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ACOUSTIC SNIPER LOCALIZATION

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Acoustic signals of small arm's fire, the muzzle blast and the shock wave generated by a supersonic bullet in air, are difficult to mask and can be exploited for localization of the hidden sniper. This paper presents the system of acoustic measurements, based on a number of both directional and omnidirectional microphones detecting the shock wave only, yielding exact solution for the sniper direction in spite of certain errors in the directional measurements. The system has a self-correcting ability concerning the sound directional measurements which contributes to the system technical feasibility. Auxiliary muzzle blast measurements would yield the sniper position.

Key words: shock waves, measurements.

1. Introduction

Recent bad experience in the former Yugoslavia and Iraq, as well as the yet earlier ones in Somalia, shows the threat posed by well-hidden snipers to peace-keepers and civilians. Equipped with relatively cheap and light arms, snipers can easily mask their position to avoid detection and elimination. Only the acoustic signals: the muzzle blast to some degree, and primarily the shock wave [1] generated by a supersonic bullet passing by, could be exploited for the effective sniper detection as they cannot be in principle masked. This requires an appropriate system to be installed in the protected area that makes necessary the acoustic measurements and evaluates the sniper position or, at least, his direction, allowing one to direct a proper counter-fire.

Such systems are studied in recent literature [2, 3], where the two above mentioned acoustic measurements are used to localize the sniper. Assuming the known bullet velocity and correct detection of the muzzle blast among possible, nearly simultaneous other blasts, these two measurements allow one to compute the sniper position with rather low accuracy; experiments show the 50% chance of successful counter-fire [4]. The substantial difficulty arises from at least two problems connected with the acoustic measurement of the muzzle blast: 1) this is the relatively low-frequency signal whose arrival time can be detected with low resolution, and 2) it propagates over a rather large distance in air of generally variable properties bending the sound propagation path and somehow enlarging the propagation time. This contributes to the evaluation error of the sniper distance (it is evaluated from the time difference between the muzzle blast and the shock wave arrival times at the measuring microphone, assuming the known bullet velocity).

Reasonably, one is interested in destroying the sniper whose fire is directed at him and posing the real threat to him, that is which fire miss-distance is small. Thus the interesting shock wave signal propagates over a rather small distance through the assumed reasonably uniform air. Moreover, this is the very characteristic N-shaped signal with the rising time much below $1\,\mu$s, thus the shock wave arrival time can be detected with high resolution by a proper wide-band microphone measuring the high-frequency signals. This makes the acoustic measurements of the shock wave much more accurate and reliable than the measurement of the muzzle blast. Even the information about the bullet dimension – thus on the used sniper's arm – is included in the shock-wave N-shaped signal, and can be exploited for defence purposes.

These peculiar properties of the shock-wave measurements, whose significance is seemingly underestimated in the existing literature, deserve high attention in the considered problem of the acoustic sniper localization. In fact, only the shock wave arrival time measurements are discussed in this paper. Naturally, using the shock wave measurements alone, one cannot evaluate the distance covered by the speeding bullet and the position of the sniper; the muzzle blast measurements must be used for that, according to the well developed method presented in [3]. However, the shock-wave measurements are able to yield, as it is shown in this paper, the accurate direction of the bullet velocity, assumed to be close to the sniper direction, that is sufficient to direct the counter-fire. Moreover, the accurate direction to the sniper allows us to detect the sniper with other than acoustic observations, the optical detection of the hot muzzle gases, for instance.

The evaluation of the bullet velocity direction requires measurements of the shock wave arrival times at a number of observation points (microphones). As known, the shock wave generated by a supersonic bullet has the form of a cone; its axis is the bullet path (assumed straight) and the conical angle depends on the bullet velocity (assumed constant). The measurement data must be sufficient to solve the geometrical problem of finding the cone parameters and thus finding the sniper direction. The problem is essentially nonlinear and quite difficult if only the shock wave arrival times are known at the given microphone positions. To ease the computation task, we propose here the directional measurements instead by two directional microphones placed in certain distance from each other; a number of additional omnidirectional measurements would improve the localization accuracy and assure the solution uniqueness. These auxiliary microphones could be planted "in field" over the safeguarded area, transmitting their raw observations to the system computer by standard radio-links.

