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COMPARISON OF DIGITAL LOUDSPEAKER – EQUALIZATION TECHNIQUES

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An overview of digital equalization techniques for loudspeaker systems is presented. The known algorithms are systematized and modified. Equalization is effected by a single digital filter for the on-axis loudspeaker response. Besides classical FIR and IIR digital filter structures, their warped versions (WFIR and WIIR) are used to build equalization systems. Equalizers of different types are compared with regard to efficiency. Also information about objective and listening-test assessments of the quality of equalization obtained by the different equalizers is provided. Besides the equalizer quality assessment techniques know from the literature, new techniques, which better represent the properties of the human auditory system, are proposed. For the new techniques better agreement between the objective results and the listening-test ratings was obtained. On the basis of objective assessments of 32 equalization systems carried out for two stereophonic pairs of loudspeaker systems algorithms yielding the best results for each of the tested filter structures (FIR, IIR, WFIR and WIIR) were selected and evaluated by listening tests.

Key words: equalizer, digital filter, loudspeaker system.

1. Introduction

In the most general terms, digital equalization of an electroacoustic system consists in the incorporation of a proper digital filter between the signal source and the system to be equalized. Ideally, the equalizing filter impulse response $f(n)$ for linear distortion equalization should be such that the following relation is satisfied:

$$h(n) * f(n) = \delta(n), \tag{1}$$

where $h(n)$ is the impulse response of the equalized loudspeaker system, $\delta(n)$ is a delta function, and * is a convolution operation. Then the loudspeaker system after equalization is characterized by a flat magnitude response and a linear phase response. It follows from relation (1) that equalizing filter transfer function $F(z)$ should be equal to:

$$F(z) = \frac{1}{H(z)}, \tag{2}$$

where $H(z)$ is the equalized system's transfer function. Since loudspeakers are band-pass devices, Eq. (1) cannot be satisfied in practice. In addition, the equalization of the loudspeaker system's magnitude response within a range of too low frequencies causes an increase in nonlinear distortion due to the peak linear displacement of the driver diaphragm. The equalization performed here is for a specific passband. The principal problem is the determination of the equalizing filter transfer function. Generally, loudspeaker systems are not minimum-phase systems and if the equalizing filter's transfer function is directly determined from Eq. (2), unstable solutions can be obtained [5].

2. Algorithm for designing equalizing filters

The general algorithm used here to design equalizing filters is shown in Fig. 1. The algorithm does not cover two essential steps: the selection of loudspeaker systems for equalization and the measurement of their response. These and the other major components of the algorithm and some practical recommendations are described in later sections. Beside some algorithm steps, the numbers of sections in which they are described are given in brackets.

2.1. Criteria for selecting loudspeaker systems for equalization

When considering equalization for a given loudspeaker system, the diffraction distortion should be taken into account. If possible, the box of the loudspeaker system to be equalized should have rounded edges. When a loudspeaker system with a sharp-edged box is equalized, the frequencies of the irregularities caused by diffraction depend on the difference between the path of the direct wave emitted by the loudspeaker and that of the wave refracted on the box's edge. Thus the frequencies of the irregularities associated with diffraction depend on the distance between the loudspeaker system and the receiver. In order to equalize such distortion, a constant distance between the listener and the loudspeaker system must be maintained and the response must be measured for exactly this distance. Otherwise, the equalizing filter may introduce additional irregularities for the diffraction frequencies associated with the measuring distance without equalizing the irregularities associated with the listening distance.

If the aim of equalization is to widen the loudspeaker system's low frequency range, one should pay attention to the peak linear displacements of the woofer's diaphragm. If a speaker with a too small diaphragm peak linear displacement is used, one can expect an increase in nonlinear distortion.

Another important criterion for selecting loudspeaker systems for equalization is the distribution and character of the resonances of their diaphragms. Taking into account a possible increase in nonlinear distortion, the system's dynamics and the stability of the equalizing filters, it is recommended that the loudspeakers' magnitude response irregu-

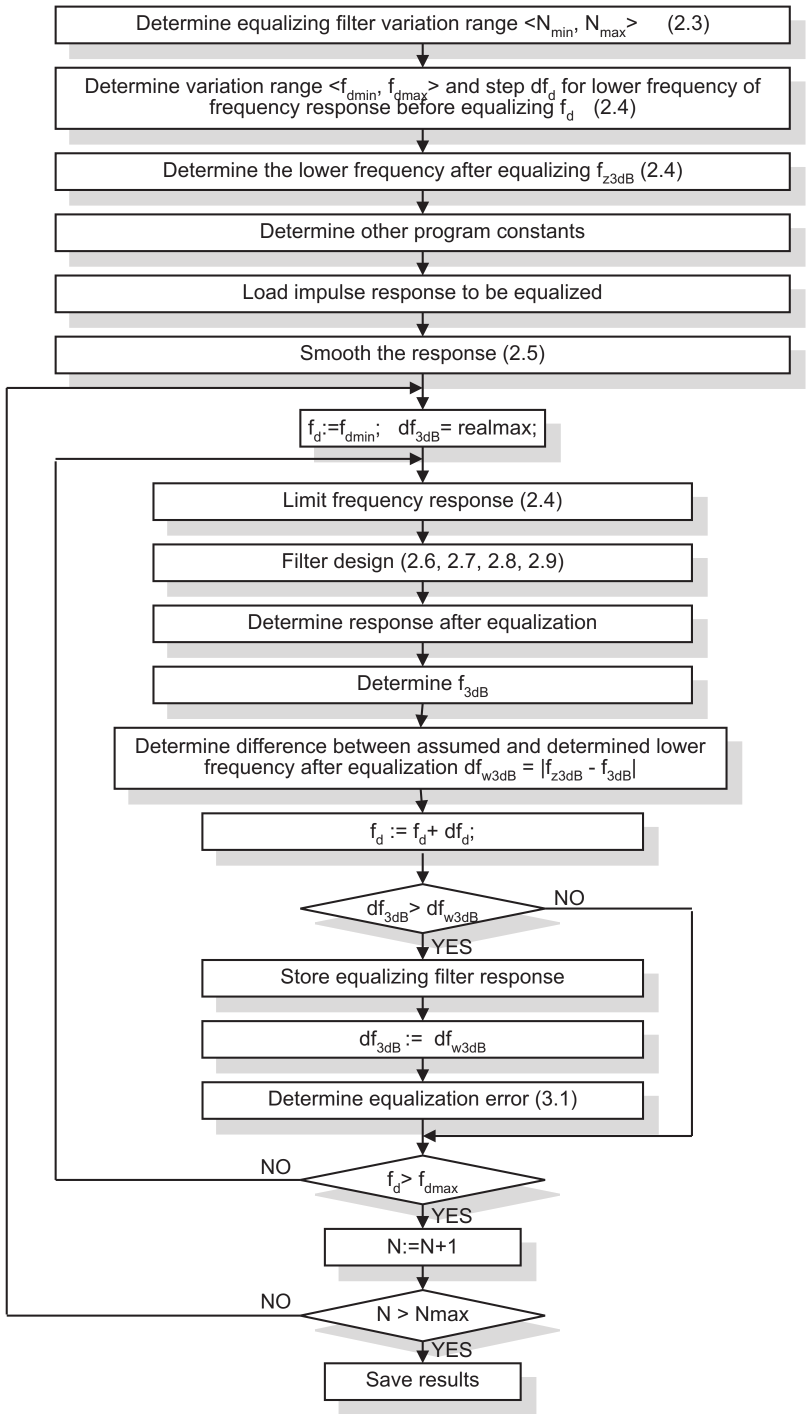

Fig. 1. Equalizing filter design algorithm.

larities should have a low Q and a small amplitude. Also the diaphragms' resonance frequencies may influence the selection of a particular equalizing filter structure. Classical filter structures better equalize the high frequency range, while warped filter structures are more suitable for low and medium frequency ranges.

