ESTIMATION OF HEAT DISTRIBUTION IN THE ACOUSTIC LENS OF AN ULTRASONIC MICROSCOPE WITH THE CARRIER FREQUENCY OF 1 GHZ

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The propagation of heat was analyzed in the lens of an acoustic microscope used for testing of living cells at the frequency of 1 GHz. Information concerning the propagation of heat is necessary for determination of thermal boundary conditions which influence the temperature increase in the tested samples representing acoustical properties of water. The time of temperature propagation from water, heated due to high absorption, to the sapphire body of the lens was estimated to be 0.77 ms. To carry out these calculations the derivations of CARSLOW and JAEGER [2] and of TAUTZ [6] were adjusted. On the other hand the propagation time of the acoustic wave in the sapphire body equalled $0.0093\,\mu$s only. The time of image formation in the microscope is rather long being equal from one to several seconds due to mechanical inertia of the support vibrating together with the tested sample. The heat capacities of the water volume and the sapphire body were found to be comparable. However, if the heat capacity of the water volume would be many time smaller then the time of the finally attained temperature would be elongated. This effect can be neglected since the time of image formation is 3 orders of magnitude longer than the time of penetration of the sapphire body by the heat supplied by water. As the result a temperature equilibrium will be obtained with the average boundary temperature of water. In such a case no heat flux will penetrate the boundary water – sapphire and the condition of the thermal insulation at the boundary will be fulfilled. This thermal boundary condition makes it possible to determine the real temperature increase in biological specimens.

1. Introduction

In the acoustic microscopy the main problem consists in obtaining images of high resolution with minimum temperature increase which may be harmful for the tested biological specimens. This problem was investigated in our previous paper (KUJAWSKA *et al.* [4]) in the case of a microscope with the carrier frequency of 1 GHz used for testing of living cells.

The intention of the present paper is to discuss hence derived thermal problems of the acoustic microscopy which are connected with the acoustic sapphire lens. The boundary thermal conditions of the lens are significant for the temperature increase of the tested biological samples. For this purpose it was necessary to determine the time of the temperature penetration from the water-sapphire boundary into the whole body of the sapphire lens (Fig. 1). The knowledge of this parameter makes it possible to determine thermal conditions of the lens, which influence the temperature increase of the tested sample.
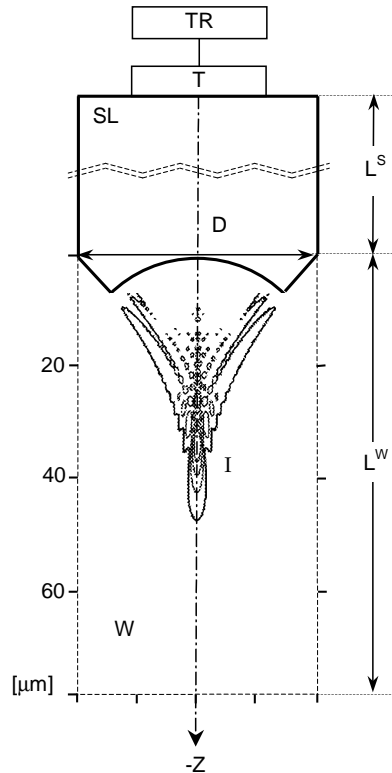
Fig. 1. The principle of the acoustic microscope system. TR – electronic transmitter and receiver, T – the transducer, SL – circular sapphire lens, W – water, $D = 86\,\mu m$ diameter of the lens, $L^S = 104\,\mu m$, $L^W = 80\,\mu m$, Z – vertical symmetry axis, I – isobars showing the pressure distributions computed in water (KUJAWSKA et al. [4]).

2. The heat exchange at the boundary water-sapphire

The heat flux from the heated water penetrates into the circular sapphire lens which is bounded by a thin layer of a transducer and mounted into a thin holder. So thermal losses at the sapphire surface are small, as well as inside the sapphire lens no thermal

losses are to be expected (BRIGGS [1]). Therefore, its temperature continually increases, tending after some insonation time to the average temperature of the boundary of water which is heated due to its very high absorption.

The above described situation corresponds to the final steady state. Let us investigate in more details the expanding process of the heat exchange on the boundary water-sapphire.

It arises a basic question what is the time of the temperature penetration from water into the sapphire lens increasing in this way its temperature. The penetration time can be roughly estimated, adjusting the results of CARSLOW and JAEGER [2] and also of TAUTZ [6] who derived equations of heat conductivity for various cases important for physical and technological purposes. For our aim thermal effects in an unbounded plate with the constant thickness L were interesting.

