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GLOBAL INDEX OF THE ACOUSTIC QUALITY OF SACRAL BUILDINGS AT INCOMPLETE INFORMATION

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The global assessment of acoustic quality of sacral building was, up to the present, performed when the full information about the object – determined by partial indices calculated from the measurements of acoustic parameters – was known. The formula for calculating the global index of acoustic quality at incomplete information by means of the Singular Value Decomposition (SVD) is given in the paper. Having only one of the indices – the reverberation index – it is possible to approximately assess the acoustic quality of the church. Application of the SVD method required certain modifications of the previously proposed index method. The technique of decomposition into singular values enabled the transformation of initial variables being in the form of correlated indices – into mutually orthogonal new variables.

The decomposition versus singular values was done on the previously built empirical model of the index observation matrix of sacral objects. Due to this operation the weight values, which were applied in the global assessment of acoustic quality of sacral objects, were obtained. The approximate theoretical models containing information on interdependency of partial indices – were developed.

The developed formula for the global assessment of acoustic quality – at incomplete information on the sacral object – was applied for comparing acoustic qualities of ten churches.

Keywords: acoustic quality, index method, sacral objects, Singular Value Decomposition (SVD).

1. Introduction

It was mentioned – in the paper published in Archives of Acoustics [11] concerning the index method of an acoustic assessment of sacral objects – that investigations on application of the Singular Value Decomposition to improve this method were carried on. The obtained results of those investigations are presented in the hereby paper.

The author, in his research performed up to day, has not found any publications concerning an application of the SVD technique in the analysis of acoustic properties of sacral objects.

Contemporaneously built in Poland sacral buildings are characterised, in majority of cases, by the lack of adequate acoustic conditions necessary for proper functioning of the object – according to its destination. There is often a poor intelligibility of speech and a bad quality of sound of music, especially the organ one. Newly built sacral objects are spatial structures with less and less fittings and equipment. Flat walls made of materials strongly sound-reflecting, significantly impair acoustic properties of the present-day churches. Acoustics of the church space is determined by its architecture, mainly by its shape and the way of finishing of interior as well as by the number of congregation present.

Designers of the sacral objects encounter several difficulties related to providing appropriate acoustic properties. The lack of the proper methods and measures allowing to determine acoustic conditions in sacral objects constitutes a lot of difficulties.

The task of developing a method, which would allow to assess explicitly acoustic conditions in the given sacral object, has been undertaken in the Chair of Mechanics and Vibroacoustics, AGH. The development of the index method of assessment the acoustic quality of interiors of sacral buildings was preceded by various investigations and analysis performed by means of the adapted methods. Those methods were intended for assessment of other types of interiors, it means – concert halls and auditoriums. The index method allows to assess an acoustic quality of a sacral building by the single-numbered global index being the function of several partial indices. The precise description of the method can be found in reference [11]. The verification of the index method was performed in a few actual objects.

Sacral buildings are of a specific type of interiors, in which – depending on the kind of religion – transmission of speech as well as of music occurs. Thus, the creation and assessment of acoustic properties of such interiors is very difficult since it usually requires the reconciliation of contradictory acoustic requirements. Therefore the proposed method is based on several assumptions and is intended for a certain group of sacral objects. Those assumptions limit, to a certain degree, the application of the method – however they were necessary for the development of the proposed method in the hitherto existing form.

The recently performed investigations on the index method proved that the SVD technique (Singular Value Decomposition) of the matrix decomposition into singular values – is a useful tool in its improvement. In order to apply the SVD to the already investigated sacral objects, certain modifications in the calculation method of the reverberation and global indices were needed (which are presented in Sec. 2).

2. Global assessment of the acoustic quality of sacral objects

2.1. Index method – global index W_{AQS}

The index method [11, 18], is based on measuring the acoustic parameters and calculating, according to the developed dependences, the values of partial indices and the global index of assessment of the acoustic quality of sacral buildings. It is schematically presented in Fig. 1. All partial indices as well as the global index assume the values from 0 to 1. The 0 value means bad acoustic properties, while 1 – very good ones.

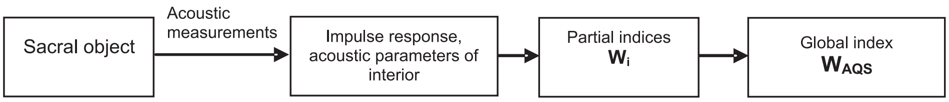

Fig. 1. Schematic presentation of the acoustic quality assessment of sacral object by means of the index method.

Global index W_{AQS} as a function of a few partial indices is determined by the equation:

$$W_{AQS} = \frac{\sum\limits_{i=1}^{n} W_i \eta_i}{\sum\limits_{i=1}^{n} \eta_i}, \tag{1}$$

where W_i – the i-th partial index, η_i – weight of the i-th partial index.

Index W_{AQS} – for the developed, up to now, five partial indices – is expressed by the formula:

$$W_{AQS} = \frac{W_r + 0.5 \cdot W_{is} + 0.3 \cdot W_{ed} + 0.3 \cdot W_{ul} + 0.2 \cdot W_m}{2.3}, \tag{2}$$

where W_r – reverberation index, W_{is} – intelligibility of speech index, W_{ed} – external disturbances index, W_{ul} – uniformity of loudness index, W_m – music sound quality index.

Details concerning the determination of partial indices are given in papers [11, 18]. The verification of the method was done in some Roman Catholic churches. Global index values are given in Fig. 2.

