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THE TOTAL SOUND POWER OF SOME FORCED VIBRATIONS OF A CLAMPED ANNULAR PLATE IN FLUID

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The energetic aspect of sound radiation by a clamped annular plate has been considered. The total sound power, active and reactive, was computed using the impedance approach, which makes possible to correctly design some acoustic systems. The plate was excited by the external pressure of an axisymmetric and time-harmonic distribution on the plate's surface. The Kelvin-Voigt theory of a visco-elastic plate was employed. The results of a modal analysis of an annular plate's in-vacuo motion were used.

1. Introduction

An accurate examination of the energetic aspect of some forced vibrations and the sound radiation produced by any surface sound sources is necessary for designing any acoustic systems correctly. The impedance and far field approaches are employed to find the complex total sound power of the source. So far, some free vibrations of rectangular [1, 2] and circular [2–4] or annular [2, 5] plates have been examined in detail. LAURA and ROMANELLI performed a vibration analysis of an annular anisotropic plate supported in different ways in [6]. AMABILI, FROSALI and KWAK used Rayleigh-Ritz approach to find the exact shapes of the higher modes of some free vibrations of annular plates coupled with fluids in [7, 8]. Additionally, the active potential and kinetic energies were presented in the form of integrals.

It is necessary to determine the modal sound power, active and reactive, if we want to find the total sound power. So far, several investigations have been carried out in this field e.g. LEVINE and LEPPINGTON have found the "effective damping" factor for a clamped circular plate [9] which is an equivalent magnitude to the standardized active sound power. RDZANEK computed the modal reactive and mutual sound power of such a plate [10] and later employed the impedance approach to determine the complex total acoustic impedance of a forced circular plate in fluid [11]. The sound radiation from forced harmonic vibrations of a clamped circular plate was also analysed by ALPER and MAGRAB [12]. Some analytical formulations for sound radiation by an annular disk in the form of multiple expansion series have been given by LEE and SINGH in [13].

Some disk rotation effects were taken into account. The complex modal sound power of a clamped annular plate was derived by RDZANEK Jr. and ENGEL [14] in the form of some asymptotic formulae valid for high frequencies. RDZANEK Jr. also considers the mutual sound power of the plate [15]. Since evaluating some integrals describing sound radiation by a source leads to some hyper-singularities (cf. [16]) it is necessary to find some procedures for their regularization, which was undertaken by many authors, e.g. SCHENCK proposed a combined Helmholtz integral equation method to circumvent some difficulties appearing while solving the Dirichlet interior problem (cf. [17]). His method was further developed by SEYBERT and RENGARAJAN in [18] and then considerably improved by CHIEN, RAJIYAH and ATLURI in [19]. The authors proposed an effective method to regularize the hyper-singular integral found in the BURTON and MILLER formulation (given in [16]). Certain identities were employed for the hyper-singular integrals arising in an associated integral equation for the Laplace equation in the interior domain. GINSBERG and CHEN employ the method of variational principles to find the sound pressure and transverse deflection of an excited disk in two different baffles – infinite and finite [20]. This method is used by many author to solve some similar problems, e.g. in [20–24].

So far, the complex total sound power of a forced and clamped annular plate in fluid has not been derived analytically. Particularly, its reactive part has not yet been discussed in the literature. This paper tries to fill the gap. The results of a modal analysis of the plate's in-vacuo modes [14, 15], such as the asymptotic and integral formulae for the standardized active and reactive sound power make the basis of the analysis. The asymptotic formulae are valid for high frequencies. For the other frequencies the integral formulae were used. The influence of some fundamental plate's geometric and material factors on the total sound power has been examined in detail, the factors taken into account being the internal friction η_T, the quotient of external and internal radii $s = r_2/r_1$, thickness h and density ϱ. The results obtained are illustrated by some frequency characteristic graphs of the modulus and the phase's cosine of the magnitudes considered.

2. Internal friction and damping from the air column

The plate examined, of internal radius r_1 and external radius $r_2 > r_1$, is forced by the axisymmetric and time-harmonic external pressure whose distribution on the plate's surface is $\operatorname{Re}\left[f(r)\exp(-i\omega t)\right]$, where $r_1 \le r \le r_2$ and $f(r) \in \mathbb{R}$. It is assumed that the vibration amplitude is small enough to describe the vibrations by a linear differential equation. The Kelvin-Voigt theory of a visco-elastic homogeneous and isotropic plate has been used. The plate fulfils the following relation between the stress σ and the deformation ε: $\sigma = E\varepsilon\,(1 - i\overline{\omega}\overline{\varepsilon}')$, where $\overline{\varepsilon}' = \omega_0\eta_T$ is a dimensionless factor of the plate's internal friction standardized by the plate's fundamental eigenfrequency ω_0, $\overline{\omega} = \omega/\omega_0$ – dimensionless frequency of the external pressure, ω_n – the plate's n-th eigenfrequency, where $n = 0, 1, 2, \ldots$. The plate's eigenvalues $x_n = k_n r_1$ have been given in [14] for several values of the geometric factor s, where $k_n^4 = \omega_n^2\varrho h/B' \in \mathbb{R}$ is the structural wavenumber, ϱ, h are the plate's density and thickness, $B' = Eh^3/\left[12(1-\nu^2)\right]$ is its stiffness, E – Young modulus, ν – Poisson ratio, $\eta_T = R'/B'$ – the plate's internal friction factor.

