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SOUND RADIATION BY A HOLLOW CIRCULAR ELASTIC CYLINDER ROTATED IN WATER WITH A VARIABLE ANGULAR VELOCITY

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In this paper the sound radiation from an elastic circular cylinder of infinite length rotated with a non-uniform angular velocity in water is studied. The cylinder is empty inside. Exact solutions of the equations which describe the hydroelastic interaction are obtained using the Fourier transform over time. Numerical examples show that the spectral structure of the sound radiation from an elastic tube is more complicated than that of a solid cylinder. In particular, the resonances of this structure are essentially dependent on the thickness of the rotating object and are subject to the phenomena of dispersion.

1. Introduction

Cylindrical bodies rotating around its axis of symmetry with a variable angular velocity are often met in practice as elements of different technical devices. Examples of these components are rotors of electric motors and hydro generators, which angular velocity of rotation in a certain regime of exploitation is varying in time varied. In the mechanics, the fact of non-uniform rotation was taken into account only for the calculation of dynamical stresses and displacements [8, 12]. At the same time, the rotating bodies, e.g. the machine elements, are often in the acoustical medium and thus the surrounded medium influences on the dynamical characteristics of the body. On the other hand, the rotating deformable solids in the acoustical medium are the sources of sound radiation [13] and, in particular, of noise [10]. Therefore, the investigation of different aspects of the problem of structure-acoustical fluid (gas) interactions are of large theoretical and practical interest.

In this article, the attention was focused mainly on the structure of the radiate wave field excited in the surrounding medium by a rotating hollow cylinder. At first we investigated the spectral characteristics of the radiated sound. In the numerical examples for the case of the steel-water interactions, the dependence of the sound pressure amplitude on the frequency and the cylindrical tube thickness was studied. It turned out that the radiated sound field consists of a series of resonances. In addition, the resonance dependence on the cylinder thickness is essentially dispersive. This effect was also illustrated by numerical calculations.

The non-constant angular velocity of the cylinder rotation is caused by the first and double sound harmonic excitations if the constant value of this velocity is modulated by the small amplitude sinusoidal oscillation over time. This is clearly illustrated by the numerical calculations of the intensity of the radiated acoustical wave for different values of the disturbance angular velocity frequency and the tube thickness. We received two series of amplitude resonances, namely, at the fundamental frequencies corresponding to the resonances of the spectrum and at frequencies two times smaller the main ones.

2. Spectral characteristics

Let us consider an elastic hollow cylinder of infinite length immersed in a compressible ideal (non-viscous) fluid. The cylinder is empty inside and rotates with a variable angular velocities around its axis of symmetry. In consequence of the non-uniform rotation, a centrifugal force varying over time arises [9]. Then in the material of the object, axially symmetric converging and diverging cylindrical elastic waves of the longitudinal and shear types are generated. Simultaneously, in the surrounding fluid medium, sound waves excited by the radial vibration of the outer cylindrical surface are radiated. The intensity of these waves depends on the frequency and the relative amplitude of the oscillation of the angular velocity.

The equation of the dynamical equilibrium of the elastic hollow cylinder rotating with variable angular velocity around its unmoved axis of symmetry has the following form [6, 8, 12]

$$(\lambda + 2\mu)\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right) + r\rho_s \Omega^2(t) = \rho_s \frac{\partial^2 u}{\partial t^2} \qquad (b \le r \le a), \tag{1}$$

where $u \equiv u(r,t)$ is the radial displacement, $\Omega(t)$ is the time-variable angular velocity of the axial rotation of the body, $\lambda,\ \mu$ are the Lamé parameters and ρ_s is the density of the elastic material, r is the radial co-ordinate with the origin on the axis of symmetry, t is the time, a and b are the outer and inner radii of the tube.

The pressure in the fluid $p \equiv p(r,t)$ is defined by the wave equation [11]

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} \qquad (a \le r < \infty), \tag{2}$$

where c is the sound velocity.

At the surfaces of the cylinder, the following boundary conditions are satisfied:

$$\sigma_r + p = 0 \qquad (r = a), \tag{3}$$

$$\frac{\partial^2 u}{\partial t^2} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0 \qquad (r = a), \tag{4}$$

$$\sigma_r = 0 \qquad (r = b), \tag{5}$$

where ρ is the density of the fluid, $\sigma_r \equiv \sigma_r(r, t)$ is the radial elastic stress connected in the following way with the displacement [8]

$$\sigma_r = (\lambda + 2\mu)\frac{\partial u}{\partial r} + \lambda\frac{u}{r} \qquad (b \le r \le a). \tag{6}$$

Here we also take into account the relation between the pressure $p(r, t)$ and the particle velocities $v(r, t)$ in the acoustical fluid [11]

$$\frac{\partial v}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0 \qquad (a \le r < \infty) \tag{7}$$

and the condition of non-discontinuity of the interacting media

$$\frac{\partial u}{\partial t} = v \qquad (r = a). \tag{8}$$

For the study of the spectral characteristics of the radiated acoustic waves in the fluid, we introduce the following integral exponential Fourier transform over time [3]

$$\widetilde{f}(\omega) = \int\limits_{-\infty}^{\infty} f(t)e^{\omega t i}dt \qquad (- \infty < \omega < \infty) \tag{9}$$

and the inverse transform

$$f(t) = \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} \widetilde{f}(\omega)e^{-\omega t i}d\omega \qquad (- \infty < t < \infty), \tag{10}$$

where ω is the circular frequency, $i = \sqrt{-1}$.