In the investigated case of the 1 GHz microscope the value $L = 104\,\mu\mathrm{m}$ is the near field distance of the transducer in sapphire taken in the direction of the wave propagation. Figure 2 shows our sapphire lens SL and the unbounded plate B. The formulae given by the mentioned authors make it possible to determine the time of penetration of the temperature T_0 from the lower plane surface of the plate to the upper one situated at the distance L. They describe also the development of the temperature T_0 in time for different distances within the plate in the form of analytical formulae and diagrams. According to our simplified assumption we expect the same effects in our lens in spite of the fact that it represents only a small bounded part of the unbounded plate under consideration.
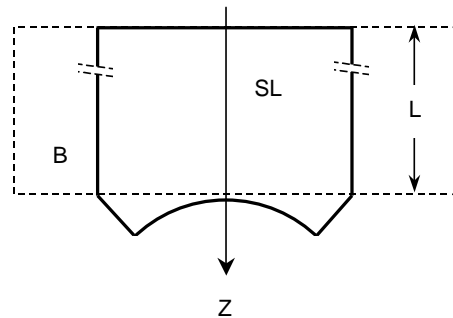

Fig. 2. The circular sapphire lens SL and the plate B (dashed lines) unbounded in the x, y directions, perpendicular to the symmetry axis z, $L = 104\,\mu\mathrm{m}$ – thickness of the plate.

The time, necessary for obtaining at the distance of L the temperature increase of 0.9 of its boundary value T_0, was demonstrated by the mention authors to be equal to

$$t_{0.9} = 1.0 L^2/a \tag{1}$$

in the centre of the plate ($X = 0$) (see Fig. 3). For obtaining the temperature increase of only $0.22\,T_0$ the necessary time is much shorter and equals $t_{0.22} = 0.2\,L^2/a$.
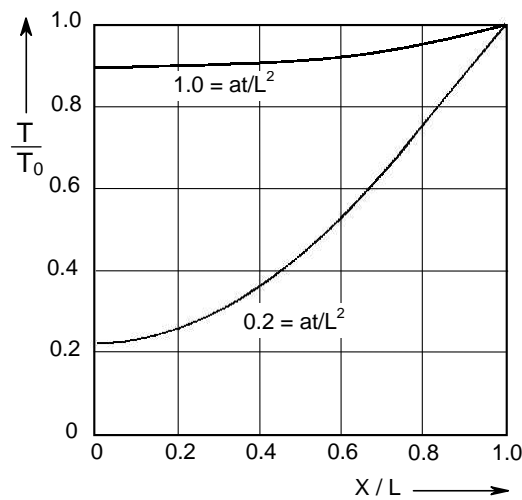
Fig. 3. The distribution of the actual temperature T in the sapphire lens calculated in relation to the temperature T_0 at the water-sapphire boundary for two values of the parameter at/L^2 equal to 1 and 0.2. X – distance from the upper surface of the lens, L – thickness of the plate, t – time (CARSLAW and JAEGER [2], TAUTZ [6]).

In the formula (1) we have the value of $a = \lambda_h/\rho c_s = 0.140 \text{ cm}^2/\text{s}$, where λ_h denotes thermal conductivity of sapphire, c_s^S – specific heat of sapphire and ρ – its density (see Table 1).

Table 1. Longitudinal wave speed c, density ρ (BRIGGS [1]), specific heat c_s and thermal conductivity λ_h of sapphire (HOLLAND [3], KYBURZ [5]) and of water.

Sapphire	$c^S = 11150$ m/s, $\rho^S = 3980$ kg/m^3 $c_s^S = 0.18$ cal/(g·°C) $= 754$ J/(kg·°C), $\lambda_h^S = 0.1$ cal/(cm·s·°C) $= 418.7$ W/(m·°C)
Water	$c^W = 1500$ m/s, $\rho^W = 1000$ kg/m^3 $c_s^W = 1$ cal/(g·°C) $= 4187$ J/(kg·°C), $\lambda_h^W = 0.0017$ cal/(cm·s·°C) $= 0.712$ W/(m·°C)

Hence the estimated time equals $t_{0.9} = 0.77$ ms, while the propagation time of the acoustic wave in sapphire, along the same path $L = 104\,\mu\text{m}$ is equal to $t = L/c^S = 0.0093\,\mu\text{s}$ (see Table 1).

Therefore, the minimum insonation time of $t_{0.9} = 0.77$ ms is necessary to enable the penetration of the water temperature to the almost whole body of the sapphire lens.