It is shown in Sec. 3 that only two directional measurements suffice to evaluate the cone parameters. The angular accuracy of the measurements is expected to be much lower than the temporal accuracy of the shock wave arrival time measurements, which can be assumed to be exact. A number of omnidirectional measurements (yielding exact data of the shock wave arrival times) helps us to overcome the problem with inaccurate directional measurements that can contribute much to the localization accuracy. Three systems are discussed in Sec. 6, based on 2 directional microphones plus 4 omnidirectional ones distributed over the safeguarded area, and $3 + 3$ or $4 + 2$ corresponding microphones; the numbers are necessary to obtain the systems of directional measurement self-correcting ability, the remedy to the technical problem of directional measurements. Numerical simulations show that the convergent (exact) solution is obtained (provided that the microphones are suitably placed with respect to the bullet path) even for directional measurement inaccuracy as large as 1% or even more.

2. The shock-wave geometry

Assuming constant velocity v and a straight bullet path, the generated weak shock wave in air has a form of a cone S_1 (Fig. 1) of axis $\mathbf{k}$ (the normalized vector directed against the bullet velocity, toward the sniper) and a tip $\mathbf{O}_1$, the position of the bullet tip at the given observation time t_1. The acoustic signal of the shock wave propagating with velocity c in the direction normal to the shock wave cone, arrives to the observers positioned at $\mathbf{r}_i$ (in given Cartesian coordinates) at the shock wave arrival times t_i. All the observers residing on the shock wave cone S_1 would detect the wave at the same time t_1 (simultaneously). Here we only consider the shock wave generated by the bullet tip moving with the supersonic velocity $v > c$. This corresponds to the front edge of the acoustic N-shaped signal [5].

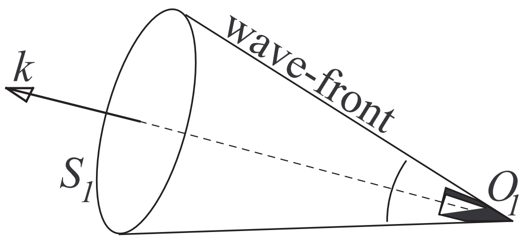

Fig. 1. The shock wave cone at the observation time t_1 when the supersonic bullet tip position is $\mathbf{O}_1$.

At the time $t_2 > t_1$, the shock wave cone tip moves to the point $\mathbf{O}_2$:

$$\mathbf{O}_2 = \mathbf{O}_1 - v\mathbf{k}(t_2 - t_1), \tag{1}$$

and the cone broadens by the distance d normal to the cone S (Fig. 2),

$$d = c(t_2 - t_1), \tag{2}$$

where c is the sound velocity in air (constant in the assumed homogeneous air). The shock wave conical angle is

$$\sin\vartheta = [c(t_2 - t_1)]/[v(t_2 - t_1)] = c/v, \tag{3}$$

assuming the supersonic bullet.

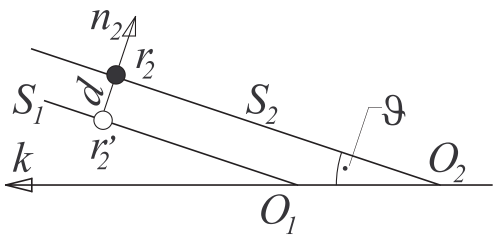

Fig. 2. The shock wave expands over time with the sound velocity in air c.

The important conclusion results from the above that shifting the observation point $\mathbf{r}_2$ (where the shock wave arrives at the time t_2) by the distance d, Eq. (2), against the outward normal $\mathbf{n}_2$ to the observed shock wave-front (the cone S_2), places the point $\mathbf{r}_2'$ on the first cone S_1 (Fig. 2); generally

$$\mathbf{r}_i' = \mathbf{r}_i - c\mathbf{n}_i(t_i - t_1), \tag{4}$$

for i-th microphone measuring the shock wave arrival time t_i at position $\mathbf{r}_i$.

3. Directional measurements

Let the two directional microphones placed at $\mathbf{r}_1$ and $\mathbf{r}_2$ detect the shock wave arrival times t_1, t_2, and simultaneously the sound propagation directions $\mathbf{n}_1$ and $\mathbf{n}_2$ (which are the outward normals to the shock wave cone), respectively. It is shown below that these two measurements suffice to evaluate 1) the cone axis k and 2) its tip $\mathbf{O}_1$, as well as 3) the conical angle ϑ, that is 4) the bullet velocity v, and finally 5) its path in space determined by $\{\mathbf{O}_1, \mathbf{k}, v\}$.