2.1. The plate's transverse deflection expansion into a series of eigenfunctions

The plate's equation of motion can be written in the amplitude form for some time-harmonic processes

$$\left(k_B^{-4}\nabla_r^4 - 1\right)\,\eta(r) - i\,\frac{\varepsilon_0}{c}\,\phi(r) = \frac{f(r)}{\omega^2\varrho h}\,, \tag{1}$$

where ω is the frequency of the plate's exciting pressure, $\nu(r) = -i\omega\eta(r)$, $k_B^4 = \omega^2\varrho h/B \in \mathbb{C}$ is the structural visco-elastic wavenumber, $B = B'\,(1 - i\,\overline{\omega}\,\overline{\varepsilon}')$ is visco-elastic plate's stiffness, $\varepsilon_0 = \overline{\varepsilon}_0/\overline{\omega}$ is the plate's damping factor from the air column, where $\overline{\varepsilon}_0 = \varrho_0 c/\varrho h\omega_0$ and ϱ_0 is the density of the air column. The plate's transverse deflection amplitude

$$\eta(r) = \sum_{n=0}^{+\infty} c_n\xi_n(r), \qquad c_n \in \mathbb{C}, \tag{2}$$

is the solution of the non-homogeneous equation of the plate's motion (1) which may be expanded into a series to an orthonormal system of eigenfunction amplitudes

$$\xi_n(r) = A_n\left[J_0(k_n r) + B_n I_0(k_n r) - C_n N_0(k_n r) - D_n K_0(k_n r)\right], \tag{3}$$

providing solutions of the homogeneous equation of the plate's motion

$$\left(k_n^{-4}\nabla_r^4 - 1\right)\xi_n(r) = 0. \tag{4}$$

The eigenfunctions ξ_n satisfy the orthogonality condition (cf. [25, 26]). They have been presented in [14] together with constants $A_n = [(s^2-1)/2]^{1/2}/g_n$, B_n, C_n, D_n where $g_n = [s^2 C_0'^2(sx_n) - C_0'^2(x_n)]^{-1/2}$, $C_\nu'(x) = J_\nu(x) - C_n N_\nu(x)$, $x \in \{x_n, sx_n\}$, $\nu \in \{0, 1\}$. Further we use the acoustic potential presented in the polar coordinates

$$\phi(r, z) = \int_S v(r_0)\,G(r, r_0, z)\,\mathrm{d}S_0, \tag{5}$$

where

$$G(r, r_0, z) = \frac{1}{2\pi}\,\frac{e^{ik|\vec{r}-\vec{r_0}|}}{|\vec{r}-\vec{r_0}|} = \frac{i}{2\pi}\int_0^{+\infty} \tau\gamma^{-1} J_0(\tau r)\,J_0(\tau r_0) e^{i\gamma z}\,\mathrm{d}\tau \tag{6}$$

is a Green function in a Hankel's representation (cf. [27, 28]), $z \geq 0$ and $\gamma = [k^2 - \tau^2]^{1/2}$.

2.1.1. The solution of the algebraic equation system

Finding the complex expansion coefficients c_n is equivalent to solving the non-homogeneous equation of motion (1) of a vibrating plate. By inserting Eq. (2) to Eq. (5) we can write the acoustic potential on the plate's surface $(z = 0)$ as

$$\phi(r) = \omega\int_0^{+\infty} \tau\gamma^{-1} J_0(\tau r) \sum_{n=0}^{+\infty} c_n\,M_n(\tau)\,\mathrm{d}\tau, \tag{7}$$

where

$$M_n(\tau) = \int\limits_{r_1}^{r_2} \xi_n(r)\, J_0(kr\sin\vartheta) r\,\mathrm{d}r. \tag{8}$$