Applying the above Fourier transform (9) to the Eqs. (1)–(6) and taking into account that all input and desired functions satisfy the causality principle [4], we obtain

$$\frac{d^2\widetilde{u}}{dr^2} + \frac{1}{r}\frac{d\widetilde{u}}{dr} + \left(k_L^2 - \frac{1}{r^2}\right)\widetilde{u} + rK_L^2 = 0 \qquad (b \le r \le a), \tag{11}$$

$$\frac{d^2\widetilde{p}}{dr^2} + \frac{1}{r}\frac{d\widetilde{p}}{dr} + k^2\widetilde{p} = 0 \qquad (a \le r < \infty). \tag{12}$$

Here $k = \omega/c$ is the wave number in the acoustical medium and $k_L = \omega/c_L$ is the wave number in the elastic body, c_L is the longitudinal wave velocity in the material of the cylinder, $c_L^2 = (\lambda + 2\mu)/\rho_s$, K_L is the additional "longitudinal wave number", introduced by the time modulation of the angular velocity of the cylinder rotation, $K_L^2 = \widetilde{\Omega}(\omega)/c_L^2$, where

$$\widetilde{\Omega}(\omega) = \int\limits_{-\infty}^{\infty} \Omega^2(t) e^{i\omega t} dt. \tag{13}$$

The solution of the Eqs. (11), (12) must obey the transformed boundary conditions:

$$\widetilde{\sigma}_r + \widetilde{p} = 0 \qquad (r = a), \tag{14}$$

$$\frac{d\widetilde{p}}{dr} = \rho\omega^2 \widetilde{u} \qquad (r = a), \tag{15}$$

$$\widetilde{\sigma}_r = 0 \qquad (r = b), \tag{16}$$

the Sommerfeld condition of the wave radiation at $r \to \infty$ [11]

$$\lim_{r\to\infty} \sqrt{r} \left(\frac{d\widetilde{p}}{dr} - ik\widetilde{p} \right) = 0 \tag{17}$$

and the relations

$$\widetilde{\sigma}_r = (\lambda + 2\mu)\frac{d\widetilde{u}}{dr} + \lambda \frac{\widetilde{u}}{r} \qquad (b \le r \le a), \tag{18}$$

$$\widetilde{v} = \frac{1}{i\omega\rho}\frac{d\widetilde{p}}{dr} \qquad (a \le r < \infty). \tag{19}$$

In the Fourier-transform space we obtain the exact solutions of the Eqs. (11), (12), (18) and (19) in the following forms:

$$\frac{1}{\lambda + 2\mu}\widetilde{p}(r,\omega) = \frac{K_L^2}{k_L^2} B H_0^{(1)}(kr) \qquad (a \le r < \infty), \tag{20}$$

$$\frac{1}{c_L}\widetilde{v}(r,\omega) = \frac{K_L^2}{k_L^2} i \frac{\kappa_s}{\kappa x_L^2} B H_1^{(1)}(kr) \qquad (a \le r < \infty), \tag{21}$$

$$\frac{1}{\lambda + 2\mu}\widetilde{\sigma}_r(r,\omega) = \frac{K_L^2}{k_L^2}\big[A_1 J_{02}^-(k_L r) + A_2\, N_{02}^-(k_L r) - 2(1-\alpha)\big] \qquad (b \le r < a), \tag{22}$$

$$\frac{1}{a}\widetilde{u}(r,\omega) = \frac{K_L^2}{k_L^2}\left[A_1 J_1(k_L r) + A_2 N_1(k_L r) - \frac{r}{a}\right] \qquad (b \le r < a), \tag{23}$$

where

$$\begin{aligned} J_{02}^-(k_L r) &= x_L\left[(1-\alpha)J_0(k_L r) - \alpha J_2(k_L r)\right], \\ N_{02}^-(k_L r) &= x_L\left[(1-\alpha)N_0(k_L r) - \alpha N_2(k_L r)\right]. \end{aligned} \tag{24}$$

In the formulas (20)–(24) A_1, A_2 and B are the constants of integration, $J_n(z)$ $(n = 0, 1, 2)$ are the Bessel functions, $N_n(z)$ $(n = 0, 1, 2)$ are the Neimann functions, $H_n^{(1)}(z)$ $(n = 0, 1)$ are the Hankel functions of the first kind, $\kappa = \rho c$ and $\kappa_s = \rho_s c_L$ are the wave resistances in the fluid medium and in the elastic material, respectively, $\alpha = c_T^2/c_L^2$, $c_T^2 = \mu/\rho_s$, where c_T is the shear wave velocity in the material of the cylinder, $x_L = k_L a$.

Satisfying the boundary conditions (14)–(16) and determining the unknown constant B, we obtained for the acoustical pressure in the fluid:

$$\widetilde{p}(r,\omega) = (\lambda + 2\mu) X_L^2 P(r,\omega) \qquad (a \le r < \infty), \tag{25}$$

where

$$P(r,\omega) = \frac{\Delta_B}{x_L^2 \Delta} H_0^{(1)}(kr), \tag{26}$$

$$\begin{aligned}
\Delta &= H_0^{(1)}(x)\Delta_0 - \frac{\kappa_s}{\kappa x_L} H_1^{(1)}(kr)\Delta_1, \\
\Delta_0 &= J_1(x_L)N_{02}^-(y_L) - N_1(x_L)J_{02}^-(y_L), \\
\Delta_1 &= J_{02}^-(x_L)N_{02}^-(y_L) - N_{02}^-(x_L)J_{02}^-(y_L), \\
\Delta_B &= -\frac{4}{\pi}(1-\alpha) + x_L\left[J_2(x_L)N_{02}^-(y_L) - N_2(x_L)J_{02}^-(y_L)\right], \\
X_L &= K_L a, \qquad y_L = k_L b.
\end{aligned} \tag{27}$$

In particular, for the far field $kr \gg 1$, using the asymptotic expression [1]

$$H_0^{(1)}(kr) \approx \sqrt{\frac{2}{\pi i k r}} e^{ikr}, \tag{28}$$

we get

$$\widetilde{p}(r,\omega) = f(k)\frac{e^{ikr}}{\sqrt{r}}, \tag{29}$$

where $f(k)$ is the amplitude of radiation

$$f(k) = \sqrt{\frac{2}{\pi i k}} (\lambda + 2\mu) \frac{X_L^2}{x_L^2} \frac{\Delta_B}{\Delta}. \tag{30}$$