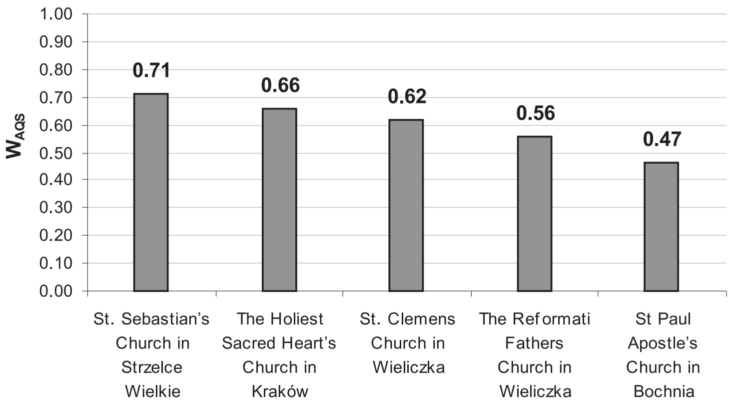

Fig. 2. Acoustic quality assessment of five churches, performed by means of the index method.

The method was applied in five churches since in those buildings the acoustic parameters, needed for determining the indices, were measured previously.

2.2. *Modified index method – global index W^*_{AQS}*

The assumption of the index method was the reduction of all indices (partial and global) to the range of values from 0 (bad acoustic properties) to 1 (very good acoustic properties). In general, we can say that this is a method comparing the measured values of parameters with the preferred ones, e.g. for good intelligibility of speech, for good sound of music etc. Initially, before other partial indices and the equation for global index W_{JAS} were developed, the acoustic quality of interiors of sacral objects was assessed by the reverberation indices. The results are given in paper [13].

The reverberation index was a function of several auxiliary reverberation indices (Eq. (3) in [11]), such as the reverberation-volume index, reverberation indices for organ music or for speech as well as the index differentiating the kinds of objects due to religion.

New studies performed recently by the author have shown that the reverberation index can be preferably reduced to the following form:

$$W^*_r = 1 - \frac{|T_{ZS} - T_p|}{3.5}, \tag{3}$$

where W^*_r – reverberation index, T_{ZS} – reverberation time corrected by the congregation presence (calculated according to Eq. (12) in [11]), T_p – reverberation time preferred for the given sacral building and its capacity (calculated from Eq. (5), (6) and (7) in [11] in dependence of the religion).

Index W^*_r (Eq. (3)) takes on values 0÷1 at the assumption that $T_{ZS} \leq T_p + 3.5$. When $T_{ZS} = T_p$ the index $W^*_r = 1$, which means that the sacral object under testing has the best acoustic properties. When $T_{ZS} = T_p + 3.5$ the index $W^*_r = 0$, which is tantamount to bad acoustic qualities of the object.

If $T_{ZS} > T_p + 3.5$ then Eq. (3) has negative values. In such case $W^*_r = 0$ should be assumed. It means, that the reverberation time, corrected by the audience presence, exceeds the preferred reverberation time by more than 3.5 s.

Presently index W^*_r takes into consideration only the kind of object due to religion, while the sound of music and intelligibility of speech are determined by separate partial indices: W_{is} – intelligibility of speech and W_m – sound of music. Due to this modification, the same information concerning speeches and music are not repeated and calculation of the reverberation index becomes less complicated.

The determined values of the modified reverberation index W^*_r calculated according to Eq. (3) and the index W_r – determined according to the previously used equations (given in [11]), for ten Roman Catholic churches – are presented in Fig. 3.

As it is shown in Fig. 3, the application of the modified equation of the reverberation index W^*_r (Eq. (3)) provides higher span of values as compared with the standard equation for W_r, which is useful at the index assessment. Thus, a good object achieves high values while a bad one – low values.

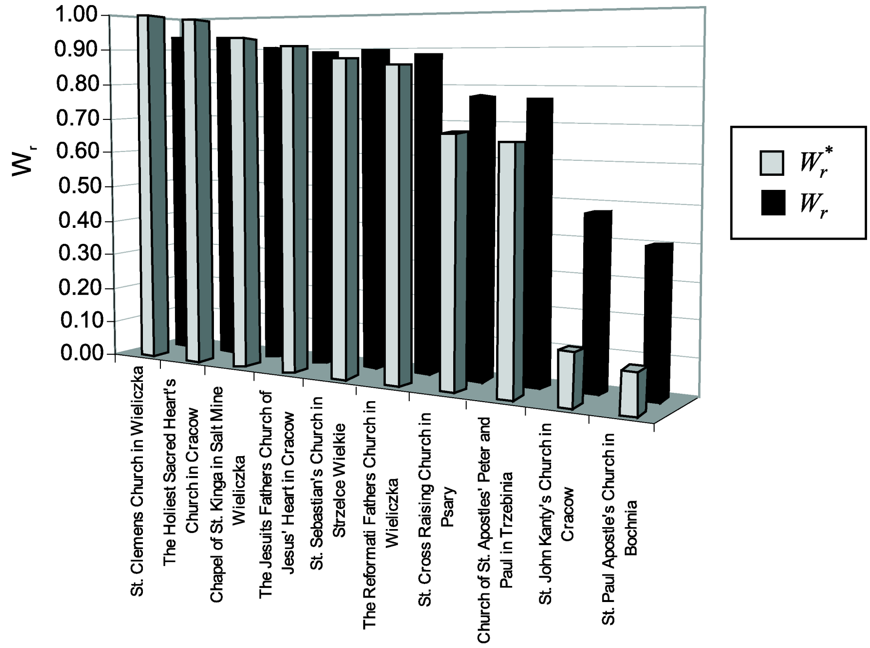

Fig. 3. The reverberation index applied for the assessment of acoustic quality of actual sacral buildings, determined according to the new equation – W_r^* and to the standard equation – W_r.