Some transformations of Eq. (1) lead to

$$c_m\,(k_B^{-4} k_m^4 - 1) - i\varepsilon_0 \sum_{n=0}^{+\infty} c_n \mathcal{P}_{nm} = f_m, \qquad n, m \in \mathbb{N}, \tag{9}$$

where

$$\mathcal{P}_{nm} = \frac{2k}{r_2^2 - r_1^2} \int\limits_0^{+\infty} \tau\gamma^{-1}\, M_n(\tau)\, M_m(\tau) \mathrm{d}\tau \tag{10}$$

is the standardized mutual sound power of a pair of two interacting in-vacuo modes of the plate $(0, n)$ and $(0, m)$, $n \neq m$. If $n = m$ we get the standardized sound power of the n-th in-vacuo mode of the plate

$$\mathcal{P}_n = \frac{2k}{r_2^2 - r_1^2} \int\limits_0^{+\infty} \tau\gamma^{-1}\, M_n^2(\tau)\,\mathrm{d}\tau \tag{11}$$

and

$$f_m = \frac{2}{\omega^2 \varrho h\,(r_2^2 - r_1^2)} \int\limits_{r_1}^{r_2} f(r)\,\xi_m(r)\, r\,\mathrm{d}r \tag{12}$$

is the m-th expansion coefficient of the external exciting pressure. The indices n, m in the equation system (9) belong to the same set of natural numbers and therefore we can interchange them. It is useful to employ some dimensionless magnitudes for our numerical computations and therefore we introduce such dimensionless coefficients as

$$\overline{c}_m = c_m\,\omega^2 \varrho h / f_{\max}, \qquad \overline{f}_m = f_m\,\omega^2 \varrho h / f_{\max}, \tag{13}$$

which transform the equation system (9) into its matrix form

$$\left\{[k_B^{-4}(\mathbf{k}\mathbf{k}^{\mathbf{T}})^2 - 1]\,\mathbf{I} - i\varepsilon_0 \mathbf{P}\right\}\,\overline{\mathbf{c}} = \overline{\mathbf{f}} \tag{14}$$

where $\mathbf{I}$ is a unity matrix, $\mathbf{k}\mathbf{k}^{\mathbf{T}}$ denotes the square of a diagonal matrix of vector $\mathbf{k}$,

$$\mathbf{k} = \begin{bmatrix} k_0 \\ k_1 \\ k_2 \\ \vdots \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} \mathcal{P}_{00} & \mathcal{P}_{01} & \mathcal{P}_{02} & \cdots \\ \mathcal{P}_{10} & \mathcal{P}_{11} & \mathcal{P}_{12} & \cdots \\ \mathcal{P}_{20} & \mathcal{P}_{21} & \mathcal{P}_{22} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \quad \overline{\mathbf{c}} = \begin{bmatrix} \overline{c}_0 \\ \overline{c}_1 \\ \overline{c}_2 \\ \vdots \end{bmatrix}, \quad \overline{\mathbf{f}} = \begin{bmatrix} \overline{f}_0 \\ \overline{f}_1 \\ \overline{f}_2 \\ \vdots \end{bmatrix}, \tag{15}$$

and vector $\overline{\mathbf{c}}$ is the solution of Eq. (14).

2.1.2. The distribution of the external exciting pressure

It has been assumed that the plate is excited by an external time-harmonic pressure of the surface density $\mathrm{Re}\left[f(r)\exp(-i\omega t)\right]$. If its amplitude distribution is

$$f(r) = \begin{cases} f_{\max} & \text{if } r \in \{a, b\}, \\ 0 & \text{otherwise}, \end{cases} \tag{16}$$

where $r_1 \le a < b \le r_2$, then we get

$$f_m = \frac{f_{\max}}{\omega^2 \varrho h} \frac{2}{r_2^2 - r_1^2} \int\limits_a^b \xi_m(r)\, r\, \mathrm{d}r \tag{17}$$

from Eq. (12). We make use of the integral formula $u \int^r \mathcal{E}_0(ur)\, r\, \mathrm{d}r = r\, \mathcal{E}_1(ur)$ [29], where $\mathcal{E}_n$ is an n-th order cylindrical function. This results in

$$\begin{aligned} f_m &= \frac{\sqrt{2}}{\sqrt{s^2-1}} \frac{f_{\max}}{\omega^2 \varrho h} \frac{1}{x_m g_m} \\ &\times \left\{ s\frac{b}{r_2}\left[J_1\left(s x_m \frac{b}{r_2}\right) - C_m N_1\left(s x_m \frac{b}{r_2}\right)\right] - \frac{a}{r_1}\left[J_1\left(x_m \frac{a}{r_1}\right) - C_m N_1\left(x_m \frac{a}{r_1}\right)\right]\right. \\ &\qquad + s\frac{b}{r_2}\left[B_m I_1\left(s x_m \frac{b}{r_2}\right) + D_m K_1\left(s x_m \frac{b}{r_2}\right)\right] \\ &\qquad \left. - \frac{a}{r_1}\left[B_m I_1\left(x_m \frac{a}{r_1}\right) + D_m K_1\left(x_m \frac{a}{r_1}\right)\right]\right\}. \end{aligned} \tag{18}$$

