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ACOUSTIC EMISSION FROM A DISLOCATION KINK ACCELERATED BY AN EXTERNAL FORCE

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The emission of elastic waves from a dislocation kink accelerated by a constant external force is presented. The dynamics of the dislocation kink interacting with longitudinal acoustic waves is described by a sine-Gordon–d'Alembert system, i.e. a sine-Gordon equation nonlinearly coupled with the d'Alembert wave equation. Within the framework of this model, the evolution of velocity of the dislocation kink altered by the applied force is determined, allowing for mechanical couplings. The total energy radiated from the dislocation kink and its spectral composition is calculated numerically and analytically. Computer simulations are presented, which graphically illustrate the analytical considerations and model the acoustic radiation.

Key words: acoustic emission, dislocations dynamics, nonlinear waves.

1. Introduction

Acoustic Emission (AE) is the phenomenon of high frequency elastic wave generation, which accompanies plastic deformations and fractures of solids when stressed. Variations of the emission characteristics are thought to be a consequence of different mechanisms responsible for their generation.

Even though the existence of AE in a wide range of metals had been known for many years, so far there was no commonly accepted quantitative understanding and no theoretical interpretation of its origin or the specific mechanisms causing it. Most of the experimental results of the AE measurements during plastic deformation of crystals are interpreted in relation to non-stationary dislocation motion ("Bremsstrahlung" type of acoustic radiation) [1–4].

In this paper, the emission of elastic waves accompanying the motion of a dislocation kink accelerated by a constant external force is investigated using a lattice dynamics approach. Since we are interested in the emission of elastic waves by dynamic processes involving solitons (dislocations), it is reasonable to investigate the interaction of elastic waves with nonlinear excitations (solitons) representing the motion of dislocations.

A model describing the interaction of dislocations with elastic waves has been elaborated in [5]. According to that paper, the interaction of dislocations with elastic waves can be described by a sine-Gordon-d'Alembert (sGdA) system, i.e. a sine-Gordon (sG) equation governing the dislocation dynamics nonlinearly, coupled with the d' Alembert wave equation. The equations, in a non-dimensional form, are as follows:

$$\begin{aligned} \varphi_{tt} - \varphi_{xx} + \sin\varphi &= f - \varepsilon_L v_x \cos\varphi, \\ v_{tt} - v_{xx} + \sin\varphi &= \varepsilon_L \left(\sin\varphi\right)_x , \end{aligned} \tag{1}$$

where φ is the displacement of the atoms constituting the dislocation, v is the longitudinal acoustic field, f is the applied constant force and ε_L is a small coupling constant. The subscripts x and t denote differentiation with respect to x and t, respectively. For the sake of simplicity, only the longitudinal elastic displacements have been considered.

In absence of the right-hand coupling terms, Eq. $(1)_2$ is a classical wave equation and Eq. $(1)_1$ is a pure sG equation for solitary waves.

In our case, the elementary solution

$$\varphi(x,t) = 4\tan^{-1}\left[\exp \pm \left(\frac{x-\xi\,(t)}{\sqrt{1-V^2}}\right)\right] , \tag{2}$$

$\xi(t) = Vt$ and V being the centre-of-mass coordinate and velocity, respectively, represents the motion of a dislocation kink or antikink, depending on the ±sign, along the dislocation line, at velocity V. Consequently, the dislocation motion as a whole, in a given direction (y-direction), may be executed as a result of a soliton motion in the perpendicular direction (x-direction) [6].

2. Emission from an accelerating dislocation kink

2.1. The equation of kink motion

On the basis of systems (1) the acoustic radiation of an accelerating dislocation kink can be studied.

The kink solution of the unperturbed sG equation is offered by Eq. (2). It is well known that in the presence of perturbations, the kink retains a form close to (2), but the equation of motion alters substantially, as the velocity V will now depend on time [7]. For the particular form of perturbation corresponding to Eqs. (1), velocity can be evaluated by means of a simple energetic approach.