Application of three indices: W_r^*, W_{is} and W_m is recommended for the global assessment by the modified index method. In such case the global index W_{AQS}^* will be as follows:

$$W_{AQS}^* = \frac{W_r^* + 0.5 \cdot W_{is} + 0.2 \cdot W_m}{1.7}. \quad (4)$$

The way of calculating indices W_{is} and W_m is identical as in the standard index method. Table 1 presents partial indices and the global index W_{AQS}^*.

Table 1. Partial indices and the global index of assessment the sacral objects.

No	Sacral object	Capacity, m^3	W_r^*	W_{is}	W_m	W_{AQS}^*
1	St. Sebastian's Church in Strzelce Wielkie	1102	0.89	0.49	0.67	**0.75**
2	The Holiest Sacred Heart's Church in Cracow	2750	0.98	0.34	0.6	**0.75**
3	St. Clemens Church in Wieliczka	6380	1	0.34	0.49	**0.75**
4	The Jesuits Fathers in Cracow	9120	0.91	0.23	0.42	**0.65**
5	The Reformati Fathers Church in Wieliczka	4455	0.86	0.33	0.48	**0.66**
6	St. Paul Apostle's Church in Bochnia	22000	0.11	0.21	0.21	**0.15**

As it has been already mentioned, the assessment of acoustic quality of sacral objects by means of the modified index method is based on three partial indices (W_r^*, W_{is},

W_m), while the traditional index method required five partial indices (Subsec. 2.1). All three indices are mutually correlated. Coefficients of linear correlation between individual partial indices are given in Table 2. Those coefficients were calculated in the STATISTICA 6.0 programme for 364 cases, it means for every measuring point in all six churches (there were altogether 364 measuring points).

Table 2. Correlations between partial indices.

	W_r^*	W_m	W_{is}
W_r^*	1.00	**0.63**	**0.37**
W_m	**0.63**	1.00	**0.69**
W_{is}	**0.37**	**0.69**	1.00

Scattering of results is shown in Figs. 4÷6. Statistical calculations performed for significantly higher numbers of churches would give more reliable and less scattered results. However, in this very moment the verification of the proposed new index method was done for six churches only. Therefore the correlation coefficients are of pictorial character only, nevertheless they convey information concerning interdependences of partial indices.

Since partial indices are correlated with each other – as it results from Table 2 and Fig. 4÷6 – we might expect that the assessment, done by means of Eq. (4), is not quite precise and slightly "smears the information". Thus, the attempt of solving this problem with application of the SVD technique was taken up.

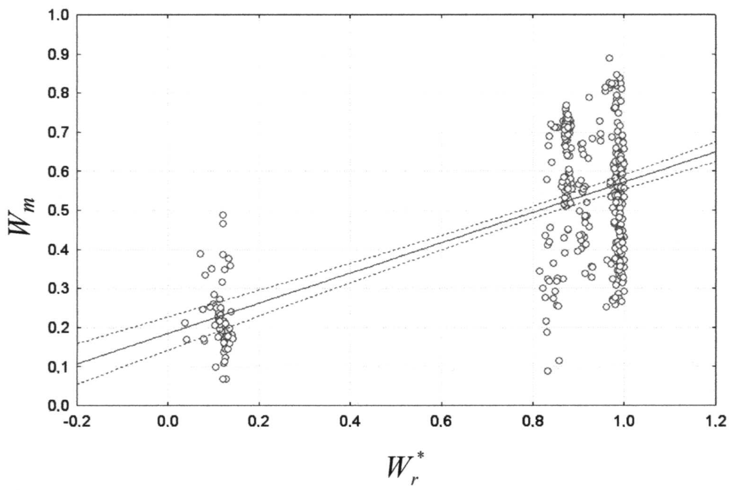

Fig. 4. Scattering of correlation coefficients of partial indices W_r andW_m.

The second weak point of equation (4) of the global assessment of acoustic quality of sacral buildings constitutes weights at indices. At the traditional index method [11] the weights were determined by checking, for which values the "acoustically bad" object obtains the minimal value of the global index and simultaneously the "acoustically good" object obtains the maximal value. Applying the determined values of weights and the equation for W_{AQS} [11] to the studied four actual objects a small difference of the global index was obtained (from 0.4 to 0.7). The problem of the weight selection (quantitative analysis of the weight values) can be solved by means of the SVD technique, which will be shown in the further subsection.

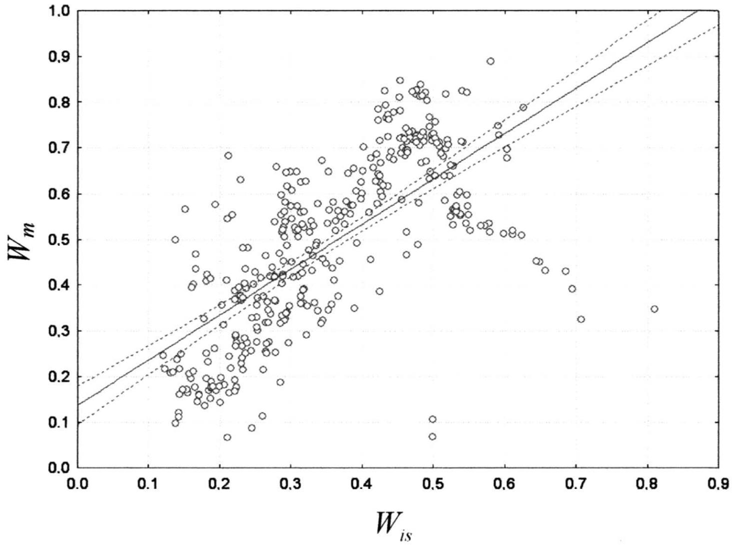

Fig. 5. Scattering of correlation coefficients of partial indices W_{is} and W_m.

