

VIBROACOUSTIC RECIPROCITY PRINCIPLE IN EXAMINATION OF MUSICAL INSTRUMENTS

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An application of the Reciprocity Principle for investigating certain acoustic parameters of the contrabass was presented in the paper. By means of direct measurements the vibroacoustic transfer function between the force occurring in the bridge of the contrabass and the acoustic pressure in the observation point was obtained. By means of reciprocity measurements the vibroacoustic transfer function between the volume velocity of the point sound source and the velocity of vibration of the element of the contrabass bridge was obtained. The compatibility of those transfer functions as well as the usefulness of the reciprocity method in acoustic examinations were assessed. Spectra of some sounds emitted by the contrabass were determined.

1. Introduction

Examination and evaluation of musical instruments are important tasks of contemporary acoustics. They are connected with the following problems:

- construction of instruments of the required acoustic properties,
- looking for new timbres and tones,
- possibility of reproduction of timbres of instruments from old centuries by instruments produced nowadays — for the needs of old-music orchestras,
- comparison and evaluation of various instruments.

There are many methods of testing musical instruments. However none of them provides complete information concerning the physical and acoustic parameters of an instrument. Therefore quite often several methods are simultaneously used. New, versatile, methods are also looked for.

Vibroacoustic reciprocity principle is one of such new methods. It can be formulated in the following way [1, 2]:

“A vibroacoustic process in a linear system — being an answer for a periodical application of an exciting force — done by an external factor at a certain point — remains without any change when the point of energy application and the point of observation are reversed”.

The reciprocity principle can be presented by the equation:

$$\iint_S \left[\frac{\partial p_1(r)}{\partial n} F_2(r) - \frac{\partial p_2(r)}{\partial n} F_1(r) \right] dS = -j\varpi\rho \iiint_V [q_1(r)p_2(r) - q_2(r)p_1(r)] dV \quad (1)$$

where: $p_1(r)$, $p_2(r)$ — sound fields generated by a system of volume sources $q_1(r)$ and $q_2(r)$ — divided by volume, V ; $F_1(r)$, $F_2(r)$ — forces acting on an elastic body — in the direction normal to the surface, S ; ω — angular frequency [rad/s] ρ — density of a medium [kg/m³].

The vibroacoustic reciprocity principle has a wide practical application. By means of this principle an identification of sources of a vibroacoustic energy in machines and equipment can be done, a sound radiation by vibrating structures can be investigated and microphones can be calibrated. With the help of the reciprocity principle the vibroacoustic transfer function showing the sound transmission from the point of vibrating structure to the point of acoustic observation can be also estimated.

2. Methods of acoustic examinations of musical instruments

In view of an immense diversity of musical instruments and their acoustic parameters several methods of examination have been introduced. Some have more universal application and provide the most complete knowledge of the properties of instruments. Those are: Long Time Averaged Spectra (LTAS) tests, investigations of the directivity and examination of the spectrum structure.

The fundamental examination constituting the bases for several further analyses is LTAS. It informs about the dependence of a sound acoustic energy emitted by a musical instrument on the frequency-averaged in time — for the adequately representative musical fragment. Musical sounds are complex acoustic signals. There is a need of introduction of a simple, universal and understandable characteristics which would encompass — without any distortion — the whole complexity of an acoustic signal in music. Such characteristics is given by the Long Time Averaged Spectra analysis. The musical fragment used for performing this analysis should be of a certain minimal length (depending on an instrument several tens of seconds up to a couple of minutes are usually enough) and should utilise — rather uniformly — all sounds which the instrument in question is able to generate. The chromatic scale played in an uniform rhythm is a good example of a representative musical fragment. Application of various, long enough, musical fragments for the LTAS analysis gives similar results with sufficient approximation. In case when different musical fragments are used for the LTAS estimation and those results are to be utilised for comparison purposes the averaged time should be prolonged as much as reasonably possible.

Investigation of the directivity allows to determine the spatial distribution of an acoustic pressure of the sound emitted by the instrument. The directivity of instruments is tested in fixed frequency ranges. One-third-octave bands are the best for that purpose, however sufficient results can be obtained from investigating the directivity in octave bands and even in bi-octave ones. The frequency band should contain the whole range of sounds, which can be generated by the instrument — from the lowest possible to a few first higher harmonic components of sounds, which limit the instruments' scale from the top.

Musical instruments emit sounds into surroundings in a highly directional way. This directivity is due to two main reasons:

- general properties of acoustic waves — known from the fundamentals of physical acoustics (a lack of the radiation directivity demonstrated by low-frequency sounds and a distinct directivity of high-frequency ones).

- structural features of instruments (location, shape and material of resonant plates, size and set-up of membranes, profile and setting of sound chambers, etc)

Obtaining information concerning the directivity of a sound radiation by musical instruments allows for:

- optimal microphone setting during professional recording sessions,
- working-out the proper positioning of instruments in bands and orchestras,
- more complete acoustic assessment of an instrument.

The spectrum structure is a three-dimensional dependence of the sound spectrum of the instrument on time. It means, it shows the changes of the sound spectrum with the changes of time. The spectrum structure can — among others — provide information on:

- structure of initial and final transients of a sound,
- structure of building up and fading out of individual harmonic components,
- choral effects in individual and concerted play,
- changes in dynamic proportions among harmonic components.

