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VIBROACOUSTIC ANALYSIS OF A SIMPLY SUPPORTED RECTANGULAR PLATE OF A POWER TRANSFORMER CASING

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This paper deals with some vibroacoustic processes occurring within the power transformer casing. The acoustic system is modeled by a simply supported rectangular plate embedded within the planar rigid baffle. The radiation directivity pattern of the system is specially focused on since it will be useful for the determination of the eigenfunctions of the system; the latters are necessary for some further analysis of the total sound power radiated by the system and the resultant distribution of the sound pressure radiated as well as for some further investigations on the noise control of the system.

1. Introduction

The noise emitted by power transformers is a significant health and environmental problem. The vibroacoustic energy is radiated by a power transformer casing in consequence of numerous factors such as some electrodynamic and magnetostriction forces of the transformer core, the cooling system, and others. The problem was discussed by the authors. The elements of a power transformer casing can be modeled using some thin rectangular plates. The sound pressure distribution in the Fraunhoffer zone of a simply supported rectangular plate embedded into a rigid baffle was analyzed in the case of the free vibrations among others by WALLACE [1]. The author expressed the radiation resistance as a spherical surface integral of the square of sound pressure. The elementary form of the expression was proposed for the very low frequencies of radiated waves only. The radiation efficiency of a vibrating rectangular plate was analyzed by LEVINE [2]. He proposed some highly efficient results valid within the high frequency range. The efficiency was also analyzed by LEPPINGTON, BROADBENT and HERON [3], [4]. Berry, Guyader and Nicolas analyzed the radiation efficiency of a rectangular panel with some arbitrary boundary conditions but presented some elementary formulations for the rigid body modes only [5]. Lomas and Hayek derived the radiation resistance of a rectangular plate in its elementary form valid within the very narrow range of low frequencies [6].

This paper deals with a theoretical analysis of the sound radiation by a simply supported rectangular plate covered by a power transformer casing. The directivity pattern in the Fraunhoffer zone is specially focused on, since its elementary formulation can be useful for any further analysis of the total sound power radiated by the acoustic system as well as for its noise control. Therefore, the results presented herein will finally be used to minimize the noise generated due to some vibroacoustic processes occurring within the power transformer casing.

2. Free vibrations of the system

The problem of some free vibrations of a thin simply supported rectangular plate, provided that the plate material is isotropic and the boundary conditions are uniform along the entire edge, was resolved and reported in the literature (cf., Refs. [7], [8]). The boundary conditions of a simply supported rectangular plate embedded within a flat rigid infinite baffle can be formulated as follows (cf., Fig. 1):

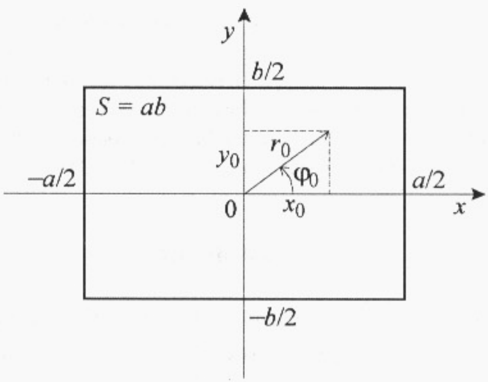


Fig. 1. A rectangular plate in the Cartesian coordinates.

$$W(x, y)|_{x=\pm a/2} = 0, \qquad \frac{\partial^2}{\partial x^2} W(x, y)|_{x=\pm a/2} = 0,$$
$$W(x, y)|_{y=\pm b/2} = 0, \qquad \frac{\partial^2}{\partial y^2} W(x, y)|_{y=\pm b/2} = 0, \tag{1}$$

where: $W(x, y)$ – transverse deflection distribution, whereas the eigenfrequencies of the system are defined by:

$$\omega_{mn} = \beta_{mn}^2 \sqrt{D/\rho h} = \pi^2 [(m/a)^2 + (n/b)^2] \sqrt{D/\rho h}, \qquad m,n = 1,2,... \tag{2}$$

where β_{mn} – corresponding eigenvalue, $D = Eh^3/[12(1-\nu^2)]$ – bending stiffness of the plate, E, h, ν, ρ, a, b – the plate's parameters such as Young modulus, thickness, Poisson ratio, density, and geometric sizes, respectively. The eigenfrequencies ω_{mn}, valid for some three different thickness values of the plate, i.e. $h = \{2, 4, 8\}$ [mm], and its geometric sizes $a \times b = 1{,}32 \times 0{,}52$ [m], have been listed in Table 1.

