ACOUSTIC STREAMING CAUSED BY MODULATED SOUND AND WAVE PACKETS

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The practical applications of sound relate to approximately periodic sounds. It is shown, that theoretical results of acoustic streaming based on a periodic everywhere sound, should be revised in spite of the experimental data demonstrating the essentially different velocities of streaming. A new approach, which allows evaluating the streaming caused by sound of every type, both periodic and non-periodic, leads to similar results. The results of numerical calculations of streaming caused by modulated sound and series of pulses are compared with those given by formulae for a periodic everywhere sound.

Key words: acoustic streaming, non-periodic sound, wavepackets.

1. Introduction

In the nonlinear viscous flows a transfer of the acoustic momentum and energy to non-acoustic types of fluid motion, – a streaming (rotational) and heating (entropy) occur. The recent point of view of both the secondary phenomena, that are widely observed, is presented by MAKAROV in a comprehensive review [1].

The classic theories of heating and streaming are based on averaged conservation equations that need an acoustic wave periodic everywhere in the space [2]. Averaging over an integer number of periods of the sound wave occurs. All temporal derivatives of acoustic values result in zero after averaging. This allows to separate non-acoustic and acoustic inputs at the level of the conservation equations [1, 2]. This procedure is not completely consequent: the energy per unit volume is a quadratic nonlinear form but during the separation the overall energy is supposed to be a sum of non-acoustic and acoustic parts. In this way governing equations for heating are derived in the classic acoustics. Averaging over an integer number of periods of the sound wave is proceeded while the number of periods should be large but provide a temporal interval much less than the characteristic scale of heating and streaming. For a periodic everywhere acoustic wave, both the secondary effects grow with time as nonlinearity develops and

their characteristic temporal scale is in fact much greater than a period of the acoustic source. A delicate temporal structure of heating and streaming is out of intertest and cannot be evaluated in principle on the basis of averaged values.

Actually, all practical applications relate to the approximately periodic ultrasound, namely to wave packets or series or pulses. Even close to a periodic sound, like a modulated infinite wave or wave packet, may be treated by the classic formulae only approximately. An infinite modulated sound is not strictly periodic if the frequencies of the carrier wave and the envelope are not divisible. Anyway, the period is larger than that of the carrier wave. A wave packet is periodic in the confined temporal and spacial areas. There are experimental evidences that the velocity of streaming caused by series of pulses differs essentially from that predicted by the classic theory. In a set of experiments, the observed values are two times less or greater than the predicted ones [3].

Recently new results on acoustic streaming and heating caused by sound of every nature were published [4, 5]. The main idea is to combine the equations of conservative laws accordingly to the specific features of the modes in order to derive evolution equations governing the flow. A specific type of projecting is applied. No other type of projecting like averaging over a period of sound is used. In this way governing equations of compressible fluids accounting all the possible interactions of modes were derived. In the quasi-plane geometry of a flow relating to a small diffraction parameter μ, $\sqrt{\mu} = \lambda/R$ (R means the radius of the transducer, and λ is a characteristic scale of sound), the formula for the transversal component of the driving force sounds [4]:

$$F_x = \sqrt{\mu}\beta \left(\frac{\partial p}{\partial x}\frac{\partial p}{\partial y} - 0.5p\frac{\partial^2 p}{\partial x \partial y} - \frac{\partial}{\partial x}\int dy \left(\frac{\partial p}{\partial y}\right)^2 \right). \qquad (1)$$