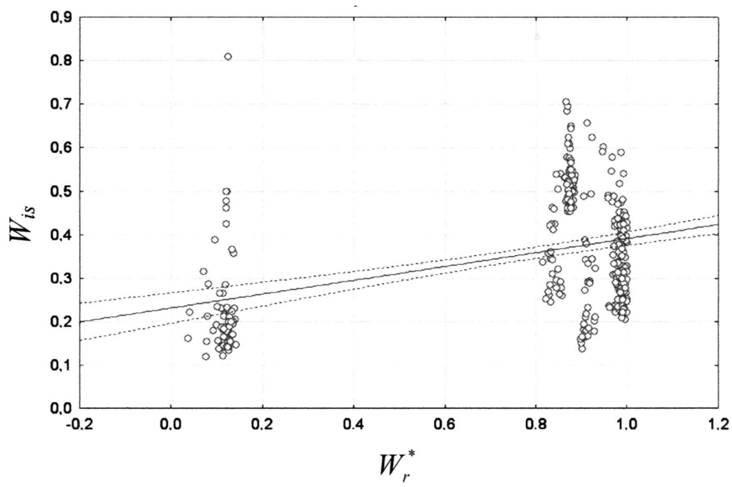

Fig. 6. Scattering of correlation coefficients of partial indices W_r and W_{is}.

3. Singular Value Decomposition – (SVD)

The Singular Value Decomposition is the matrix decomposition into singular values. It is one of the most widely used decompositions in the numerical linear algebra [7]. The mathematic SVD method is applied, among others, in statistical analysis and is used in reduction of the matrix size.

The decomposition into singular values was discovered and later developed by five mathematicians: Eugenio Beltrami (1835–1899), Camille Jordan (1838–1921), James Joseph Sylvester (1814–1897), Erhard Schmidt (1876–1959) and Hermann Weyl (1885–1955). History of the SVD method is presented in [22].

This decomposition is applied by many researchers. One of the first, who applied this decomposition in diagnostics, was CEMPEL [2–5] and [6].

The SVD method has many applications in various fields of science. It is used in the reduction of matrix sizes, compression of data and analysis of interdependences of variables. It is also applied in conversion of sound signals in robotics and automatics.

The decomposition into singular values is being applied in investigations of various vibroacoustic processes e.g. at identification of sound sources, in analysis of sound radiation by vibrating surfaces, at acoustic assessment of machines and devices – on the basis of the analysis of acoustic field parameters [7]. Such research has been carried on by Engel's team.

The SVD is used in inversion methods, at the analysis of regularisation methods – especially Tichonow's method and in Lanwerber's iteration analysis.

According to the SVD theory, each matrix $A \in R^{m \times n}$ can be presented in the decomposed form:

$$A = U \cdot \Sigma \cdot V^{\mathrm{T}}, \tag{5}$$

where U – orthonormal matrix $m \times n$, Σ – diagonal matrix $n \times n$, V – orthonormal matrix $n \times n$.

Expression (5) can be presented in the following form:

$$[A] = [U]^{[\Sigma][V^{\mathrm{T}}]}, \tag{6}$$

where Σ – singular values of matrix A, U – singular left vectors of matrix $A - u_i$, V^{T} – singular right vectors of matrix $A - v_i$.

Σ – diagonal matrix:

$$\sigma_{ij} = \begin{cases} \sigma_i > 0 & \text{for} \quad i = 1...n \\ \sigma_i = 0 & \text{for} \quad i > n, \end{cases} \tag{7}$$

Diagonal elements fulfill the condition: $\sigma_1 \geq \sigma_2 \geq ...\sigma_n$.

Singular A values are not negative square roots of eigenvalues $A^{\mathrm{T}}A$. Singular left vectors u_i are eigenvectors $A^{\mathrm{T}}A$, while singular right vectors v_i are eigenvectors AA^{T}. It follows that singular left vectors u_i:

$$A^{\mathrm{T}}Au_i = \sigma_i^2 u_i \tag{8}$$

and singular right vectors v_i:

$$AA^{\mathrm{T}}v_i = \sigma_i^2 v_i. \tag{9}$$

4. The SVD analysis applied in the acoustic assessment of sacral objects

The Principal Components Analysis (PCA) is used for analysis of main components. It is a set of statistic procedures, which by transformation of initial variables in mutually orthogonal new variables, builds the theoretical model describing the dependence structure in between the tested properties [21]. Other formulation determines the PCA as a method using the covariance matrix for performing transformation of original correlated components into the set of new non-correlated variables [1]. By means of the SVD we can analyse singular components (singular values). Application of the SVD method is more convenient since we obtain three matrices: U, Σ, V^{T} in one operation, without determination of the covariance and correlation matrix and without squaring the σ value.

As it was mentioned in the previous section, the SVD analytical technique seems to be a useful tool for removal of weak points of the index method. The application of the method will be illustrated on the example of the acoustic analysis of sacral objects.

The graphical presentation of the SVD technique applied in analysis of acoustic properties of sacral objects is shown in Fig. 7 (Eq. (5) was used).