Table 1. Eigenfrequencies ω_{mn} of simply supported rectangular plate for $m, n = 1, 2, ...$; and three sample values of the plate's thickness h (cf., Eq. (2)).

h = 2 [mm]	n = 1	n = 2	n = 3	n = 4
m = 1	142.48	227.97	370.45	569.92
m = 2	484.43	569.92	712.40	911.87
m = 3	1054.35	1139.84	1282.32	1481.79
m = 4	1852.24	1937.73	2080.21	2279.68

h = 4 [mm]	n = 1	n = 2	n = 3	n = 4
m = 1	284.96	455.94	740.89	1139.84
m = 2	968.86	1139.84	1424.80	1823.74
m = 3	2108.70	2279.68	2564.64	2963.58
m = 4	3704.47	3875.45	4160.41	4559.35

h = 8 [mm]	n = 1	n = 2	n = 3	n = 4
m = 1	569.92	911.87	1481.79	2279.68
m = 2	1937.73	2279.68	2849.60	3647.48
m = 3	4217.40	4559.35	5129.27	5927.16
m = 4	7408.95	7750.90	8320.82	9118.71

The mode shape of the mode mn can be formulated as

$$W_{mn}(x, y) = A_{mn} \sin(m\pi x/a)\, \sin(n\pi y/b), \tag{3}$$

where the modenumbers are $m, n = 1, 2, ...$, for $0 \le x \le a$, and $0 \le y \le b$ (cf., Refs. [7], [9]). Substituting $x = x_0 + a/2$ and $y = y_0 + b/2$ results in the following form of the mode shape of the system:

$$W_{mn}(x_0, y_0) = A_{mn} \sin\frac{m\pi}{a}(x_0 + a/2) \sin\frac{n\pi}{b}(y_0 + b/2),$$
$$m, n = 1, 2,... \; |x_0| \le a/2 \; |y_0| \le b/2 \tag{4}$$

valid in the coordinate system presented in Fig. 1. The integral constant A_{mn} has been computed analytically from the normalization condition $\int_S W_{mn}^2(P)\,dS = S$, where P – a point of the plate surface of (x, y) coordinates, $S = ab$ – the plate's area. The condition assumes the form of

$$A_{mn} \int_{-a/2}^{a/2} \sin^2\frac{m\pi}{a}\left(x_0 + \frac{a}{2}\right) dx_0 \int_{-b/2}^{b/2} \sin^2\frac{n\pi}{b}\left(y_0 + \frac{b}{2}\right) dy_0 = ab \tag{5}$$

and after integration results in $A_{mn} = 4$. Some sample mode shapes of the system are presented in Fig. 2.