Here, x and y are dimensionless co-ordinates related to the dimensional X, Y in the following way: $x = \sqrt{\mu}X/\lambda$, $y = Y/\lambda$, $\beta = \dfrac{\zeta + 4\eta/3}{\rho_0 c\lambda} + \dfrac{\chi(1 - 1/\gamma)}{\rho_0 c\lambda C_v}$ is the attenuation coefficient accounting loses due to the thermal conductivity as well, ζ, η are bulk and shear viscosities, χ is thermal conductivity. The acoustic beam propagates along the y-axis. The value p is the dimensionless pressure related to the dimensional P as follows: $p = P/(c^2\rho_0)$ with c being a small signal sound velocity and ρ_0 is the unperturbed density. Another transversal component of the driving acoustic force F_z has the form analogous to (1). The formula is valid with an accuracy of the order $\sqrt{\mu}\beta$ and all terms of higher order are omitted. A formal limit of the force given by the periodic everywhere acoustic pressureas an example taken from the paper of RUDENKO and GUSEV [6]

$$p(x, y, t) = P_0\theta(x)\exp(-\beta y/2)\sin(t - y) \qquad (2)$$

has been traced in [4]. Here $\theta(x)$ is the cross section of the acoustic beam which is supposed to be non-diffracting and is usually treated as the Gauss function. The governing equation for the transversal component of the velocity V_x includes hydrodynamic nonlinearity and viscosity:

$$\frac{\partial V_x}{\partial t} - \delta \frac{\partial^2 V_x}{\partial y^2} + (\mathbf{V}\boldsymbol{\nabla})V_x = F_x. \tag{3}$$

The value V is dimensionless, a dimensional velocity divided by the small signal sound velocity, t is the dimensionless time related to the dimensional T in the following way: $t = Tc/\lambda$; δ is the dimensionless viscous coefficient $\delta = \eta/\rho_0 c\lambda$ with η being shear viscosity. Equation (3) is rather complicated for an analytical solution. As the first approximation, the velocity may be found by simple integration of the driving force over time:

$$V_x(x, y, t) = \int\limits_{o}^{t} F_x(x, y, \tau) d\tau. \tag{4}$$

In the case of a periodic sound, the essential disease of formula (4) with the averaged value in place of the driving force $\Phi_x = \langle F_x(x, y, t) \rangle$ is that it results in an infinite growth of velocity, while the hydrodynamic nonlinearity actually provides a constant limit of the velocity when $t \to \infty$. Large values of the streaming velocity are hardly expected when the driving sound is a single pulse or a bounded wave packet, so the hydrodynamic nonlinearity is rather insignificant. Without this term, Eq. (3) is a linear equation of thermal conductivity with an acoustic source on the right-hand side. All other components of streaming may be found by the use of the condition of vortex flow:

$$\boldsymbol{\nabla}\mathbf{V} = 0. \tag{5}$$

2. Streaming caused by a non-periodic sound. Calculations and comparison with results predicted by the theory based on averaged values

In this section some non-periodic acoustic sources (modulated sound and wave packet) that are not periodic everywhere, but are actually widely used in the experiments, are treated by the formulae (1), (4) suitable for every type of sound, and by the formulae of the classic theory based on averaged values. The results of numerical calculations are compared.

2.1. Modulated sound

The pressure of two-dimensional modulated non-diffracting sound beams at the transducer is given by the following formula:

$$p(x, y = 0, t) = p_0 \exp(-x^2)(1 + m \cdot \cos(Nt)) \sin(t), \tag{6}$$

where p_0 is the amplitude, m is the depth of modulation, N is the frequency of the envelope, both being dimensionless. In the leading order, the sound in all the space is:

$$p(x, y, t) = p_0 \exp(-x^2 - \beta y/2) \sin(t - y)$$
$$+ 0.5 m p_0 \exp(-x^2 - \beta(1 - N)^2 y/2) \sin((1 - N)(t - y))$$
$$+ 0.5 m p_0 \exp(-x^2 - \beta(1 + N)^2 y/2) \sin((1 + N)(t - y)). \qquad (7)$$