3. Acoustic testing of a contrabass

3.1. Contrabass

A contrabass has been in use since the 16th century, at first in Germany and later on in other European countries. Its predecessor was a viol, the five-string instrument of the size intermediate between a violoncello and a contrabass (it means approximately 1.25 m long). Through the ages contrabasses were constructed in various ways and differed by size, number of strings, tuning and a shape of a resonant chamber. They appeared in a wide range of heights from 1.8 m to more than 4 m (the so called octobass — a rare instrument applied e.g. in the Hector Berlioz orchestra). Contrabasses usually had 4 or 5 strings tuned in fourth; their tuning was lower or higher depending on the size of an instrument. Just tuning in fourths differs contrabasses from other stringed instruments used nowadays (violins, violas, violoncellos — are tuned in fifths). Tuning in fourths is caused by the large measure of the instrument (the length of the vibrating part of the string) and significant spacing of grips on the neck of a contrabass.

The predecessors of all contemporary stringed instruments were viols. Viols, stringed instruments, quite common in the Renaissance belonged to two kinds: *viola da gamba* — a knee viol and *viola da braccio* — an arm viol. They are now very rarely used (and only for performing old-time music).

A contrabass belongs to the group of bowed chordophones. It is the largest stringed instrument used nowadays. Its height reaches 1.8 to 2.0 meters. The general structure of a contrabass is presented in Fig. 1. The main elements responsible for the sound are: strings, bridge and two plates: upper and bottom. The upper plate is made of spruce

wood while the bottom one of sycamore wood. Two resonant openings of the shape of the letter "f" are being cut in the upper plate. The bridge transmits the vibration of strings onto the upper plate and by means of the sound post onto the bottom plate. Strings, fixed at the one side in the holes of a tailpiece, are supported by the bridge and run above the ebony neck of a contrabass. Their other ends are attached to the tuning pegs equipped with worm gears.

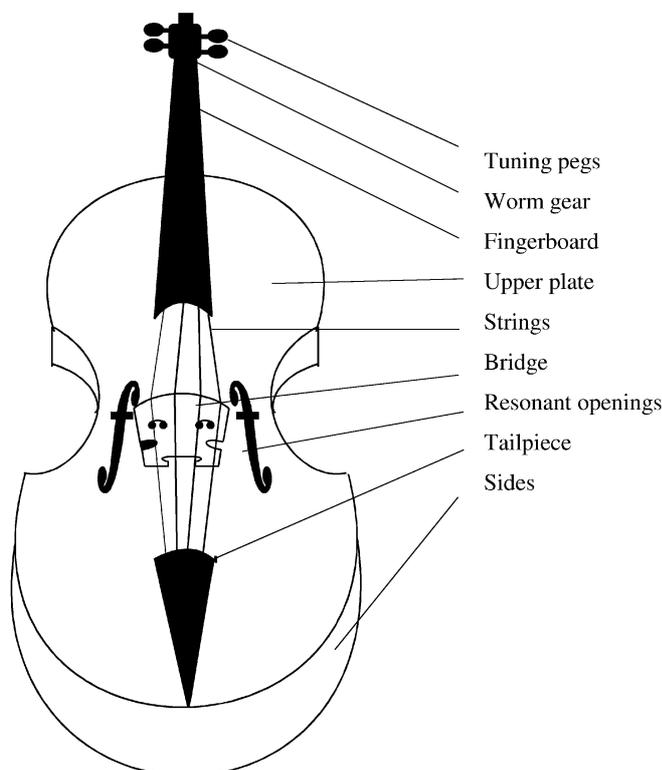


Fig. 1. General structure of a contrabass.

An integral part of a contrabass is a bow used to excite strings' vibrations. It consists of a wooden shaft, horse hair and the so called "frog" equipped with a screw enabling to tighten horse hair on a shaft. The cross-section of a contrabass at the height of a bridge is presented in Fig. 2.

An asymmetrical support of the bridge, which is due to the sound post, forces its torsional vibrations. By generating corresponding forms of vibrations in upper and bottom plates those vibrations shape the sound of the instrument. Strings of higher tuning are placed above the supported part of the bridge. Since this part transfers more sounds of high frequencies it is called the high base of the bridge. Strings of lower tuning are placed above the freely vibrating part of the bridge, which transfers sounds of low frequencies.

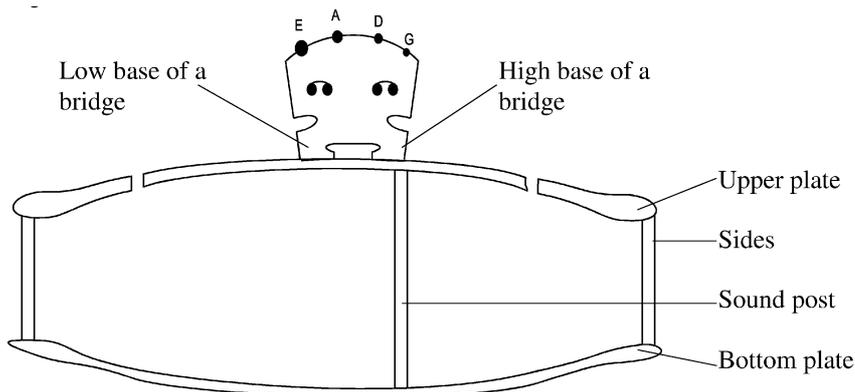


Fig. 2. The cross-section of a contrabass – at the height of a bridge.