The Lagrangian density of the system, based on Eqs. (1), is

$$\mathcal{L} = \frac{1}{2}\varphi_t^2 - \frac{1}{2}\varphi_x^2 - 2\sin^2\frac{\varphi}{2} + \frac{1}{2}v_t^2 - \frac{1}{2}v_x^2 - \varepsilon_L v_x \sin\varphi. \tag{3}$$

The full Lagrangian $L = \int\limits_{-\infty}^{\infty} \mathcal{L}(x)\, dx$, can be calculated by inserting the kink waveform (2) into (3). The result is

$$L = \frac{-8 + 8V^2 + \frac{4}{3}\varepsilon_L^2}{(1 - V^2)^{1/2}}. \tag{4}$$

The canonical momentum corresponding to the Lagrangian (4) is

$$P \equiv \frac{dL}{dV} = \frac{8V\left[\left(1 - V^2\right) + \frac{\varepsilon_L^2}{6}\right]}{(1 - V^2)^{3/2}}. \tag{5}$$

The kink's equation of motion under the action of a constant force f takes the form

$$\frac{dP}{dt} = f. \tag{6}$$

Inserting Eq. (5) into (6) and integration (assuming $1 - V^2 \ll 1$) yields, at $t \to \infty$,

$$V = 1 - \frac{1}{2\,(\alpha\, t)^{2/3}}, \tag{7}$$

with $\alpha = \dfrac{f}{4{\varepsilon_L}^{2/3}}$. Finally, in consideration of Eq. (7), the equation of motion of the accelerating kink assumes the form:

$$\xi(t) = t - \frac{3}{2}\left(\frac{t}{\alpha^2}\right)^{1/3}. \tag{8}$$

2.2. *Emission of acoustic waves*

The longitudinal acoustic wave field emitted by the accelerating kink can be represented as follows:

$$v(x,t) = (2\pi)^{-1} \int\limits_{-\infty}^{\infty} \overline{v}(k,t)\, e^{ikx}\, dk, \tag{9}$$

$\overline{v}(k,t)$ being the Fourier amplitude.

If $F_L(x, t)$ stands for the right-hand side of Eq. $(1)_2$ (i.e. the expression $\varepsilon_L(\sin\phi)_x$), the equation of motion for the Fourier amplitude $\overline{v}\,(k,t)$ can be formulated as:

$$\frac{dq\,(k,t)}{dt} = -ikq\,(k,t) + \varepsilon_L \overline{F}_L\,(k,t)\,, \tag{10}$$

where $q\,(k,t) = \partial\,\overline{v}\,(k,t)/\partial t - ik\,\overline{v}\,(k,t)$, and $\overline{F}_L\,(k,t)$ is the Fourier transform of $F_L(x,t)$.

Insertion of the kink waveform (2) into the right-hand side of Eq. $(1)_2$ results in the following Fourier transform, to be inserted into Eq. (10):

$$\overline{F}_L\,(k,t) = 2\pi\,\varepsilon_L k^2 \left(1 - V^2\right) e^{-ik\xi} \operatorname{sec h}\left(\pi k \frac{\sqrt{1-V^2}}{2}\right). \tag{11}$$

In consideration of a new amplitude

$$B\,(k,t) = q\,(k,t)\;e^{ikt}, \tag{12}$$

the evolution equation for the amplitude reads

$$\frac{dB}{dt} = \varepsilon_L\, e^{ikt} \overline{F}_L\,(k,t)\,. \tag{13}$$

The equation of kink motion (8) is now substituted into (11), and the result is inserted into Eq. (13). The following expression is obtained:

$$\frac{dB}{dt} = \frac{2\pi\varepsilon_L k^2}{(\alpha t)^{2/3}} \operatorname{sec h}\left[\frac{\pi k}{2\,(\alpha t)^{2/3}}\right] \exp\left[\frac{3ik}{2}\left(\frac{t}{\alpha^2}\right)^{1/3}\right]. \tag{14}$$

