ARCHIVES OF ACOUSTICS
29, 3, 435–445 (2004)

ESTIMATION OF THE TIME DELAY OF HYDROACOUSTIC SIGNALS FOR PASSIVE LOCATION OF UNDERWATER OBJECTS

A. MAKAR

Naval University
Institute of Navigation and Hydrography
Śmidowicza 69, 81-103 Gdynia, Poland
e-mail: Artur.Makar@amw.gdynia.pl

(*received 8 May 2003; accepted 3 December 2003*)

In this paper a method of time delay estimation of acoustic signals radiated in water has been shown. The source of these signals is a moving ship. The gradient adaptive method of minimizing the error has been used. An algorithm of estimation using this method and the results of estimation the real signals radiated by a moving ship have been presented.

Key words: estimation, time delay, underwater navigation.

1. Introduction

The problem of estimating and tracking the delay between two signals arises in many purposes such as speed measurement, localization and tracking of signal sources and sonar or radar detection. The adaptive methods for solving this problem gained great popularity because they generally do not require a priori statistics of the signals.

For location of underwater objects and the determination of parameters of their movement a passive system has been chosen. This system uses only acoustic waves radiated by the source, which can be a ship, a submarine, a torpedo, or a diver. The advantage of this system is the fact that the monitored object is not receiving sounding pulses. The acoustic signal, radiated by the moving object is received by the antenna array with a specific configuration.

The coordinate system can be treated as a 2D providing the ship is detected from a long distance or is moving on a shallow water. When the detected object is moving under the sea surface, for example a submarine, or is detected from a short distance, the coordinate system should make possible the determination of the position of this object in the 3D space (additionally, it should make possible the determination of the draught of the object).

The basic assignment of the antenna arrangement is receiving signals originated from the sources of underwater movement of acoustic waves. These signals are brought to the time delay estimation arrangement in order to delimitate the delay among them, and then for the delimitations of navigational parameters, such as: bearings, distance, speed. These parameters are surrendered to a filtering process in the time filtering block.

The antenna arrangement consists of unidirectional hydrophones seated along a straight line. Measuring the delay makes possible the delimitation of bearings and distances to detected object.

2. Description of the problem

The functional scheme of the system consists of two components: the sum function and the variable time delay system. The input signals are in the form of a sequence of signals' samples. The operation of the system is based on the minimizing of the $e(k)$ error which is achieved by a modification of the variable time delay to a value that corresponds to the minimum error $e(k)$ [5, 6]. Then, the value of the variable time delay is an estimated time delay between the input signals.

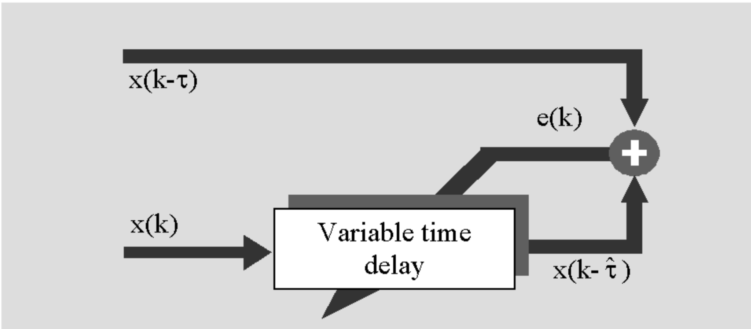

Fig. 1. Functional scheme of the time delay estimation using the gradient adaptive algorithm.

On the basis of the above [3, 4]:

$$e(k) = x(k-\tau) - x(k-\hat{\tau}), \tag{1}$$

where k – time index; τ – time delay between signals; $\hat{\tau}$ – estimated value of the time delay.