In the case of a solid cylinder when $b \to 0$ $(1/\varepsilon \to \infty,\ \varepsilon = b/a)$, we obtained from (27)

$$\begin{aligned}
&\Delta_0 \approx J_1(x_L)N_{02}^-(y_L), \qquad \Delta_1 \approx J_{02}^-(x_L)N_{02}^-(y_L), \\
&\Delta_B \approx x_L J_2(x_L)N_{02}^-(y_L), \qquad \Delta = -\Delta^\infty N_{02}^-(y_L), \\
&\Delta^\infty = J_1(x_L)H_0^{(1)}(x) - \frac{\kappa_s}{\kappa x_L} J_{02}^-(x_L)H_1^{(1)}(kr),
\end{aligned} \tag{31}$$

and

$$P(r,\omega) = P^{\infty}(r,\omega) = -\frac{J_2(x_L)}{x_L\Delta^{\infty}} H_0^{(1)}(kr). \tag{32}$$

3. The time characteristics

Let the oscillation of the angular velocity $\Omega(t)$ has the form:

$$\Omega(t) = \Omega_0(1 + \varepsilon_0 \sin \omega_0 t) \qquad (-\infty < t < \infty), \tag{33}$$

where Ω_0 is the constant angular velocity of the cylinder rotation, ε_0 is a small non-dimensional parameter characterizing the amplitude of the disturbance of this velocity, ω_0 is the circular frequency. For example, if the source putting the cylinder in the rotatory movement is electrical current, the oscillations of the angular velocity are caused by variations of this electrical current near its constant value. Then the Fourier-transform (13) for $\Omega^2(t)$ is obtained as [3]

$$\begin{aligned} X_L^2 = 2\pi X_{L0}^2 \{ &(1 + 0.5\varepsilon_0^2)\delta(\omega) - i\varepsilon_0[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)] \\ &- 0.25\varepsilon_0^2[\delta(\omega + 2\omega_0) + \delta(\omega - 2\omega_0)]\}, \end{aligned} \tag{34}$$

where $\delta(z)$ is the Dirac function, $X_{L0} = \Omega_0 a/c_L$.

The inverse Fourier-transform (10) applied to Eqs. (25) and (26) results in

$$\begin{aligned} \frac{1}{(\lambda + 2\mu)X_{L0}^2} p(r,t) = {} & (1 + 0.5\varepsilon_0^2) \lim_{\omega \to 0} P(r,\omega) \\ & - i\varepsilon_0[P(r, -\omega_0)\exp(i\omega_0 t) - P(r,\omega_0)\exp(-i\omega_0 t)] \\ & - 0.25\varepsilon_0^2[P(r, -2\omega_0)\exp(2i\omega_0 t) - P(r, 2\omega_0)\exp(-2i\omega_0 t)]. \end{aligned} \tag{35}$$

Using the asymptotical formulas [1]

$$\begin{aligned} J_n(z) &\approx \frac{1}{n!}\left(\frac{z}{2}\right)^n \qquad (n = 0,\ 1,\ 2), \\ H_0^{(1)}(z) &\approx -\frac{2i}{\pi} \ln \frac{2}{\gamma z}, \\ H_1^{(1)}(z) &\approx -\frac{2i}{\pi z} \qquad (z \to 0) \end{aligned} \tag{36}$$

and the properties of the cylindrical functions

$$\begin{aligned} J_n(-z) &= (-1)^n J_n(z) \qquad (n = 0,\ 1,\ 2), \\ H_0^{(1)}(-z) &= -H_0^{(1)^*}(z), \\ H_1^{(1)}(-z) &= H_1^{(1)^*}(z), \end{aligned} \tag{37}$$

we obtained

$$\lim_{\omega\to 0} P(r,\omega) = 0, \qquad P(r,-\omega)\exp(i\omega t) = [P(r,\omega)\exp(-i\omega t)]^*, \tag{38}$$

where γ is the Euler constant; the asterisk denotes the complex conjugated functions.

Thus the formula for the acoustical pressure generated by the rotatory movement of the hollow cylinder with modulated angular velocity may be presented in the form

$$\begin{aligned}\frac{1}{(\lambda+2\mu)X_{L0}^2}p(r,t) = &-2\varepsilon\,\mathrm{Im}[P(r,\omega_0)\exp(-i\omega_0 t)] \\ &- 0.5\varepsilon^2\mathrm{Re}[P(r,2\omega_0)\exp(-2i\omega_0 t)] \qquad (r \ge a).\end{aligned} \tag{39}$$

4. Analysis of the numerical results

For numerical calculations it is conveniently to introduce the following functions:

$$\begin{gathered} B = \frac{J_1(x)}{N_1(x)}, \qquad D = \frac{N_0(x)}{N_1(x)}, \\ E = \frac{J_0(x)}{N_0(x)}, \qquad A_j = \frac{J_1(x_L)}{J_2(x_L)}, \qquad A_n = \frac{N_1(x_L)}{N_2(x_L)}, \\ F_j(k_L r) = \frac{J_0(k_L r) - J_0(x_L)}{J_2(x_L)}, \qquad F_n(k_L r) = \frac{N_0(k_L r) - N_0(x_L)}{N_2(x_L)}, \\ G_j(k_L r) = \frac{J_2(k_L r)}{J_2(x_L)}, \qquad G_n(k_L r) = \frac{N_2(k_L r)}{N_2(x_L)}. \end{gathered} \tag{40}$$