To find the total amplitudes of the emitted acoustic waves, it is assumed that the free waves were absent prior to the motion of the kink, i.e. at $t = -\infty$, and the final amplitude is defined as

$$B\,(k) = \int\limits_{-\infty}^{\infty} \frac{dB\,(k,t)}{dt} dt. \tag{15}$$

This quantity determines the spectral density $\mathcal{E}$ of the emitted energy E, according to the relation

$$\mathcal{E}\,(k) = \frac{1}{4\pi}\left|B\,(k)\right|^2. \tag{16}$$

By substituting Eq. (14) into Eq. (15) and performing the integration by means of the residue theorem, we arrive, after some lengthy calculations, at the following expression of the radiation amplitudes:

$$B\,(k) = 24\pi\,\varepsilon_L \frac{k^3}{\alpha} \sum_{\rho \geq 1}^{\infty} (2\rho - 1)^{-2}\,(-1)^{\rho} \exp\left(-\frac{3k^2}{2\alpha\,(2\rho-1)}\right). \tag{17}$$

Finally, the total emitted energy can be expressed as

$$E = \int_{-\infty}^{\infty} \mathcal{E}\,(k)\,dk. \tag{18}$$

Using Eqs. (17) and (18), the power of spectral density and the total energy of the emitted elastic waves have been investigated.

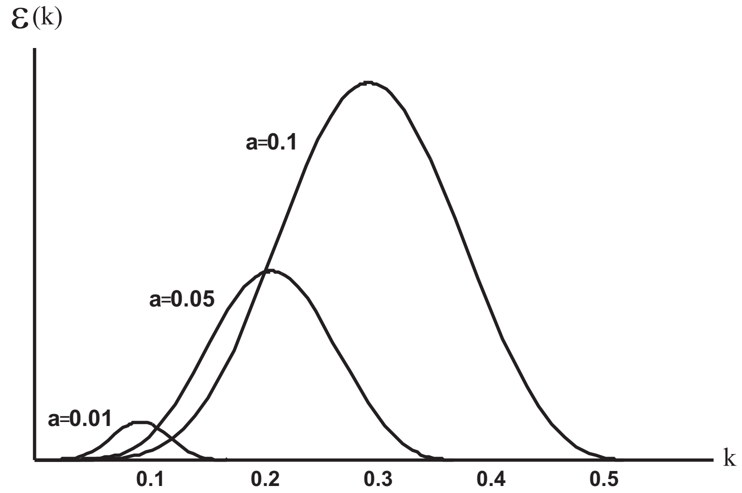

Fig. 1. The spectral density of the energy emitted by the accelerated dislocation kink for different values of the driving force.

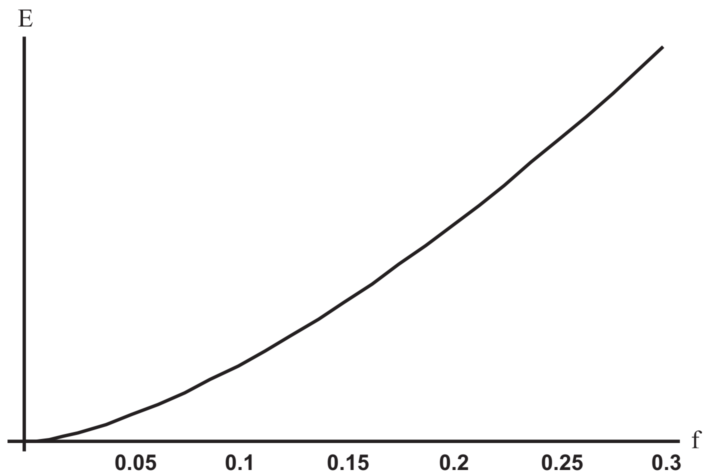

Fig. 2. The total emitted energy as a function of the driving force.

In Fig. 1, the power of spectral density is shown as a function of the wavenumber for different values of the driving force. It can be seen that there exists a value $k_{\max}$ corresponding to the maximum of the spectral density. Furthermore, with increasing driving force, there is a shift of the maximum emission towards higher frequencies.