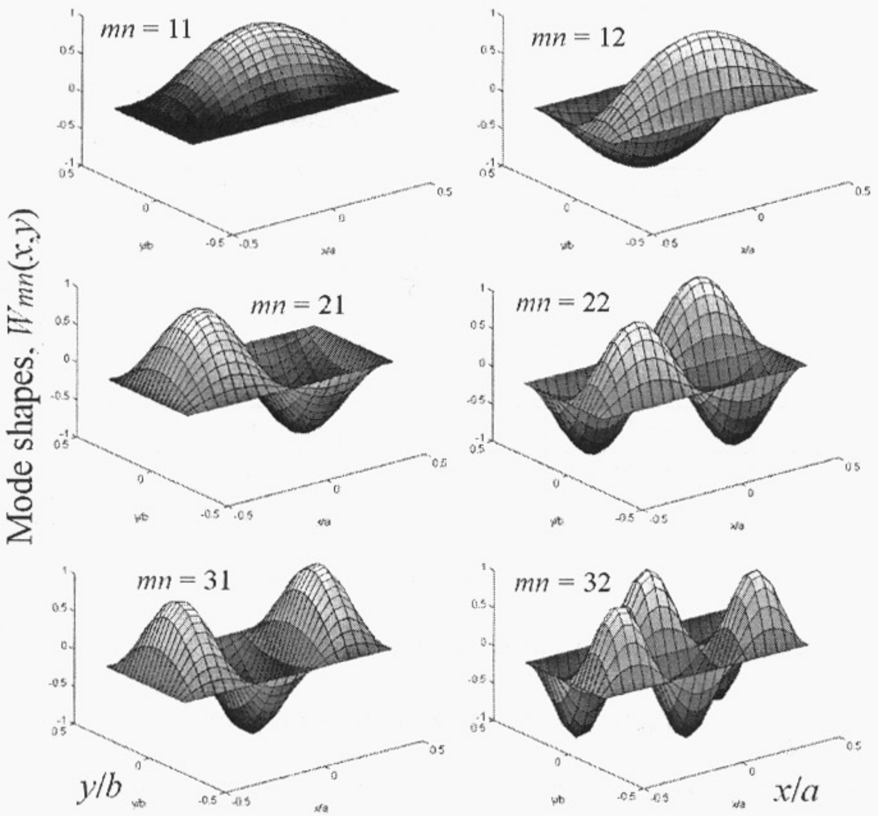


Fig. 2. Mode shapes of a simply supported rectangular plate for some sample modenumbers *mn*.

3. Sound pressure distribution in the Fraunhoffer zone

The acoustic potential for a surface source can be formulated in the Fraunhoffer zone as (cf., Ref. [9]):

$$\phi(\mathbf{r},t) = \phi(R, \upsilon, \varphi, t) = \frac{1}{2}\frac{\exp[i(\omega t - kR)]}{R}\int_{S_0} v(\mathbf{r}_0)\exp[ikr_0\cos(\mathbf{r},\mathbf{r}_0)]\,dS_0, \qquad (6)$$

where: $\cos(\mathbf{r},\mathbf{r}_0) = \sin\upsilon\cos(\varphi - \varphi_0)$, $\mathbf{r} = (R, \upsilon, \varphi)$ – radius vector of a field point in the spherical coordinates, $\mathbf{r}_0 = (r_0, \varphi_0) = (x_0, y_0)$ – radius vector of a point on the plate's surface in the Cartesian coordinates (cf., Fig. 3). In the case of some time harmonic processes, the following time dependencies for the distributions of vibration velocity of an acoustic particle and sound pressure radiated can be formulated as:

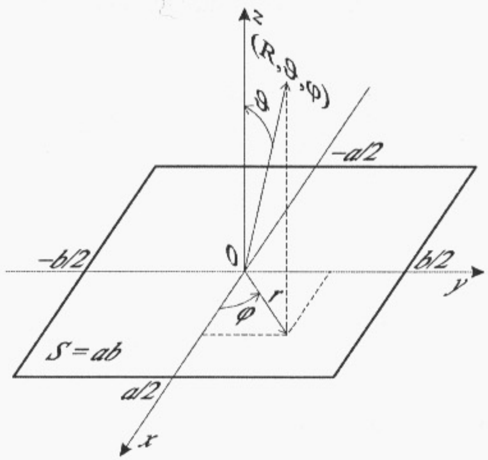


Fig. 3. The use of spherical coordinates to describe the sound field radiated by a thin rectangular plate.

$$v(\mathbf{r}, t) = \rho_0\frac{\partial}{\partial t}w(\mathbf{r}, t) = i\omega\, w(\mathbf{r}, t), \qquad w(\mathbf{r}, t) = W(\mathbf{r})\exp(i\omega t),$$
$$p(\mathbf{r}, t) = \rho_0\frac{\partial}{\partial t}\phi(\mathbf{r}, t) = i\omega\,\phi(\mathbf{r}, t), \qquad (7)$$

for any field point determined by the vector radius $\mathbf{r}$. Thus the sound pressure radiated in the Fraunhoffer zone may be formulated as follows:

$$p(R, \upsilon, \varphi, t) = -\rho_0\omega^2\frac{\exp[i(\omega t - kR)]}{2\pi R} \qquad (8)$$

$$\cdot \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} W(x_0, y_0) \exp\left[ik \sin \upsilon (x_0 \cos\varphi + y_0 \sin\varphi)\right] dx_0\, dy_0. \tag{8}$$

[cont.]