Next Eq. (26) can be written in the form

$$P(r,\omega) = \frac{\widetilde{\Delta}_B}{x_L N_1(x)\widetilde{\Delta}} H_0^{(1)}(kr), \tag{41}$$

where

$$\begin{aligned} \widetilde{\Delta}_B &= \varphi_n^-(y_L) - \varphi_j^-(y_L), \\ \widetilde{\Delta} &= D(E+i)\widetilde{\Delta}_0 - \frac{\kappa_s}{\kappa}(B+i)\widetilde{\Delta}_1, \\ \widetilde{\Delta}_0 &= (1-\alpha)A_{jn} + \left[A_j\varphi_n^-(y_L) - A_n\varphi_j^-(y_L)\right], \\ \widetilde{\Delta}_1 &= -\frac{2\alpha(1-\alpha)}{x_L}A_{jn} + \left[\psi_j^-(x_L)\varphi_n^-(y_L) - \psi_n^-(x_L)\varphi_j^-(y_L)\right], \\ A_{jn} &= -A_j + A_n = \frac{2}{\pi x_L J_2(x_L) N_2(x_L)}, \end{aligned} \tag{42}$$

$$\varphi_s^-(k_L r) = (1-\alpha)F_s(k_L r) - \alpha G_s(k_L r), \qquad \varphi_s^-(x_L) = -\alpha,$$
$$\psi_s^-(k_L r) = \varphi_s^-(k_L r) + \chi_s, \qquad \chi_s = (1-\alpha)\left(\frac{2}{x_L}A_s - 1\right) \quad (s = j,\ n). \tag{42 [cont.]}$$

Similarly, the spectral distribution of the particle velocity in the acoustical medium $\widetilde{v}(r,\omega) = -i(\omega\rho)^{-1}(\partial\widetilde{p}/\partial r)$ can be expressed as follows:

$$\widetilde{v}(r,\omega) = c_L(K_L a)^2 V(r,\omega) \qquad (r \geq a), \tag{43}$$

where

$$V(r,\omega) = i\frac{\kappa_s}{\kappa}\frac{\widetilde{\Delta}_B}{N_1(x)\widetilde{\Delta}}H_1^{(1)}(kr), \qquad \lim_{\omega\to 0} V(r,\omega) = 0. \tag{44}$$

At the same time, in the pulse situation

$$\frac{1}{c_L X_{L0}^2}v(r,t) = -2\varepsilon_0\,\mathrm{Im}\,[V(r,\omega_0)\exp(-i\omega_0 t)] - \frac{1}{2}\varepsilon_0^2\mathrm{Re}[V(r,\,2\omega_0)\exp(-2i\omega_0 t)] \qquad (r \geq a). \tag{45}$$

For the estimation of the sound energy radiated in the acoustical medium it is necessary to calculate the time average of the power over the period $T_0 = 2\pi/\omega_0$

$$I = \frac{1}{T_0}\int_0^{T_0} p(r,t)v(r,t)dt \qquad (r \geq a). \tag{46}$$

Then, substituting $p(r,t)$ and $v(r,t)$ from the Eqs. (39), (45) in the Eq. (46) and taking into account that

$$\frac{1}{T_0}\int_0^{T_0}\exp(in\omega_0 t)dt = \begin{cases} 1, & n = 0, \\ 0, & n = \overline{-4,\,4}; \quad n \neq 0 \end{cases} \tag{47}$$

we obtain

$$I = 2\varepsilon_0^2\kappa_s c_L^2 X_{L0}^4\mathrm{Re}\,[P(r,\omega_0)V^*(r,\omega_0) + 0.0625\varepsilon_0^2 P(r,\,2\omega_0)V^*(r,\,2\omega_0)] \qquad (r \geq a). \tag{48}$$

The numerical calculations were carried out for the case of an Armco iron hollow cylinder ($\rho_s = 7700\ \mathrm{kg/m^3}$, $c_L = 5960\ \mathrm{m/s}$, $c_T = 3240\ \mathrm{m/s}$ [14]), immersed in the water ($\rho = 1000\ \mathrm{kg/m^3}$, $c = 1493\ \mathrm{m/s}$ [2]).

Figure 1 shows the modulus of the function $P(r,\omega)$ (in dB), characterizing the amplitude of acoustical pressure as function of the non-dimensional frequency $x = ka$ (the wave outer radius of the cylinder) and the geometrical parameter $\varepsilon = b/a$ (the relative

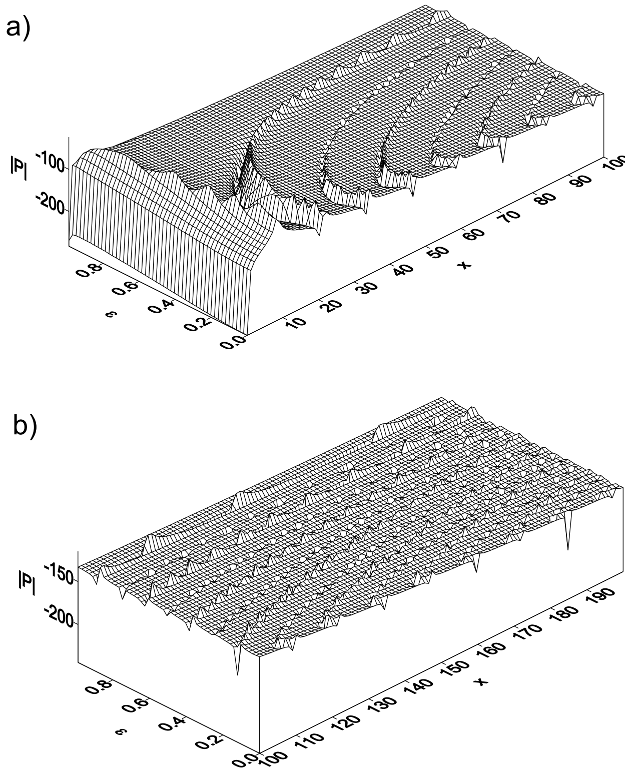


Fig. 1. The modulus of the acoustical pressure amplitude $P(r,\omega)$ (in dB) radiated by the metallic cylinder in the water for $r/a = 1$.