Therefore it is called the low base of the bridge. Above both bases there are special holes into which two segments of a converter of the contrabass are fitted.

3.2. Characteristics of a transducer

A transducer being used for acoustic investigations is a typical double-segment transducer placed by a slight pressing into special holes in the bridge. The force characteristic of the transducer was determined and the verification of its linearity was performed. Separate measurements were done for each segment of the transducer. To take a measurement an exciting force was applied to the segment while the electric voltage was measured at its output.

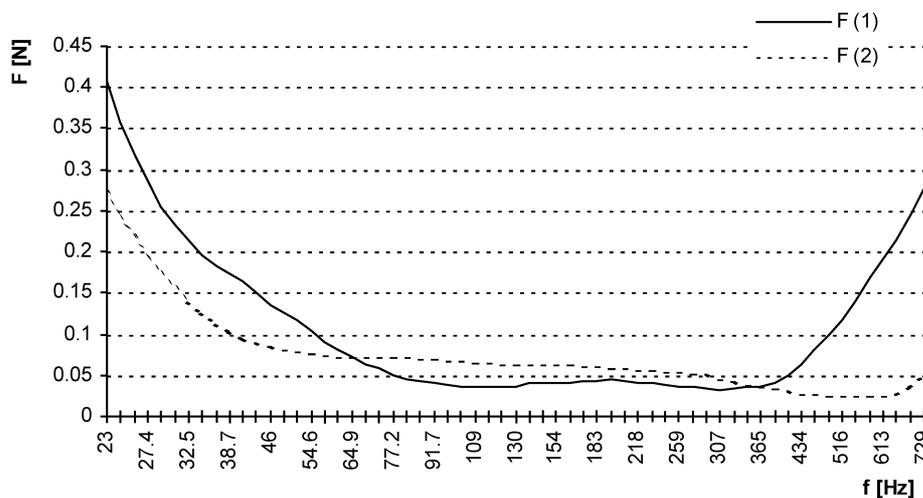


Fig. 3. The exciting force vs the frequency at the constant output voltage — for two segments of the transducer of the contrabass.

The dependence of the exciting force, $F[N]$ on the frequency, at the constant output voltage, $U = 20$ [mV] was determined. The obtained plot is presented in Fig. 3. The characteristics concerning the low and the high base are marked by numbers 1 and 2 — respectively. One can easily see the difference in the transmitting bands of the two segments of the transducer. The transmitting band of the segment marked for the high base is shifted in the direction of higher frequencies as compared to the other segment.

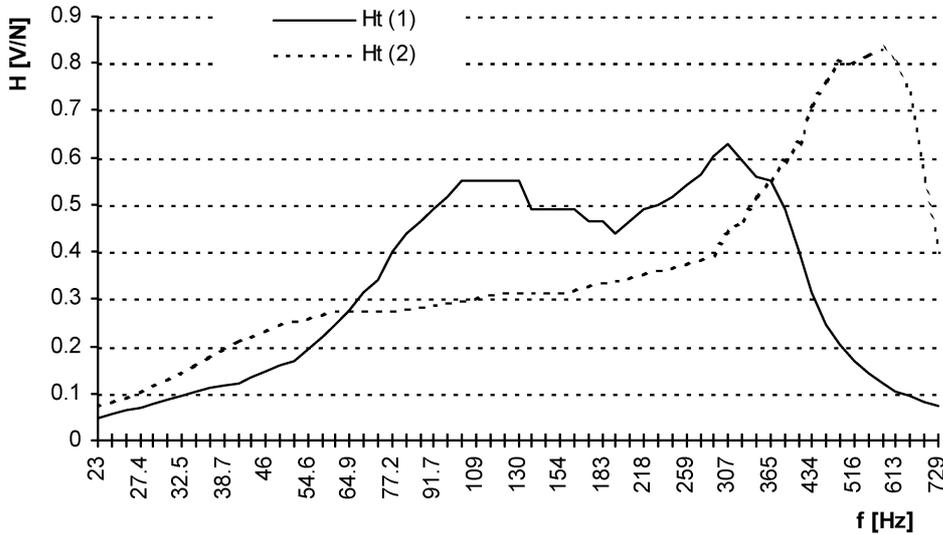


Fig. 4. Transfer functions, H_T for two segments of the transducer of the contrabass.