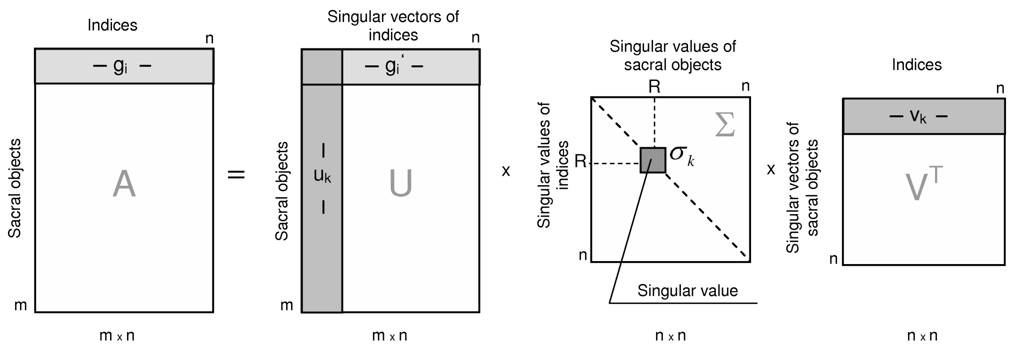

Fig. 7. Graphical presentation of the SVD of the index observation matrix A, with information concerning acoustic properties analysis of sacral objects, (R – matrix rank).

Values of three indices: W_r^*, W_{is} and W_m, determined on the basis of the measured acoustic parameters in actual churches, constituted grounds for creating the empirical model of the index matrix observation A (indices and sacral objects – Fig. 7). Rows of matrix A constitute six churches, in which acoustic tests were performed, arranged (according to the global assessment done by means of W_{AQS} index) from the acoustically best to the worst interior of the sacral building. Columns of matrix A were created using indices of: reverberation W_r^*, sound of music W_m and inteligibility of speech W_{is} – determined for individual churches.

Matrix A can be considered as the set of data forming a certain structure of dependences. All indices (data) written in matrix A have the same unit (they are dimensionless since they take values from 0 to 1), thus there is no need of standardisation of variables prior to performing the SVD.

By means of the decomposition of matrix A into singular values, the selection of the most informative singular components describing the analysed object can reduce the empirical model.

Decomposition into singular values (SVD) applied in acoustic analysis of churches was performed in the MATLAB environment. It looks as follows:

$$\overset{A}{\begin{bmatrix} 0.9 & 0.67 & 0.49 \\ 0.98 & 0.6 & 0.34 \\ 1 & 0.49 & 0.34 \\ 0.91 & 0.42 & 0.23 \\ 0.86 & 0.48 & 0.33 \\ 0.11 & 0.21 & 0.21 \end{bmatrix}}$$

$$= \overset{U}{\begin{bmatrix} -0.4756 & -0.5802 & 0.049147 & 0.14977 & -0.2008 & -0.60993 \\ -0.4727 & 0.03423 & 0.73042 & -0.3768 & -0.035453 & 0.31402 \\ -0.4584 & 0.26369 & -0.58687 & -0.40923 & -0.44433 & 0.10504 \\ -0.4024 & 0.45725 & 0.068747 & 0.77716 & -0.094989 & 0.10637 \\ -0.41 & 0.04287 & -0.25535 & -0.10177 & 0.86706 & -0.052151 \\ -0.1014 & -0.6178 & -0.22301 & 0.23191 & -0.013164 & 0.71015 \end{bmatrix}}$$

$$\times \overset{\Sigma}{\begin{bmatrix} 2.5331 & 0 & 0 \\ 0 & 0.29883 & 0 \\ 0 & 0 & 0.0611 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}} \times \overset{V^{\mathrm{T}}}{\begin{bmatrix} -0.81908 & -0.47924 & -0.31533 \\ 0.55492 & -0.52248 & -0.64736 \\ -0.14549 & 0.70522 & -0.6939 \end{bmatrix}}. \tag{10}$$

Initial space of partial indices written in matrix A was decomposed into three matrices U, Σ, V^{T}. After removal of zero rows from matrix Σ and corresponding with them columns of matrix U we obtain the reduced SVD, looking as follows:

$$\overset{A}{\begin{bmatrix} 0.89 & 0.67 & 0.49 \\ 0.98 & 0.6 & 0.34 \\ 1 & 0.49 & 0.34 \\ 0.91 & 0.42 & 0.23 \\ 0.86 & 0.48 & 0.33 \\ 0.11 & 0.21 & 0.21 \end{bmatrix}} = \overset{U^1}{\begin{bmatrix} -0.4756 & -0.5802 & 0.049147 \\ -0.4727 & 0.03423 & 0.73042 \\ -0.4584 & 0.26369 & -0.58687 \\ -0.4024 & 0.45725 & 0.068747 \\ -0.41 & 0.04287 & -0.25535 \\ -0.1014 & -0.6178 & -0.22301 \end{bmatrix}}$$

$$\times \overset{\Sigma^1}{\begin{bmatrix} 2.5331 & 0 & 0 \\ 0 & 0.29883 & 0 \\ 0 & 0 & 0.0611 \end{bmatrix}} \times \overset{V^{\mathrm{T}}}{\begin{bmatrix} -0.81908 & -0.47924 & -0.31533 \\ 0.55492 & -0.52248 & -0.64736 \\ -0.14549 & 0.70522 & -0.6939 \end{bmatrix}}. \tag{11}$$

Three matrices (including two of the shortened dimensions – U^1 and Σ^1) multiplied by each other give exactly the matrix A, which means that after the decomposition process the valid information is preserved.

Percentage participation of explanation of information on independent indices via the subsequent principal components is illustrated in Fig. 8. They were calculated from the equation:

$$W_k = \frac{\sigma_k}{\sum\limits_{k=1}^{R} \sigma_k} \cdot 100\%, \tag{12}$$

where σ_k – the k-th singular value.

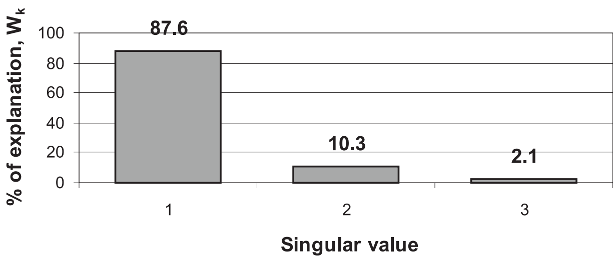

Fig. 8. Information on independent indices in the observation matrix A.