If all the plate's surface is excited by the pressure of the homogeneous amplitude distribution, i.e. $r_1 = a < b = r_2$, then the excitation coefficients will be reduced to

$$f_m = \overline{f}_m \frac{f_{\max}}{\omega^2 \varrho h}, \qquad \text{where} \quad \overline{f}_m = \frac{2\sqrt{2}}{\sqrt{s^2-1}} \frac{s^2 {C_1'}^2(s x_m) - {C_1'}^2(x_m)}{x_m g_m}. \tag{19}$$

3. The total sound power

3.1. Analytical formulations

The total sound power Π will be further expressed by the expansion coefficients c_n and the standardized mutual sound power $\mathcal{P}_{nm}$. The insertion of $\nu(r_0) = -i\omega\eta(r_0)$ and (2) to formula (8) results in

$$W(\tau)\, W^*(\tau) = \omega^2 \sum_{n=0}^{+\infty} \sum_{m=0}^{+\infty} c_n c_m^* \, M_n(\tau)\, M_m(\tau). \tag{20}$$

The total sound power in the Hankel's representation (cf. [14, 15])

$$\Pi = \pi \varrho_0 c k \int\limits_0^{+\infty} \tau \gamma^{-1}\, W(\tau)\, W^*(\tau)\, \mathrm{d}\tau = \pi \omega^2 \varrho_0 c \frac{r_2^2 - r_1^2}{2} \sum_{n=0}^{+\infty} \sum_{m=0}^{+\infty} c_n c_m^* \mathcal{P}_{nm} \tag{21}$$

is derived from Eqs. (10), (11), (20) and (21). The reference sound power, when $k\to\infty$, is

$$\Pi^{(\infty)} = \pi\omega^2\varrho_0 c\sum_{n=0}^{+\infty}\sum_{m=0}^{+\infty} c_n c_m^* \int_{r_1}^{r_2} \xi_n(r)\,\xi_m(r)\,r\,\mathrm{d}r = \pi\omega^2\varrho_0 c\,\frac{r_2^2-r_1^2}{2}\sum_{n=0}^{+\infty} c_n^2, \tag{22}$$

which is used to standardize the total sound power as follows

$$\mathcal{P} = \frac{\Pi}{\Pi^{(\infty)}} = \frac{\sum\limits_{n=0}^{+\infty}\sum\limits_{m=0}^{+\infty} c_n c_m^* \mathcal{P}_{nm}}{\sum\limits_{n=0}^{+\infty} c_n^2}. \tag{23}$$

The expansion coefficients c_n are determined by solving the algebraic equation system (14). The standardized mutual sound power $\mathcal{P}_{nm}$ is computed with the asymptotic or integral formulae given in [14, 15].

The total sound power Π can also be expressed by a single expansion series. For that purpose the equation system (14) must be multiplied by c_m^* and summed up for index m from 0 to $+\infty$

$$\sum_{n=0}^{+\infty}\sum_{m=0}^{+\infty} c_n c_m^* \mathcal{P}_{nm} = \frac{i}{\varepsilon_0}\sum_{m=0}^{+\infty} c_m^* \left[f_m - c_m\left(k_B^{-4}k_m^4 - 1\right)\right] \tag{24}$$

which, inserted to formula (20), gives

$$\Pi = \pi\omega^2\varrho_0 c\,\frac{i}{\varepsilon_0}\,\frac{r_2^2-r_1^2}{2}\sum_{m=0}^{+\infty} c_m^* \left[f_m - c_m\left(k_B^{-4}k_m^4 - 1\right)\right]. \tag{25}$$

In one particular case i.e., when the influence of the air column on the plate's motion can be neglected, and $\varepsilon_0 = \varrho_0(\varrho kh)^{-1} \ll 1$, we get a basic relation

$$c_n = f_n \Big/ \left(k_B^{-4}k_n^4 - 1\right) \tag{26}$$

from the algebraic equation system (14).