Considering all the above factors, equalization of the response of loudspeaker systems which are to work with a digital equalization system seems to be most sensible.

2.2. Response measurement

The loudspeaker system response should be measured in accordance to the following rules: most preferably in an anechoic chamber, using high-class measuring equipment and proper standards and taking into account the way in which the loudspeaker systems are to be used (the measuring distance should be similar to the typical loudspeaker system-listener distance and the power supply similar to the typical loudspeaker system power supply). Because of the diffraction distortion mentioned above, it is recommended to perform the measurements for a few distances and compare the responses to check if no irregularities appear in the magnitude response in the frequency ranges which change with the distance.

If the equalization is to cover the very-low frequency range, it may be difficult to find an anechoic chamber with a properly low frequency limit and therefore approximate techniques are usually used [4, 19, 30, 33]. But in the latter case, one must make sure that the measurements are precise and analyse the effect of a change in the values characteristic for the particular techniques (e.g. for the Keele technique [19] the linking frequency and the parameters that affect the response level in the near field should be analysed).

In the case of some design algorithms it is necessary to eliminate completely any group delay resulting from the distance between the loudspeaker system and the measuring microphone.

For the purpose of this research the impulse responses of the loudspeaker systems to be equalized were measured in an anechoic chamber. Because of the chamber's limitations (the low frequency limit much higher than the minimum frequency covered by equalization) the responses of the loudspeaker systems were determined by the Keele technique, i.e. by combining the frequency response determined in the near field with the frequency response determined in the far field.

2.3. Determination of the equalizing filter order variation range

The maximum order of an equalizing filter is connected directly with the latter's structure and the signal processor's computing power. The equalizing filter order does not always have to be the maximum order realized for a particular computing power. It is recommended to carry out objective analyses of equalization results also for lower orders because the errors (e.g. the maximum irregularities of the magnitude response) obtained for the latter can be slightly smaller.

2.4. Frequency response limitation

The aim of equalization band limitation is to reduce the probability of introducing nonlinear distortion by the equalizing filter and to increase the filter's dynamics. The equalization band was limited using a function which makes it possible to limit the magnitude response below the lower frequency and above the upper frequency. The limitation consists in replacing the magnitude response samples below the lower frequency and above the upper frequency with the value of the nearest sample from the equalization band. The selection of a frequency to limit the equalized response in the low-frequency range is particularly critical. This frequency is often different from the desired lower frequency of the response after equalization, and so its selection should be combined with an analysis of the loudspeaker system's response after equalization. In this paper the algorithm is simplified by assuming the minimum lower frequency after equalization to be equal to the box's resonance frequency. It would be especially helpful to measure the effect of limiting the equalized response's lower frequency on the changes in nonlinear distortion after equalization. More information can be gained if, instead of the total harmonic distortion, an analysis of the Volterra series [10] or the NARMAX model [25] is performed. Another approach is to carry out listening tests to determine the effect of this frequency on the parameters associated with the loudspeaker system's nonlinearities.

2.5. Smoothing response to be equalized

The responses used for digital equalization should be smoothed to make it possible to equalize mainly the distortion perceptible by the human being. The equalization of the smoothed response may contribute to a reduction in the equalizing filter order and an increase in the filter dynamics; it also reduces the probability that the equalizing filter will introduce nonlinear distortion. Smoothing with a width of 1/16 octave was applied here. The proper choice of the equalizing filter width remains unresolved. The best solution seems to be to construct a complex algorithm based, among others, on the works of Green *et al*. [11] and Toole *et al*. [34].

2.6. Separation of the minimum-phase part

As mentioned in the Introduction, some equalizing filter design techniques allow one to obtain stable designs only for minimum-phase systems. Then the system's minimum-phase part is separated in one of the design stages. The techniques of separating the minimum-phase part from the impulse response of the loudspeaker system to be equalized are presented later in this section.

2.6.1. Complex cepstrum

The use of the complex cepstrum for the separation of the minimum-phase part of the system response was described, among others, by Oppenheim and Schafer [23].

DZIECHCIŃSKI [7] proposed to apply this technique to equalization systems. Here the MATLAB's *rceps* function is used to determine the minimum-phase part by the complex cepstrum method. The function's algorithm is described by this relation [29]:

$$h_{\min}(n) = \mathrm{Re}\big(\mathrm{IFFT}\big(\exp(\mathrm{FFT}(u(n) \cdot h_c(n)))\big)\big), \tag{3}$$

where $h_{\min}(n)$ is the minimum-phase part of the system response, Re( ) is the real part, IFFT is the inverse fast Fourier transform, $h_c(n)$ is the cepstrum of sequence $h(n)$ described by relation (4):

$$h_c(n) = \mathrm{Re}\big(\mathrm{IFFT}\big(\log(\mathrm{FFT}(h(n)))\big)\big), \tag{4}$$

where log is the natural logarithm, $u(n)$ stands for windowing in the cepstral domain, described by relation (5):

$$u(n) = \begin{cases} 1 & n = 0, \\ 2 & 1 \le n < N/2, \\ 1 & n = N/2, \\ 0 & N/2 \le n \le N-1. \end{cases} \tag{5}$$

2.6.2. The Hilbert transform

HAWKSFORD [15] proposed to use the Hilbert transform to separate the minimum-phase part in equalization systems. The minimum-phase part is determined from the folowing relation:

$$h_{\min}(n) = \mathrm{Re}\big(\mathrm{IFFT}\,(\exp\,((\mathrm{HILBERT}\,(\log|\mathrm{FFT}(h(n))|))^{*}))\big) \tag{6}$$

where HILBERT is the Hilbert transform, * is a complex conjugate.