inner radius of the cylinder) at $r/a = 1$. On account of the reflection of the strain waves between the cylindrical surfaces, the frequency spectrum of the sound generated in the surrounding water have a brightly resonance character. Therefore, the resonance locations depend sufficiently on the thickness of the cylindrical objects. Namely, the sound waves of resonance frequencies are subject to geometrical dispersion when the tube becomes thinner. This means that the resonance locations are generally non-monotonic functions of the parameter ε. This effect is well illustrated in Fig. 2, where curves of identical levels of the sound spectrum amplitudes are plotted. As one can see, the first low frequency resonance is shifted towards the lower frequency range when the parameter ε decreases. This phenomenon also appears for the resonances of higher orders, however only for ε values not too large. The range of ε values for which the resonance frequency becomes lower is rapidly narrowed with increasing resonance order. Moreover, there are values of the cylindrical tube thickness for which the direction of the shift of the resonance curves motion changes, i.e. for continuously increasing ε, the resonances are shifted quickly toward high frequencies. As a matter of fact, we observe the appearance of radiated sound waves with a negative group velocity. These plots also show that the resonance amplitudes decrease rapidly with increasing resonance order.

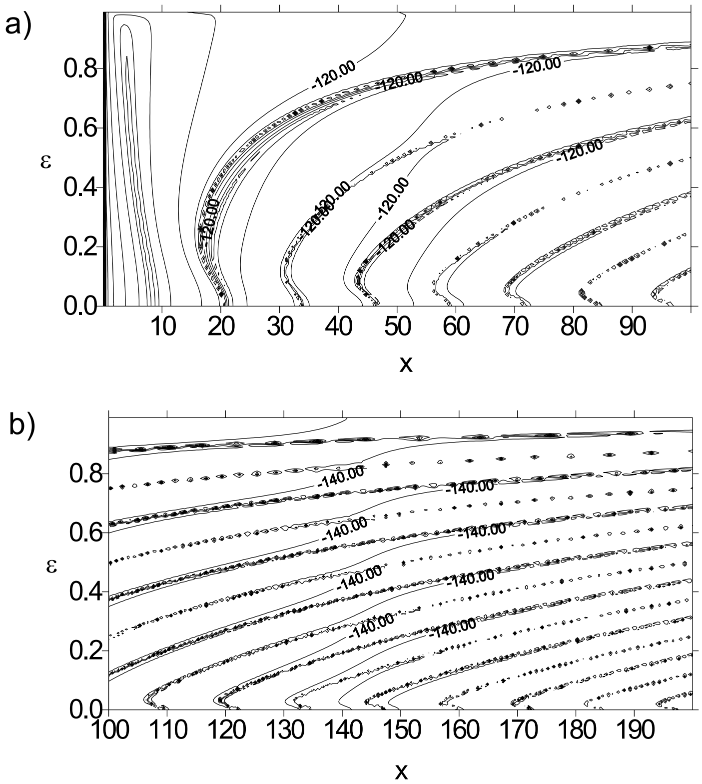
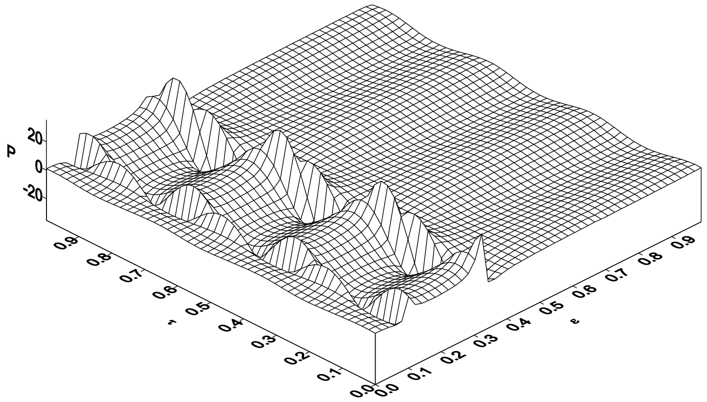


Fig. 2. The curves of the constant levels of the sound spectrum amplitudes for $r/a = 1$ (in dB).

In the case of pulses, we can see from Eq. (39) that the acoustical waves of two frequencies ω_0, and, $2\omega_0$ are radiated. Figure 3a shows the time dependence of the sound pressure (in Pa, $\tau = ct/a$) for different values of the thickness parameter ε and $x_0 = 20$ ($x_0 = k_0 a$, $k_0 = \omega_0/c$). The calculations are carried out for a pressure value far from the cylindrical surface, $r/a = 10$. The relative angular velocity is $X_{L0} = 0.001$ (e.g. a cylinder of the outer radius $a = 0.25\,\mathrm{m}$ rotates with the angular velocity $\Omega_0 = 1256\ \mathrm{rad/s}$) and the relative amplitude of the angular velocity modulation is $\varepsilon_0 = 0.1$. These plots illustrate the space resonances. In fact, as long as the values of ε are outside the resonance positions (cf. Figs. 1 and 2), the acoustical signals have low sinusoidal amplitudes. The picture is sharply changed when the ε parameter crosses the dispersive curves. Then the oscillations of the signals become good noticeable although for small ε_0 values the amplitudes scarcely reach 20 Pa. Figure 3b shows only for the second term of the expression (39), i.e. it displays the component with the double frequency. Here the above mentioned effect of space resonance is also demonstrated, but, it arises of course for other values of ε. The amplitudes of these oscillations are of two order lower because the parameter ε_0 is small again. In fact, such an additional signal is masked on the phone of the signal of frequency ω_0. However, its existence provides evidence that, a)



b)

