally. The theoretical method used was the Method of Source Simulation Technique. This method has been frequently used in the last decade for the solution of purely theoretical and/or numerical radiation and scattering problems. Very few works are found in the literature which allow a comparison between the numerical results obtained with the source simulation technique and experimental data. The contribution of the present work is then to present a comparison between numerical results and experimental data for the theoretical basis and the practical use of the source simulation technique. An important pratical property of the source simulation technique is the controlled accuracy: the error is directly determined as a discrepancy in the boundary conditions on the surface of the body in each specific case. This property is very important especially if analytical solutions are not available.

The principle of the method is very simple. However, research should be done to investigate the influence of the type of source, type of surface over which the sources are positioned (single-layer method), the existence or not of resonance frequencies as is the case with BEM, and the applicability of the method for more complex surfaces. The great disavantage in the use of the Method of Source Simulation Technique is in the fact that rules for the positioning of the source surface are not known a *priori*. The positioning of the source surface and in consequence of the sources themselves is based on the experience of the programmer.

References

[1] Lord Rayleigh, *The Theory of sound*, The Mamillan Company London 1877 or Dover Publication, New Yoyrk 1945.

[2] L. Cremer, *Die Synthese de Schallfeldes eines beliebigen festen Körper in Luft mit beliebieger Schnelleverteilung aus Kugel schallfelder*, Acustica, **55**, 44–46 (1984).

[3] R. Kress and A. Mohsen, *On the simulation source technique for exterior problems in acoustics*, Math. Meth. in the Appl. Sci., **8**, 585–597 (1986).

[4] M. Heckl, *Bemerkung zur Berechnung der Schallabstrahlung nach der Methode der Kugelfeldsynthese*, Acustica, **68**, 251–257 (1989).

[5] M. Heckl, *Modern methods in analytical acoustics*, Springer-Verlag, London 1992, capitel 10.

[6] M. Ochmann, *The source simulation technique for acoustic radiation problems*, Acustica, **82**, 512–527 (1995).

[7] W. Wiliams, N. Parke, D. Moran and C. Scherman, *Acoustic radiation from a finite cylinder*, J. Acoust. Soc. of Am., **36**, 2310–2322 (1964).

[8] L. Cremer and M. Heckl, *Köperschall*, Springer-Verlag, Berlin 1996.

[9] P.R. Stepahishen, *A generalized internal source density method for the forward and backward projection of harmonic pressure fields from complex bodies*, J. Acous. Soc. Am., **101**, 3270–3272 (1997).

[10] M. Ochmann, *Die Multipolstrahlersyntese – ein effektives Verfahren zur Berechnung der Schallabstrahlang von schwingenden Strukturen beliebiger Oberflächenimpedanz*, Acustica, **72**, 177–190 (1990).

[11] L. Cremer and M. Wang, *Die Synthese eines von einem beliebigen Körper in Luft erzeugten Feldes aus Kugelschallfeldern und deren realisierung in Durchrechnung und Experiment*, Acustica, **65**, 53–74 (1988)

[12] M. Ochmann, *The full-field equations for acoustic radiation scattering*, J. Acous. Soc. of Am., **105**, 2574–2584 (1999).

[13] A.D. Pierce, *Acoustics – An Introduction to its physical principles and applications*, published by the Acoustical Society of America, second printing (1991).

[14] P.M. Morse and H. Fesbach, *Morse, Methods of theoretical physics*, MacGraw-Hill, New York 1953.

[15] R.H. Hackman, *The transition matrix for acoustic and elastic wave scattering in prolate spheroidal coordinates*, J. Acoust. Soc. Amer., **75**, 35–45 (1984).

[16] M. Abramowitz and I. Stegun, *Handbook of mathematical functions*, Dover Publ., 1972.

[17] P.M. Morse, *Vibration and Sound*, Phublished by the Acoustical Society of America, fourth printing (1991).

[18] M. Heckl, *Methoden zur Feldberechnung*, 20. Deutsche Jahrestagung für Akustik, Dresden 1994.