3.2. The frequency characteristics of the plate's response

The frequency characteristics have been determined in the domain of the angle frequency standardized by the plate's fundamental eigenfrequency, i.e.

$$\overline{\omega} = \omega/\omega_0 \tag{27}$$

which is dimensionless. We introduce some of the plate's material parameters

$$\overline{\varepsilon}_0 = \frac{\varrho_0 c}{\varrho h\omega_0}, \qquad \overline{\varepsilon}' = \omega_0\eta_T, \tag{28}$$

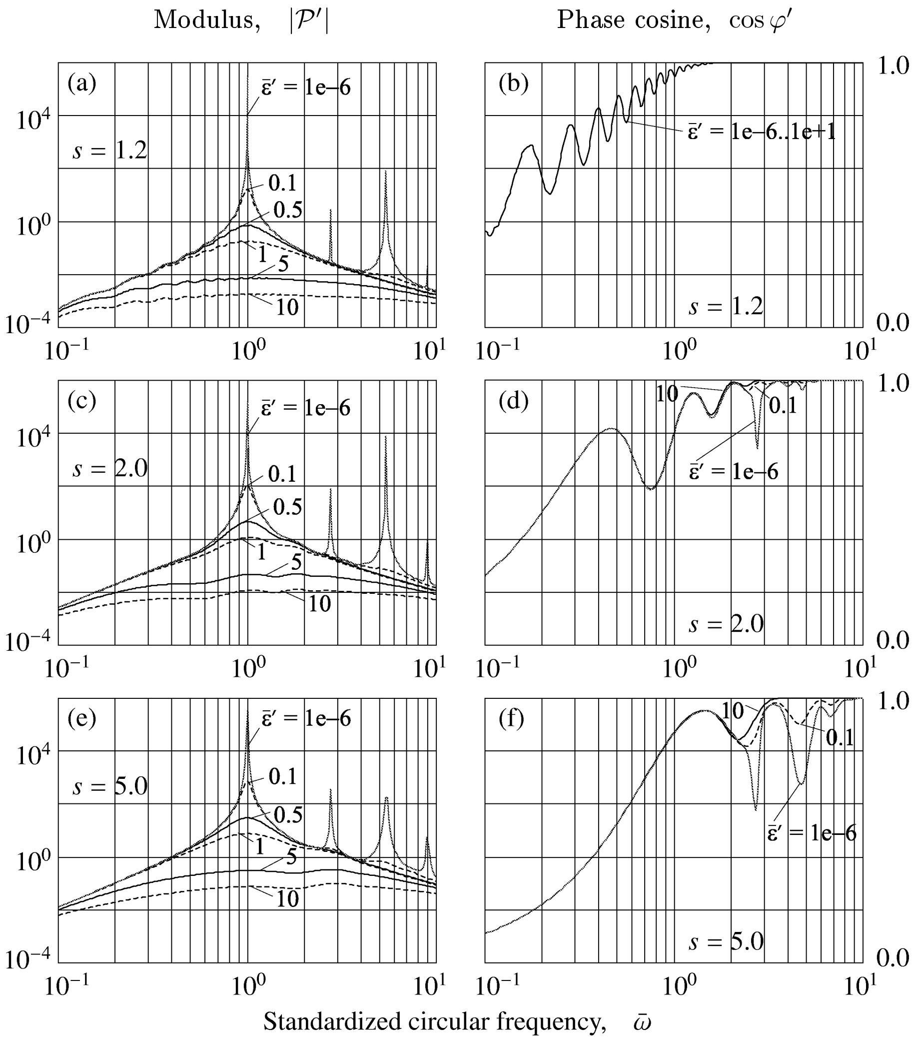

Fig. 1. The modulus $|\mathcal{P}'|$ and phase cosine $\cos\varphi'$ of the standardized total sound power of an annular plate, where $h = 1\mathrm{e}-3$ [m].

independent from frequency $\overline{\omega}$ together with some further relations to be used to plot the graphs in Figs. 1–3:

$$k_B^{-4}k_m^4 - 1 = -1 + x_0^{-4}x_m^4\,\overline{\omega}^{-2}\left(1 - i\overline{\varepsilon}'\overline{\omega}\right) \quad \text{and} \quad \omega_0 = x_0^2 r_1^{-2}\sqrt{B'/\varrho h}. \tag{29}$$

The total sound power Π, determined by Eq. (20), is standardized by the conventional magnitude

$$\Pi' = \pi r_1^2 \varrho_0 c \left(\frac{f_{\max}}{\varrho h \omega_0}\right)^2, \tag{30}$$