Therefore:

$$e(k)^2 = [x(k-\tau) - x(k-\hat{\tau})]^2 = x(k-\tau) - 2 \cdot x(k-\tau) \cdot x(k-\hat{\tau}) + x(k-\hat{\tau})^2. \tag{2}$$

Using the operator of the expected value for both sides of the equation, we obtain:

$$\begin{aligned} E[e(k)^2] &= E[x(k-\tau)^2] - 2E[x(k-\tau) \cdot x(k-\hat{\tau})] + E[x(k-\hat{\tau})^2] \\ &= E[x(k-\tau) \cdot x(k-\tau)] - 2E[x(k-\tau) \cdot x(k-\hat{\tau})] \\ &\quad + E[x(k-\hat{\tau}) \cdot x(k-\hat{\tau})] = 2R_{xx}(0) - 2R_{xx}[\tau - \hat{\tau}], \end{aligned} \tag{3}$$

where R_{xx} ( ) – function of autocorrelation of the signal X.

The mean square error MS $E[e(k)^2]$ is determined by the function of autocorrelation of the input signal. The form of Eq. (3) imposes the existance of one minimum of the value of $E[e(k)^2]$ with respect to the variable $\hat{\tau}$. This means that the autocorrelation function has to have one maximum with respect to the $\hat{\tau}$ values.

It is necessary to define the iterational algorithm, which will modify the variable time delay $\hat{\tau}$ until achieving the minimum of the error. The issue which is represented by Eq. (3) can be treated as a problem of synchronization of two signals, where the accuracy of the synchronization is determined by $E[e(k)^2]$.

The solution of the problem of obtaining a minimum of the error is based on a gradient type adaptive algorithm. The synchronization process begins from a value of the time delay, which is set before setting in motion the process. In next step, the gradient of the function of the error is measured, which, in another step of iteration, increases the accuracy of the determination of the $\hat{\tau}$ value. The procedure is consequently repeated until the minimum error is obtained.

The current value of the time delay is signed as $\hat{\tau}$, but the integer number nearest to $\hat{\tau}$ determines the time delay. The universal formula for optimization the algorithm, which uses the gradient type adaptive algorithm, can be written in the form [3, 4]:

$$\hat{\tau}(k+1) = \hat{\tau}(k) - \mu \cdot \nabla(k), \tag{4}$$

where μ – convergence coefficient, $\mu > 0$, $\nabla(k)$ – gradient (derivative) of the error (MS) with respect to $\hat{\tau}$.

$$\nabla(k) = \frac{\partial E[e(k)^2]}{\partial \hat{\tau}} = -2 \cdot \frac{\partial R_{xx}(\tau T - \hat{\tau} T)}{\partial \hat{\tau}}, \tag{5}$$

where T – period of sampling.

The approximation of the gradient is determined as:

$$\begin{aligned}\hat{\nabla}(k) &= \frac{\partial e(k)^2}{\partial \hat{\tau}} = 2 \cdot e(k) \cdot \frac{\partial e(k)}{\partial \hat{\tau}} \\ &= 2 \cdot e(k) \cdot \frac{\partial [x(k-\tau) - x(k-\hat{\tau})]}{\partial \hat{\tau}} = -2 \cdot e(k) \cdot \frac{\partial x(k-\hat{\tau})}{\partial \hat{\tau}}.\end{aligned}$$

The derivative of the input signal with respect to $\hat{\tau}$ is determined from the values of the input signal in adjoining steps of iteration. Then:

$$\begin{aligned}\hat{\nabla}(k) &= -2 \cdot e(k) \cdot \frac{x(k-\hat{\tau}-1) - x(k-\hat{\tau}+1)}{(\hat{\tau}+1) - (\hat{\tau}-1)} \\ &= -e(k) \cdot [x(k-\hat{\tau}-1) - x(k-\hat{\tau}+1)].\end{aligned} \tag{6}$$

The above equation is the estimator of the gradient of the error's function.