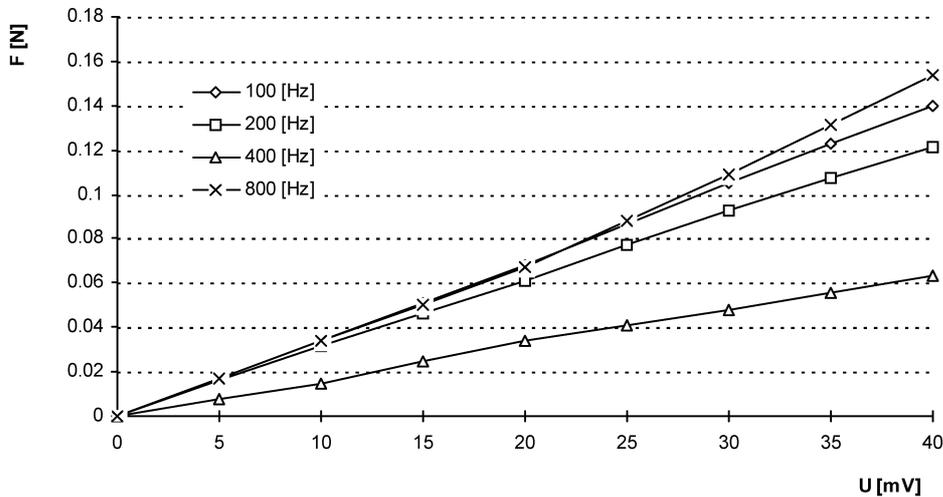


Fig. 5. The exciting forces vs input voltage (at 4 frequencies) measured for the first segment of the transducer.

On the bases of the presented above measurements the transfer function, H_T of the transducer (Fig. 4) was calculated:

$$H_T(f) = \frac{U(f)}{F(f)} \quad (2)$$

To verify the linearity of the measurements the dependence of the exciting force on the input voltage at various frequencies (100, 200, 400 and 800 Hz) was checked. As can be seen from Fig. 5. the tested transducer expresses the linear dependency of those values – in the frequency and force ranges occurring actually during the contrabass play.

3.3. Reciprocity Investigations

The reciprocity relations in the sound emission by the contrabass can be presented by the following equation:

$$\left(\frac{p_2}{F_1}\right)_{Q_2=0} = \left(\frac{v_1}{Q_2}\right)_{F_1=0} \quad (3)$$

where: F_1 — force occurring in the bridge when strings are being excited for vibrations, p_2 — pressure at the observation point when strings are being excited for vibrations, Q_2 — volume velocity of the point sound source, v_1 — velocity of vibrations of the bridge when the point sound source influences the contrabass.

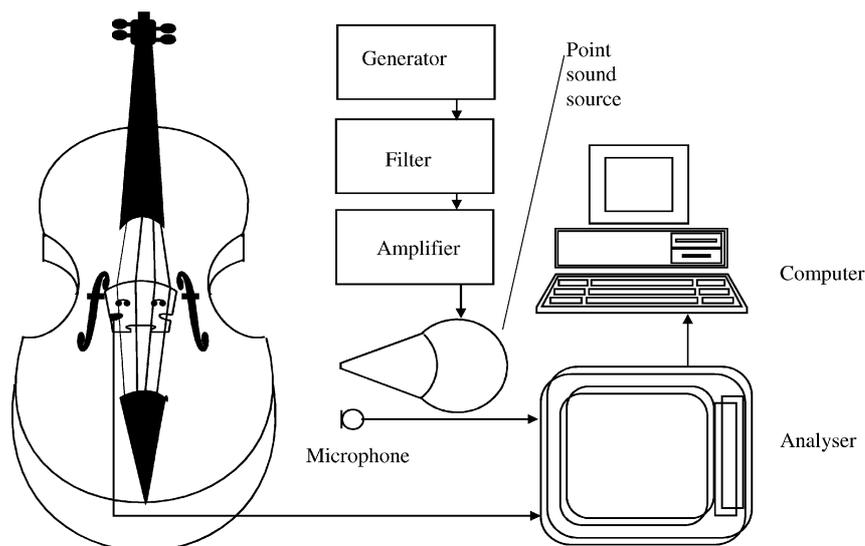


Fig. 6. Equipment set-up for the reciprocity investigations of the contrabass.

The special equipment set-up — schematically presented in Fig. 6. — was prepared for the tests. The bridge of the contrabass with the attached transducer is presented in Fig. 7.

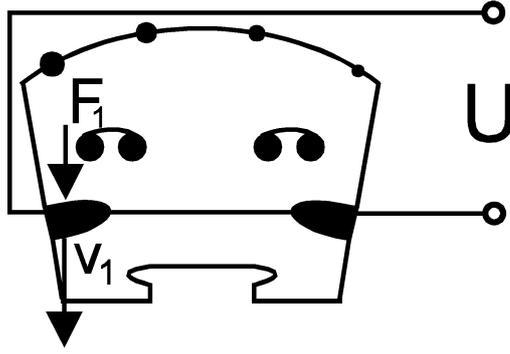


Fig. 7. Bridge of the contrabass with the attached transducer.

Measurements were performed in two different modes:

- mode of direct measurements
- mode of reciprocity measurements.

During the direct measurements strings of the contrabass were being excited in a continuous way by playing a chromatic scale in the full range of the instrument. The acoustic pressure, (p_2) was measured at the observation point while the voltage (U) at the output of the transducer. The observation point was located in the plane of symmetry of the instrument at the height of the bridge, approximately 1 m from the upper plate.