Let us introduce denotations: $\alpha_a = m\pi / a$, $\beta_a = ka \sin\upsilon \cos\varphi$, to compute the values of two integrals. The first integral is:

$$\begin{aligned} u_1 &= \int_{-a/2}^{a/2} \sin \alpha_a (x_0 + a/2) \exp(ix_0 \beta_a / a)\, dx_0 \\ &= ie^{-im\pi/2} \int_{-a/2}^{a/2} \cos(\alpha_a - \beta_a / a) x_0\, dx_0 - ie^{-im\pi/2} \int_{-a/2}^{a/2} \cos(\alpha_a + \beta_a / a) x_0\, dx_0. \end{aligned} \tag{9}$$

If, moreover, $\alpha_a \neq \beta_a / a$, it is possible to formulate it briefly as

$$u_1 = \frac{i\alpha}{m\pi} \left[e^{-m\pi/2} \frac{\sin\frac{1}{2}(m\pi - \beta_a)}{1 - \beta_a/(m\pi)} - e^{-m\pi/2} \frac{\sin\frac{1}{2}(m\pi + \beta_a)}{1 + \beta_a/(m\pi)} \right], \tag{10}$$

and finally as:

$$u_1 = -i \frac{2a}{m\pi} \frac{\psi_m(\beta_a/2)}{1 - (\beta_a/m\pi)^2}, \tag{11}$$

where $\psi_m = \begin{cases} i\cos & \text{for } m = 1,3,5,\ldots, \\ \sin & \text{for } m = 2,4,6,\ldots. \end{cases}$

If, however, $\alpha_a = \beta_a / a$, then

$$u_1 = \int_0^{a/2} \sin^2 \frac{m\pi}{a} \left(x_0 + \frac{a}{2} \right) e^{im\pi x_0/a}\, dx_0 = -\frac{1}{2} a (-i)^{m+1}. \tag{12}$$

An analogous method leads to the value of the second integral:

$$u_2 = \int_{-b/2}^{b/2} \sin \alpha_b \left(y_0 + \frac{b}{2} \right) \exp\left(i \frac{\beta_b}{b} y_0 \right) dy_0 = -i \frac{2b}{n\pi} \frac{\psi_n(\beta_b/2)}{1 - (\beta_b/n\pi)^2}, \tag{13}$$

provided that the following denotations have been introduced: $\alpha_b = m\pi / b$ and $\beta_b = kb \sin\upsilon \sin\varphi$, for $\alpha_b \neq \beta_b / b$. The sound pressure amplitude $p_{mn}(R, \upsilon, \varphi)$, based on Eq. (8), and on the values of the integrals (11) and (13) can now be formulated as:

$$p_{mn}(R,\upsilon,\varphi) = -\rho_0 c \frac{ke^{-ikR}}{R}\omega_{mn}A_{mn}\frac{ke^{-ikR}}{R}\frac{u_1 u_2}{2\pi}$$
$$= 2\rho_0 c\,\omega_{mn}A_{mn}\frac{k\,ab}{\pi^3 nm}\frac{e^{-ikR}}{R}\frac{\psi_m(\beta_a/2)}{1-(\beta_a/m\pi)^2}\frac{\psi_n(\beta_b/2)}{1-(\beta_b/n\pi)^2}. \tag{14}$$

