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THE TOTAL SOUND POWER RADIATED BY A CLAMPED–GUIDED ANNULAR PLATE EXCITED FOR VIBRATIONS BY AN EXTERNAL SURFACE PRESSURE

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A problem of sound radiation by a clamped-guided annular plate with a hole in its center has been considered which is considerably more complex than it is in the case of a circular plate. The plate is subjected for vibrations by an external pressure. The vibrating system is controlled by a shaker clamped into the internal edge of the plate which make it possible to control vibrations of the plate as well as the total sound power radiated. The plate's volume displacement has been presented which make it possible to roughly rate the compensation quality. An amplitude-phase compensation system make it possible to observe both resonance and antiresonance phenomenas.

1. Introduction

A problem of sound radiation by a circular-annular plate with a hole in its center is still important because of its numerous application for industry, communication, transportation and many others. However, any theoretical analysis of the problem is considerably more complex and arduous than it is in the case of a circular plate. Considering an annular plate implies many possible axisymmetric boundary conditions with their big influence on the corresponding mode shapes shown earlier in [1, 2]. Given that the plate is subjected for vibrations by an external pressure of a measured amplitude and phase distribution and driven by a backward loop control-shaker system the resultant total sound power radiated can also be controlled. An amplitude-phase compensation system make it possible to observe both resonance and antiresonance phenomenas.

The total sound power radiated by a clamped circular plate was investigated analytically, among the others, by CZARNECKI, ENGEL and PANUSZKA who presented an approximated equivalent area method [3] whereas ENGEL and STRYCZNIEWICZ analysed the magnitude using an exact method [4]. LEVINE, LEPPINGTON and RDZANEK presented some highly efficient high frequency asymptotics to derive the sound radiation power and efficiency of a clamped circular plate using a closed contour integral method [5, 6]. RDZANEK Jr., RDZANEK and ENGEL focused on the sound radiation by annular plates using analogous methods [7, 8, 9, 10]. Some vibration of annular plates were in focus of analysis, among the others, by AMABILI, FROSALI and KWAK [11] or KWAK and KIM [12]. LEE and SINGH investigated sound radiation and radiation efficiency of a rotating vibrating computer annular disk. The authors used structural modes reaching a good agreement with their measurements [13].

In the case of some rectangular plates, the total sound power was analyzed, among the others, by BERRY, GUYADER and NICOLAS who presented some efficient elementary acoustic impedance expressions for the rigid body modes of the plates with some arbitrary boundary conditions. The authors presented the impedance values for the elastic body modes in their elementary forms using the approximate Rayleigh-Ritz method [14].The total sound power of a simply supported rectangular plate was presented by RDZANEK Jr. and ZAWIESKA. Their investigations were motivated by controlling the noise produced by a power transformer casing [15].

The problem of the total sound power radiated amplitude-phase controlling was analyzed, among the others, by RDZANEK Jr., ENGEL and RDZANEK in the case of a clamped annular plate with one or two edges driven by a shaker system. Their analysis was dedicated to compensate an external sound pressure. The plate's volume displacement was also presented to make it possible to roughly rate the compensation quality [16, 17]. The authors reached a good agreement with their measurements presented in [18].

This paper mainly deals with the sound radiation by a clamped-guided annular plate. The internal clamped edge of the plate is driven by a shaker system coupled with a measuring amplitude-phase controlling system dedicated for compensation of an external sound pressure. The problem was not yet considered analytically for a clamped-guided annular plate.

2. Analysis assumptions

A circular-annular plate of its internal and external radii a and b is subjected for some time-harmonic axisymmetric vibrations by an external pressure with a known distribution $f(r,t)$ where $a < r < b$, and by shaking the edge for $r = a$ with a known transverse deflection amplitude $W(a,t)$. The external plate's edge is guided by a flat and rigid baffle for $r = b > a$. We assume that the roller located within the plate does not move being also the baffle component.

The plate's equation of motion assumes the form of

$$\left(D_E \nabla_r^4 + \varrho h \frac{\partial^2}{\partial t^2}\right) W(r) e^{-i\omega t} = f(r) e^{-i\omega t}, \tag{1}$$

where $W(r) \in \mathbb{C}$ is the surface amplitude transverse deflection distribution, ϱ, h, E, ν are the plate's density, thickness, Young modulus and Poisson ration, respectively, ω – circular frequency, $D_E = Eh^3 / 12\,(1-\nu)^2$ – bending stiffness, t – time, $\nabla_r^2 = \dfrac{1}{r}\dfrac{\mathrm{d}}{\mathrm{d}r}\left(r\dfrac{\mathrm{d}}{\mathrm{d}r}\right)$ – biharmonic operator for an axisymmetric problem.