The force in the base of the bridge (F_1) was calculated according to the equation (4):

$$F_1(f) = \frac{U(f)}{H_T(f)} \quad (4)$$

The results of direct measurements allowed to estimate the vibroacoustic transfer function (H_D) between the force occurring in the base of the bridge and the acoustic pressure at the observation point:

$$H_D \left(\frac{p_2}{F_1} \right)_{Q_2=0} \quad (5)$$

In the mode of reciprocity measurements the omnidirectional, point sound source was located at the observation point. The white noise, filtered in octave bands, was fed to this source. The acoustic pressure (p) for that signal was measured in the axis of the source at the distance $r = 1$ m from its output. Then the volume velocity of the source was calculated as:

$$Q_2 = \frac{p}{f} \frac{r}{\rho} \quad (6)$$

Simultaneously the vibration velocity (v_1) in the base of the bridge perpendicular to the resonant plate was measured. Reciprocity measurements were performed in the frequency range 54 to 730 [Hz] in 1/12 octave bands.

On the bases of the obtained results the transfer function H_R was estimated:

$$H_R = \left(\frac{v_1}{Q_2} \right)_{F_1=0} \quad (7)$$

All above mentioned measurements were performed separately for the high and the low base of the bridge of the contrabass.

4. Analysis of the results

The obtained results are presented as graphs in Figs 8–12. Fig. 8. illustrates the comparison of voltages at the output of a transducer's segments placed in the low and high base of the bridge — during the direct mode of measurements. Fig. 9. gives the comparison of the acoustic pressure at 1 meter distance from the contrabass and the voltage at the output of the transducer's segment situated in the low base of the bridge. Fig. 10. enables to compare the level of the volume velocity of the point sound source with the voltage level at the output of the transducer's segment situated in the high base of the bridge of the contrabass subjected to the influence of that source.

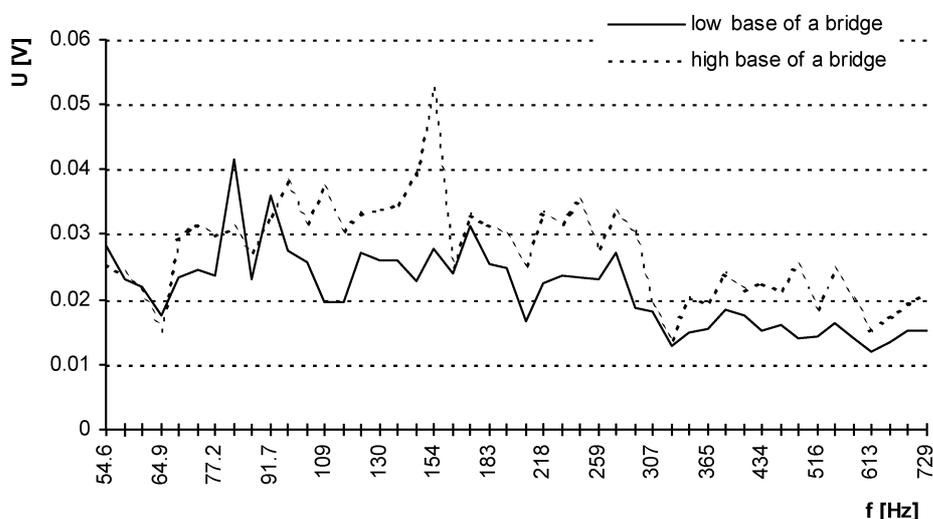


Fig. 8. Comparison of voltages U at the output of the transducer's segment placed in the low and the high base of the bridge — during the direct mode of measurements.

The direct transfer function H_D of the p/F type and the reciprocity transfer function H_R of the v/Q type for the low base (1) of the bridge of the contrabass are presented in Fig. 11, while the same functions for the high base (2) are presented in Fig. 12.

During the investigations the voltage at the transducer output was compared with the acoustic pressure measured simultaneously in the plane of symmetry of the contrabass at 1 m distance.

The contrabass used in the hereby described examinations was a high quality instrument; it did not have audible formants in the frequency range of the basic sounds from the whole scale of the instrument. On account of the above and due to the characteristics of the transducer the obtained dependencies of the pressure at the observation point and

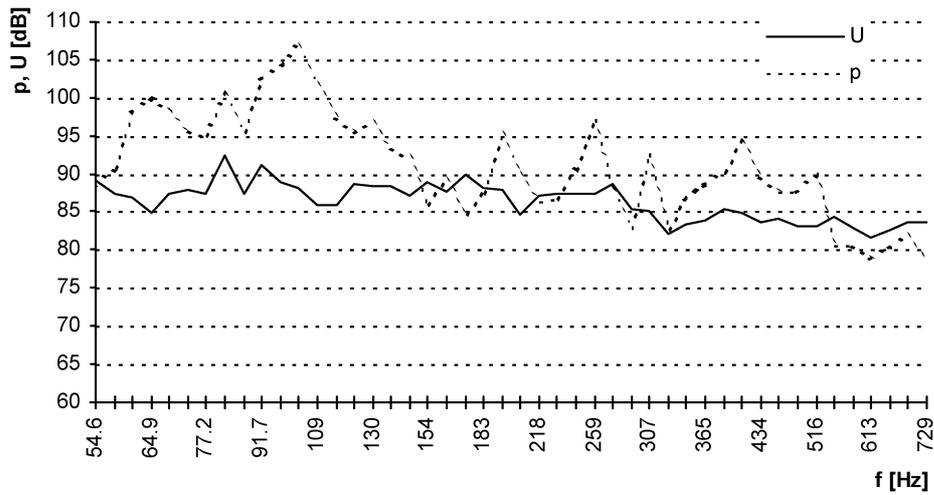


Fig. 9. Comparison of the acoustic pressure p at the observation point and the voltage U at the output of the segment of the transducer, placed in the low base of the bridge.