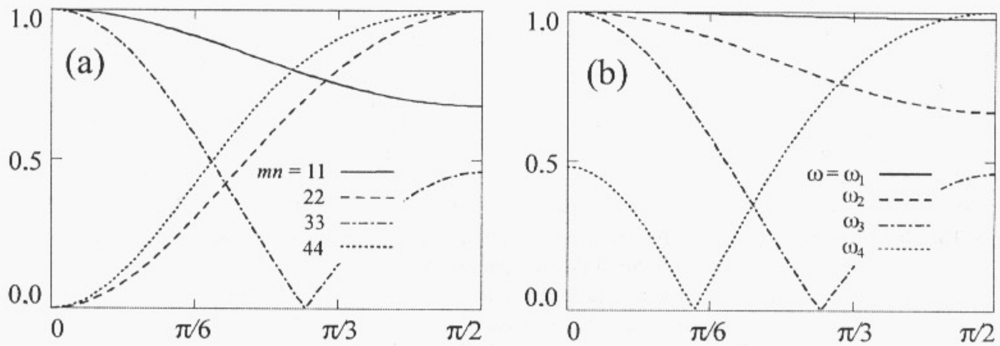
The directivity pattern $\Re(\upsilon,\varphi)$ has been defined by normalizing the sound pressure modulus from Eq. (14) by the sound pressure modulus in the (υ_0, φ_0) direction for which the modulus assumes its maximal value

$$\Re_m(\upsilon,\varphi) = \frac{|p_m(R,\upsilon,\varphi)|}{\max\limits_{\upsilon,\varphi}|p_m(R,\upsilon,\varphi)|}, \tag{15}$$

which is valid in the Fraunhoffer zone.

4. Numerical results

The numerical analysis for the directivity pattern of the acoustic system has been performed basing on Eqs. (14) and (15). Some sample curves for the modal values of the magnitude have been plotted and presented in Figs. 4 and 5. Figure 4(a) shows that the radiated sound pressure assumes its maximal values in the plate's main direction $\upsilon = 0$ for the odd modenumbers $m = n = 1,3,...$, and an inverse situation for the even modenumbers $m = n = 2,4,...$. In Fig. 4(b), the directivity of a single mode is plotted for some different radiation frequencies, whereas Figs. 4(c) and 4(d) illustrate the directivity for some different values of the plate's thickness and modenumbers $m = n = 3,4$. The radiation directivity arises with an increase in the plate's thickness value as well as with an increase in the radiation frequency, as it should. The radiation directivity for some different asymmetric modenumbers $m \neq n$ has been plotted in Figs. 5(a) and 5(b). The dependence of the magnitude on the angle φ has been illustrated in Figs. 5(c) and 5(d) for the sample modes of the plate (3,2) and (3,3), respectively.



[Fig. 4 a, b]

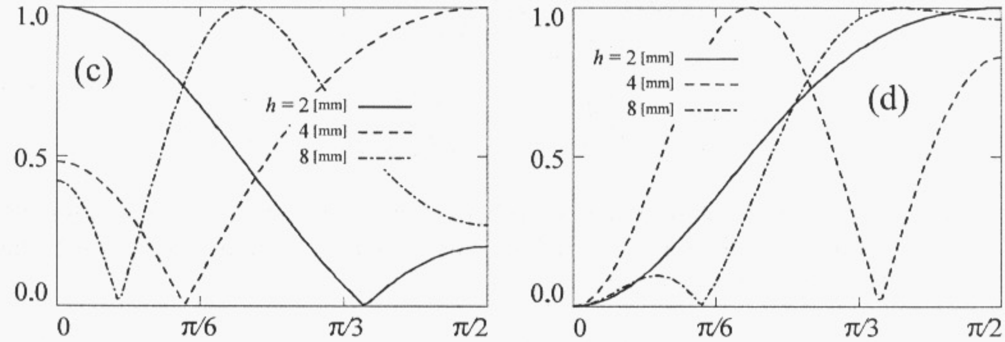


Fig. 4. The directivity radiation pattern for some sample four mode shapes of a simply supported rectangular plate for the following parameter values:

(a) $m = n = 1,2,3,4$; $\omega = \omega_{33}$; $\varphi = \pi/2$; $h = 4$ [mm];

(b) $m = n = 3$; $\omega = \omega_{11},\ldots,\omega_{44}$; $\varphi = \pi/2$; $h = 4$ [mm];

(c) $m = n = 3$; $\omega = \omega_{44}$; $\varphi = \pi/2$; $h = \{2,4,8\}$ [mm];

(d) $m = n = 4$; $\omega = \omega_{44}$; $\varphi = \pi/2$; $h = \{2,4,8\}$ [mm].