For some time-harmonic processes, the non-homogeneous equation of motion (1) can be written in its amplitude form

$$(k_D^{-4} \nabla_r^4 - 1)\, W(r) = f(r)/\varrho h \omega^2, \tag{2}$$

where $k_D^4 = \omega^2 \varrho h / D_E$ is the plate's structural wavenumber with a solution expected as a sum of some two particular solutions

$$W(r) = W_1(r) + W_S(r). \tag{3}$$

First of them – general – written in its amplitude form as

$$W_1(r) = A\mathrm{J}_0(k_D r_0) + B\mathrm{I}_0(k_D r_0) + C\mathrm{N}_0(k_D r_0) + D\mathrm{K}_0(k_D r_0) \tag{4}$$

is the solution for the plate's homogeneous equation of motion

$$(k_D^{-4} \nabla_r^4 - 1)\, W_1(r) = 0. \tag{5}$$

The second is the particular solution for the non-homogeneous equation of motion (2) and can be formulated as

$$W_S = -f_0/\varrho h \omega^2, \tag{6}$$

if we assume a uniform distribution for the external pressure $f(r) = f_0 = \text{const}$ for $a \le r \le b$.

We assume low fluid loading. The plate is made of steel with its internal friction small enough to be neglected.

The boundary conditions in Eq. (2) are non-homogeneous (cf. [2, 19]).

$$\begin{gathered} W(a) = W_a \mathrm{e}^{-i\varphi}, \qquad \frac{\mathrm{d}}{\mathrm{d}r} W(r)\Big|_{r=a} = 0, \qquad W_a, \varphi \in \mathbb{R}, \\ \frac{\mathrm{d}}{\mathrm{d}r} W(r)\Big|_{r=b} = 0, \qquad \frac{\mathrm{d}}{\mathrm{d}r} \nabla_r^2 W(r)\Big|_{r=b} = 0, \end{gathered} \tag{7}$$

because the normal vibration component for the plate's internal edge is non zero and assumes the value of $v_a = -i\omega W(a)$ known or measured.

Inserting solution (3) into the boundary conditions (7) we get the following complex constant values:

$$\begin{aligned}
A &= \overline{W}_a \mathrm{N}_1(s\lambda)V_0, \qquad B = \lambda\overline{W}_a \frac{\mathrm{K}_1(s\lambda)}{\mathrm{I}_1(s\lambda)}[\mathrm{I}_1(\lambda) - V_0 M_S],\\
C &= -\overline{W}_a \mathrm{J}_1(s\lambda)V_0, \qquad D = \lambda\overline{W}_a[\mathrm{I}_1(\lambda) - V_0 M_S],\\
M_S &= S(\lambda)\mathrm{N}_1(s\lambda) - T(\lambda)\mathrm{J}_1(s\lambda),\\
M_N &= N(\lambda)\mathrm{N}_1(s\lambda) - R(\lambda)\mathrm{J}_1(s\lambda),\\
V_0 &= \frac{\mathrm{I}_1(\lambda)\mathrm{K}_1(s\lambda) + \mathrm{K}_1(\lambda)\mathrm{I}_1(s\lambda)}{\mathrm{K}_1(s\lambda)M_S + \mathrm{I}_1(s\lambda)M_N},
\end{aligned} \tag{8}$$

where

$$S(x) = \mathrm{J}_1(x)\mathrm{I}_0(x) + \mathrm{J}_0(x)\mathrm{I}_1(x), \qquad T(x) = \mathrm{N}_1(x)\mathrm{I}_0(x) + \mathrm{N}_0(x)\mathrm{I}_1(x),$$
$$N(x) = \mathrm{J}_1(x)\mathrm{K}_0(x) - \mathrm{J}_0(x)\mathrm{K}_1(x), \qquad R(x) = \mathrm{N}_1(x)\mathrm{K}_0(x) - \mathrm{N}_0(x)\mathrm{K}_1(x),$$

and $\lambda = k_D a$ – structural wavelength, $s = b/a$ – plate's geometric parameter, $\overline{W}_a = W_a \mathrm{e}^{-i\varphi} - W_S$ – excitation amplitude, and $W_a, W_S, \varphi \in \mathbb{R}$.