The expected value $\hat{\nabla}$ of the estimator is

$$E[\hat{\nabla}(k)] = -[R_{xx}(\tau T - \hat{\tau} T - T) - R_{xx}(\tau T - \hat{\tau} T + T)], \tag{7}$$

$$\begin{aligned}\nabla(k) &= -2 \cdot \frac{\partial R_{xx}(\tau T - \hat{\tau}T)}{\partial \hat{\tau}} \\ &= -2 \lim_{T \to 0} \frac{R_{xx}[\tau T - (\hat{\tau}+1)T] - R_{xx}[\tau T - (\hat{\tau}-1)T]}{(\hat{\tau}+1)-(\hat{\tau}-1)} \\ &= -\lim_{T \to 0} R_{xx}[\tau T - (\hat{\tau}+1)T] - R_{xx}[\tau T - (\hat{\tau}-1)T]\end{aligned} \tag{8}$$

From the two last equations results that the estimate is non-weighted, when $T \to 0$. For non-zero T:

- the gradient achieves zero for $\tau T = \hat{\tau}T$, i.e. a maximum of the function of autocorrelation,
- $E[\hat{\nabla}(k)]$ achieves zero for the sake of symmetry of the function of autocorrelation with respect to the maximum.

Joining the expression for optimizing the delay and using the gradient type adaptive algorithm with the estimator of the gradient we obtain:

$$\hat{\tau}(k+1) = \hat{\tau}(k) + \mu \cdot e(k)[x(k-\hat{\tau}-1) - x(k-\hat{\tau}+1)]. \tag{9}$$

In this recursive algorithm the sign of the gradient keeps the modification of the current value $\hat{\tau}$ in the correct direction.

3. Mathematical model of the method

The gradient type adaptive algorithm does not require a knowledge of the density of the signals' spectra. This method is based on correlation characteristics of the signal. The algorithm procedure can be shown in the following steps:

1. Calculation of the mean square signal of the error in the k-th step of iteration:

$$e(k) = x_1(k) - x_2(k - \hat{\tau}(k)), \tag{10}$$

where $x_1(k)$ – value of the signal deleted in the k-th step of the iteration, $x_2(k)$ – value of the signal not deleted in the k-th step of the iteration, $\hat{\tau}(k)$ – value of the time delay in the k-th step of the iteration.

2. Calculation of the gradient of the error's function in the k-th step of the iteration:

$$\nabla(k) = -e(k) \cdot [x_2(k - \hat{\tau}(k) - 1) - x_2(k - \hat{\tau}(k) + 1)]. \tag{11}$$

3. Estimation of the time delay in another step of the iteration:

$$\hat{\tau}(k+1) = \hat{\tau}(k) + \mu \cdot (-\nabla(k)). \tag{12}$$

The value of the gradient in k-th step of iteration can be calculated as the following derivative:

$$\nabla(k) = \left.\frac{\partial e(\hat{\tau})}{\partial \hat{\tau}}\right|_{\hat{\tau}=\hat{\tau}_k} = 2\chi\left(\hat{\tau}(k) - \hat{\tau}^*\right), \tag{13}$$

where χ – inclination of the error's function.

The transitional process from the beginning value $\hat{\tau}_0$ to the optimal one $\hat{\tau}^*$ can be analyzed by means of the equation obtained on the basis of Eqs. (12) and (13):

$$\hat{\tau}(k+1) = (1 - 2\mu\chi)\hat{\tau}(k) + 2\mu\chi\hat{\tau}^*. \tag{14}$$

This equation is a linear homogeneous differential equation of the 1-st order with a constant coefficient. It can be calculated using the induction method on the basis of several first iterations. The value of the k-th iteration is:

$$\hat{\tau}(k) = \hat{\tau}^* + (1 - 2\mu\chi)^k(\hat{\tau}_0 - \hat{\tau}^*). \tag{15}$$

This result is the solution of the gradient searching algorithm. The equation below is only convergent, when the following condition is fulfilled:

$$|r| = |1 - 2\mu\chi| < 1. \tag{16}$$

The latter condition can be transferred into the following one:

$$0 < \mu < \frac{1}{\chi}. \tag{17}$$

The decreasing speed depends on the value r. In Fig. 2 typical relationships, which occur in the correction process, for different values r have been shown.