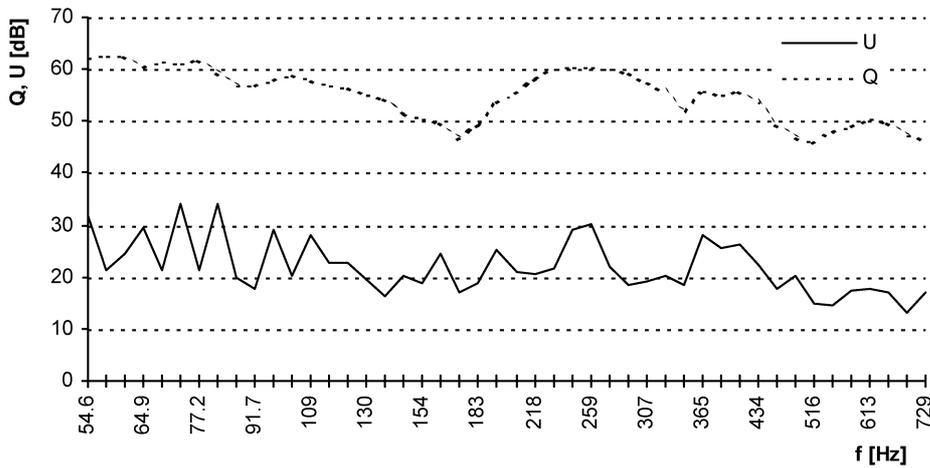


Fig. 10. Comparison of the volume velocity Q level of the point sound source and the voltage U level at the output of the transducer' segment placed in the high base of the bridge during the reciprocity measurements.

the voltage at the transducer's output for the selected test sample (chromatic scale) — were similar to the constant function in quite wide frequency range.

Voltages at the output of the transducer's segments placed in the low and in the high base of the bridge obtained in direct as well as in reciprocity measurements were also compared. In addition the volume velocity of the point sound source was compared to the voltage at the transducer's output for both bases of the contrabass bridge. The differences in voltages obtained for two segments of the transducer in direct and reci-

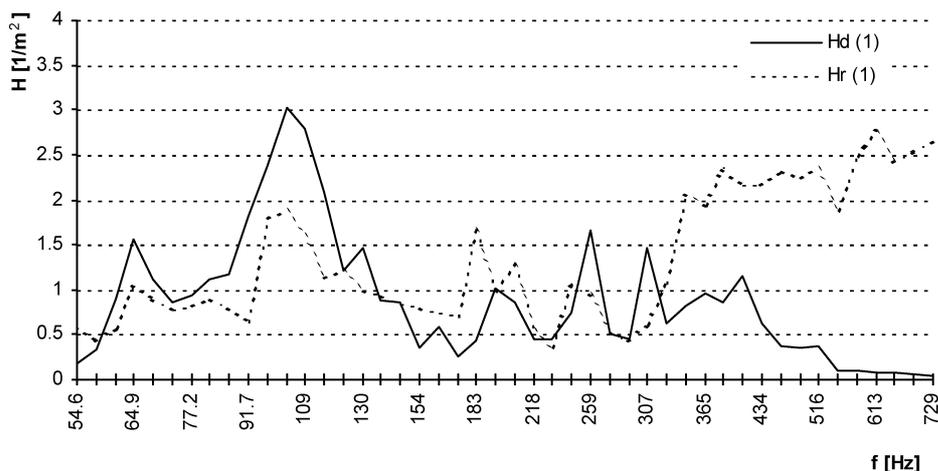


Fig. 11. Direct transfer function H_D of the p/F type and the reciprocity transfer function H_R of the v/Q type for the low base (1) of the bridge of the contrabass.

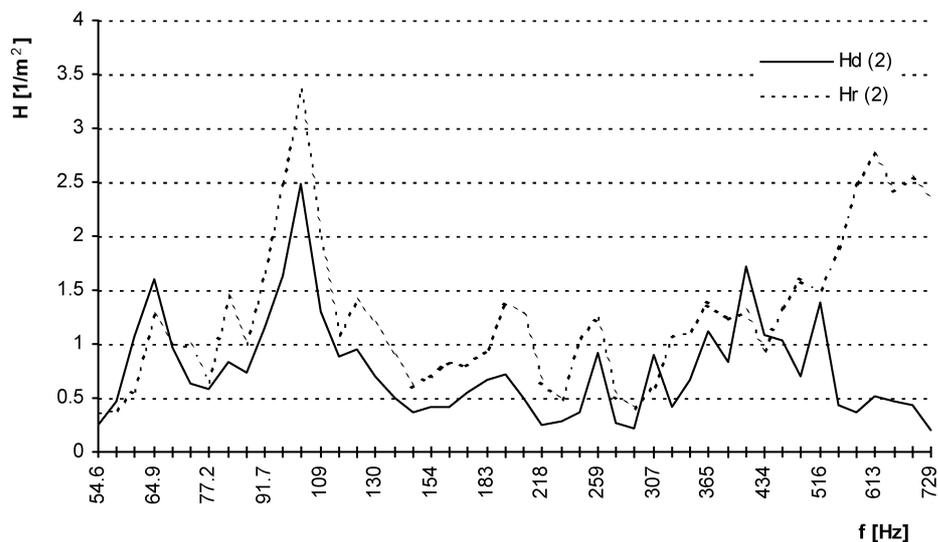


Fig. 12. Direct transfer function H_D of the p/F type and the reciprocity transfer function H_R of the v/Q type for the high base (2) of the bridge of the contrabass.