3. The sound power in its integral form

The total sound radiation power can be formulated as (e.g. [18, 10])

$$\varPi = \varPi_a - i\varPi_r = \pi\varrho_0 c k^2\omega^2 \int_0^\infty M^2(kx)\,\frac{x\mathrm{d}x}{\gamma_0}, \tag{9}$$

where $\varPi_a$ and $\varPi_r$ are the active and reactive components, respectively, if computed within the corresponding limits $(0,1)$ and $(1,\infty)$ with

$$\gamma_0 = \begin{cases} \sqrt{1-x^2} & \text{for} \quad 0 \le x \le 1,\\ i\sqrt{x^2-1} & \text{for} \quad 1 \le x < +\infty. \end{cases}$$

The sound radiation characteristic function for a vibrating surface source (cf. [20])

$$M(kx) = \int_a^b W(r_0)\mathrm{J}_0(kxr_0)r_0\mathrm{d}r_0, \tag{10}$$

assumes the form of

$$M(kx)=\frac{a^2}{\beta}\left\{\frac{sx\mathrm{J}_1(s\beta x)(A_0\delta^2-B_0x^2)}{\delta^4-x^4}\right.\\ +\frac{x\mathrm{J}_1(\beta x)(C_0\delta^2+D_0x^2)-2\delta^3G_1(\lambda)\mathrm{J}_0(\beta x)}{\delta^4-x^4}\\ \left.+\frac{W_S}{x}\Big[s\mathrm{J}_1(s\beta x)-\mathrm{J}_1(\beta x)\Big]\right\}, \tag{11}$$

in the case of a vibrating annular plate with the vibration amplitude distribution given by Eq. (3) where $\beta=ka$ – dimensionless interference parameter, $k=2\pi/\lambda_0$ – acoustic wavenumber, $\delta=\lambda/\beta=k_D/k$ – dimensionless parameter, $G_\mu(x)=A\mathrm{J}_\mu(x)+C\mathrm{N}_\mu(x)$ for $\mu\in\{0,1\}$, $x\in(\lambda,s\lambda)$, and

$$\begin{aligned}&A_0=\overline{W}_aV-G_0(s\lambda), \qquad B_0=\overline{W}_aV+G_0(s\lambda),\\ &C_0=2G_0(\lambda)-\overline{W}_a, \qquad D_0=\overline{W}_a, \qquad V=[\mathrm{I}_1(\lambda)-V_0M_S]/s\mathrm{I}_1(s\lambda).\end{aligned} \tag{12}$$

The sound radiation characteristic function square $M^2(kx)=M(kx)M^*(kx)$ within the integral (9) is a real function, and $M^*(kx)$ is its conjugate value for $M(kx)$. Let us introduce the following denotation

$$M(kx)M^*(kx)=\frac{a^4}{\beta^2}\left\{\frac{M_0(kx)M_0^*(kx)}{(\delta^4-x^4)^2}+\left[W_S\,\frac{s\mathrm{J}_1(s\beta x)-\mathrm{J}_1(\beta x)}{x}\right]^2\right.\\ \left.+W_S\frac{M_0(kx)+M_0^*(kx)}{\delta^4-x^4}\,\frac{s\mathrm{J}_1(s\beta x)-\mathrm{J}_1(\beta x)}{x}\right\}, \tag{13}$$

to get

$$\begin{aligned}M_0(kx)M_0^*(kx)&=s^2x^2(A_0\delta^2-B_0x^2)(A_0^*\delta^2-B_0^*x^2)\,\mathrm{J}_1^2(s\beta x)\\ &+sx^2\Big[(A_0\delta^2-B_0x^2)(C_0^*\delta^2+D_0^*x^2)\\ &+(A_0^*\delta^2-B_0^*x^2)(C_0^*\delta^2+D_0^*x^2)\Big]\mathrm{J}_1(s\beta x)\mathrm{J}_1(\beta x)\\ &-2\delta^3sx\Big[G_1^*(\lambda)(A_0\delta^2-B_0x^2)+G_1(\lambda)(A_0^*\delta^2-B_0^*x^2)\Big]\,\mathrm{J}_1(s\beta x)\mathrm{J}_0(\beta x)\\ &-2\delta^3x\Big[G_1^*(\lambda)(C_0\delta^2+D_0x^2)+G_1(\lambda)(C_0^*\delta^2+D_0^*x^2)\Big]\,\mathrm{J}_1(\beta x)\mathrm{J}_0(\beta x)\\ &+x^2(C_0\delta^2+D_0x^2)(C_0^*\delta^2+D_0^*x^2)\mathrm{J}_1^2(\beta x)+4\delta^6G_1(\lambda)G_1^*(\lambda)\,\mathrm{J}_0^2(\beta x),\end{aligned} \tag{14}$$

$$
\begin{aligned}
M_0(kx) + M_0^*(kx) = sx \Big[(A_0 + A_0^*)\delta^2 - (B_0 + B_0^*)x^2\Big] \, \mathrm{J}_1(s\beta x) \\
+ x \Big[(C_0 + C_0^*)\delta^2 + (D_0 + D_0^*)x^2\Big] \, \mathrm{J}_1(\beta x) \\
- 2\delta^3 \Big[G_1(\lambda) + G_1^*(\lambda)\Big] \, \mathrm{J}_0(\beta x). \qquad (15)
\end{aligned}
$$