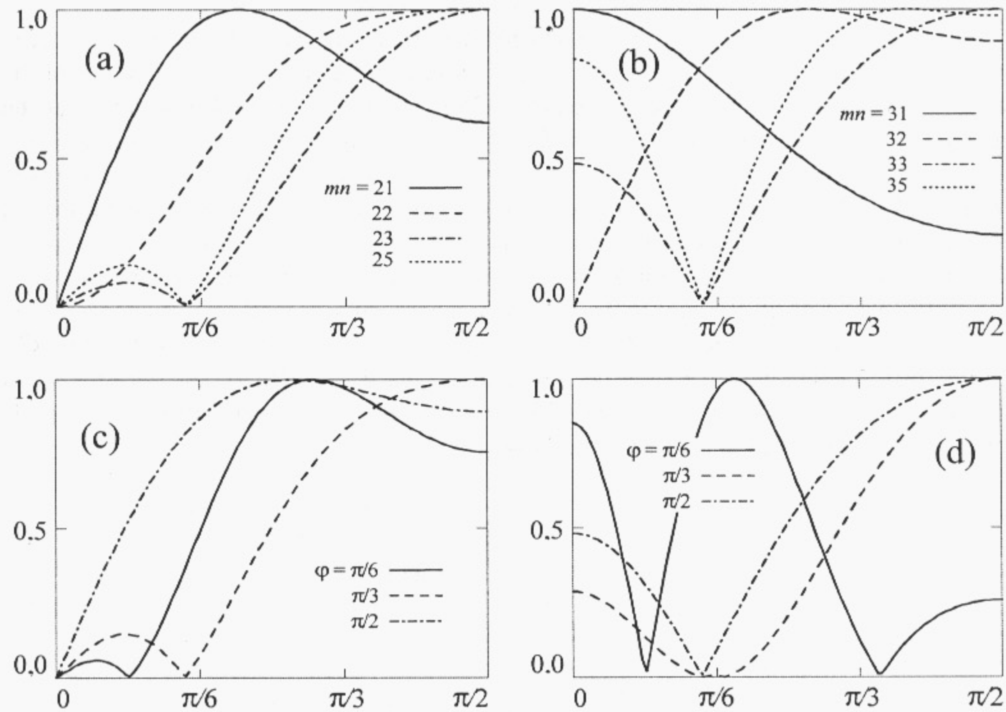


Fig. 5. The directivity radiation pattern for some sample four mode shapes of a simply supported rectangular plate for the following parameter values:

(a) $m = 2$, $n = 1,2,3,5$; $\omega = \omega_{44}$; $\varphi = \pi/2$; $h = 4$ [mm];

(b) $m = 3$, $n = 1,2,3,5$; $\omega = \omega_{44}$; $\varphi = \pi/2$; $h = 4$ [mm];

(c) $m = 3$, $n = 3$; $\omega = \omega_{44}$; $\varphi = \{\pi/6, \pi/3, \pi/2\}$; $h = 4$ [mm];

(d) $m = n = 3$; $\omega = \omega_{44}$; $\varphi = \{\pi/6, \pi/3, \pi/2\}$; $h = 4$ [mm].

5. Concluding remarks

The radiation directivity pattern and the radiated sound pressure distributions have been derived for the Fraunhoffer zone. The elementary formulations presented in this paper are necessary to determine the eigenfunctions of the acoustic system discussed and will further be used to analyze the radiated sound pressure distribution of the system as well as the total sound power radiated by the excited plate. The results presented herein will also be useful for the noise control of the system.

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