procity measurements are caused by different transfer functions of those segments and the asymmetrical build of the instrument — which was discussed in the previous part of the paper.

The dependence of the volume velocity of the point sound source and the voltage at the transducer's output for the low and the high base (in reciprocity measurements) on the frequency show quite good agreement.

Vibroacoustic transfer functions obtained in direct and reciprocity measurements express compatibility in the frequency and amplitude up to 345 [Hz] — for the low base, and up to 546 [Hz] — for the high base. Discrepancies between direct and reciprocity transfer functions appearing above the mentioned frequencies are due to the limited transmission band of individual segments of the transducer and too low volume velocity of the point sound source in this frequency range. This volume velocity is not sufficient to excite the instrument to such an extend as to enable the execution of proper measurements.

As a supplement to the reciprocity examinations the measurements of spectra of sounds emitted by idle, not shortened strings of the contrabass were performed. The results are presented in Figs 13–16.

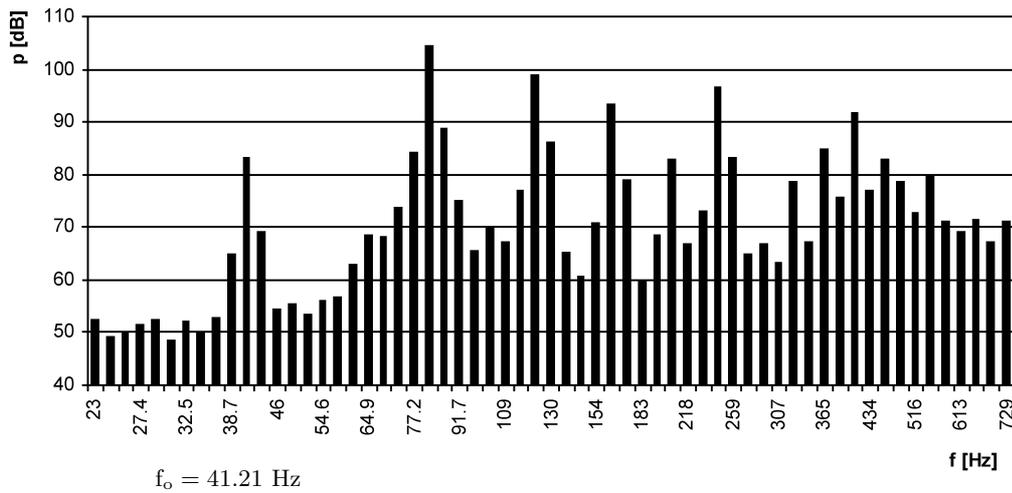


Fig. 13. Spectrum of sound E_1 — emitted by an idle string of the contrabass.

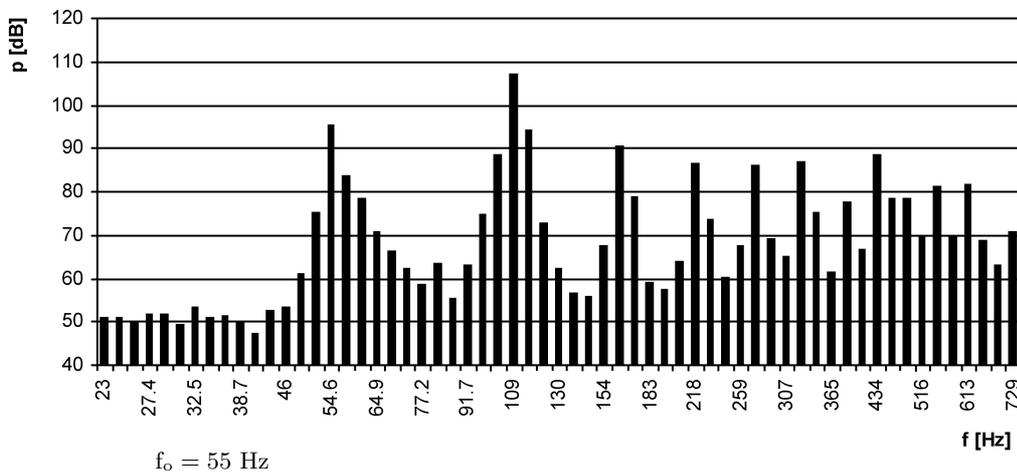


Fig. 14. Spectrum of sound A_1 — emitted by an idle string of the contrabass.