The formula (7) accounts for the nonlinear distortion of every harmonics but does not account their nonlinear interaction. In a set of calculations of transversal velocity of the sound beam based on formulae (1) and (4) with a pressure of sound in the form of (7), the following values were accepted: $\gamma = 1.4$, $\beta = 0.004$, $N = 1/33$, $m = 1$, $c = 331.45$ m/sec (sound speed in the air at $T = 0°C$). The value β corresponds to the propagation of a strongly attenuated sound of frequency $f = 2$ MHz : $f = \dfrac{\pi \beta}{c(\alpha/f^2)}$ with the attenuation α/f^2. It is a constant available in the literature: for air $\alpha/f^2 = 1.85 \cdot 10^{-11} s^2/m$ [7]. The unperturbed density of air at $T = 0°C$ is $\rho_0 = 1.29$ kg/m^3. The formula (7) is correct beyond some vicinity of the transducer with an amplitude which does not depend on the amplitude of the pressure at the transducer $p_0 = 4\beta/(\gamma + 1)$. The velocity obtained by integration of the driving force of streaming is averaged, in accordance with formula (4), over one period of the carrier sound in order to compare the results with those given by the classic formulae on periodic sound. The other set of calculations relates to the classic formula for the driving force if the sound were periodic in the whole space:

$$\Phi_x = \sqrt{\mu} \frac{\partial}{\partial x} \langle p^2(x, y, t) \rangle. \qquad (8)$$

The velocity is then calculated according to (8) and (4). The results of calculations are shown in the figures below. Figure 1 shows the pressure of modulated sound at the transducer $y = 0$. All pictures below relate to a transversal co-ordinate $x = 0.5$.
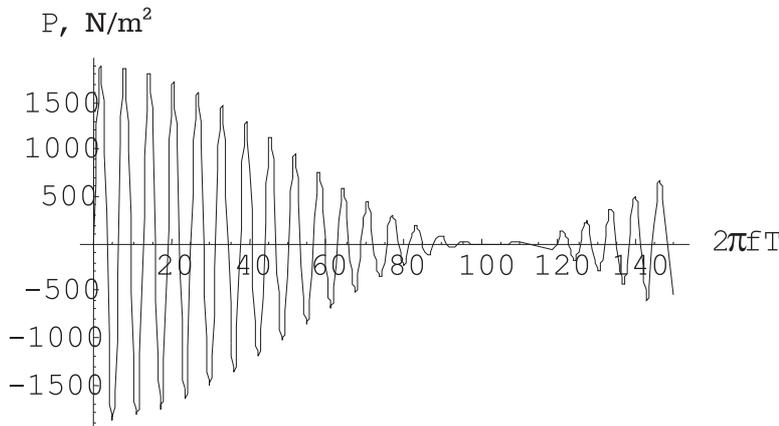
Fig. 1. Sound pressure at the transducer $y = 0$ ($x = 0.5$).