2.6.3. Decomposition of the factored form

The ARMA model of the factored form decomposition technique consists in expressing the model of the system to be equalized in the product form (formula (7)) [23], where the components $(1 - c_r z^{-1})$ and $(1 - d_p z^{-1})$ correspond to respectively zeros and poles, and exploit the property that in the case of a minimum-phase sequence all the zeros and poles of its transfer function lie within a unit circle [12].

$$H(z) = A \cdot \frac{\prod\limits_{r=1}^{R} (1 - c_r z^{-1})}{\prod\limits_{p=1}^{P} \big(1 - d_p z^{-1}\big)}, \tag{7}$$

where A is a factor of proportionality. The technique was applied to equalization systems by GREENFIELD and HAWKSFORD [12].

2.7. *Design of FIR equalizing filters*

2.7.1. *Direct inversion of magnitude response*

This technique represents the most intuitive approach to loudspeaker system magnitude response equalization. First the loudspeaker system magnitude response is inversed and then an equalization filter with this magnitude response is designed using one of the techniques of designing digital filters with a magnitude response set. In the literature one can find several techniques which can be used to design a FIR (finite impulse response) filter, realizing a specified magnitude response. In this research the following techniques were used:

- windowing [23],
- the Parks–McClellan algorithm [23] – MATLAB's function *remez* [29],
- the least square linear-phase method [24] – MATLAB's function *firls* [29],
- CLS (constrained least square) [28] – MATLAB's function *fircls* [29].

2.7.2. *Autoregressive (AR) modelling*

If the transfer function of a loudspeaker system to be equalized is modelled as an autoregressive system (AR), the following description of the loudspeaker system is obtained:

$$H(z) = \frac{G}{1 - \sum\limits_{n=1}^{N} a_n z^{-n}}, \tag{8}$$

where a_n are model coefficients and G is the gain. Thus the equalizing filter transfer function can be obtained directly from this relation:

$$F(z) = \frac{1}{H(z)} = \frac{1 + \sum\limits_{n=1}^{N} a_n z^{-n}}{G}. \tag{9}$$

The AR model can be determined using the LPC technique [2, 26, 29]. This technique has several versions, the most popular ones are the autocorrelation method and the covariance method. These and a few other methods are presented in a paper of RUTKOWSKI [26].

2.7.3. *The least-squares (LS) algorithm*

The least-squares algorithm for designing equalization filters was proposed by Clarkson *et al*. [5, 22], WILSON [38] and others. The algorithm was also employed in a commercial equalization system [27]. The minimized error is expressed by the following formula:

$$E = \sum_{n=0}^{N-1} e^2(n) = \sum_{n=0}^{N-1} [\delta(n) - y(n)]^2, \tag{10}$$

where E is the equalization error, N is the length of the equalized loudspeaker system's impulse response, $e(n)$ is an equalization error vector and $y(n)$ is the loudspeaker system response after equalization. If the least-squares algorithm were applied to the error defined above, the equalizing filter for the mixed-phase response would be infinite and noncausal. The algorithm can be used to determine a filter for equalizing the minimum-phase part of the loudspeaker's response. But a modification allowing one to equalize the mixed-phase impulse response $h(n)$ is possible. This is done through the delay *δ(n)* by k samples. Then the minimized error has the follwing form:

$$E = \sum_{n=0}^{N-1} e^2(n) = \sum_{n=0}^{N-1} [\delta(n-k) - y(n)]^2. \tag{11}$$

In order to determine the filter, one must solve a set of N linear equations described by this relation:

$$\mathbf{R} \cdot f(n) = g(n), \tag{12}$$

where **R** is an autocorrelation matrix of the measured impulse response (formulas (13) and (14)),

$$\mathbf{R} = \begin{bmatrix} r_0 & r_1 & r_2 & \cdots & r_{M-1} \\ r_1 & r_0 & r_1 & \cdots & r_{M-2} \\ r_2 & r_1 & r_0 & \cdots & r_{M-3} \\ \vdots & \vdots & \vdots & \cdots & \\ r_{M-1} & r_{M-2} & r_{M-3} & \cdots & r_0 \end{bmatrix}, \tag{13}$$

$$r_j = \sum_{t=0}^{M+N-2} h_{t+j} h_t, \qquad -(N-1) \leq j \leq N-1 \tag{14}$$

$f(n)$ is the equalization filter response, $g(n)$ is a cross correlation matrix of the measured impulse response and the desired output signal. If the desired system response is equal to $\delta(n-k)$, the matrix $g(n)$ is described as follows:

$$g(n) = \begin{bmatrix} h_k \\ h_{k-1} \\ h_{k-2} \\ \\ h_0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{15}$$

The equalization error E is a function of delay k and reaches a minimum value in a wide range of delays above a certain value j which follows from the trace of the measured impulse response. Hence it is possible to search for an equalizing filter at a predefined equalizing error level and stop the search process when the error reaches the preset value. It is also possible to allow an admissible error if the critical value is the smallest possible signal delay after equalization [38]. In the present paper, a set of filter responses for the whole range of delays is determined. Then a filter with a response which gives the smallest error for the criterion defined by relation (11) is chosen.

2.7.4. *Fast deconvolution*

The fast deconvolution algorithm has been presented by KIRKEBY *et al*. [20] and FARINA *et al*. [9]. KIRKEBY *et al*. describe the application of the fast deconvolution algorithm to a multichannel sound reproduction. The algorithm can be used to design both the minimum-phase filters and the mixed-phase ones. The relation given by KIRKEBY *et al*. [20] can be simplified to a single-channel assuming that the desired response $W(z) = 1$. Then the equalizing filter transfer function can be determined from the following relation [9]:

$$F(z) = \frac{H^*(z)}{H^*(z) \cdot H(z) + \beta}, \tag{16}$$

where β is a regularization parameter [20].

The fast deconvolution algorithm used in this research consists of four steps:

1. determine the N-point FFT of the equalized impulse response,
2. determine the equalizing filter transfer function $F(z)$ using relation (16) given by FARINA *et al*. [9],
3. determine the initial equalizing filter impulse response $f(n)$ by determining the N-point inverse FFT from $F(z)$,
4. implement the m-point delay through cyclical shifts of $f(n)$.

KIRKEBY *et al*. [20] recommends $m = N/2$. In this paper m was experimentally determined in the range $0 - N$ and then a filter whose response gave the smallest error for the criterion described by relation (11) was selected.

2.8. *Design of IIR equalizing filters*

Only minimum-phase filters can be realized using filters with an infinite impulse response. The design process can be divided into the following three main stages:

- the separation of the minimum-phase part from the system response,
- the design of a filter (model),
- the determination of the inverse response.

The ARMA model can be determined for both the loudspeaker system response and the inverse response. Hence the order of the stages may be different.

2.8.1. *Design with inverse response modelling*

In the case of designing an IIR equalizing filter with modelling the inverse response, the design algorithm is as follows:

1. determine the module of the loudspeaker system's frequency response,
2. determine the inverse response for one element at a time

$$H_i(k) = 1/H(k), \tag{17}$$

3. separate the minimum-phase part (Sec. 2.6),
4. design the filter by the Prony's method [21, 24, 29] or by the Steiglitz–McBride method [29, 32].