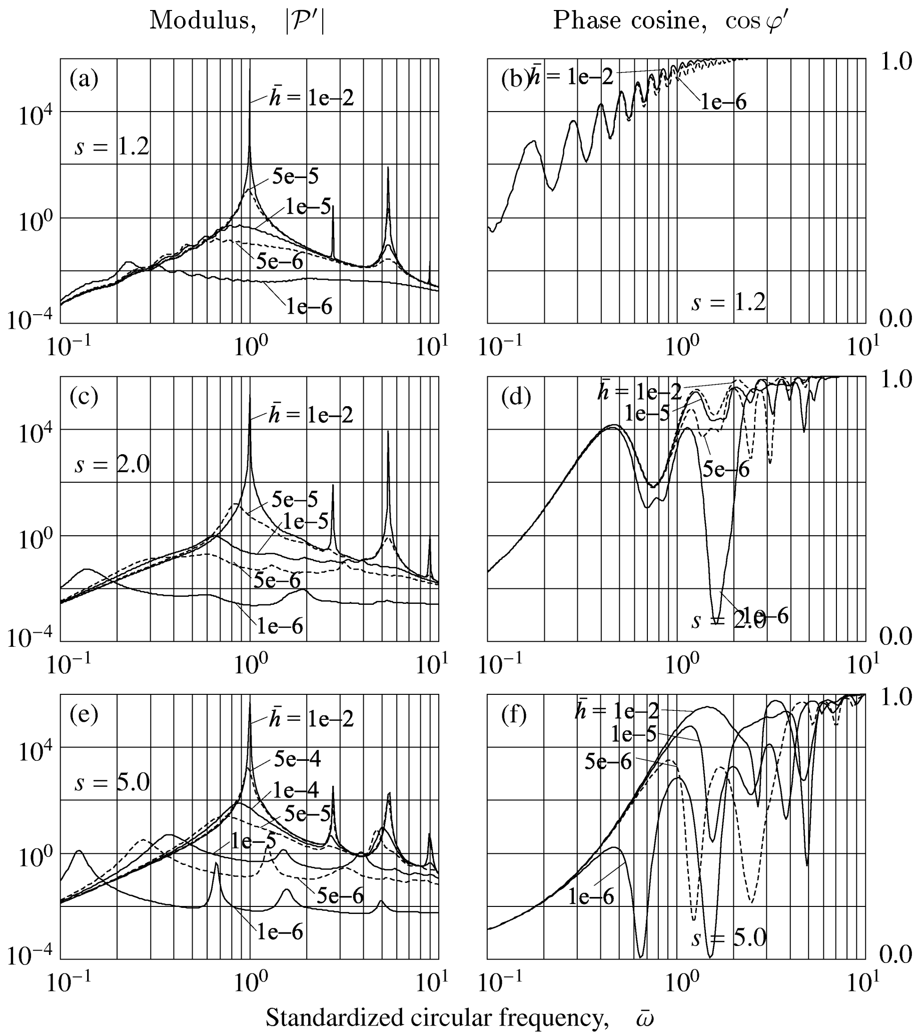

Fig. 2. The modulus $|\mathcal{P}'|$ and phase cosine $\cos\varphi'$ of the standardized total sound power of an annular plate, where $\bar{\varepsilon}' = 1\mathrm{e}-6$.

which leads to the dimensionless sound power

$$\mathcal{P}' = \Pi/\Pi' = \frac{s^2-1}{2\overline{\omega}^2} \sum_{n=0}^{+\infty} \sum_{m=0}^{+\infty} \overline{c}_n \overline{c}_m^* \mathcal{P}_{nm}. \tag{31}$$

The magnitude is a function of the plate's excitation frequency $\overline{\omega}$ and some parameters independent from $\overline{\omega}$ such as s, $\overline{\varepsilon}_0$, $\overline{\varepsilon}'$.