As an example, a non-diffracting beam is taken, so the velocity in another transversal co-ordinate may be calculated taking into account that $V_x(x, y, t) \sim x \cdot \exp(-2x^2)$. This function achieves a maximum of the absolute value at $x = \pm 0.5$. At the upper half-space, the velocity is negative: a flow turns to the axis of propagation. Figure 2 presents spatial distribution of the transversal velocity of streaming in different times, Fig. 3 presents the temporal behavior of velocity at two different distances from the transducer. As for the spatial distribution, the difference between both the sets is noticeable at the beginning of the evolution and becomes less with time passing. The temporal behavior exhibits a growth of the velocity in different ways. All the figures present the dynamics of $V_x(x, y, t)/\sqrt{\mu}$, so the values of velocity vary with the radius of the transducer and with the characteristic wavenumber.
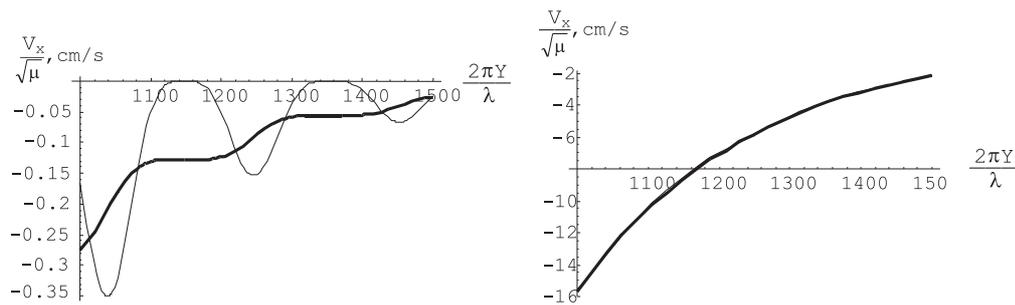
Fig. 2. Transversal velocity of streaming via dimensionless distance from the transducer $2\pi Y/\lambda$, thin line marks calculations according to the formula (8) suitable for periodic sound, bold line marks calculations according to the general ones (1), (4): a) at $T = 2\pi/f$, b) at $T = 2\pi \cdot 100/f$, curves are undistuingishable.
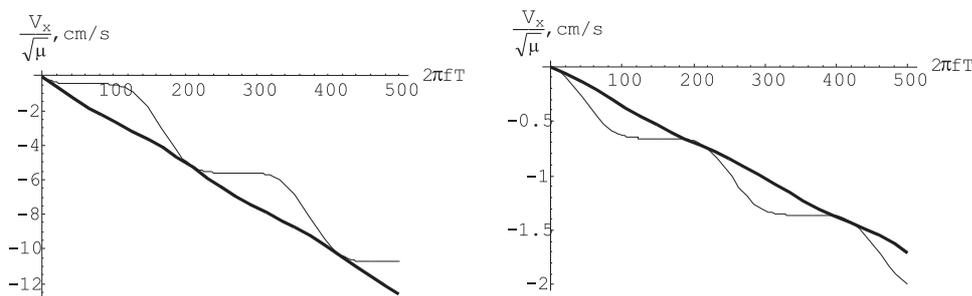
Fig. 3. Transversal velocity of streaming via dimensionless time $2\pi f T$, thin line marks calculations according to the formula (8) suitable for periodic sound, bold line marks calculations according to the general ones (1), (4): a) at $Y = \lambda \cdot 1000/2\pi$, b) at $Y = \lambda \cdot 1500/2\pi$.

2.2. Wave packet

As an example of an wave packet, a set of single positive two-dimensional pulses propagating along the y-axis is considered. Every pulse is a solution of the Burgers equation multiplied by the transversal Gauss function $\exp(-x^2)$. A non-diffracting

beam is considered as well. The pressure of a single pulse is therefore defined by the formula:

$$p_n(x, y, t) = -\sqrt{\frac{2\beta}{\pi}} \exp(-x^2) \frac{\exp(-(\tau + \tau_n)^2/2\xi)}{\varepsilon\sqrt{\xi/\beta}\left(C - \mathrm{Erf}\left((\tau + \tau_n)/\sqrt{2\xi}\right)\right)}, \qquad (9)$$

where C is a constant responsible for the shape and the polarity of the curve, $\tau = t - y$ is retarded time, $\xi = \beta y$, $\varepsilon = (\gamma + 1)/2$. A choice of τ_n allows to form a set of pulses with a delay between every two ones. In spite of the nonlinearity of the flow, it is important to consider the spatial domain and temporal intervals, where pulses do not cover each other. Series of ten pulses $\{p_n\}_{n=1}^{10}$ with $\tau_n = \tau_0 n$, $\tau_0 = 100$ are considered. The driving force of streaming is calculated as a superposition of forces caused by every pulse. The same set of pulses gives an averaged driving force if it were treated as a periodic field. The transversal component of this force looks as follows:

$$\Phi_x = \sqrt{\mu}\frac{\partial}{\partial x} \sum_{n=1}^{n=10} \langle p_n^2(x, y, t)\rangle. \qquad (10)$$

Here, square brackets mean averaging over the temporal delay τ_0. Calculations based on (10) are compared with calculations based on formula which does not need a periodic everywhere acoustic source (1). The driving acoustic source given by (1) is averaged over τ_0 for a consequent comparison with the results obtained by formula (10). A set of acoustic pulses at a non-dimensional distance from the transducer $Y = \lambda \cdot 100/2\pi$ is shown in Fig. 5. Here, λ and f should be understood as effective the wavenumber (width) and frequency ($f = c/\lambda$) of a single pulse These are rather approximate values since a pulse is widening during its propagation. All calculations below relate to the value $\beta = 0.1$.