Consider a line described by its point $\mathbf{r}_i$ and the vector $\mathbf{n}_i$ along it. The line is the sound ray generated at $\mathbf{P}_i$ and arriving in $\mathbf{r}_i$ at the time t_i (Fig. 3). The bullet path described by the point $\mathbf{O}_1$ and the vector $\mathbf{k}$ crosses the rays $(\mathbf{r}_i, \mathbf{n}_i), i = 1, 2$, at points $\mathbf{P}_i$ at the same angle

$$\vartheta' = \pi/2 - \vartheta, \tag{5}$$

as it results from the geometry shown in Fig. 2. This yields the following vector equations (a dot meaning the scalar product)

$$\begin{aligned} \mathbf{r}_i - \alpha_i\mathbf{n}_i &= \mathbf{P}_i, \qquad i = 1, 2, \\ \mathbf{k} &= \pm(\mathbf{P}_1 - \mathbf{P}_2)/\,\|\mathbf{P}_1 - \mathbf{P}_2\|, \\ \mathbf{k}\cdot\mathbf{n}_1 &= \mathbf{k}\cdot\mathbf{n}_2 < 0, \end{aligned} \tag{6}$$

where the last inequality helps us to choose the correct sign to $\mathbf{k}$; α_i are unknown constants (scalars).

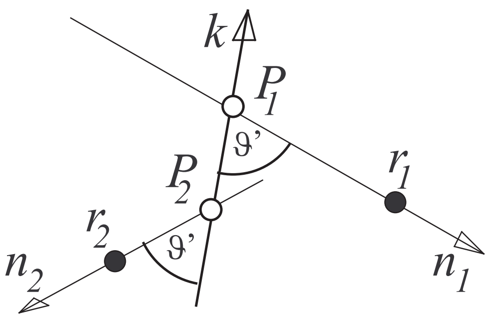

Fig. 3. The cone axis $\mathbf{k}$ crosses two normals to the cone surface $n_{1,2}$ at the same angle ϑ'.

On the strength of Eq. (2),

$$(\mathbf{r}_2 - \mathbf{P}_2) \cdot \mathbf{n}_2 - (\mathbf{r}_1 - \mathbf{P}_1) \cdot \mathbf{n}_1 = d = c(t_2 - t_1), \tag{7}$$

and the second of Eqs. (6) multiplied by $\| \mathbf{P}_1 - \mathbf{P}_2 \|$:

$$(\mathbf{r}_1 - \alpha_1 \mathbf{n}_1 - \mathbf{r}_2 + \alpha_2 \mathbf{n}_2) \cdot \mathbf{n}_1 = (\mathbf{r}_1 - \alpha_1 \mathbf{n}_1 - \mathbf{r}_2 + \alpha_2 \mathbf{n}_2) \cdot \mathbf{n}_2, \tag{8}$$

one obtains:

$$\begin{aligned} \alpha_1 &= \frac{(\mathbf{r}_2 - \mathbf{r}_1) \cdot \mathbf{n}_2 + c(t_1 - t_2)}{1 - \mathbf{n}_1 \cdot \mathbf{n}_2}, \\ \alpha_2 &= \frac{(\mathbf{r}_1 - \mathbf{r}_2) \cdot \mathbf{n}_1 + c(t_2 - t_1)}{1 - \mathbf{n}_1 \cdot \mathbf{n}_2}, \\ \sin\vartheta &= -\mathbf{n}_1 \cdot \mathbf{k} > 0, \qquad i = 1, 2, \end{aligned} \tag{9}$$

where $\mathbf{n}_i, \mathbf{k}$ are normalized vectors: $\| \mathbf{n}_i \| = \mathbf{n}_i \cdot \mathbf{n}_i = 1$, similarly $\mathbf{k} \cdot \mathbf{k} = 1$. The solution does not exist if $\mathbf{n}_1 \cdot \mathbf{n}_2 = 1$, which particular case (where both $\mathbf{r}_i$ reside on the same generatrix of the cone) is neglected in this study.

The cone tip $\mathbf{O}_1$ can be evaluated from the right-angled triangle $(\mathbf{O}_1, \mathbf{P}_1, \mathbf{r}_1)$, Fig. 4, where both ϑ and

$$\| \mathbf{P}_1 - \mathbf{r}_1 \| = (\mathbf{r}_1 - \mathbf{P}_1) \cdot \mathbf{n}_1 = \alpha_1 \tag{10}$$

are known, yielding

$$\begin{gathered} \mathbf{O}_1 = \mathbf{r}_1 + \alpha_1 \left(\frac{\mathbf{k}}{\mathbf{n}_1 \cdot \mathbf{k}} - \mathbf{n}_1 \right) = \mathbf{P}_1 - \mathbf{k} \frac{\alpha_1}{\sin\vartheta}, \\ v = -c/(\mathbf{n}_1 \cdot \mathbf{k}). \end{gathered} \tag{11}$$

This concludes the searched solution.