The first singular value σ_1 is the most informative one and contains 88% of the total information (explanation of the phenomenon) in the observation matrix A. The second singular value σ_2 constitutes 10%, while the third one σ_3 only 2% of the information.

According to Eq. (5), the decomposition equation SVD for g_i, which is a linear combination of v_k, has the form:

$$g_i = \sum_{k=1}^{R} u_{ik}\sigma_k v_k, \tag{13}$$

$i = 1, 2, \ldots, m$.

We can write that g_i' (i-th row of matrix U), shown in Fig. 7, contains coordinates of the i-th sacral object in the coordinate system (base) of the weighted eigenvalues of sacral objects $\sigma_k v_k$.

If $R < n$, transmission of responses of eigenvalues can be obtained from g_i'. This property of the decomposition is called "rank reduction".

It is easy to notice that, since the component No 1 is large as compared to the remaining ones, the empirical model can be reduced to the first – rank approximation. Thus, we will obtain the approximate model (containing a certain error).

The advantage of application the SVD method is the fact that by a simple strategy, the optimal approximation with the use of smaller matrices is possible. The empirical model can be reduced by applying approximations of rank R. Approximations for the ranks $R = 1$ to $R = 3$ are presented below.

$$A' = \sigma_1 u_1 v_1^{\mathrm{T}}, \tag{14}$$

$$A' = 2.533 \times \begin{bmatrix} -0.4756 \\ -0.4727 \\ -0.4584 \\ -0.4024 \\ -0.41 \\ -0.1014 \end{bmatrix} \times \begin{bmatrix} -0.82 & -0.48 & -0.32 \end{bmatrix} = \begin{bmatrix} 0.99 & 0.58 & 0.38 \\ 0.98 & 0.57 & 0.38 \\ 0.95 & 0.56 & 0.37 \\ 0.83 & 0.49 & 0.32 \\ 0.85 & 0.5 & 0.33 \\ 0.21 & 0.12 & 0.08 \end{bmatrix} = A^*, \tag{15}$$

$$A'' = \sigma_1 u_1 v_1^{\mathrm{T}} + \sigma_2 u_2 v_2^{\mathrm{T}}, \tag{16}$$

$$A'' = A' + 0.2988 \times \begin{bmatrix} -0.58 \\ 0.0342 \\ 0.2637 \\ 0.4573 \\ 0.0429 \\ -0.618 \end{bmatrix} \times \begin{bmatrix} 0.55 & -0.52 & -0.65 \end{bmatrix} = \begin{bmatrix} 0.89 & 0.67 & 0.49 \\ 0.99 & 0.57 & 0.37 \\ 0.99 & 0.52 & 0.32 \\ 0.91 & 0.42 & 0.23 \\ 0.86 & 0.49 & 0.32 \\ 0.11 & 0.22 & 0.20 \end{bmatrix}, \tag{17}$$

$$A''' = \sigma_1 u_1 v_1^{\mathrm{T}} + \sigma_2 u_2 v_2^{T} + \sigma_3 u_3 v_3^{\mathrm{T}}, \tag{18}$$

$$A''' = A'' + 0.0611 \times \begin{bmatrix} 0.0491 \\ 0.7304 \\ -0.587 \\ 0.0687 \\ -0.255 \\ -0.223 \end{bmatrix} \times \begin{bmatrix} -0.15 & 0.71 & -0.69 \end{bmatrix}$$

$$= \begin{bmatrix} 0.89 & 0.67 & 0.49 \\ 0.98 & 0.6 & 0.34 \\ 1 & 0.49 & 0.34 \\ 0.91 & 0.42 & 0.23 \\ 0.86 & 0.48 & 0.33 \\ 0.11 & 0.21 & 0.21 \end{bmatrix} = A. \tag{19}$$

It results from the above equations, that the rank $R = 3$ approximates matrix A in the best way, since then $A''' = A$.

Absolute errors Δ', Δ'' corresponding to matrices A' and A'' are described by Eq. (20) and (21). When the rank of approximation increases, the error values decrease.

$$\Delta' = \begin{bmatrix} 0.1 & 0.09 & 0.11 \\ 0 & 0.03 & 0.04 \\ 0.05 & 0.07 & 0.03 \\ 0.08 & 0.07 & 0.09 \\ 0.01 & 0.02 & 0 \\ 0.1 & 0.09 & 0.13 \end{bmatrix} \tag{20}$$

$$\Delta'' = \begin{bmatrix} 0 & 0.00 & 0 \\ 0.01 & 0.03 & 0.03 \\ 0.01 & 0.03 & 0.02 \\ 0 & 0 & 0.00 \\ 0 & 0.01 & 0.01 \\ 0 & 0.01 & 0.01 \end{bmatrix}. \tag{21}$$

Matrix of absolute errors for the third rank of approximation equals 0 (matrix is a null matrix). Considering dependences (14) and (15) it can be stated, that the matrix of data A^* obtained as the result of the first rank approximation is similar to matrix A and contains the absolute error Δ'.

5. Selection of weights at the acoustic assessment of churches by means of the global index W_{AQS} with the SVD application

At excellent correlation of matrix A^* coefficients (correlation coefficients between indices $r = 1$) there are certain numbers (shown in Eq. (15) and obtained from the SVD – components of vectors u_1 and v_1), which allow us to obtain matrix A^*. They contain the same information as matrix A^*, however it is obtainable via orthogonal vectors u_1 and v_1. Since the components of vector v_1 are – at this rank of approximation ($R = 1$) – the same for all sacral buildings for individual indices, they can be assigned to these indices as weights, to be later used at the global assessment.