By analogy to the automatic regulation systems, if the absolute value $|r| < 1$, the speed increases with the decreasing value of r and achieves a maximum value for $r = 0$,

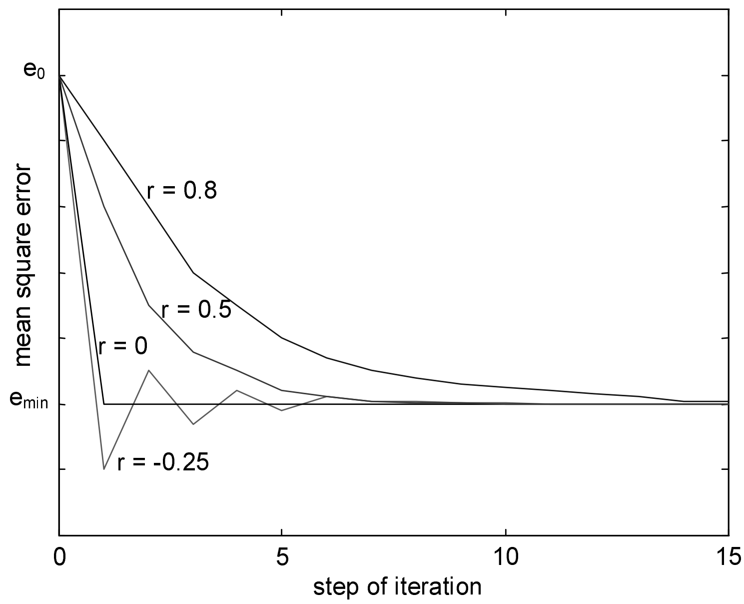

Fig. 2. Adaptive curves – dependence of the mean square error $e(k)$ on the step of iteration k.

when the optimal value is achieved in the 1-st step of the iteration. Besides, for positive values $r < 1$ there are no oscillations of the temporary values of the weight coefficient. For negative values of r, temporary values of the coefficient are non-optimal ones and achieve the value $\hat{\tau}^*$ according to the smother oscillation. In the 1-st case, the process is over-regulation, in the 2-nd one, the process is under-regulation. For $r = 0$ the process is equivalent to the Newton's method and it is critical. For $r \geq 1$ the process is divergent.

4. Results

The operation of the estimation algorithm has been tested during simulation tests using real hydroacoustic signals radiated by the sailing ship. For the simulation tests generated signals with the specific function of the autocorrelation with additive noise have been used.

Algorithms of modeling the stationary processes with a normal distribution are based on the linear transformation of the stationary sequence $x[n]$ of independence random numbers with a normal distribution (discreet white noise) to the sequence $\varsigma[n]$ with the specific function of the correlation. The operator of the linear transformation can have the form of the slidable summation (with the weight $c_k = c(k)$) [1]:

$$\varsigma[n] = \sum_{k=1}^{N} c_k x[n-k] \tag{18}$$

or recurrent equation:

$$\begin{aligned} \varsigma[n] &= a_0 x[n] + a_1 x[n-1] + \cdots \\ &\quad + a_1 x[n-l] - \cdots - b_l \varsigma[n-1] - \cdots - b_m \varsigma[n-m] \\ &= \sum_{k=0}^{l} a_k x[n-k] - \sum_{k=1}^{m} b_k \varsigma[n-k]. \end{aligned} \tag{19}$$

The required shape of the function of correlation of the random process modeled using these algorithms is obtained by a correct selection of the values of the weight coefficients a_k, b_k, c_k, and their number. These algorithms allow on forming discrete realization of random processes of any length.

The algorithm of generation of the random distribution $N(0,1)$ is based on the method of elimination and superposition of processes. It can be described by the following equation [8]:

$$\Phi(x) = a_1 F_1(x) + a_2 F_2(x) + a_3 F_3(x) + a_4 F_4(x), \tag{20}$$

where F_1, F_2, F_3 – probability density functions in the range $(-a,\ a)$, F_4 – density connected with the "tail" of the normal distribution, $a_1 = 0.8638$, $a_2 = 0.1107$, $a_3 = 0.0228002039$, $a_4 = 0.0026997691$.