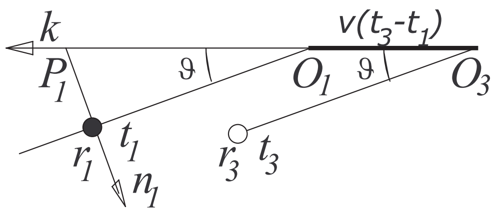

Fig. 4. The consistency condition of omnidirectional measurement at $\mathbf{r}_3$ results from comparison of the conical angle ϑ.

4. The measurement consistency criterion

Having three or more directional measurements of the same shock wave, the same $\mathbf{k}, v$ and other parameters should result from Eqs. (9), (11), evaluated from different pairs of data but including the same $(\mathbf{r}_i, \mathbf{n}_i, t_i)$. For example, Eqs. (9) applied to the pairs $(1,2)$ and $(2,3)$, should yield the same α_2

$$\alpha_2 = \frac{(\mathbf{r}_1 - \mathbf{r}_2) \cdot \mathbf{n}_1 + c(t_2 - t_1)}{1 - \mathbf{n}_1 \cdot \mathbf{n}_2} = \frac{(\mathbf{r}_2 - \mathbf{r}_3) \cdot \mathbf{n}_3 + c(t_2 - t_3)}{1 - \mathbf{n}_1 \cdot \mathbf{n}_3} \tag{12}$$

in order to obtain the same $\mathbf{P}_2$ and thus the same $\mathbf{k}$. This consistency condition (applied for different α_i) will be exploited later in order to reduce the measurement errors causing the above equation to fail.

Now consider an omnidirectional measurement of the shock wave arrival time t_3 by a microphone placed at point $\mathbf{r}_3$ (Fig. 4). It is assumed here that the same shock wave (the same bullet) is detected by the directional microphones yielding the data $(\mathbf{r}_i, \mathbf{n}_i, t_i), i = 1, 2$, and the evaluated shock wave tip $\mathbf{O}_1$ at the time t_1. The shock wave cone tip at the time t_3 is $\mathbf{O}_3$:

$$\mathbf{O}_3 = \mathbf{O}_1 - v\mathbf{k}(t_3 - t_1), \tag{13}$$

according to Eq. (1). Naturally, the cone has the same conical angle $\vartheta = \arcsin(c/v)$, hence

$$\frac{(\mathbf{r}_3 - \mathbf{O}_3) \cdot \mathbf{k}}{\| \mathbf{r}_3 - \mathbf{O}_3 \|} = \cos\vartheta. \tag{14}$$

If the measurements are exact then

$$\xi = \sqrt{1 - \left[\frac{(\mathbf{r}_3 - \mathbf{O}_3) \cdot \mathbf{k}}{\| \mathbf{r}_3 - \mathbf{O}_3 \|} \right]^2} + \mathbf{k} \cdot \mathbf{n}_1 \tag{15}$$

equals zero ($\mathbf{k} \cdot \mathbf{n}_1 = -\sin\vartheta$), otherwise $\xi \neq 0$ indicating the incorrect measurement data. The condition $\xi = 0$ is the consistency condition of omnidirectional measurement data with respect to the pair of directional measurements which can include certain errors concerning the directions $\mathbf{n}_i$. According to the earlier assumption that the arrival time is measured exactly, the omnidirectional measurements are considered to be exact.

Introducing matrix notations where vectors $\mathbf{r}_i, \mathbf{O}_i, \mathbf{k}, \mathbf{n}_i$ are row matrices and $\mathbf{r}', \mathbf{k}'$ etc. are their transposed (column) matrices, the above equation can be conveniently rewritten in the form

$$\xi^{(3)} = \sqrt{\frac{\mathbf{z}(\mathbf{I} - \mathbf{k}'\mathbf{k})\mathbf{z}'}{\mathbf{z}\mathbf{z}'}} + \mathbf{k}\mathbf{n}_1', \qquad \mathbf{z} = \mathbf{r}_3 - \mathbf{O}_3 \tag{16}$$

($\mathbf{I}$ is a unitary matrix). All the earlier vector equations can be rewritten and evaluated in a similar manner, and the same notations are applied to both vectors and the corresponding matrices.

The measurement consistency condition $\xi = 0$ will help us to reduce the directional measurement inaccuracy. Assuming $\mathbf{r}_i, t_i$ to be known exactly and admitting certain errors $\boldsymbol{\delta}_i$ in the measured shock wave propagation direction $\mathbf{n}_i$, this condition yields an equation for $\boldsymbol{\delta}_i$. Several omnidirectional measurements (at points $\mathbf{r}_3, \mathbf{r}_4, \dots$) are necessary to obtain a sufficient number of equations in order to evaluate all the components of vectors $\boldsymbol{\delta}_i$ of interest. Regretfully, Eqs. (16), (9) and others are highly nonlinear and their solution may not be unique, in general. It is assumed here that the measurement can be only slightly inaccurate which allows us to apply the perturbation analysis with respect to $\boldsymbol{\delta}_i$. The resulting linear equations for the measurement errors $\boldsymbol{\delta}_i$ can be easily solved. It is a matter of numerical testing how large $\boldsymbol{\delta}_i$ can be admitted to obtain convergent solution for given microphone positions $\mathbf{r}_i$, and the bullet miss-distance and direction $-\mathbf{k}$.