2.8.2. Design with loudspeaker system response modelling

In the case of designing an IIR equalizing filter with equalized loudspeaker system response modelling, the design algorithm is as follows:

1. separate the minimum-phase part (Sec. 2.6),
2. determine the ARMA model of the loudspeaker system by the Prony's method [21, 24, 29] or by the Steiglitz–McBride method [29, 32],
3. inverse the rational function which describes the model and normalize it by the first numerator coefficient (formula (18)).

$$F(z) = \frac{\dfrac{1 - \sum\limits_{k=1}^{N} a_k z^{-k}}{b_0}}{\dfrac{\sum\limits_{k=0}^{M} b_k z^{-k}}{b_0}} = \frac{\sum\limits_{k=0}^{N} b_k^{'} z^{-k}}{1 - \sum\limits_{k=1}^{M} a_k^{'} z^{-k}}, \tag{18}$$

where a_k, b_k are model coefficients, $b_0^{'} = 1/b_0$, $b_k^{'} = a_k/b_0$, $a_k^{'} = b_k/b_0$.

2.9. *Design of warped filters*

The idea of warped filters consists in replacing the unit delay z^{-1} of a filter by first-order all-pass filter $D_1(z)$ described by relation (19) [18]:

$$D_1(z) = \frac{z^{-1} - \lambda}{1 - \lambda z^{-1}}, \tag{19}$$

where λ is a warping parameter and $D_1(z)$ is a warped delay element. As a result, filtration with a nonuniform frequency scale, whose distortion is defined by the parameter λ, becomes possible. Information on warped filters can be found, among others, in papers of KARJALAINEN *et al.* [18], TYRIL *et al.* [35] and HÄRMÄ *et al.* [14]. In contrast to the design of classical digital filters, prior to the design of warped filters with a constant parameter λ the scale of the designed filter's frequency response has to be converted. The further steps in the design are the same as in the case of classical filters and depend on the algorithm.

2.9.1. Choice of values of the warping parameter λ

An appropriate warping parameter λ must be chosen for the conversion of the frequency scale. In the case of band filters, the choice of the λ parameter depends on the filter's limit frequencies in relation to the sampling frequencies. Positive values of λ increase the low frequencies resolution while negative values increase the high frequencies resolution. When designing filters for loudspeaker system equalization, such a warping parameter can be chosen that the warped frequency scale will approximate one of the pitch scales. The warping parameter λ for the Bark scale and the ERB scale can be determined from the formula (20) and (21) [31].

$$\lambda = 1.0674\sqrt{\frac{2}{\pi}\arctan(0.06583 f_s)} + 0.1916, \tag{20}$$

$$\lambda = 0.7446\sqrt{\frac{2}{\pi}\arctan(0.1418 f_s)} + 0.03237, \tag{21}$$

where f_s is the sampling rate (kHz).

2.9.2. Conversion of frequency scale

Having determined the warping parameter, one should convert the frequency scale of the designed equalizing filter. This can be done in several ways.

One way [35] consists in distorting the response directly in the frequency domain. For each point of the linear frequency scale f, one should find the corresponding frequency f according to the formula:

$$f_w(f,\lambda) = f + \frac{f_s}{\pi}\arctan\frac{\lambda\sin\dfrac{2\pi f}{f_s}}{1-\lambda\cos\dfrac{2\pi f}{f_s}}. \tag{22}$$

Having obtained the response from the formula (22), one interpolates the samples for the linearly distributed frequency values.

Another way consists in converting the frequency scale by means of a WFIR filter [35]. The WFIR filter's coefficient is the loudspeaker system impulse response $h(n)$. The delta function $\delta(n)$ is fed to the system's input. If the impulse response is to be warped with the parameter λ, the filtration is performed using a warping parameter value of $-\lambda$. The output signal is the loudspeaker system's warped impulse response.

A still another way was proposed by KARJALAINEN *et al.* [17, 18]. The system impulse response is filtered by means of first-order all-pass filters (with the transfer function defined by formula (19)) connected in cascade. As in the case of the previous technique, if the impulse response is to be warped with the λ parameter, the filtration is performed using a warping parameter value of $-\lambda$.

The second technique seems to be most effective and it is used in this paper.

3. Assessment of equalization results

The equalization results were assessed: 1) objectively by running proper computer simulations in the MATLAB environment, 2) through measurements using a real-time equalizer and 3) by means of listening tests. Commercial vented-box loudspeaker systems were equalized. The market value of the system no. 1 was twice lower than that of the system no. 2.

3.1. Objective measures of the equalization quality

As the measure of the equalization quality the average ripple of magnitude response in a specified frequency band E_a, defined as a standard deviation (formula (23)), is given in the literature on this subject [5, 22]. Another measure is the square of the Euclidean distance between the ideal response having the form of the function $\delta(n)$ and the system response after equalization, which can be expressed in per cents (formula (25)) or decibels (formula (26)) [16]. To be consistent with the literature [5, 22], a biassed estimator is used in formulas (23)–(26). For *Mo* and M higher than 1000 the difference between the biassed estimator and the unbiassed one is negligible:

$$E_a = \sqrt{\frac{1}{Mo} \sum_{i=0}^{Mo-1} (10 \log |Y(k_i)| - r)^2}, \tag{23}$$

where Mo is the number of samples in the analysed band,

$$r = \frac{1}{Mo} \sum_{i=0}^{Mo-1} 10 \log |Y(k_i)|, \tag{24}$$

$Y(k_i)$ is the i-th sample of the loudspeaker system frequency response after equalization.

$$E_i = \frac{1}{M} \sum_{n=1}^{M} [\delta(n-k) - y(n)]^2 \cdot 100\%, \tag{25}$$

where M is the impulse response length, $\delta(n-k)$ is the function $\delta(n)$ delayed by k samples, $y(n)$ is the loudspeaker system impulse response after equalization.

$$E_{il} = 10 \log \left[\frac{1}{M} \sum_{n=1}^{M} [\delta(n-k) - y(n)]^2 \right] \quad \text{[dB]}. \tag{26}$$

The maximum irregularities of the magnitude response and the standard deviation were determined for two frequency bands. One band was associated with the assumed lower frequency after equalization (1.26*$f_{d-3\ \mathrm{dB}}$ – 20 kHz), while the other one (100 Hz – 8 kHz) is associated with the useful frequency range for which the smallest magnitude of the response irregularities are required by the standards [37]. The average magnitude frequency response defined by relation (23) does not exactly correspond to the characteristics of human hearing because of the linear representation of the frequency scale. This problem was raised, among others, by DOBRUCKI *et al.* who proposed to determine the standard deviation using the logarithmic frequency scale [6]. In contrast to the literature reports, samples determined for the ERB scale and not for the linear scale (the direct result obtained by FFT) were used to determine the standard deviation. The WFIR filter was used for conversion to the ERB scale.

The limitation of the average magnitude response ripple as a measure of error is that it does not take into account the phase distortion. Whereas the square of the Euclidean distance between $\delta(n)$ and the system impulse response after equalization in the linear (formula (25)) or logarithmic (formula (26)) measure does not include the impulse response time spread after equalization.