The stationary levels of $V_x(x, y, t)/\sqrt{\mu}$ after passing a set of pulses are quite different when calculated either on the basis of the formula for periodic sound (10) or on the
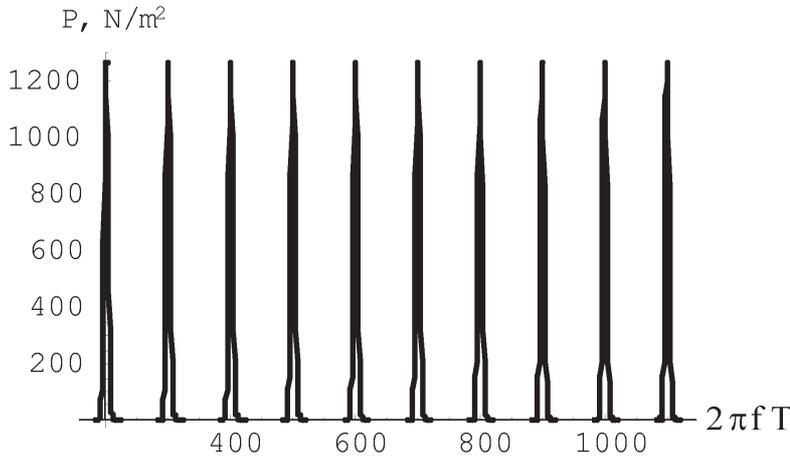
Fig. 4. Acoustic pressure as a function of dimensionless time $2\pi fT$ at $Y = \lambda \cdot 100/2\pi$, $x = 0.5$.
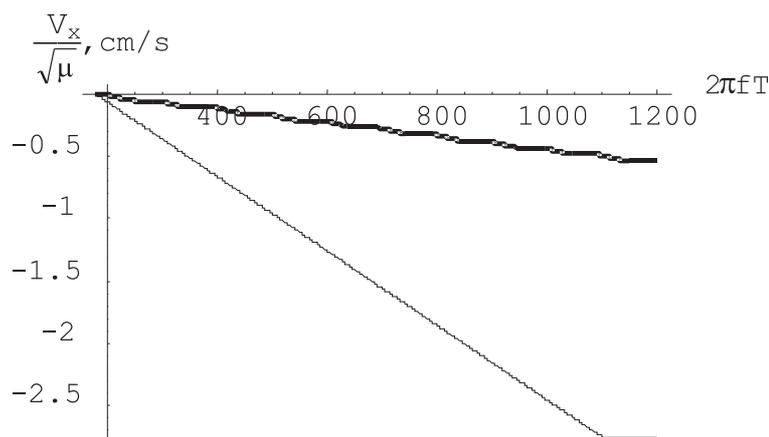
Fig. 5. Transversal velocity of streaming via dimensionless time $2\pi fT$ at $Y = \lambda \cdot 100/2\pi$, $x = 0.5$. Thin line marks calculations according to the formula (10) suitable for periodic sound, bold line marks calculations according to the general ones (1), (4).

general basis (1). The more sound differs from the periodic everywhere one, the more results of calculations differ. In many practical applications of non-periodic sound, the predictions of streaming and heating by the classic formulae of periodic sound should be revised.

3. Conclusions

Examinations of secondary processes caused by sound in lossy media reveal the importance of using formulae based on instantaneous values. Examples of calculations due to the original formula (1) governing the acoustic streaming compared to the calculations based on the well-used formula for averaging values (8) reveal a noticeable difference. As acoustic sources, a set of pulses and a modulated waveform are considered. Moreover, there exists a great variety of acoustic sources that in principle can not be treated by the well-used formula, including single pulses or a wavepacket. The advance in the theory of secondary phenomena in attenuating flow – acoustic streaming and heating – becomes possible by the application of projecting at the level of initial conservation equations which essentially deals with instantaneous values.

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