5. Perturbation analysis

Assuming the measured (normalized) direction $\mathbf{n}_i + \boldsymbol{\delta}_i$ instead of the correct $\mathbf{n}_i$, it is evident that the equality

$$\boldsymbol{\delta}_i \cdot \mathbf{n}_i = 0 \tag{17}$$

results from the normalization condition $(\mathbf{n}_i + \boldsymbol{\delta}_i) \cdot (\mathbf{n}_i + \boldsymbol{\delta}_i)$, neglecting higher order terms. Equation (17) shows that $\boldsymbol{\delta}_i$ has only two independent components orthogonal to $\mathbf{n}_i$. We may choose them in directions of two orthogonal vectors:

$$\begin{aligned} \mathbf{e}_i^{(1)} &= \mathbf{n}_i \times \mathbf{r}_i, \\ \mathbf{e}_i^{(2)} &= \mathbf{n}_i \times \mathbf{e}_i^{(1)}, \end{aligned} \tag{18}$$

again normalized after evaluation of the vector products denoted here by $\times$. In matrix notations:

$$\begin{aligned} \mathbf{e}_i &= [\mathbf{e}_i^{(1)}; \mathbf{e}_i^{(2)}], \\ \boldsymbol{\delta}_i &= \mathbf{d}_i \mathbf{e}_i, \end{aligned} \tag{19}$$

where $\mathbf{d}_i$ is the row matrix with two components fully characterizing the measurement errors (for the already chosen $\mathbf{e}_i$).

In perturbation analysis $\mathbf{d}_i$ is infinitesimal, but in real computations, for $\boldsymbol{\delta}_i$ small but finite, the corrected vectors $\mathbf{n}_i \leftarrow \mathbf{n}_i + \boldsymbol{\delta}_i$ must be always normalized in order to keep the earlier equations, like Eq. (15), valid. The perturbation analysis (for infinitesimal $\mathbf{d}_i$) of Eqs. (9) yields in the matrix notations using the summation convention:

$$\delta\alpha_i = \boldsymbol{\delta}_j a_{ji},$$
$$[a_{ji}] = \frac{1}{1 - \mathbf{n}_1\mathbf{n}_2'} \begin{bmatrix} \alpha_1\mathbf{n}_2' & \alpha_2\mathbf{n}_1' + (\mathbf{r}_1 - \mathbf{r}_2)' \\ \alpha_1\mathbf{n}_2' + (\mathbf{r}_2 - \mathbf{r}_1)' & \alpha_2\mathbf{n}_1' \end{bmatrix}, \tag{20}$$

where $\alpha_{1,2}$ are unperturbed scalars evaluated from Eq. (9) within zero-order accuracy with respect to $\boldsymbol{\delta}_i$. Note that $\boldsymbol{\delta}_j \cdot \mathbf{n}_i'$ is a scalar. To indicate the set of data: $(\mathbf{n}_1, t_1, \mathbf{r}_1)$ and $(\mathbf{n}_2, t_2, \mathbf{r}_2)$ used for evaluation of these coefficients, the superscript $(1, 2)$ will be introduced in the subsequent analysis like $a_i^{(1,2)}, i = 1, 2$.

Similarly, one can obtain the perturbation equations for

$$\delta\mathbf{O}_i = \boldsymbol{\delta}_j\mathbf{O}_{ji}, \qquad \delta\mathbf{k} = \boldsymbol{\delta}_j\mathbf{k}_j, \quad \text{and} \quad \delta\xi = \boldsymbol{\delta}_j\boldsymbol{\xi}_j \tag{21}$$

(the unperturbed ξ has zero value). Note that again, $\delta\mathbf{k}$ is a vector orthogonal to $\mathbf{k}$:

$$(\delta\mathbf{k})\mathbf{k}' = 0 \tag{22}$$

because of normalization of $\mathbf{k}$. It can be evaluated from the perturbation of $\mathbf{P}_1 - \mathbf{P}_2$, Eqs. (6); the perturbation of ξ is evaluated from Eqs. (15) provided that the perturbations to $\mathbf{O}_i$, Eqs. (11)–(13) are evaluated first. The explicit formula are too long to be presented here; note only the introduced perturbation matrices $\mathbf{O}_{ji}, \mathbf{k}_j, \boldsymbol{\xi}_j$ in Eqs. (21) which will be applied in further analysis.