On the other hand the image formation time in a typical microscope can be equal from one to several seconds (see for example BRIGGS [1]). This results from the limitations caused by the mechanical inertia of the support vibrating together with the tested

sample (Fig. 4). Therefore the transient process of the temperature elevation will reach the steady state in a very short time in relation to the time of the image formation.
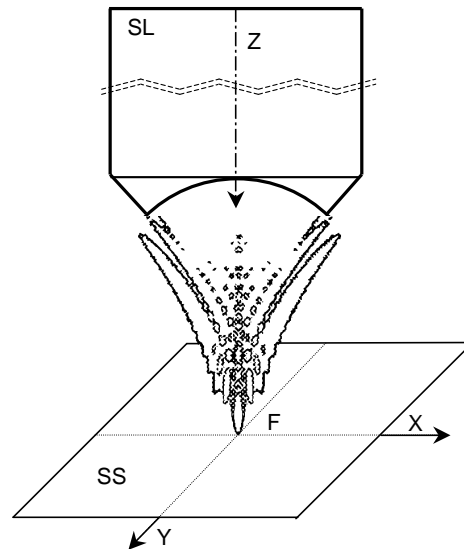

Fig. 4. The scanning system of the microscope. SL – circular sapphire lens, SS – support with the tested sample, Z – direction of the ultrasonic beam, F – focal region of the ultrasonic beam, X – direction of the relatively rapid scanning translations, Y – direction of the relatively slow translation. For illustrative reasons the maximum pressure point of the focus, corresponding to the highest resolution, was shifted a little upwards. The -6 dB width of the focus was equal to 2.2 μm at the frequency of 1 GHz (KUJAWSKA *et al.*, [4]).

One should take into consideration also heat capacities C_h of the sapphire lens and of the water volume shown in Fig. 1. This value is equal to $C_h = V\rho c_s$ where V denotes volume, ρ – density, c_s – specific heat. The ratio of the volumes of the sapphire V^S and of water V^W is dependent on their thickness equal to $L^S = 104$ μm in the first and $L^W = 80$ μm in the second case, since the radius of both media was assumed to be the same (Fig. 1). So we received finally the ratio of heat capacities of the sapphire lens C_h^S to the water volume C_h^W equal to

$$C_h^S/C_h^W = L^S\rho^S c_s^S/L^W\rho^W c_s^W = 0.94, \tag{2}$$

where the corresponding values for sapphire and water were taken from Table 1.

The obtained result (2) should be considered as a rough estimation because many factors were here simplified as for example the exact shape of the lens, temperature distributions etc. Nevertheless according to this estimation the heat capacities of the sapphire lens and of the water volume are comparable, therefore the thermal balance can be easily obtained.

3. Discussion and conclusions

The estimated time of temperature propagation from the water boundary to almost the end of the sapphire lens equals $t_{0.9} = 0.77$ ms. This time period is by about 5 orders of magnitude longer than the propagation time of the acoustic wave in sapphire being there equal to $0.0093\,\mu s$.

The image formation time in a typical microscope can be equal from one to several seconds. This results from the limitations caused by rapid movements of the mechanical support with the tested sample. Therefore, the transient process of the temperature elevation will reach the steady state in a very short time in relation to the time of the image formation.

The heat capacities of the sapphire lens and of the water volume were estimated to be comparable and even equal in the case under consideration. The volume of water may be by many times smaller, especially if it is used only as a coupling medium. However, such a case exceeds our basic problem of testing living cells representing acoustical properties of water. If the heat capacity of the water volume would be many times smaller than of the sapphire, then the transport of heat from water to the end of the sapphire lens would last a longer time to reach the temperature of water. However, this fact can be neglected since the time for the image formation is almost 3 orders of magnitude longer than the time of penetration the body of the sapphire lens by the heat supplied from water.

In the conclusion we can say that in the steady-state, corresponding to the formation time of the microscopic image, the temperature flux generated in water could reach the end of the sapphire lens many times. As the result a temperature equilibrium can be obtained with the average boundary temperature of water. In such a case no heat flux will penetrate the water-sapphire boundary and the condition of thermal insulation at this boundary is fulfilled. This situation is most important for us because then the possible temperature increase is maximum, corresponding to the worst critical circumstances for example for the living cell, which should be avoided.

Therefore the following thermal insulation condition for the plane half-space boundary, which was assumed in our previous calculations (KUJAWSKA *et al.* [4]), namely

$$\partial T/\partial z|_{z=0} = 0 \tag{3}$$

has been in that way verified.

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