The sound radiation power Π is normalized by

$$
\Pi_0 = \frac{1}{2}\pi a^2(s^2 - 1)\,\varrho_0 c v_a^2,
$$

where $v_a^2 = \omega^2 W_a^2$. The normalized sound radiation power assumes the form of

$$
\begin{aligned}
\mathcal{P} = \Pi/\Pi_0 = \mathcal{P}_a - i\mathcal{P}_r = \frac{2}{s^2 - 1}\,\frac{1}{W_a^2}\Bigg\{ \int_0^\infty \frac{M_0(kx)M_0^*(kx)x\,\mathrm{d}x}{(\delta^4 - x^4)^2\gamma_0} \\
+ W_S \int_0^\infty \frac{M_0(kx) + M_0^*(kx)}{\delta^4 - x^4}\Big[s\mathrm{J}_1(s\beta x) - \mathrm{J}_1(\beta x)\Big]\frac{\mathrm{d}x}{\gamma} \\
+ {W_S}^2 \int_0^\infty \Big[s\mathrm{J}_1(s\beta x) - \mathrm{J}_1(\beta x)\Big]^2 \frac{\mathrm{d}x}{x\gamma_0}\Bigg\}, \qquad (16)
\end{aligned}
$$

where

$$
Q = \mathrm{J}_1(s\lambda)\mathrm{N}_0(\lambda) - \mathrm{J}_0(\lambda)\mathrm{N}_1(s\lambda), \qquad E = \mathrm{J}_1(s\lambda)\mathrm{N}_1(\lambda) - \mathrm{J}_1(\lambda)\mathrm{N}_1(s\lambda), \qquad (17)
$$

$$
G_0(s\lambda) = -\frac{2}{\pi s\lambda}\overline{W}_a V_0, \qquad G_0(\lambda) = -\overline{W}_a Q V_0, \qquad G_1(\lambda) = -\overline{W}_a E V_0, \qquad (18)
$$

$$
\begin{aligned}
\psi &\equiv \overline{W}_a\overline{W}_a^* = W_a^2 + W_S^2 - 2W_S W_a\cos\varphi, \\
\phi &\equiv -\frac{1}{2}(\overline{W}_a + \overline{W}_a^*) = W_S - W_a\cos\varphi.
\end{aligned} \qquad (19)
$$

All the functions defined above are real with some real arguments and they can be useful for some numerical computations using Eqs. (14)–(16).

4. The volume displacement

The volume displacement $Q_W e^{-i\omega t}$ of the vibrating surface was presented in a considerable detail e.g. in [21]. The magnitude of amplitude formulated as

$$
Q_W = \mathrm{Re}\left\{2\pi \int_a^b W(r)\,r\,\mathrm{d}r\right\}, \qquad (20)
$$

makes it possible to preliminary rate the external pressure compensation. It assumes the form of

$$Q_W = \frac{2\pi a^2}{\lambda}\left\{(s^2-1)\,\lambda W_S - \frac{1}{2}\left[G_1(\lambda)+G_1^*(\lambda)\right]\right\} \tag{21}$$

after integrating the vibration velocity amplitude $W(r)$ given by (4).

Using Eqs. (17)–(19) we get

$$Q_W = \frac{2\pi a^2}{\lambda}\Big[(s^2-1)\,\lambda W_S - EV_0\,(W_S - W_a\cos\varphi)\Big]. \tag{22}$$

In the case, when the plate's excitation frequency ω is equal to one of the plate's eigenfrequencies simultaneously with the plate's volume displacement Q_W equal to zero we get

$$s^2-1 = (1+\varepsilon\cos\varphi)\,EV_0/\lambda,$$

where

$$\varepsilon = -W_a/W_S = \varrho h\omega^2 W_a\,/\,f_0.$$

This situation results in the minimal value of total sound power radiated which is known as the antiresonance phenomena.

5. Concluding remarks

The total sound power radiated by a clamped-guided annular plate with a driven internal edge has been presented in its zero order Hankel representation which has made it possible to derive an elementary form for the sound radiation characteristic function valid at the Fraunhoffer's zone.

The formulations obtained for the active and reactive sound radiation power make a basis for some further analyses in the case of the low and high frequencies to get some highly efficient asymptotic formulations.

The formulations presented for the volume displacement make it possible to perform some further analyses of the antiresonance phenomena. They also enable us to derive the optimal conditions for the most efficient amplitude-phase compensation of the external exciting sound pressure.

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