During the simulation tests, the delay between the generated signals was constant. Operating the algorithm has been tested for different signal to noise ratios. For each ratio three SNR convergence coefficients have been used. The results of the tests have been presented in Figs. 3 and 4.

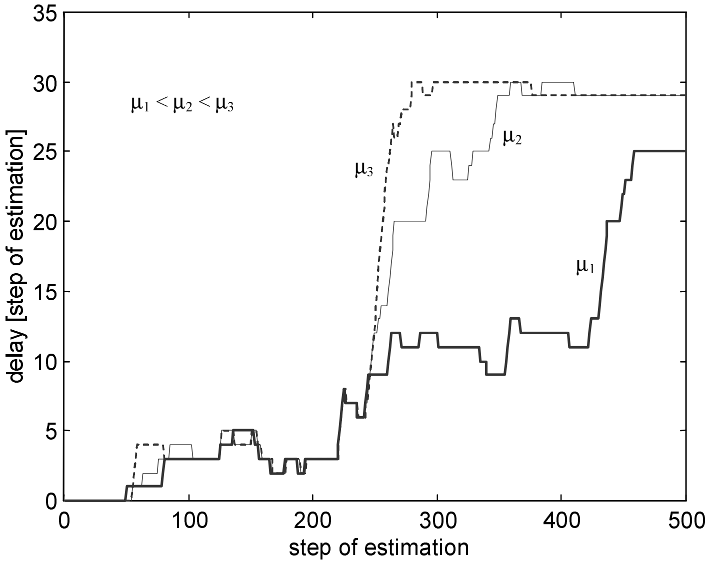

Fig. 3. Time delay estimation of simulated signals with the SNR $= 0.2$.

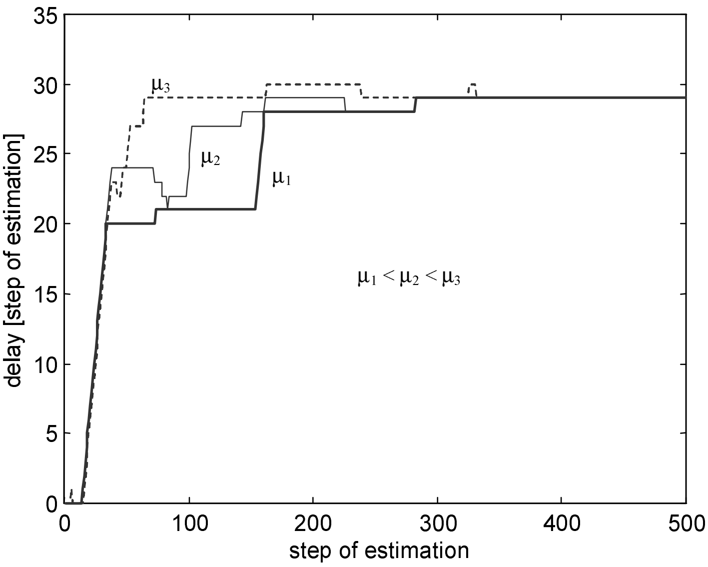

Fig. 4. Time delay estimation of simulated signals with the SNR $= 3$.

Hydroacoustic signals radiated by a sailing ship have been recorded in a measurement area of the system, which has been shown in a lower panel of Fig. 5. The ship was sailing in the waterway between buoys and its trajectory was well known. In the local navigation system the distance and the bearing to the ship were determined. These parameters were obtained in the hyperbolic navigation system using three hydrophones. The operation of the system was based on the determination of differences between the distances from the ship to the hydrophones. These differences were obtained on the basis of the determination of the time delay between signals received by the hydrophones.