6. Self-correcting systems

Two directional measurements introduce four unknown errors: $\mathbf{d}_i^{(1,2)}, i = 1, 2$ (each $\boldsymbol{\delta}_i$ has two independent components $\mathbf{d}_i^{(1,2)}$). Four independent conditions from the measurement consistency conditions, for instance, are needed to evaluate $\mathbf{d}_i^{(j)}, i, j = 1, 2$ and to retrieve the correct values of $\mathbf{n}_i$.

This shows that two directional and four omnidirectional (introducing no extra errors) measurements of the shock wave generated by the passing-by bullet are sufficient for evaluation of correct $\mathbf{n}_i$ in spite of the measurement errors. The correct values of $\mathbf{n}_i$ allow one to evaluate the correct bullet path parameters, $\mathbf{k}, \mathbf{O}_1$ and v, that is to obtain correct sniper localization. One may conclude that the directional measurements yield only the first guess of these parameters to ease the corresponding computation task based on the perturbation analysis.

Another measurement systems can be proposed as well. For example, three directional measurements $(\mathbf{n}_i, t_i, i = 1, 2, 3)$ introducing six unknowns $\mathbf{d}_j^{(1,2)}, i = 1, 2, 3$, and three omnidirectional ones (at different $\mathbf{r}_i, i = 4, 5, 6$) which can be exploited for formulation of six consistency conditions $\boldsymbol{\xi}^{(i,j)}$, evaluated using different pairs of directional data: $(\mathbf{n}_1, \mathbf{n}_3)$ and $(\mathbf{n}_2, \mathbf{n}_3)$, for instance. The other possibility is to formulate three consistency conditions like in Eq. (12) for α_i, appended by three ξ_i chosen to obtain the best conditioned system of equation. Yet another system uses four directional measurements and two omnidirectional ones; they will be discussed below in some details. Note however that directional measurements are much more expensive than the omnidirectional ones, thus the first above mentionad system, "$2+4$", is preferred over two other: "$3+3$" and "$4+2$".

At the first glance, the system of four directional measurements seems to be self-correcting without omnidirectional measurements. Namely, we can formulate the sufficient number of consistency conditions like Eq. (12) using different pairs of the directional measurements only. Regretfully, the rank of such system appears to be only six, indicating that two other equations are necessary, namely resulting from independent omnidirectional measurements.

6.1. The system "2+4"

The system of equations resulting from two directional measurement data: $t_i, \mathbf{n}_i + \boldsymbol{\delta}_i$ at positions $\mathbf{r}_i, i = 1, 2$, and four omnidirectional measurements: t_j at different $\mathbf{r}_j, j = 3, \ldots, 6$, results from four consistency criteria $\boldsymbol{\xi}^{(j)}$, Eq. (16). Explicitly, according to Eq. (21):

$$\boldsymbol{\delta}_i \boldsymbol{\xi}_i^{(j)} = x_j, \qquad i = 1, 2, \qquad j = 3, 4, 5, 6, \tag{23}$$

what can be further transformed using Eq. (19) to obtain the complete system of equations for the unknown $d_i^{(l)}, i, l = 1, 2$:

$$\sum_{i,l=1}^{2} d_i^{(l)} q_{ij}^{(l)} = x_j, \qquad q_{ij}^{(l)} = e_i^{(l)} \xi_i^{(j)}, \tag{24}$$

where the values of x_j are $\xi^{(j)}$ evaluated from Eq. (16) using the measurement data (that is the values of $\mathbf{n}_i$ including certain error $\boldsymbol{\delta}_i$, what causes $\xi_j \neq 0$).

This solved, yields the measurement errors $\boldsymbol{\delta}_i$ which – subtracted from the measured data-yield the correct directions $\mathbf{n}_i$. In practice, $\boldsymbol{\delta}_i$ are not infinitesimal and the values of $\xi_i^{(j)}$ are evaluated from inaccurate values $\overline{\mathbf{n}}_i = \mathbf{n}_i + \boldsymbol{\delta}_i$. Although the $\overline{\mathbf{n}}_i - \boldsymbol{\delta}_i$ is considered to be closer to the correct $\mathbf{n}_i$, it is evident that the correct solution can be obtained repeating the calculations in a recursive manner. If convergent, they yield the searched correct $\mathbf{n}_i$, and finally the correct bullet path parameters, particularly the most important k.