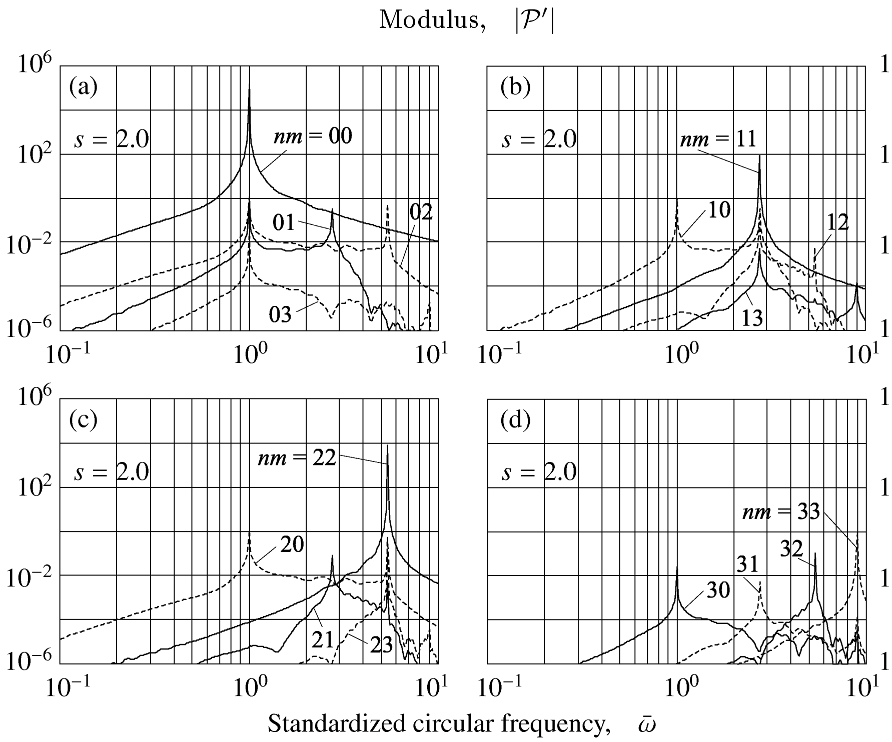

Fig. 3. The modulus $|\mathcal{P}'|$ of the standardized mutual sound power of an annular plate, where $h = 1e-3$ [m], $\bar{\varepsilon}' = 1e-6$.

3.3. Numerical computations and their discussion

The equation system (14) was solved by a combined method of reduction and successive approximations to derive the frequency characteristics of the sound power. The reduction consists in making the assumption that $n, m < \infty$, i.e. the number of equation in the system and the number of terms taken into account in all the expansion series occurring in the equations are finite.

The total sound power is standardized by the sound power of a model plate, which is produced of elastic steel (cf. Table 1). This approach makes it possible to find some frequency characteristics independent from the plate's material and geometric parameters. If we wanted to employ the characteristics to determine the sound power of a plate of any parameters, we would have to multiply their values by the values adequate for the plate (cf. Eq. (31)).

The frequency characteristics represent the modulus $|\mathcal{P}'|$ (cf. Figs. 1(a), (c), (e)) and phase cosine $\cos\varphi'$ (cf. Figs. 1(b), (d), (f)) of the standardized total sound power $\mathcal{P}$ in the domain of the external excitation frequency $\overline{\omega}$. The values of the model plate's material and geometric parameters have been presented in Tabs. 1 and 2. The eigenfrequencies of the model plate ω_n (cf. Table 3) are derived from the eigenvalues x_n, given in several papers (e.g. [2, 5, 14]), by $\omega_n = x_n^2 r_1^{-2} (B'/\varrho h)^{1/2}$.

Table 1. The material and geometric parameters of the model plate.

parameter	value
area	$S_0 = 13.8\,\mathrm{e} - 3\,\mathrm{m}^2$
thickness	$h = 1.0\,\mathrm{e} - 3\,\mathrm{m}$
Poisson ratio	$\nu = 0.3$
sound velocity	$c = 340\,\mathrm{m/sec}$
Young modulus	$E = 205.0\,\mathrm{e} + 9\,\mathrm{Pa}$
air density	$\varrho_0 = 1.293\ \mathrm{kg/m^3}$
steel density	$\varrho = 7700\ \mathrm{kg/m^3}$
excitation amplitude	$f_{\max} = 10\ \mathrm{Pa}$

Table 2. The plate's radii r_1, r_2 for its area $S_0 = 0.0138\ \mathrm{m}^2$ and different values of the geometric parameter s.

s	1.1	1.2	1.5	2.0	3.0	5.0
r_1	0.1447	0.1000	0.0593	0.0383	0.0235	0.0135
r_2	0.1592	0.1200	0.0890	0.0766	0.0704	0.0677

Table 3. The eigenfrequencies ω_n of the model plate.

s	1.1	1.2	1.5	2.0	3.0	5.0
	$\times 10^3$ [rad./sec.]					
ω_0	166.722	87.318	39.659	23.754	15.784	11.790
ω_1	459.588	240.719	109.375	65.564	43.624	32.616
ω_2	900.987	471.926	214.465	128.611	85.639	64.097
ω_3	1489.386	780.134	354.557	212.661	141.663	106.095
ω_4	2224.893	1165.399	529.675	317.730	211.703	158.610
ω_5	3107.500	1627.717	739.819	443.814	295.754	221.638

Table 4. The plate's thickness $\overline{h}$, which is standardized by the square root of its area $S_0 = 13.8\,\mathrm{e} - 3\ [\mathrm{m}^2]$.