In Fig. 2, the results for the total energy emitted as a function of the driving force are shown. It can be seen that the emitted energy increases with increasing driving force.

3. Numerical illustrations and conclusions

To illustrate the problem of acoustic radiation by an accelerating dislocation kink, the sGdA system $(1)_1$ and $(1)_2$ is treated numerically. The numerical scheme adopted is a simple finite difference technique called the "leap-frog" method.

a)

b)

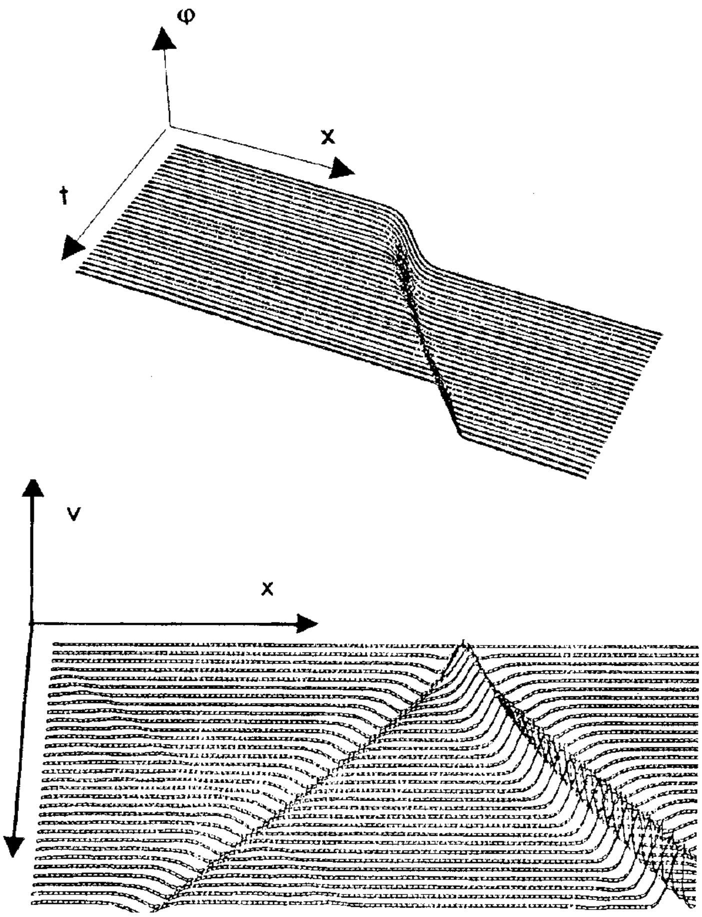

Fig. 3. Energy radiation from an accelerated dislocation kink. (a) Motion of a dislocation kink. (b) Associated radiation of the elastic waves.

First, the medium is considered to be originally at rest, containing a dislocation kink (i.e. at $t = 0$, $v = 0$, $\left(\dfrac{\partial v}{\partial t}\right)_{t=0} = 0$ and $\varphi(x,t) = 4\tan^{-1}(\exp(x - x_o))$. Then the dislocation kink is suddenly put into motion by a constant force, moving from left to right, as shown in Fig. 3a. Due to the coupling between the dislocation kink and the substrate, the motion of the kink induces nonlinear elastic waves in the substrate. In Fig. 3b (which represents the longitudinal elastic displacements), two types of waves may be noticed: a hump-like "shadow", moving with the dislocation from left to right at the speed of the dislocation, and the emitted acoustic radiation, moving from right to left at the speed of sound. Thus, the results of a numerical simulation of the sGdA system illustrate a situation which is related to energy emission by an accelerating dislocation kink.

The method presented here allows not only for the calculation of the energy emitted, but also for investigation of its spectral characteristics. The results can be used to provide a better physical insight into acoustic emission due to non-stationary dislocation motion occurring at the time of plastic deformation of crystals.

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