3.2. Results of objective assessment of the equalization errors as function of the equalizing filter order

Using the algorithms described in Secs. 2.6–2.9, each class of equalization filters can be designed by 8 methods, which gives the total number of 32 algorithms for designing equalization systems. The particular error measures versus the equalizing filter order were analysed. It was assumed that an equalization system operating in real time was to be realized using a modern medium-class signal processor. In the case of a stereophonic sound system and a sampling frequency of 44.1 kHz such processors allow one to realize FIR filters with an order of about 400. An FIR filter order range of 128–512 was adopted for comparisons. The IIR, WFIR and WIIR filter orders were such that the computing power needed to realize the filters corresponding to the computing power of the FIR filter. The conversion factor depends on the signal processor architecture and the filter structure. The filters were to be realized using a Motorola DSP56000 family signal processor. The IIR filter was to be realized using a structure consisting of parallel second-order, type-I sections (the structure recommended for high-quality audio applications for the DSP56000 family [3]) while the structures proposed by KARJALAINEN *et al.* were to be used for the WFIR and WIIR filters [18]. For such filter structures and signal processors the orders of the particular filter types can be determined from the following relations:

$$N_{\mathrm{IIR}} = \frac{2N_{\mathrm{FIR}} - 21}{5}; \qquad N_{\mathrm{WFIR}} = \frac{N_{\mathrm{FIR}}}{3}; \qquad N_{\mathrm{WIIR}} = \frac{N_{\mathrm{FIR}}}{4}. \qquad (27)$$

In the first stage, the algorithms were compared within each filter class and an algorithm characterized by the smallest magnitude of the response equalization error was determined for each kind of filters. For the FIR filter the best results were obtained by AR modelling, but for the loudspeaker systems no. 1 and no. 2, respectively, the LPC technique and the Prony's method proved to be slightly better. Good results were also obtained at lower equalization orders (< 150) for the system no. 2 by inverting the magnitude response and designing the filter by the windowing. In the case of IIR filters, the best results were obtained by modelling the loudspeaker system response by the Steiglitz–McBride method. The separation of the minimum-phase part by the Hilbert transform gave the same results as when the complex cepstrum was used for this purpose. In the case of WFIR filters, the best results were obtained when AR modelling by the Prony's method was used, but the LPC algorithm yielded similar results for the loudspeaker system no. 2. In the case of WIIR filters, the smallest equalization error was obtained by modelling the loudspeaker system response by the Steiglitz–McBride method. The separation of the minimum-phase using the complex cepstrum and the Hilbert transform gave similar results, but for orders higher than 120, slightly better results were obtained when the Hilbert transform was used.

The design techniques were also compared with regard to impulse response equalization. A significant effect of the design technique on impulse response equalization, in agreement with theory, was observed only for the FIR filters. As regards the phase response equalizing algorithms (LS and fast deconvolution), the best results were obtained in most cases for the LS algorithm, but for the loudspeaker system no. 1, in a quite large range of equalizing filter orders a smaller impulse response error was obtained for the filter designed by means of the fast deconvolution algorithm.

In the second stage, comparisons were made for each class of the filters designed in the first stage by the selected techniques:

1. FIR – LS;
2. FIR – AR modelling (LPC for the loudspeaker system no. 1 and the Prony's method for the loudspeaker system no. 2);
3. IIR – loudspeaker system response modelling by the Steiglitz–McBride method, minimum-phase part separation by the complex cepstrum method;
4. WFIR – AR modelling by the Prony's method;
5. WIIR – loudspeaker system response modelling by the Steiglitz–McBride method, minimum-phase part separation by the Hilbert transform;

The results obtained by the five techniques mentioned above for the loudspeaker system no. 2 are shown in Figs. 3–6. For the loudspeaker system no. 1 only the maximum magnitude response irregularities are shown in Fig. 2.

Comparing the maximum magnitude response irregularities in the full frequency range for the loudspeaker system no. 1, the best results were obtained using the IIR filter, but for the computing powers corresponding to FIR filter orders below and above 360, comparable or better results were obtained for the filters WFIR and FIR AR. Comparing the maximum magnitude response irregularities in the narrowed frequency band for computing powers corresponding to FIR filter orders above 240, the best results were obtained for the WFIR filter and slightly worse for the filters IIR and WIIR. At small computing powers, the WIIR filter gave the best results. As regards the standard deviation in the full frequency range, the best results at higher computing powers were obtained for the WFIR filter and slightly worse ones for the filters IIR and FIR AR, while at lower computing powers the results were similar for all the filters, except for the FIR LS filter. In the narrowed frequency band, the smallest error was obtained for warped filters WIIR and WFIR at respectively lower and higher computing powers.

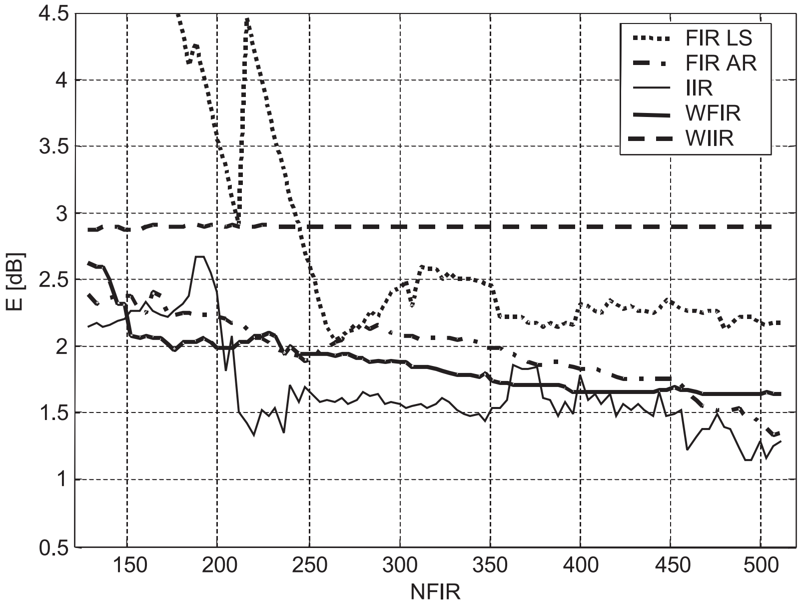

Fig. 2 Maximum magnitude response irregularities versus equalizing filter order. Analysed band: 77 Hz–20 kHz. Loudspeaker system no. 1.