$\overline{h}$	1.0e − 2	5.0e − 4	2.0e − 4	1.0e − 4	5.0e − 5
h [m]	1.2e − 3	5.9e − 5	2.4e − 5	1.2e − 5	5.9e − 6
$\overline{h}$	2.0e − 5	1.0e − 5	5.0e − 6	2.0e − 6	1.0e − 6
h [m]	2.4e − 6	1.2e − 6	5.9e − 7	2.4e − 7	1.2e − 7

We can see some "oscillations" of the modulus and phase cosine in the excitation frequency domain when s approaches unity (cf. Figs. 1(a), (b), 2(a), (b), where $s = 1.2$, and [13, 14]), which does not appear in the case of a circular plate (cf. [7–9, 11, 12, 20]). Let us look at the phenomena closer. When s approaches infinity the plate becomes similar in shape to a circular plate and the "oscillations" do not appear or are small enough to be ignored. In the opposite case, when s approaches unity, the plate approaches the shape of an annulus and its clamped edges are very close each other, which results in some very strong mechanical interactions between them and some apparent "oscillations" in the frequency characteristics. Also the number of "oscillations" in the frequency domain depends on the geometric parameter s. The "oscillation" number increases with the decrease in s.

The dimensionless parameter $\overline{\varepsilon}'$ represents the internal friction of the plate. If it has a considerably small value the maxima of the modulus around the successive eigenfrequencies are sharp and strong (cf. Figs. 1(a), (c), (e)). The power phase cosine is almost independent from $\overline{\varepsilon}'$ (cf. Figs. 1(b), (d), (f)).

The dependance of the total sound power on the plate's thickness standardized by the square root of the plate's area

$$\overline{h} \equiv \frac{h}{\sqrt{S_0}} = \frac{h}{r_1\sqrt{\pi(s^2-1)}} \tag{32}$$

has also been illustrated. It is easy to notice that an increase in the plate's thickness results in stronger and sharper maxima of the power modulus around the plate's successive eigenfrequencies (cf. Figs. 2(a), (c), (e)). The power phase cosine is almost independent from the plate's thickness for small values of s and the dependance becomes stronger with an increase in value of s (cf. Figs. 2(b), (d), (f)). If the plate's thickness decreases then the maxima progressively vanish and the level of the modulus decreases, as expected, and the maxima are shifted towards the low frequencies. This may be accounted for by the regularity that the thicker plate, the greater its stiffness and the stronger and sharper the maxima of the total sound power modulus. On the other hand, a considerably thin plate cannot produce any strong and sharp maxima and some deviations such as a decrease in frequency of the maxima locations can appear.

Figure 3 shows some sample values of the modulus of the mutual sound power separated from the total sound power. Let us insert the Kronecker delta values $\delta_{\nu n}$ and $\delta_{\mu m}$ to Eq. (31)

$$\mathcal{P}'_{nm} = \Pi_{nm}/\Pi' = \frac{s^2-1}{2\overline{\omega}^2}\sum_{\nu=0}^{+\infty}\sum_{\mu=0}^{+\infty}\delta_{\nu n}\delta_{\mu m}\overline{c}_\nu\overline{c}_\mu^*\mathcal{P}_{\nu\mu} \tag{33}$$

to perform the separation, which gives the formula for the separated mutual sound power

$$\mathcal{P}'_{nm} = \frac{s^2-1}{2\overline{\omega}^2}\,\overline{c}_n\overline{c}_m^*\mathcal{P}_{nm}. \tag{34}$$

4. Concluding remarks

If the stream of the sound power density $p\vec{v}$ passing through the hemisphere around the sound source is positive within some ranges of frequency the source losses some energy. In the opposite case, the source absorbs some energy for the remaining frequencies. Those phenomena are related with some reciprocal interactions between the two different modes of the plate (cf. [10, 15]), which results in the positive or negative gain of the plate's total sound power by its modes for some frequency ranges.

It is easy to see that the sound power of the plate's successive modes gives the main contribution to the plate's total sound power, which is particularly clear around the plate's successive eigenfrequencies (cf. Fig. 3) while the contribution from the mutual sound power is considerably smaller. These findings are valid for light fluids like the air, i.e. when $\varrho_0 \ll \varrho$. The greatest contributions come from the plate's modes of the lowest indices and from the modes of which the eigenfrequency ω_n is close to the plate's excitation frequency ω. The contribution from the mutual sound power of a pair of two different modes increases when their indices are close to each other. The contribution is also considerably bigger if both indices are either odd or even. Otherwise, i.e. if one index is odd and the other is even, the contribution is considerably smaller (cf. Fig. 3), the phase cosine characteristic is strongly non-uniform, and the "non-oscillating" parts of the mutual sound power vanish (cf. [15]). The contribution from the higher modes is considerably smaller, because the expansion series into the plate's eigenfunctions in Eqs. (21), (23) and (25) are rapidly convergent. That is why that contribution can be ignored with no considerable error introduced to the total sound power. This is useful for some numerical computations, because for practical use a finite number of equations and a finite number of terms in expansion series can be taken into account when the equation system (14) is to be solved numerically.

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