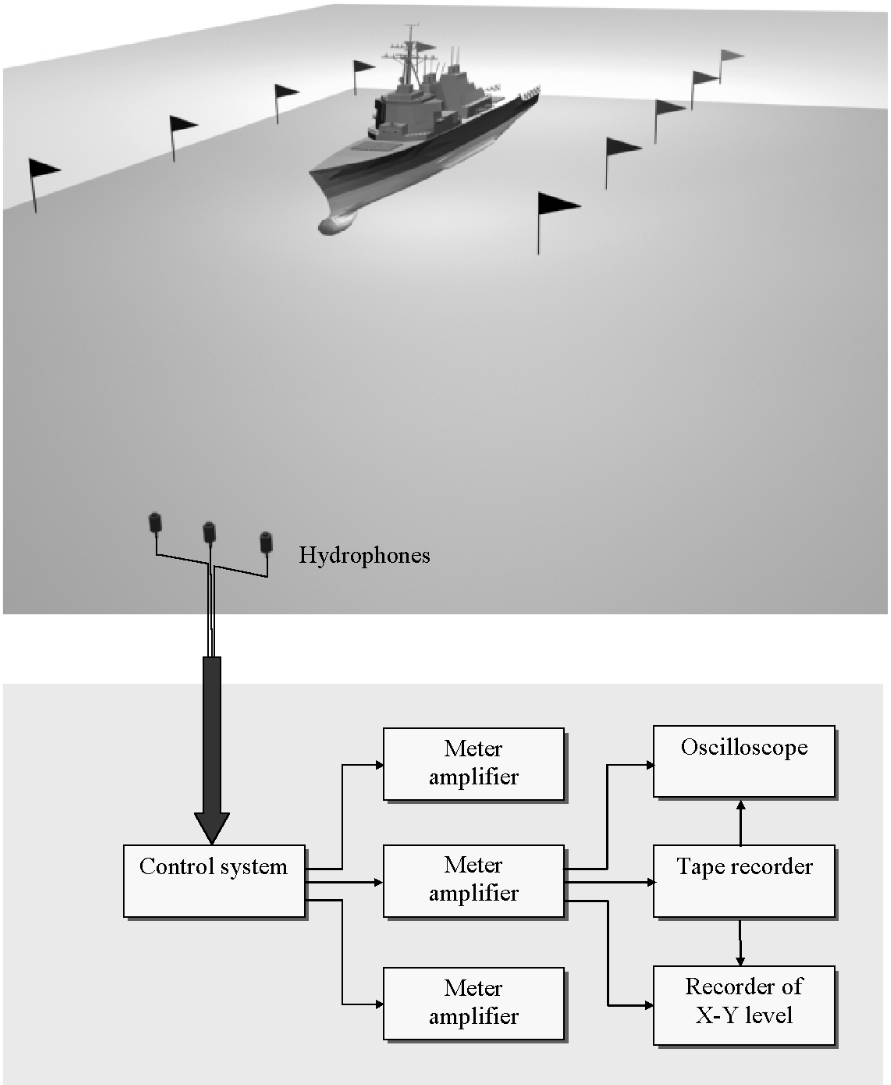

Fig. 5. The system of recording the hydroacoustic signals.

The system of recording the hydroacoustic signals consists of the Bruel & Kjaer equipment: the hydrophone type 8101, the amplifiers type 2636, recorder of the level type 2307, recorder X-Y type 2308 and recorder of the level type 2317. The system above enables the recording of signals in the band from 2 Hz to 50 kHz.

The analogue technique of recording signals using the tape recorder type 7006 with modules ZM 0053 working in the band from 20 Hz to 60 kHz has been used. The recorded signals have been analyzed in laboratory conditions using analog and digital techniques of the signal processing.

The hydroacoustic signals radiated by the sailing ship have been recorded using two hydrophones for the determination of the time delay between them. The analog signals have been sampled using the D/A device. Recorded signals have been shown in Figs. 6. Figure 7 presents a typical pattern of the autocorrelation function.