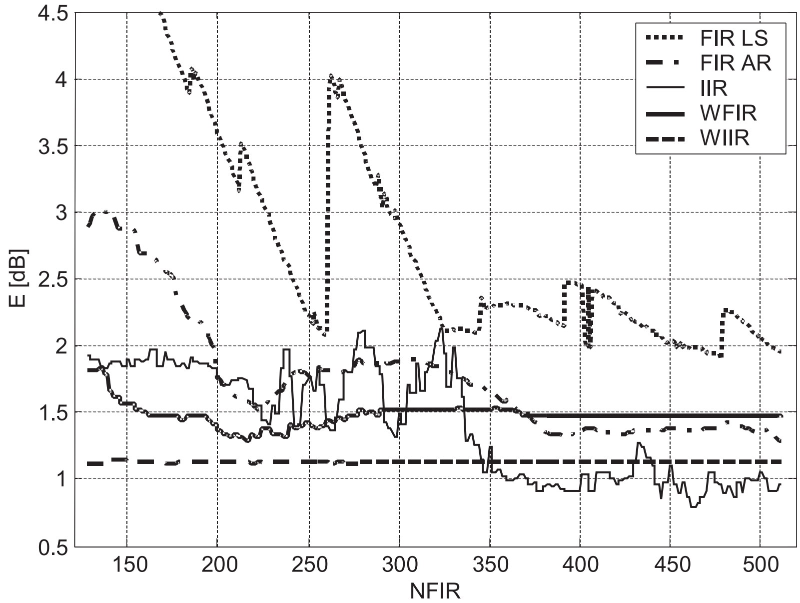

Fig. 3 Maximum magnitude response irregularities versus equalization filter order. Analysed band: 63 Hz–20 kHz. Loudspeaker system no. 2.

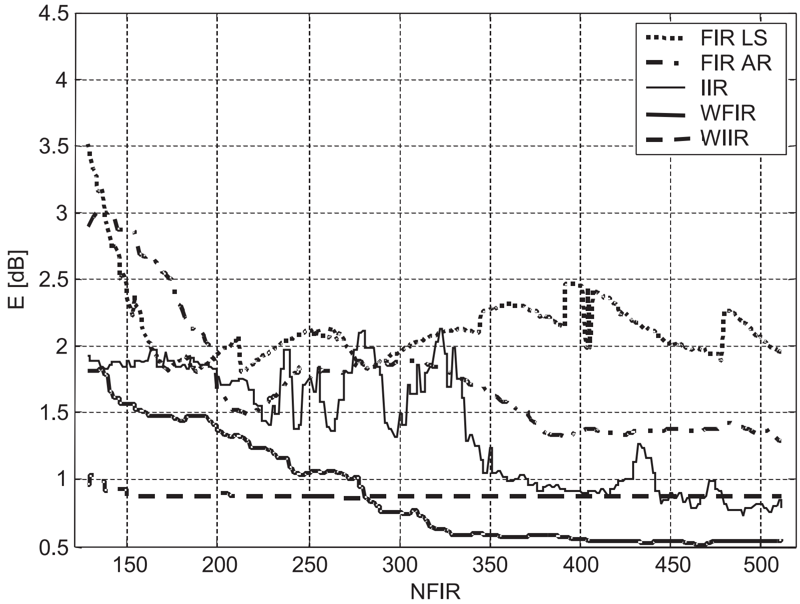

Fig. 4 Maximum magnitude response irregularities versus equalization filter order.
Analysed band: 100 Hz–8 kHz. Loudspeaker system no. 2.

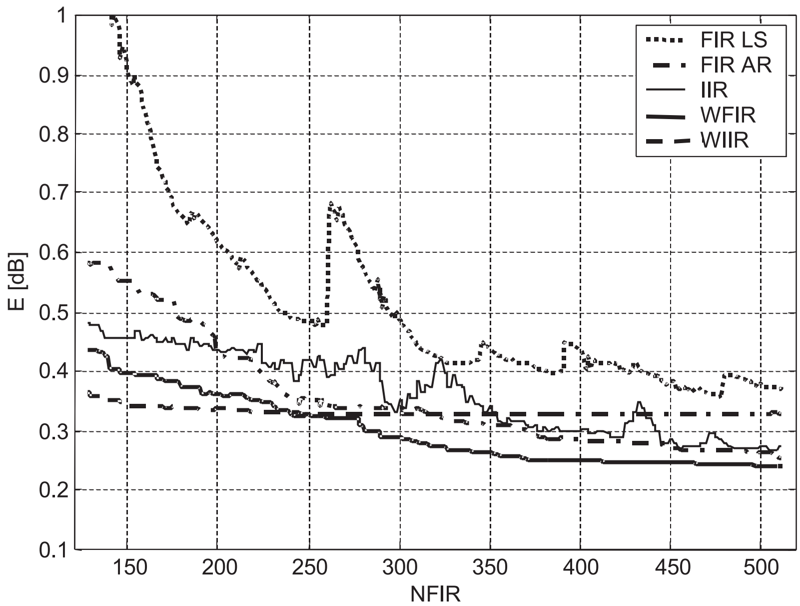

Fig. 5 Standard deviation of magnitude response versus equalizing filter order.
Analysed band: 63 Hz–20 kHz. Loudspeaker system no. 2.

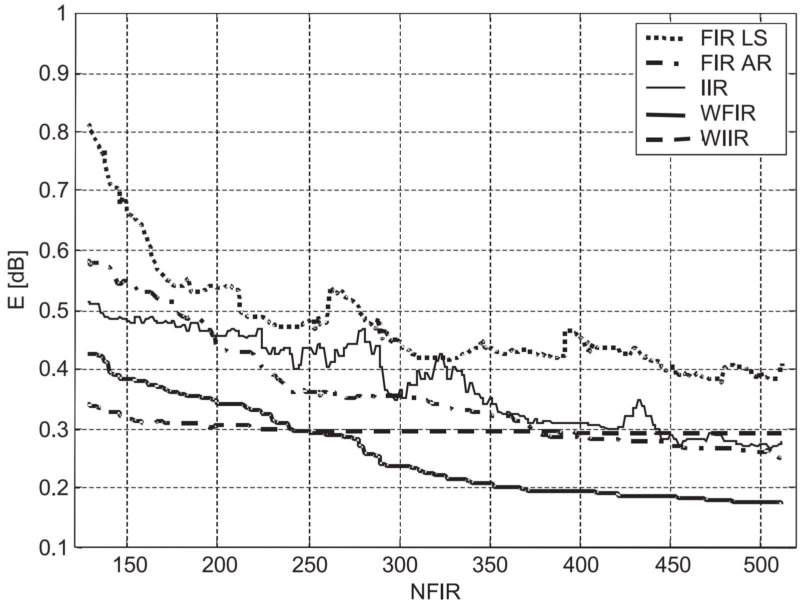

Fig. 6 Standard deviation of magnitude response versus equalizing filter order. Analysed band: 100 Hz-8 kHz. Loudspeaker system no. 2.