6.2. The system "3+3"

Three directional measurement data, $(\mathbf{n}_i, t_i)$ at microphone positions $\mathbf{r}_i, i = 1, 2, 3$, substituted into Eqs. (9), yield different values of the same α_j due to inaccurate $\mathbf{n}_i$.

The perturbation expansion yields three equations resulting from comparison of the same $\alpha_i, i = 1, 2, 3$, Eq. (12):

$$\delta_k[a_{k1}^{(i,m)} - a_{ki}^{(i,n)}] = x_i, \tag{25}$$

where $x_i = \alpha_i^{(i,n)} - \alpha_i^{(i,m)}$ is the difference of the two values of α_i computed from Eqs. (9) using different pairs of measurements: $(i, m), m \neq i$, and $(i, n), n \neq i$; the same i is indicated in the superscripts of the perturbed coefficients evaluated from Eqs. (20).

Another three equations can be chosen from several possible consistency conditions (16) concerning three $(l = 1, 2, 3)$ omnidirectional measurements and evaluated using different pairs of directional measurements. One should choose those which yield the best conditioned final system of equations. The chosen equations of the form:

$$\delta_k \xi_k^{(l;n,m)} = x_l^{(n,m)}, \tag{26}$$

appended to the earlier formulated Eqs. (25) yield a complete system of equations for six unknown components $\mathbf{d}_k^{(1,2)}, k = 1, 2, 3$, cf. Eq. (24). Like in the previous section, the localization problem is solved iteratively, with each step improving the values of $\mathbf{n}_i$ used for evaluation of x_i for the next step. One may also seek the least-square solution to all possible equations (25), (26), but this usually reduces the condition factor of the equation matrix and thus worsens the convergence of iterations mentioned above.

6.3. The system "4+2" and higher

The last system discussed here is based on four directional and two omnidirectional measurements. The corresponding equations are formulated in the way presented in the previous sections, using some of the multiple consistency conditions (12) and (16). They yield a still larger system of equations, here of dimension eight, that is to be solved iteratively, if the iterations converge.

Naturally, the system of equations can be formulated neglecting directional measurement in one point and using the method "$3 + 3$" (or even "$2 + 4$" if two $\mathbf{n}_i$ are neglected). This makes it evident that six directional measurements alone (without extra omnidirectional ones) suffice for the solution of the sniper localization problem. Such system is not, however, much technically attractive taking into account the cost of directional measurements.

7. Numerical examples and conclusions

The first considered system, "$2 + 4$", is the cheapest one. Numerical results show that fortunately, it performs also better than others, yielding a convergent system of equations for larger domains of bullet path parameters and larger directional measurement inaccuracy.

In the numerical example presented here for the system "$2 + 4$", the microphones are placed on the ground (this is also the sniper's post level) and distributed over the protected area about 20 m long (Fig. 5; squares represent directional microphones, circles – omnidirectional ones). The bullet miss-distance is assumed to be 2 m above the ground, and its velocity is $3c$. The directional inaccuracy is modelled by performing calculations for 20 random directions $\mathbf{n}_i + \boldsymbol{\delta}_i$ within 1% limit off the correct $\mathbf{n}_i$. If all calculations converge to correct $\mathbf{n}_i$ then the corresponding bullet path is plotted with dashed, otherwise with solid line. The example shows that the system fails in only few cases of sniper's fire aimed at different points from different sniper's posts.

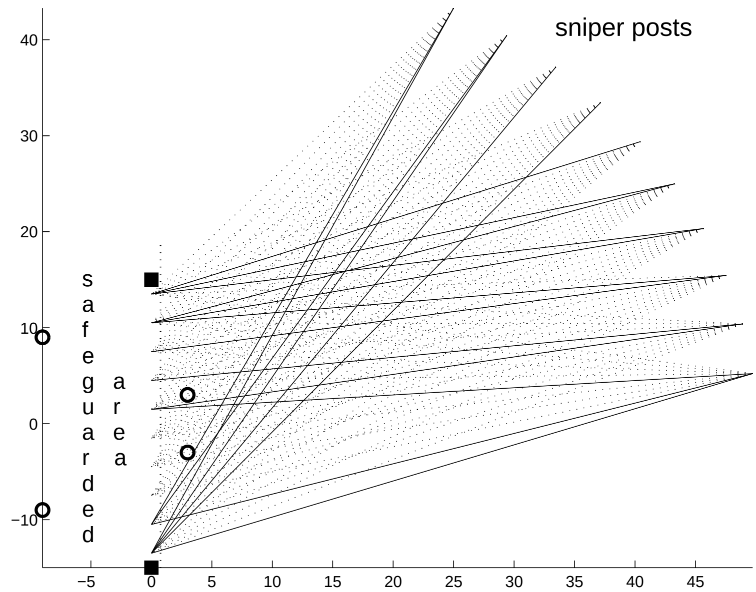

Fig. 5. The simulation of the system "2+4" for 1% measurement inaccuracies of directional measurements. Axes units are 1 [m]. Dot lines shows successful evaluation of the sniper direction $\mathbf{k}$, and solid lines indicate cases of not convergent iterations.