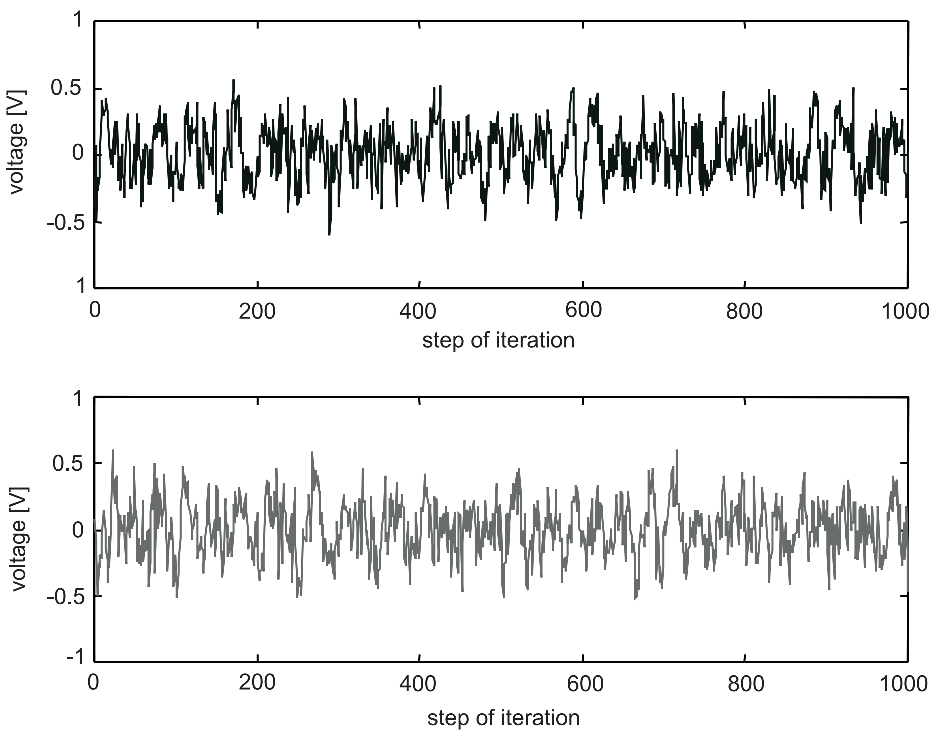

Fig. 6. Hydroacoustic signals radiated by a moving ship.

On the basis of the function of correlation the delay between signals can be determined – 100 steps of iteration. For 4 kHz the frequency of sampling the delay was 25 ms.

Using the gradient type adaptive method the time delay was estimated. The results have been shown in Fig. 8 for a constant (a) and a changeable (b) delay between signals.