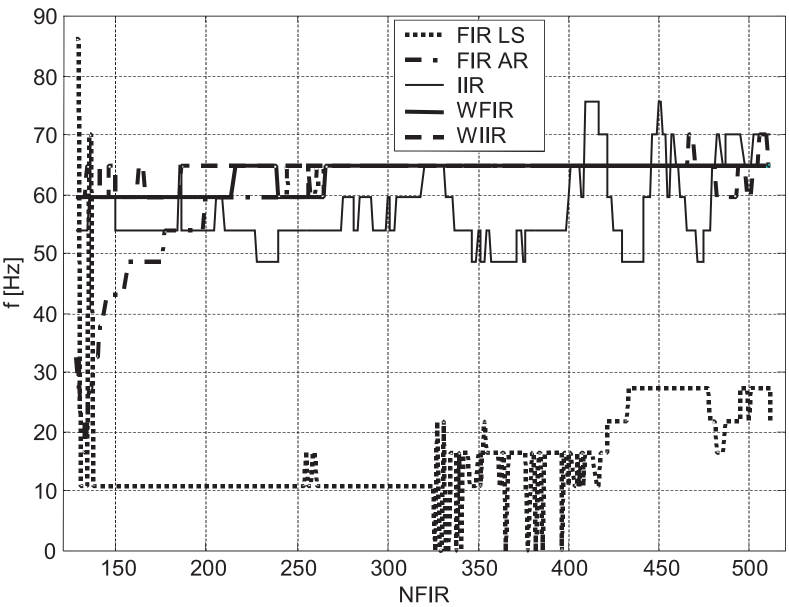

Fig. 7 Lower response frequency during preprocessing, allowing one to obtain an assumed lower frequency-range frequency after equalization. Loudspeaker system no. 2.

For the loudspeaker system no. 2, the smallest maximum magnitude response irregularities were obtained for the filter WIIR at lower computing powers and for the filter IIR at higher computing powers (Fig. 3). In the case of the three other measures of the magnitude response error, the warped filters WIIR and WFIR gave the best results at respectively lower and higher computing powers (Fig. 4–6).

As regards the impulse response equalization quality (formula (26)), the best results for the two loudspeaker systems were obtained, as expected, for the filter FIR LS, whereas in the case of the other filters, the results were similar and did not represent a significant improvement over the quality before equalization.

As part of the analyses, the effect of the lower frequency of the equalized response on the lower frequency of the loudspeaker system after equalization was determined. As follows from Fig. 7, in order to obtain a post-equalization frequency as close as possible to the assumed one, the equalized response has to be limited for widely ranging frequencies which depend on the design algorithm and the equalizing filter order.

3.3. Comparison of pre- and post-equalization responses

The magnitude frequency responses of the loudspeaker system no. 2 before and after equalization by means of the filters described in Sec. 3.2 are shown in Fig. 8. The responses were determined for filters with orders which were subsequently used in the subjective tests. In accordance with the assumptions made in Sec. 3.2, the order of the FIR filter did not exceed 400 and the maximum orders of the other filters were determined from relation (27). The equalizing filter orders were precisely determined from

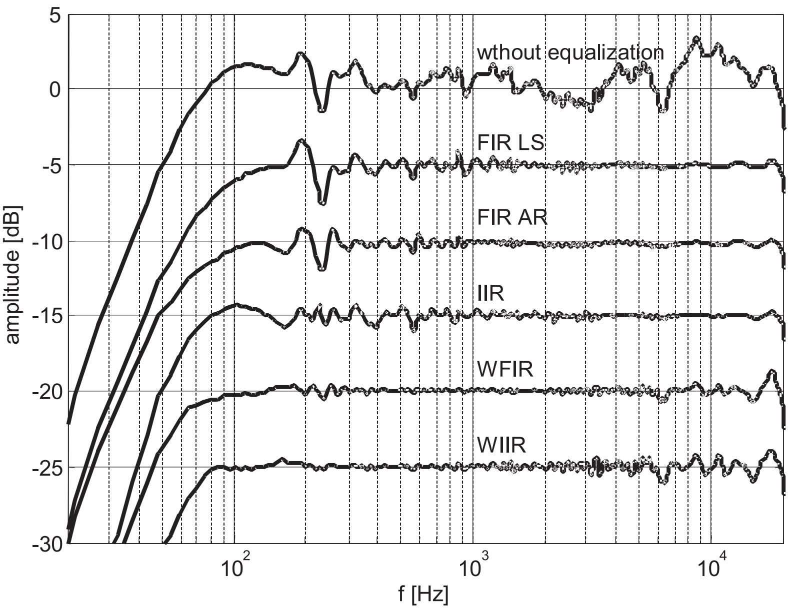

Fig. 8. Comparison of magnitude responses before and after equalization for the loudspeaker system no. 2.

the responses shown in Sec. 3.2 as it became apparent that in some cases by lowering the filter order one could reduce some of the equalization errors. The properties of classical and warped filters are shown in Fig. 8. Classical filter structures reduce response irregularities in the high frequency range, while the warped filter structures reduce the irregularities in the low frequency range.

The impulse response of the loudspeaker system no. 1 before and after equalization by means of the FIR filter designed by the LS algorithm is shown in Fig. 9. It is clear that the equalization shortens the impulse response and introduces symmetry (linear phase response).

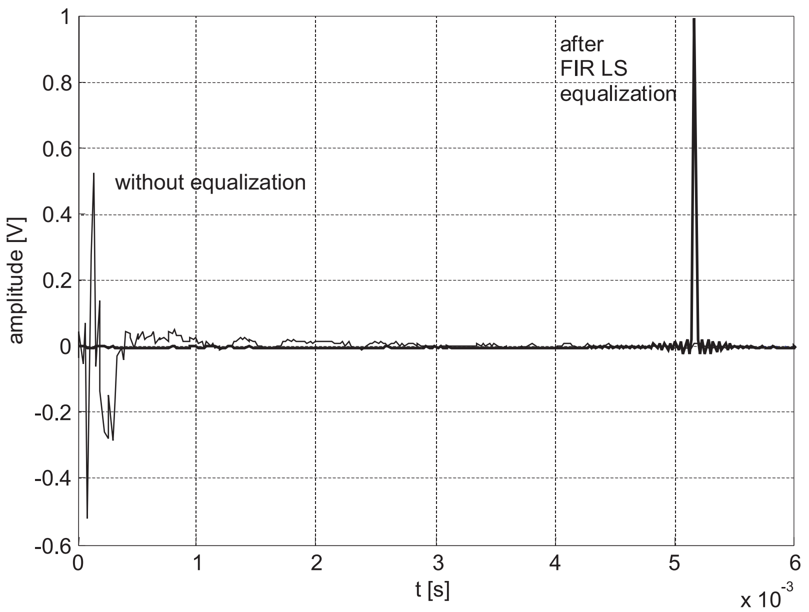

Fig. 9 Impulse response of loudspeaker system no. 1 before and after equalization by means of filter FIR LS.

3.4. Real time equalization

An equalization system operating in real time was implemented to check the agreement between the simulation results and the measured responses of the loudspeaker systems after equalization. Considering the fact that the computing powers of modern x86 processors are high, a decision was made to realize a digital real-time filter based on an IBM PC equipped with a proper sound card. Computations are performed for *double* variables, i.e. 64-bit floating-point representation of numbers is used whereby quantization problems during processing are practically negligible. The differences between the simulated responses and the measured ones in most cases do not exceed $\pm$ 0.5 dB and they are mainly due to measuring errors.