Table 3 includes comparison of the tested churches according to the global assessment of acoustic quality W^*_{AQS} (with traditional weights – Eq. (4)) and W^{**}_{AQS} (with weights obtained by means of the SVD technique) determined by the formula:

$$W^{**}_{AQS} = \frac{0.82 \cdot W^*_r + 0.32 \cdot W_{is} + 0.48 \cdot W_m}{1.62}. \tag{22}$$

The results obtained from Eq. (22) – with new values of weights – are nearly identical with the ones obtained from the traditional equation (4).

As it can be noticed, the weight values from the approximation of the first rank were used. More precise weights can be obtained by performing the consecutive approximations. However, there is no need of doing that since the first component – as shown in Fig. 8 – is the most informative. In addition, after applying new weights in the equation for the global index (22), the information describing interdependence of indices was preserved, which is indicated by the results shown in Table 3. Thus, the accuracy at $R = 1$ is sufficient.

Table 3. Comparison of the global assessment of acoustic quality of the tested churches by means of indices W^*_{AQS} and W^{**}_{AQS}.

No	Sacral objects	Indices			Weight values						W^*_{AQS}	W^{**}_{AQS}
					Traditional			Obtained by means of the SVD				
		W^*_r	W_{is}	W_m	η_1	η_2	η_3	η'_1	η'_2	η'_3		
1	St. Sebastian's Church in Strzelce Wielkie	0.89	0.49	0.67	1	0.5	0.2	0.82	0.32	0.48	**0.75**	**0.75**
2	The Holiest Sacred Heart's Church in Cracow	0.98	0.34	0.6							**0.75**	**0.74**
3	St. Clemens Church in Wieliczka	1	0.34	0.49							**0.75**	**0.72**
4	The Jesuits Fathers in Cracow	0.91	0.23	0.42							**0.65**	**0.63**
5	The Reformati Fathers Church in Wieliczka	0.86	0.33	0.48							**0.66**	**0.64**
6	St. Paul Apostle's Church in Bochnia	0.11	0.21	0.21							**0.15**	**0.16**

6. Finding of the global index of acoustic quality at incomplete information concerning the object

Approximate empirical model obtained at the first rank of approximation ($R = 1$) according to (15) can be presented as:

$$A' = \overset{\sigma_1}{2.533} \times \underset{\text{sacra objects}}{\overset{u_1}{\begin{bmatrix} -0.4756 \\ -0.4727 \\ -0.4584 \\ -0.4024 \\ -0.41 \\ -0.1014 \end{bmatrix}}} \times \overset{v_1^{\mathrm{T}}\ \text{(weights)}}{\begin{bmatrix} -0.82 & -0.48 & -0.32 \end{bmatrix}}$$

$$= \begin{bmatrix} 0.99 & 0.58 & 0.38 \\ 0.98 & 0.57 & 0.38 \\ 0.95 & 0.56 & 0.37 \\ 0.83 & 0.49 & 0.32 \\ 0.85 & 0.5 & 0.33 \\ 0.21 & 0.12 & 0.08 \end{bmatrix} = A^*. \tag{23}$$

Singular value σ_1 – at this rank of approximation – is the same for all sacral objects. Components of vector v_1^{T} are identical numbers for all churches for individual indices. They were assumed as weights (of individual partial indices) at the assessment done by the global index. Components of vector u_1 are changing depending on the sacral building. The dependence illustrated in Fig. 9 occurs between vector u_1 and vector a_{i1} (the first column of the index observation matrix A formed by the reverberation index W_r^*).

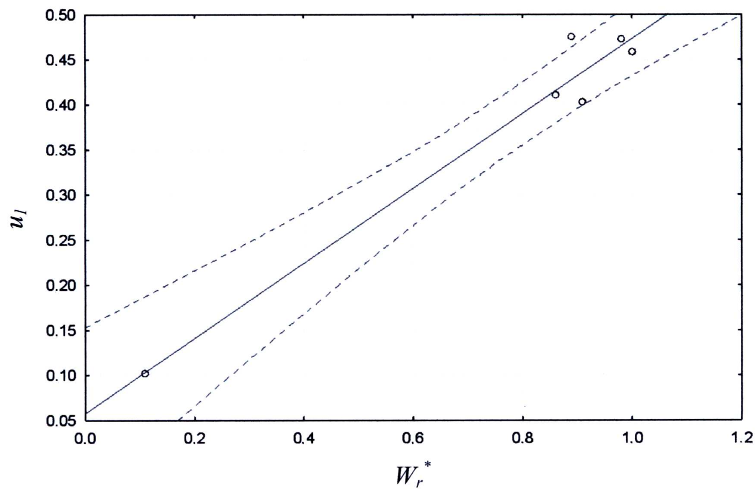

Fig. 9. Index W_r^* versus vector u_1.

The correlation coefficient between W_r^* and u_1 equals $r = 0.98$, which means nearly full correlation. This dependence is determined by the equation:

$$u_1 = 0.058 + 0.416 \cdot W_p^*. \tag{24}$$

After inserting dependence (24) into Eq. (14) and performing conversions, the dependences for components of matrix A^* (values of indices from the first approximation) dependent on only one data (input parameter) – reverberation index W_p^* determined by Eqs. (25)–(27) – were obtained:

$$W_r' = 0.146 + 0.86W_r^*, \tag{25}$$

$$W_m' = 0.146 + 0.5W_r^*, \tag{26}$$

$$W_{is}' = 0.146 + 0.33W_r^*. \tag{27}$$

Insertion of Eqs. (25), (26) and (27) into the formula for the global index W_{AQS}^{**} (22) allowed to obtain the equation for the acoustic quality global index at not complete information, being the function of the reverberation index:

$$W_{AQS}^{W_r^*} \cong 0.651 \cdot W_r + 0.1462. \tag{28}$$

This equation seems to be very useful in the case when we have incomplete information. It should be mentioned that the global assessment by means of Eq. (28) is an approximate one, depending on the assumed approximation rank of matrix A (in this case of the first rank – $R = 1$). Having information of the reverberation time we can calculate the reverberation index W_r^* and then – on the basis of Eq. (28) – we determine the global index. It means, that we assess acoustic conditions of the interior of the sacral building on the basis of one parameter only. In addition, having the same incomplete information – index W_r^* – we are able to determine (on the basis of (26) and (27)) approximate values of the remaining indices, such as: the sound of voice index and intelligibility of speech index of the given building.