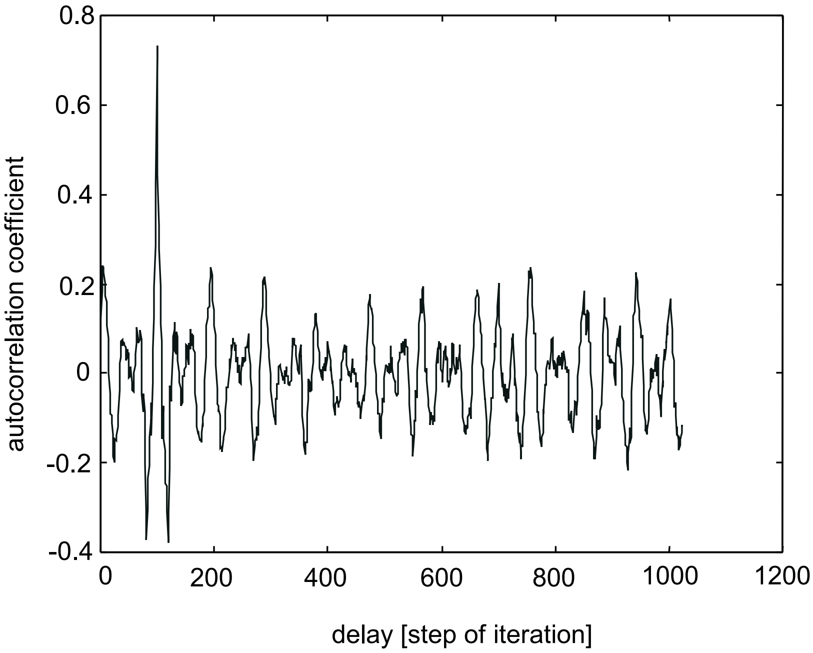

Fig. 7. The function of correlation.

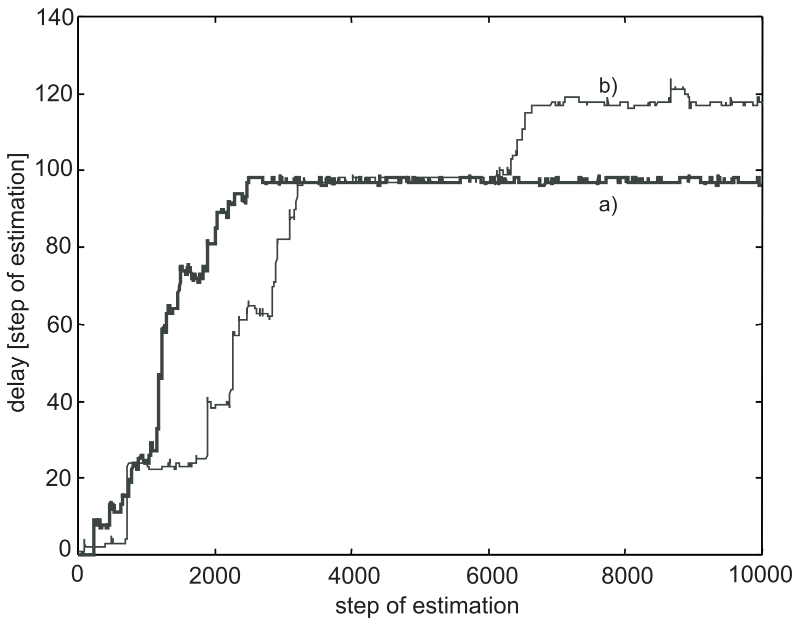

Fig. 8. Constant (a) and changeable (b) time delay between signals obtained using the gradient type adaptive method.

5. Conclusions

The presented method of estimation the time delay between signals is based on the correlation characteristics of these signals. One of the algorithm parameters is the convergence coefficient influencing the speed of estimation. It depends on SNR and works correctly for SNR < 1. On the basis of tests, it can be affirmed that the method can be used for the estimation of the time delay between hydroacoustic signals radiated by a moving ship in order to localize underwater sources and to determine navigation parameters such as: the speed and the course.

References

[1] GRZESIAK J., *Digital models of radiolocation signals and processing systems*, WAT, Warszawa 1984.

[2] MAKAR A., *Influence of SNR on determination of location of underwater object by means of acoustics method*, Hydroacoustics, **4**, 157–160 (2001).

[3] MAKAR A., *The method of time delay estimation of acoustic signals on the basis of hydroacoustic measurements*, Bulletin AMW, 4, 43–57, Gdynia 2001.

[4] PIENIĘŻNY A., MAKAR A., *Adaptive method of time delay estimation between narrow-band signals*, VIII International Scientific and Technical Conference "The Part of Navigation in Support of Human Activity on the Sea", Gdynia 1992.

[5] VASILEV V. N., AIDEMIRSKI P. P., *Time delay estimation with gradient type adaptive algorithm*, Electronics Letters, **26**, 21, 1797–1798 (1990).

[6] WIDROW B., STEARNS S. D., *Adaptive signal processing*, Prentice-Hall, Englewood Cliffs, New York 1989.

[7] ZIELIŃSKI R., *Generators of random numbers*, WNT, Warszawa 1972.