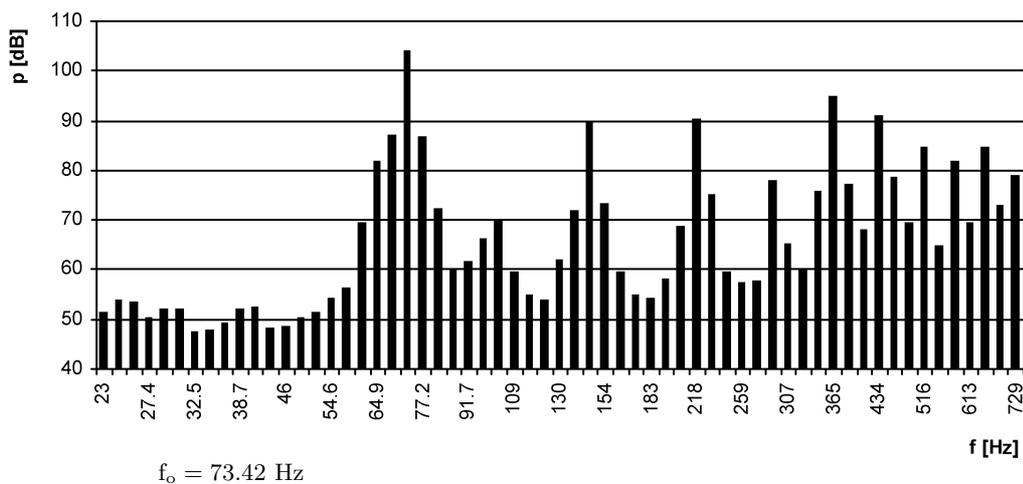


Fig. 15. Spectrum of sound D — emitted by an idle string of the contrabass.

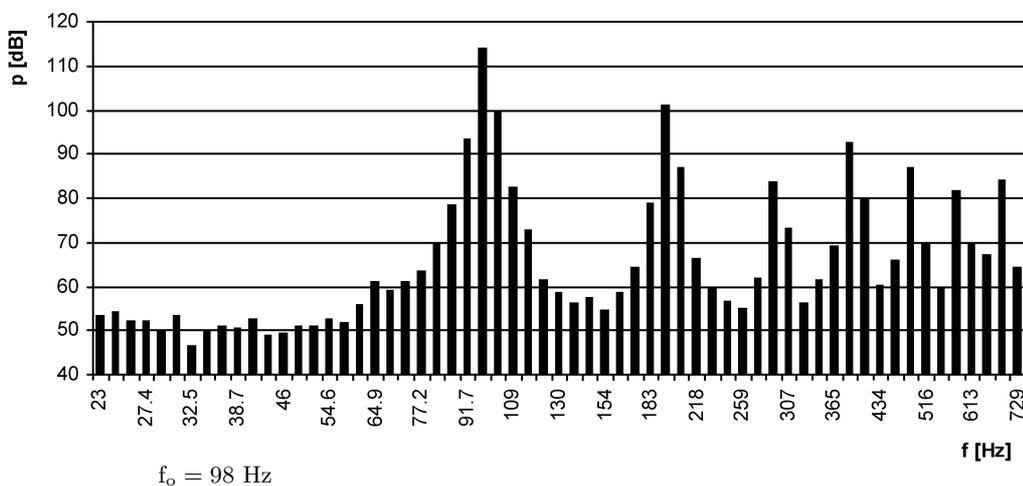


Fig. 16. Spectrum of sound G — emitted by an idle string of the contrabass.

Differences in a harmonic structure of individual sounds of the contrabass can be observed in those Figures. An interesting phenomenon is also noticeable: amplitudes of basic components of E_1 and A_1 sounds are much smaller than amplitudes of their low harmonic sounds. Probably frequencies of the basic components of the lowest sounds are behind the frequency range of the resonant chamber of the contrabass. However, regardless of that, the basic components of the lowest sounds of the contrabass are quite well heard. Such situation occurs due to one of the psycho-acoustic abilities of a hearing organ, which causes that the impression of hearing basic components is being formed in a human brain on the bases of the higher harmonic sounds. The basic component is the

dominant component in spectra of the sound D and of the higher sounds emitted by a contrabass.

Frequencies of the basic components of sounds emitted by idle strings of the contrabass are as follows: E_1 — 41.21 Hz, A_1 — 55 Hz, D — 73.42 Hz, G — 98 Hz and C — 130.82 Hz.

5. Conclusions

An application of the Reciprocity Principle for the examination of certain acoustic parameters of the contrabass was presented in the paper. Vibroacoustic transfer functions obtained from direct and reciprocity measurements show quite good agreement in the frequency range concerning sounds from the basic part of the instrument scale. Verification of reciprocity relations of vibroacoustic transfer functions between the bridge of the contrabass and the observation point enables investigation of several acoustic parameters of the instrument. However, since the investigations were performed on one contrabass only it is not possible — at the moment — to compare its acoustic properties with the ones of other instruments. In order to be able to utilise the obtained results for the acoustic estimation of musical instruments it is necessary to examine several instruments and apply many variants of a testing procedure as well as gaining considerable experience in the interpretation of results. Nonetheless it can be stated that the acoustic examination of musical instruments is an additional practical application of the vibroacoustic Reciprocity Principle. It would be reasonable to utilise this Principle in cases where investigations performed by means of other methods are more difficult and time consuming.

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