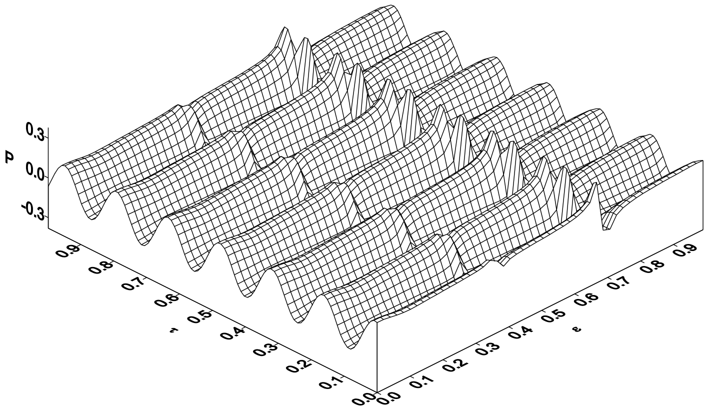


Fig. 3. The time dependence of the sound pressure (in Pa) calculated for the different values of the thickness parameter ε and for $r/a = 10$, $x_0 = 20$, $X_{0L} = 0.001$, $\varepsilon_0 = 0.1$ (a – the total signal; b – the signal at the frequency $2\omega_0$).

in our case, the sound radiation has the character of a wave field of the second harmonic. This is clearly shown in the Fig. 4 a which illustrates the sound radiation intensity [5]

$$N = 10 \lg(I/I_0), \qquad I_0 = 10^{-12}\ \mathrm{W/m^2}. \tag{49}$$

as the function of x_0 and ε. The calculations have been performed on the basis of Eq. (48) with $r/a = 1$, $X_{L0} = 0.001$, and $\varepsilon_0 = 0.3$. Figure 4a depicts the total intensity, while Fig. 4b represents only a part of it corresponding to the second component of the expression for I. These plots disclose also both the resonances of the radiation amplitudes and the dispersive character of the wave formation. More details concerning the structure of the radiation intensity as function of frequency are displayed in Fig. 5 (also in dB) for discrete values of ε (all other parameters are the same as in the case of Fig. 4). These illustrations show that the resonances are of high quality with fairly intensive amplitudes. The resonances of the double frequency are hard to notice; they can be

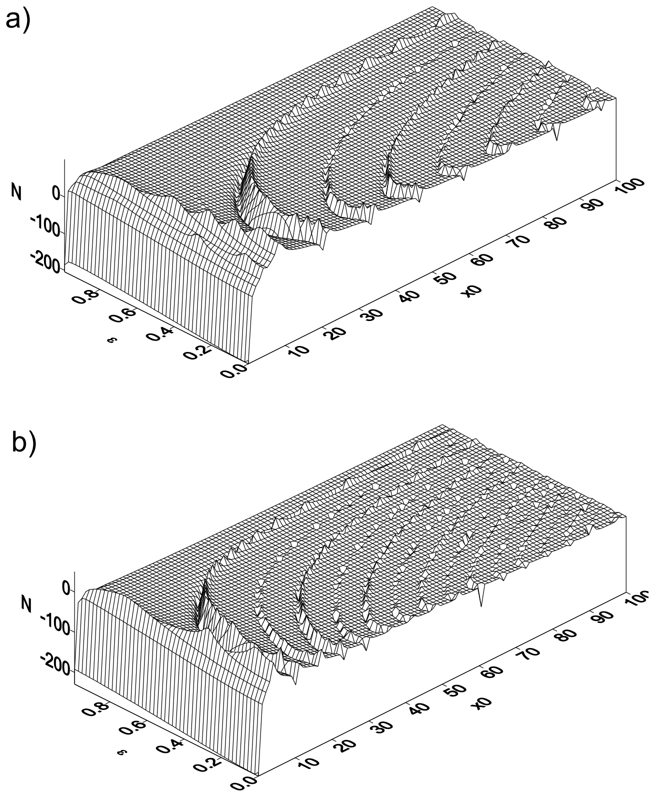


Fig. 4. The intensity of the sound waves as function of the frequency x_0 and the geometrical parameter ε for $r/a = 1$, $X_{L0} = 0.001$, $\varepsilon_0 = 0.3$ (a – the total intensity; b – the second component of the expression for I).

observed only in the low frequency range. The plots describe very well the motion of the resonance locations with the change of the geometrical parameter ε. As one can see, the first resonance line is shifted toward lower frequencies, extending and decreasing. All the other resonance lines diverge quickly and are shifted toward higher frequencies. This is connected with the reflection of the elastic waves from the boundary surfaces of the hollow cylinder. In Fig. 6, analogous curves for the intensity at secondary frequencies are drawn. The corresponding low-level resonances are practically masked in Fig. 5.

Finally, Fig. 7 are shows the distributions of the sound wave intensity near the cylindrical surface, $1 \le r/a \le 2$, when the elastic tube thickness is the continuously variable. The pictures are obtained for $x_0 = 5,\ 10,\ 25$ and 50. These plots are interesting examples of the clear expression for both the prime resonances (with x_0) and the secondary resonances (with $2x_0$). In other words, it is demonstrates the fact that continuous variation of the parameter ε results in a visible splash of the sound intensity for arbitrary frequencies of the radiation.