The other two systems are substantially inferior, failing in much more cases of the similar simulations. This is caused by a generally larger condition factor of the matrix of equations. For smaller measurements errors however, all three systems perform well; Fig. 6 presents the case "$4 + 2$" assuming measurement errors hundred times smaller that in the case "$2 + 4$". More extensive simulations, beyond the scope of this paper, for realistic cases of the microphone distribution, sniper's positions and bullet path orientation with respect to the protected area, would reveal the true value of the above proposed systems.

Having the bullet path evaluated (characterized by $\mathbf{O}_1, \mathbf{k}, v$), one can easily exploit the other acoustic information about the fire – the acoustic signal of the muzzle blast. Assuming a small miss-distance of the sniper's fire and sufficiently large distance (L)

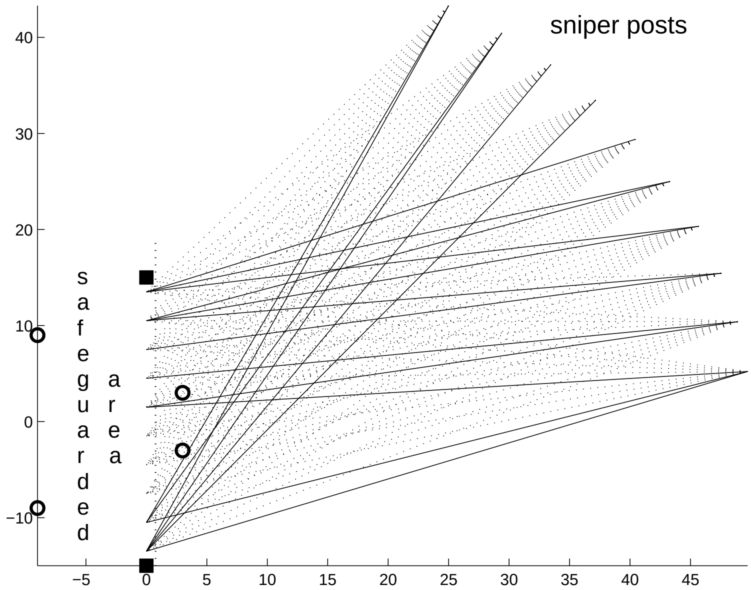

Fig. 6. The simulation for the system "$4+2$" with directional errors within 0.01% only. Dot lines present successful computations; cases where they failed are represented by solid lines.

to the sniper, the muzzle blast signal propagates nearly along the bullet path. This gives the approximation concerning the time difference between the measured shock wave (t_1) and muzzle blast (t_o) arrival times:

$$L/c - L/v = t_o - t_1 \tag{27}$$

($t_1, t_2, \ldots$ are assumed close), from which one easily finds L and thus the sniper's position measured along the evaluated bullet path.

Concluding, the acoustic system is proposed for the localization of the sniper position fully exploiting the most reliable [6] and impossible to mask information delivered by a supersonic bullet. Making two or more directional, and a number of supplemental omnidirectional measurements of the shock wave signal with adequate accuracy (note that this is a "single-event" measurement that cannot be repeated to improve it), one can evaluate the bullet path parameters (particularly the most important bullet velocity direction $-\mathbf{k}$ pointing at sniper's post) and the bullet velocity with improved accuracy. The proposed system has the ability to correct the directional measurement inaccuracies. The numerical examples based on perturbation linearization of highly nonlinear equations governing the geometry of the considered problem show that this self-correcting ability works well for the measurement errors as large as 1%, which can be expected to be technically feasible. Fully nonlinear analysis would certainly even lower this requirement admitting still larger directional measurement inaccuracy and making the presented concept of the sniper localization even more attractive for implementation, saving precious life of peace-keepers and innocent civilians from the snipers' threat.

Instantaneous evaluation of the sniper's direction ($-\mathbf{k}$) is essential also for this reason that it may enable us to apply the other countermeasures. For example, properly directed optical (infrared) sensor can pick-up the cloud of hot muzzle gases detecting precisely the sniper's post. In many cases, fast response is necessary to prevent the change of the sniper's post; perhaps automatic counter-fire is necessary. Knowing known of such countermeasures, the sniper would try to act fast, which would surely contribute to degradation of his fire, making it less lethal in any case.

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