3.5. Listening tests

Listening tests were carried out to determine whether the use of the equalization systems brings about a subjective improvement in the loudspeaker system's quality and to select the equalizer which gives the best results. Comparisons were made for the five filters mentioned in Sec. 3.2.

3.5.1. Listening test procedure

The listening tests were carried out in accordance with the standard AES20 [1]. Although there is a Polish standard consistent with international standard IEC 268-13 [36], the standard AES was used because it contains a description of the evaluated parameters. The loudspeaker systems-listener geometry conformed to the EBU recommendations [8]. The loudspeaker systems' direction angle and height were set so that the tweeter's axis was directed towards the listener's ears. Thirteen parameters divided into four groups were chosen from the twenty nine parameters recommended by the standard AES [1]. The parameter groups were as follows:

G1. Spectral uniformity

P1. Uncoloured;
P2. Balance, bass to treble.

G2. Sound-stage imaging

P3. Stage evenness left-right;
P4. Stage width;
P5. Image left-centre-right localization;
P6. Open, transparent;
P7. Stability of stage or image,
P8. Separation of images.

G3. Ambience reproduction

P9. Spacious, diffuse;
P10. Size-of-space rendition.

G4. Dynamics and distortion

P11. Transient impact or punch;
P12. Not modulated, compressed;
P13. Pianissimo clarity.

A proper sample was prepared for each of the parameters. Sound samples were copied from CDs onto the computer's hard disk where they were properly edited, filtered and normalized (the original sample's and the processed sample's loudness levels were equalized). From the hard disk the samples were copied onto the RDAT tape. Filtration was performed for both channels using filters designed on the basis of the response of the loudspeaker system processing the particular channel. SPDIF digital input and output were used to communicate with the computer in the input and output process. A program *FiltrWav* for filtering files in the **.wav* format was created.

The samples on the tape were arranged in pairs: original sample–processed sample. The test consisted in comparing and evaluating the sound samples on a scale of 0 ± 3. Negative ratings represented quality deterioration, rating 0 meant no discernible difference, and positive ratings represented quality improvement. The listening team consisted of 11 persons. Each person performed the test twice with a one-day break in between.

3.5.2. Test results and their statistical analysis

The obtained results were analysed using a two-way ANOVA (ANalysis Of VAriance) [13] and the MS Excel 200 *Data Analysis* tool (Anova: Two-Factor Without Replication). The hypothesis about the positive influence of digital equalization on the subjectively perceived quality of sound emitted by test loudspeaker systems and the effect of the type of the equalizing filter on the quality of equalization were tested by means of statistical analyses. It could also be determined which of the parameters or groups of parameters were most affected by equalization. The obtained results are different in the two tested loudspeaker systems, but generally it has been established that digital equalization of linear distortion results in a subjectively perceptible improvement of the quality.

For the loudspeaker system no. 2, assuming the type of filter and the averages of all 13 parameters as the source of variation, the type of the equalizing filter was found to have no significant effect on the quality of equalization. However the equalization was found to have a widely different effect on the particular parameters. Equalization improved mainly the following parameters:

- spacious, diffusive;
- transient impact or punch;
- pianissimo clarity.

Equalization was also found to cause an increase in nonlinear distortion. But despite this fact equalization results in a subjectively perceptible improvement in quality.

For the case in which the source of variation were the filters and the averages for the parameter groups, the type of the filter was found to affect the quality of equalization. The highest average was obtained for the WIIR filter, but the averages for the filters FIR, LS, FIR AR and WFIR are only slightly lower. ANOVA showed that the four filters provide a comparable equalization quality. The group of ambience reproduction parameters was found to have the strongest effect on the equalization improvement.

In the case of the loudspeaker system no. 1, when the averages for both the parameters and the groups were the source of variation, the analyses showed the effect of the kind of filter on the quality improvement. The biggest improvement in quality was recorded for the following parameters:

- open, transparent;
- spacious, diffuse;
- stage width.

Like in the case of the loudspeaker systems no. 2, equalization adversely affected the nonlinear distortion. The WFIR filter, followed by the filters WIIR and FIR AR, received the highest rating when the source of variation were the parameters and the groups. The same improvement in quality was recorded for the three filters. It was also found that the equalization by the FIR LS filter does not bring about any improvement in quality. Filter IIR improves the equalization quality when all the parameters are the source of variation, but no improvement is noticeable when the source of variation are the parameter groups.

4. Conclusions

The advantages of warped filters as applied to digital equalization of the linear distortion of the loudspeaker systems have been demonstrated. Both the objective and subjective results obtained using filters of this class were found to be comparable with or better than those obtained for classical digital filter structures. Objective comparisons show that WIIR filters should be used at lower computing powers while WFIR filters ought to be used at higher computing powers.

It was surprising to find that the filters (FIR LS) had a negligible effect on the subjectively perceived quality of the phase response equalization. In the case of one of the equalized loudspeaker systems, the FIR filter designed by the LS algorithm did not bring about any noticeable improvement, while in the other loudspeaker system the improvement was similar to that brought about by the filters equalizing only the magnitude response. This, however, does not mean that phase response equalization is useless. The obtained results indicate that the subjectively perceived effect of phase response equalization will be greater for loudspeaker systems characterized by more even magnitude response before equalization.

Particularly large discrepancies between the subjective and objective equalization quality ratings were observed for the filters IIR which in the objective assessment of equalization received good ratings, whereas in the subjective evaluation of one of the loudspeaker systems (no. 2) they did not bring about any improvement in quality while there was only a slight improvement in the other system (no. 1).

The proper preparation of the response of the loudspeaker system to be equalized is a crucial element of the design process. This includes: precise measurements and response preprocessing consisting in smoothing the magnitude response and limiting the range of equalization by smoothing the magnitude response outside the band and, for some design algorithms, in eliminating group delay.

The best equalization results are not always obtained for the largest realizable equalizing filter order.

In the case of warped filters with an infinite impulse response, the increasing of the filter order above a certain boundary value does not result in a significant reduction of the equalization error, which is due to the fact that equalization has a little effect on the high frequency range. This can be probably changed if lower values than those recommended by Smith, are used for the warping coefficient λ.

The objective and subjective comparisons have shown that the equalizer realized through the finite impulse response filter designed by the LS algorithm or the fast deconvolution technique gives the best results for both magnitude and phase distortion. The WFIR filter designed by means of AR modelling has been found to be most suitable for magnitude distortion equalization at higher computing powers while the WIIR filter designed through modelling the minimum-phase part of the equalized loudspeaker system response by means of the Steiglitz–McBride algorithm, inverting the rational function describing the model and normalizing it by the numerator's first coefficient, is suitable for lower computing powers.

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