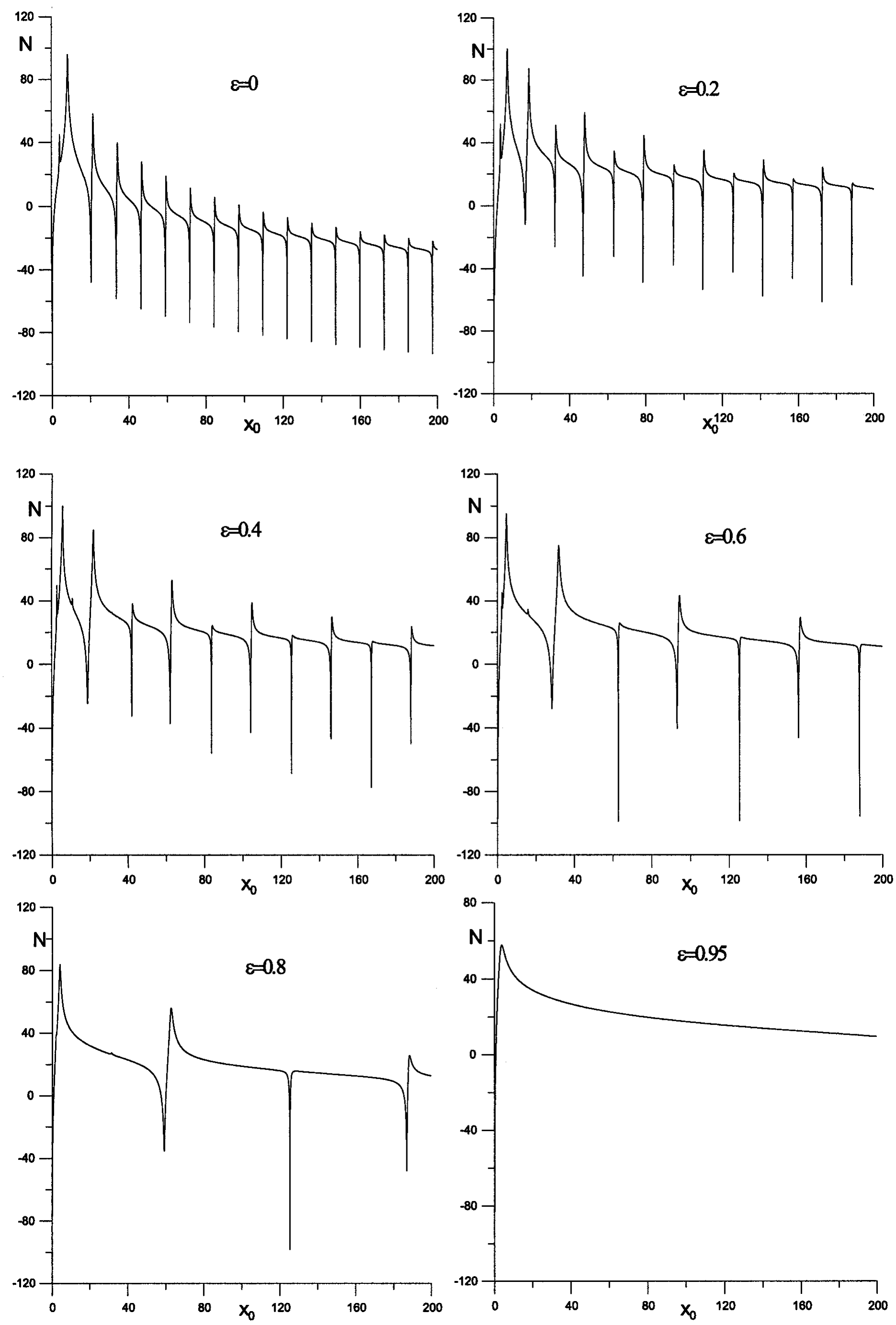


Fig. 5. The structure of the radiation intensity as function of the frequency for the different values of ε and for $r/a = 1$, $X_{L0} = 0.001$, $\varepsilon_0 = 0.3$.