The application of Eq. (28) for the acoustic quality assessment – when only one parameter, the reverberation index is at our disposal – is shown in Table 4. The obtained results are compared with the ones obtained from the previously developed equations for the global index. Data collected in Table 4 indicate that Eq. (28) is correct and burdened with smaller errors than the remaining equations.

Due to the developed formula for the global index, at incomplete information – $W_{AQS}^{W_r^*}$ – it was possible to perform the global assessment of the acoustic properties of other churches too. The data were collected from M. Eng. Thesis performed in the Chair of Mechanics and Vibroacoustics of AGH, in which the students not always had the possibility to determine, apart from the reverberation time, other acoustic parameters e.g. intelligibility of speech etc. Comparison of the acoustic quality of churches by means of the global index $W_{AQS}^{W_r^*}$, (calculated from Eq. (28) on the grounds of one partial index W_r^*) is given in Fig. 10.

Table 4. Comparison of the global acoustic assessment obtained on the basis of the developed dependences.

No	Sacral object	Capacity, m^3	W_r^*	W_{is}	W_m	W_{AQS}^*
1	St. Sebastian's Church in Strzelce Wielkie	1102	0.89	**0.75**	**0.75**	**0.73**
2	The Holiest Sacred Heart's Church in Cracow	2750	0.98	**0.75**	**0.74**	**0.78**
3	St. Clemens Church in Wieliczka	6380	1	**0.75**	**0.72**	**0.80**
4	The Jesuits Fathers in Cracow	9120	0.91	**0.65**	**0.63**	**0.74**
5	The Reformati Fathers Church in Wieliczka	4455	0.86	**0.66**	**0.64**	**0.71**
6	St. Paul Apostle's Church in Bochnia	22000	0.11	**0.15**	**0.16**	**0.22**

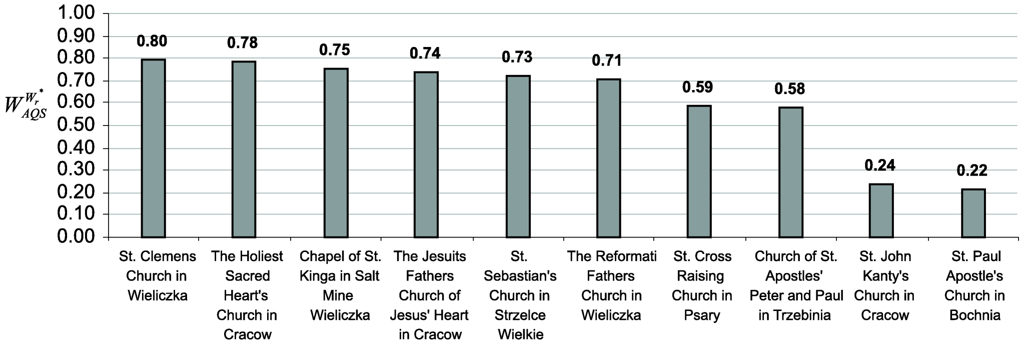

Fig. 10. Comparison of acoustic quality of churches by means of the global index $W_{AQS}^{W_r^*}$ determined at the incomplete information.

As it can be seen from Fig. 10, the best acoustic properties appear in the churches of: St. Clemens, the Holiest Sacred Heart's and the Chapel of St Kinga in the Salt Mine "Wieliczka". Acoustically the worst – according to the global assessment – are the churches built on a circular lay-out: St. John Kanty's Church and St. Paul's Church.

7. Conclusions

The author's efforts in development of the index method of acoustic assessment of sacral objects [11] by means of the singular value decomposition technique (SVD) are presented. The author did not find any application of the SVD technique in the analysis of acoustic properties of churches, in his up-to-date research. In order to apply the SVD technique, the index matrix of the sacral objects observation containing three mutually correlated indices in six churches (determining acoustic quality of the given interior: reverberation, intelligibility of speech and sound of music indices), was developed. Application of this method required certain modifications in the index method. The technique of decomposition into singular values enabled the transformation of initial variables into indices, which were correlated in mutually orthogonal new variables. It was shown, that the empirical model can be reduced by using approximation of the appropriate rank.

The quantitative analysis of the weight values was performed by means of the SVD technique. Those values were determined for three partial indices used for the acoustic assessment done by the global index.

The equation for the global assessment of the acoustic quality at incomplete information about the object was developed. Having measured the reverberation time of the interior we can calculate the reverberation index and then – on the basis of this only information – it is possible to forecast an approximate global assessment of the acoustic quality of the church. Formulas for calculating approximate values of the indices of intelligibility of speech and sound of music on the basis of the reverberation index only – are also useful.

Application of the equation for the acoustic quality global assessment at incomplete information for ten actual churches provided an approximate information on their acoustic quality. As it was shown, two of the churches built on circular and ellipsoidal plans, have very low values of the global index (0.2 in the range from 0 to 1), which is in agreement with subjective opinions of congregations that the acoustic quality is there very bad.

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