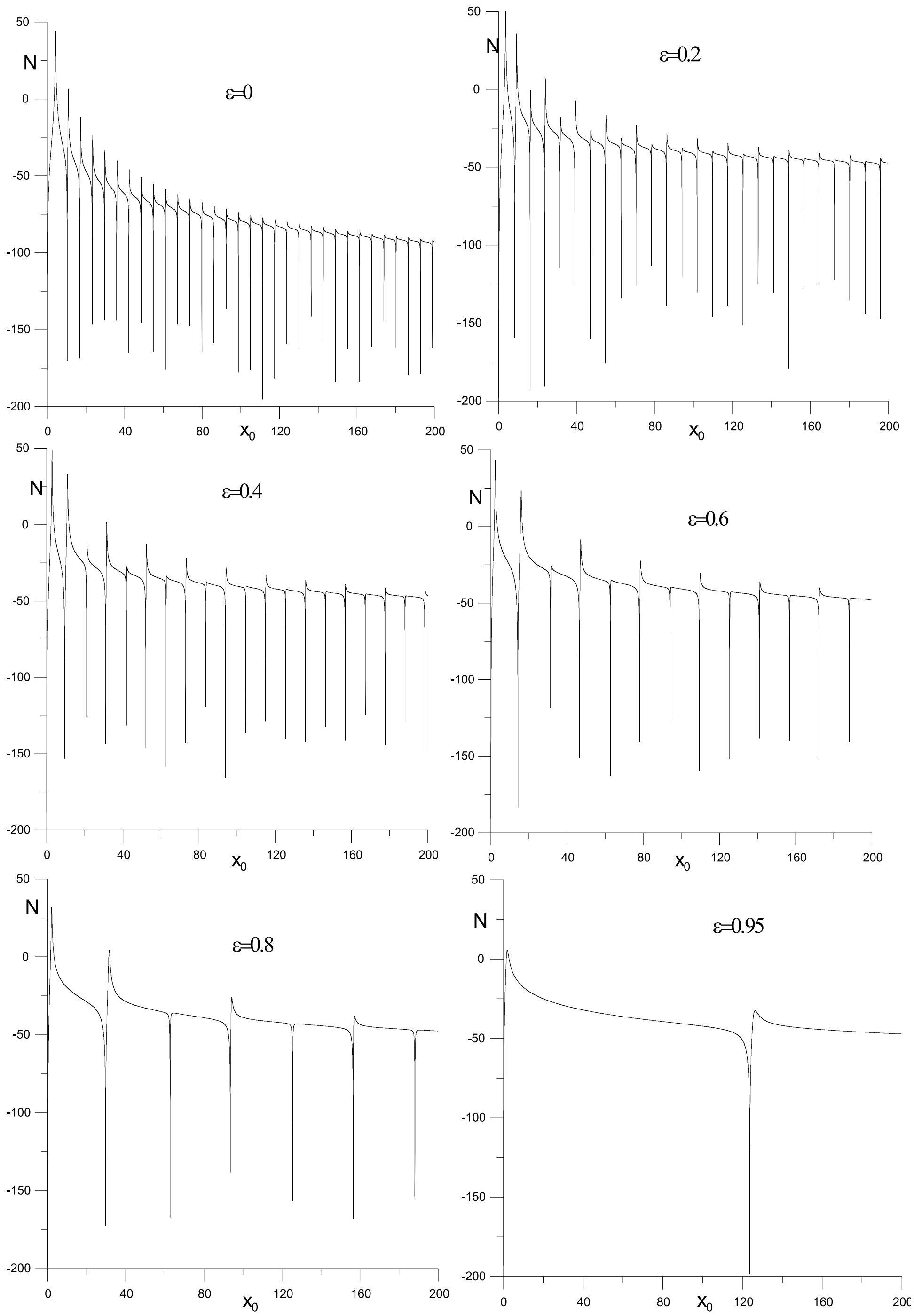


Fig. 6. The frequency characteristic of the radiation intensity component with $2x_0$ for different values of ε (all other parameters as in the Fig. 5).

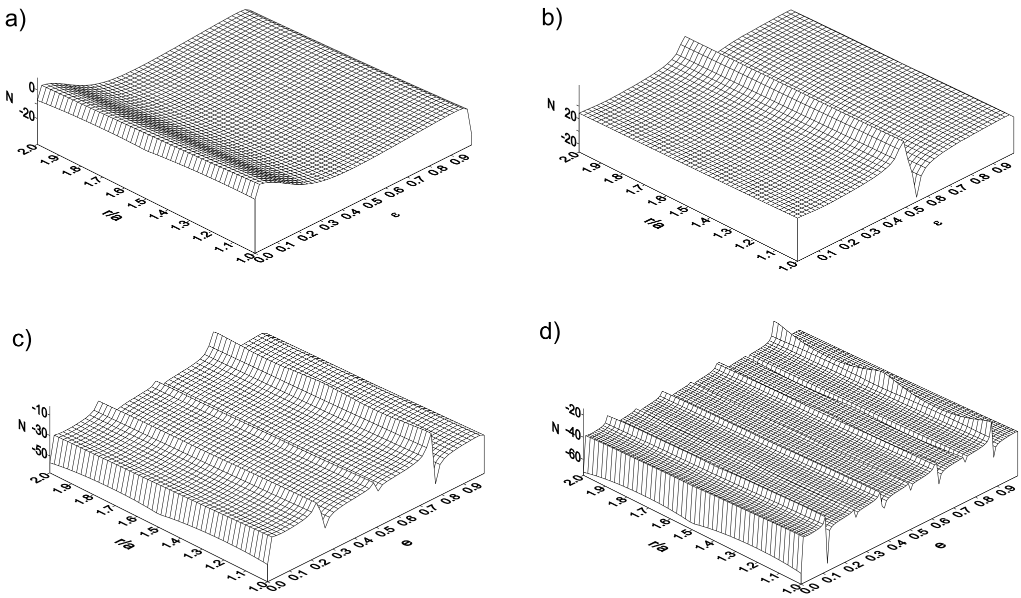


Fig. 7. The distribution of the sound wave intensity near the cylindrical surface, $1 \leq r/a \leq 2$ for continuously changing of the elastic tube thickness for different $x_0 (a - x_0 = 5$; $b - x_0 = 10$; $c - x_0 = 25$; $d - x_0 = 50)$ and for $X_{L0} = 0.001$, $\varepsilon_0 = 0.3$.

5. Conclusions

The rotation of the hollow circular elastic cylinder with varying angular velocity causes acoustical radiation into the surrounding medium. More precisely, the source of the wave propagation in the cylinder and in the compressible fluid is a mass force, namely, the centrifugal force varying with time and excited by the rotatory movement of the elastic body. As the result of the time modulation of this motion, sound waves with a complicated spectral structure of a clear expressed resonance character are generated. The resonance properties are also transferred to the stationary excited sound signals. The generated signal contains the first and second oscillation harmonics, because the centrifugal force is proportional to the second power of the angular velocity. Thus we propose to consider the rotating elastic cylinder as an all-directional transducer of the sound waves generated simultaneously on the fundamental and on the double harmonics. On the other hand, the rotating object can be considered as the source of the undesirable sound radiation (noise) in the water. The investigation of the thin structure of the frequency characteristics is a necessary precondition of the sound radiation control.

The analysis of the numerical calculations shows the following major peculiarities of the sound wave structure:

1. The amplitudes of the radiated acoustical pressure or wave intensity has the sequence of resonances caused by the superposition of outgoing and ingoing cylindrical waves in the elastic material of the rotated tube.

2. The resonance locations are connected with the phase velocities $v_j^{\rm ph} = cx/x_j^{\rm res}(x)$ $(j = 1, 2, \ldots)$ of the resonance wave propagation. The phase velocities as well as the group velocities $v_j^{\rm gr} = dv_j^{\rm ph}/dx$ $(j = 1, 2, \ldots)$ of these waves are subject to the influence of the dispersion phenomenon caused by the varying cylinder thickness parameter.

3. The resonance lines are distinguished by the good quality and high intensity.

4. The resonances corresponding to the solid cylinder are of constructive type. In the case of the hollow cylinder, the resonances are divided in the constructive and destructive classes. This effect is visible very well for thin elastic cylindrical shells.

5. In contrast to the positions of all other resonances, the first resonance is a particular one since its location is little movable with the variation of the cylindrical wall thickness.

6. The series of low-level resonances, demonestrated in the acoustical intensity and masked on the phone of the high-amplitude resonances, are well disclosed when the frequencies of the oscillation of the angular velocity are fixed and the cylindrical tube thickness is continuously changed.

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